1. To show surjectivity, we must show that for any $\forall s \in \mathbb{N}, f(m,n) = s$. We let $s \in \mathbb{N}$. Then, s is even or s is odd.
   Case s even: If s is even, we can take s to be written in the form $2^a(b)$ where $a \geq 1$ and b is an odd number. Since b is an odd number, b can be written as $2n-1$. Then, we can take $a+1=m$. Thus, $2^{m-1}(2n-1) = 2^a b = s$. Thus if s is even, f is surjective.

   Case s odd: If s is odd, we can simply take $s = 2n-1$ for some $n \in \mathbb{N}$. With this and $m=1$, we find $2^0(2n-1) = s$. Thus if s is odd, f is surjective.

   Thus, in all cases, f is surjective.

2. (a) We will proceed by cases.
   Case $x, y > 0$: Assume $f(x) = f(y)$. Since $f(x) = f(y)$ and $x, y > 0$, $2x+1 = 2y+1$. By algebra, $x = y$. Thus, if $x, y > 0$, f is injective.

   Case $x, y \leq 0$: Assume $f(x) = f(y)$. Since $f(x) = f(y)$ and $x, y \leq 0$, $3x = 3y$. By algebra, $x = y$. Thus, if $x, y \leq 0$, f is injective.

   Since f is injective in all cases, f is injective.

   (b) Above, we proved f is injective. Thus, it suffices to show f is not surjective from $\mathbb{R} \to \mathbb{R}$. Let $3x = \frac{1}{2}$ and $x \leq 0$. Then, $x = \frac{1}{6}$. However, this contradicts the fact that $x \leq 0$. Thus it must be the case that $2x + 1 = \frac{1}{2}$ for some $x > 0$. Then, $2x = -\frac{1}{2}$ and $x = -\frac{1}{4}$. However, this contradicts that $x > 0$. Thus, we have found an element in $\mathbb{R}$ such that there is no $f(x) = \frac{1}{2}$. Thus, f is not surjective.

   Since f is not surjective, f is not bijective.

3. (a) Let $x = y^2 + 1$. Then $y = \pm\sqrt{x-1}$. Since $y \in [0, \infty]$, $y = \sqrt{x-1}$. Let $x \in [1, \infty)$. Then, $y = \sqrt{x-1}$ is a real number and $x = (\sqrt{x-1})^2 + 1$. Since $x \geq 1$, $y = \sqrt{x-1} \geq 0$. Thus, there exists a $y \in [0, \infty)$ such that $(x, y) \in R$. Assume $(x, u) \in R$ and $(x, v) \in R$. Then, $\sqrt{x-1} = u$ and $\sqrt{x-1} = v$. Thus, $u = v$. Therefore, R is a function from $[1, \infty) \to [0, \infty)$.

   (b) Assume $x_1, x_2 \in [1, \infty)$ and $x_2 > x_1$. Then, $x_2 - 1 > x_1 - 1$ and $\sqrt{x_2 - 1} > \sqrt{x_1 - 1}$. Since $\sqrt{x_2 - 1} > \sqrt{x_1 - 1}$, $y_2 > y_1$.

   (c) Let $x \in [1, \infty), y \in (-\infty, \infty)$. Since $x = y^2 + 1$, $y = \pm\sqrt{x-1}$. Thus, $f(x) = \pm\sqrt{x-1}$. Since x maps to $\pm\sqrt{x-1}$, $f(x) = y$ is not unique. Therefore, f is not a function from $[1, \infty)$ to $(-\infty, \infty)$.

4. (a) No, since every y except zero has two x which maps to it, namely $\pm\sqrt{y}$.
   (b) Yes, since every y in $[0,\infty)$ is mapped to by some x namely $\sqrt{y}$.
   (c) Yes, since every y value is mapped to exactly one x value.
   (d) Yes, since every $y \in [0,\infty)$ is mapped to by some x namely $\frac{y}{y+1}$.
   (e) $f \circ g = f(g(x)) = (\frac{x}{1-x})^2$. Domain $= [0,1)$. Codomain $= [0,\infty)$ This function is bijective from $[0,1)$ to $[0,\infty)$.
   (f) $g \circ f = g(f(x)) = \frac{x^2}{1-x^2}$. Domain $= (-\infty,-1) \cup (-1,1) \cup (1,\infty)$ Codomain $= (-\infty,\infty)$. This function is surjective, but not injective.
   (g) i. Since f was not injective, we also see that $g \circ f$ is also not injective which we could have guessed from Theorem 4.3.3.
      ii. In our case, f is not bijective yet $f \circ g$ is bijective. This is because g is only defined on $[0,1)$ making the domain of the composite function also $[0,1)$. Since f is bijective on this limited domain, $f \circ g$ is also bijective.
   (h) i. Inverse function of g: $\frac{x}{x+1}$. Domain: $[0,\infty)$. Codomain: $[0,1)$.
      ii. Inverse function of $f \circ g$: $\frac{\sqrt{x}}{\sqrt{x}+1}$. Domain: $[0,\infty)$ Codomain: $[0,1)$.