

Math 200 University of Connecticut

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1. Introduction

This handout discusses the preparation of mathematical documents using LaTeX. Look at the file \math200format.tex with your favorite text editor to see how mathematical (and typesetting) commands described below are written.

This file can also be used as a template for the preparation of your paper. With this in mind, open the file \math200paper.tex with your favorite text editor and save it as math200lastname.tex (where you write your last name instead in the name of the file). If you need any LATEX construction, an equation for example, just find a similar one in this or other LATEX file, copy, paste and edit.

If you use Apple computers in the Math department, you can open any LaTeX file by clicking on it. There is a "typeset" button which you use to see the results of your editing, and then to print the paper. While the edited changes you make go through after typesetting once, sometimes you have to typeset two or three times to see all the corresponding changes in equation numbers, the table of contents, and cross references. When equations get moved around they have to be renumbered, but on the first pass at typesetting LaTeX won't fix all the cross-references (for instance) but only notices they need to be changed. It will fix things up better the second (or third) time.

If you use UNIX or LINUX, then you can open a LATEX file with a text editor. Also you can open a terminal window and use the command latex math200 lastname to typset the edited and saved file. Typically the command xdvi math200 lastname is used to see the results of your editing, and dvips math200 lastname is used for printing.

There are free versions of LATEX for Windows. See http://www.ams.org/tex/ for more on LATEX and http://www.cs.queensu.ca/home/gradseries/help/stuff/symbols.pdf for a very large list of LATEX symbols. (Warning: Not all the symbols there work without having special packages installed, so try a symbol out once to see LATEX recognizes it before deciding to use it.) Online tutorials at http://www.comp.leeds.ac.uk/andyr/misc/latex/ might be helpful, especially tutorials 4, 6, 8, and 10.

A little history. This typesetting system was originally designed by Donald Knuth, who named it TeX, or TeX. Leslie Lamport made improvements on TeX and slapped the first two initials of his last name on the result, thus giving us LaTeX. If you want to get TeXnical about it, the system is pronounced like this: TeX = tek and LaTeX = "lay-tek" (or "la-tek"), but not "teks" or "lay-teks". This ends our little history. For more information, see http://www.tug/org/whatis.html.

2. Typesetting equations

In LaTeX, math in a line of text is entered and exited using a single dollar sign: x > 1 comes out as x > 1. This is the cardinal rule!

To type a mathematical expression on its own line, there are a few options. If you don't want the expression to be numbered (for instance, you won't be referring back to it later on), then use double dollar signs: f(x) = 2x - 5 comes out on its own line as

$$f(x) = 2x - 5.$$

But maybe you want the equation to have a number on the side, like this:

(2.1)
$$\int_{a}^{b} f'(x) \, \mathrm{d}x = f(b) - f(a).$$

This equation was typeset using the commands \begin{equation} and \end{equation}. A number on the side is produced. **NOTE**: All LaTeX commands begin with a backslash \.

How do you refer back to a displayed equation? The command to use is \ref but we have to label the equation to give it a name to refer back to. ¹ You do *not* want to refer to the above equation by actually typing in an equation number in your file, since the equation may get moved and then the number changes. Let LaTeX handle that problem for you! The above equation was labelled eqn-one by adding \label{eqn-one} after typing \begin{equation} above. You can refer back to the equation by typing (\ref{eqn-one}). The result: (2.1).

There is no reason at all to call your equations eqn-one, eqn-two, and so on. ² You could just as well call them something else. For instance, consider the following famous formula:

(2.2)
$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

The label for this equation is holycow, but you can't see that from the output in the formula. The reference command (\ref{holycow}) produces something innocent-looking: (2.2). A more reasonable label for (2.2) might have been, say, qformula or qf. Anyway, the point of using labels is that even when things get moved around (and they sure will) this referencing system will regenerate correct numbers for everything.

Quite generally, the command \label{name} is used to label equations, theorems, sections, and anything else you want to refer back to later, which you do as \ref{name}. Put whatever you want in "name" to be the label you need when making references elsewhere to that item. For instance, right now we are in Section 2. The section was (arbitrarily) called tsetsec by typing \label{tsetsec} on the line in the file where the section starts. The command \ref{tsetsec} anywhere in the file produces the number of this section.

The commands \begin{eqnarray*}, \end{eqnarray*} get used for unnumbered stacked equations such as

$$(x+y)^{2} = (x+y)(x+y)$$

$$= x^{2} + xy + yx + y^{2}$$

$$= x^{2} + 2xy + y^{2}$$

$$> 0.$$

If we want to number one of those equations, say to get

(2.3)
$$(x+y)^2 = (x+y)(x+y)$$

$$= x^2 + xy + yx + y^2$$

$$= x^2 + 2xy + y^2$$

$$> 0.$$

use \begin{eqnarray}, \end{eqnarray} - without the asterisks - and add \label{name} at the end of the line you want labeled (calling it "name" or whatever else you want) while putting \nonumber at the end of the lines you don't want labeled. Now (2.4) is available for referencing purposes.

¹Here's a footnote, produced with \footnote.

²Here's another footnote, using \edit. It leaves a number in the margin. Use this when making editing notes to yourself.

Sometimes things are defined in cases, like the absolute value function

$$|x| := \begin{cases} x, & \text{if } x \ge 0, \\ -x, & \text{if } x < 0. \end{cases}$$

The brace is generated using \begin{cases},\end{cases}. All text in math mode, by default, is put in italics. To tell LaTeX to keep certain text in math mode unitalicized, like "if" in the definition above of the absolute value, write \text{if}. If you don't do this, then "if" gets italicized and the output is ugly:

$$|x| := \begin{cases} x, & if x \ge 0, \\ -x, & if x < 0. \end{cases}$$

Finally, you might want to make something on a line of text appear as large as it won a displayed equation. Use \displaystyle{}. For instance, $\frac{a}{b+c}$ is obtained using \displaystyle{\frac{a}{b+c}}. Without using displaystyle, the fraction in text looks like $\frac{a}{b+c}$.

3. User-Defined Commands and Operators

How do you make a new command? For instance, let's suppose you are going to be typing the notation for real numbers, \mathbf{R} , a lot. You could type \mathbf R (in math mode, e.g., inside dollar signs) each time. That is tedious. It's better to create your own little command to abbreviate this. Look at the top of the file and you'll see \RR is introduced as a shortcut for \mathbf{R} with the command \newcommand{\RR}{\mathbf R}. Also included are shortcut commands for $\mathbf{C}, \mathbf{Z}, \mathbf{Z}/m\mathbf{Z}$, and \mathbf{Q} . Shortcuts for the alternate fonts $\mathbb{R}, \mathbb{C}, \mathbb{Z}, \mathbb{Z}/m\mathbb{Z}$, and \mathbb{Q} are at the top of the file as well. If you are going to be writing \mathcal{M} a lot and you don't want to write out \mathcal M each time, the line \newcommand{\calM}{\calM} \mathcal M} at the top of the file lets you instead write \calM to produce what you expect: \mathcal{M} . \mathcal{M} arvelous.

In addition to user-defined commands, there are user-defined operators. An operator is something like sin or det which is applied to something else: $\sin \pi$, det A. To ensure the right amount of spacing after the operator, it should be defined as an operator rather than as a command if it is not already part of LaTeX. While \sin and \det are already part of LaTeX, consider the following examples.

Example 3.1. The default LaTeX real and imaginary part operators are \Re and \Im, which come out like \Re and \Im , e.g., $\Im(2+3i)=3$. How ugly! See the top of the file for new operators \re and \im. Now we can write Im(2+3i)=3. That's better.

Example 3.2. Although LaTeX has an operator \det for the determinant of a matrix, it has no default operator for the trace! Look at the top of the file to see how the new operator \Tr is defined, which lets us write things like $Tr(\frac{2}{0}\frac{1}{4}) = 6$.

Example 3.3. We want to refer to a probability using the shorthand "Prob" while in math mode. At the top of the file a user-defined operator \Prob has been defined, so we can write $\operatorname{Prob}(a \leq x \leq b) = \frac{1}{\sqrt{2\pi}} \int_a^b e^{-x^2/2} \, \mathrm{d}x$ and Prob is not in italics. If we wrote plain "Prob" there without a special code for it, we'd get $\operatorname{Prob}(a \leq x \leq b) = \frac{1}{\sqrt{2\pi}} \int_a^b e^{-x^2/2} \, \mathrm{d}x$. Yucko.

The final topic for this section is commands with variable entries. Here's the idea. Suppose you will often need to write something like $\int_a^b f(x) dx$ with a lot of different functions. You

could type $\int_a^b \sin x\, \int_a^b e^x\, \and so on each time. But wouldn't it be nicer to type only the new function each time and have the surrounding material automated? Here's how: A new command <math>\int f$ is defined at the top of the file in the following way:

$$\label{limit_a^b{\#1}\\,\dd{x}}.$$

The [1] after \intf tells LATEX that this command allows one variable input, and the structure of the command tells LATEX where to put the variable. (Look for the #1 in the command.) For instance, typing just \intf{\sin x} returns $\int_a^b \sin x \, dx$ and typing \intf{e^x} returns $\int_a^b e^x \, dx$.

If you want the bounds of integration adjustable too, use a 3-variable command \intfb, defined at the top of the file as

 $\label{limit} $$ \operatorname{limit}_{#1}^{#2}_{#3}\, \dd{x}$.$

Then $\inf \{0\}\{5\}\{e^x\} \text{ returns } \int_0^5 e^x dx \text{ and } \inf \{3\}\{b\}\{\cos x\} \text{ returns } \int_3^b \cos x dx.$

4. Theorems and Definitions

Your paper probably will include theorems, corollaries, definitions, remarks, and so on. The file math200paper.tex has included special commands at the top which will properly number and format all of these *together* according to the section of your paper.

Use \begin{theorem},\end{theorem} when starting and ending your theorems. Theorems are automatically typset in italics. Here's one.

Theorem 4.1 (Pythagoras). If a, b, and c are the sides of a right triangle, with c the hypotenuse, then $c^2 = a^2 + b^2$.

The name Pythagoras is tacked on in square brackets [,] after \begin{theorem}. Of course you don't have to attribute your theorems.

All theorem-like environments (which means lemmas, theorems, corollaries, and conjectures) are typeset the same way. Here is a lemma.

Lemma 4.2. Let W and W' be subspaces of \mathbb{R}^n . Then $\dim(W + W') = \dim W + \dim W'$ if and only if $W \cap W' = \{0\}$.

In addition to theorem-like environments, there are definition-like environments. That means: definitions, examples, and remarks In these environments text is unitalicized by default. Produce a definition, for instance, using \begin{definition},\end{definition}.

Definition 4.3. A function is called *smooth* when it is infinitely differentiable.

Remark 4.4. Remarks are easy to produce, using \begin{remark},\end{remark}. Use this environment if there is something unusual (perhaps an easy misunderstanding) you want to draw to the reader's attention.

When you want to start and end a proof, use \begin{proof},\end{proof}. Let's see how it looks as output. The box at the end of the proof is generated by the \end{proof} command.

Theorem 4.5. If x and y are real and $x^2 - xy + y^2 = 0$ then x = 0 and y = 0.

Proof. We complete the square: $x^2 - xy + y^2 = (x - \frac{1}{2}y)^2 + \frac{3}{4}y^2$. This expression is always non-negative, so it vanishes only when $x - \frac{1}{2}y = 0$ and y = 0. Then x = 0, so both x and y vanish.

5. Relations

Here is an assortment of mathematical things you might want to type. Check out the commands which produce them in the file.

$$<>\leq\geq \leq \ll \gg \cong \sim \equiv \approx \stackrel{\text{def}}{=}\stackrel{?}{=}\subset \subseteq \times$$

$$\mapsto \leadsto \to \leftarrow \longleftrightarrow \longrightarrow \longleftarrow \longleftrightarrow \Longrightarrow \Longleftrightarrow \Longleftrightarrow \Longleftrightarrow$$

$$\downarrow \; \rfloor \; \infty \; \hat{a} \; \widehat{abc} \; \vec{a} \; \overrightarrow{abc} \; \underline{abc} \; \tilde{a} \; \widehat{abc} \; \imath \; \jmath \; \hat{\imath} \; \hat{\jmath}$$

$$\text{sin cos tan arctan log ln exp dim lim}$$

The dotless i and j are available since decorated i's and j's like \hat{i} and \hat{j} look weird with the dot.

Example 5.1. You can choose between $\lim_{\theta\to 0}(\sin\theta)/\theta=1$ or $\lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1$. The tiny fraction is hard to read. Maybe it's better in the displayed version

$$\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1.$$

If you leave off the backslash on the sine function, say you write $sin \theta$ in haste, it will come out like this: $sin\theta$. Ugh. If you forget both backslashes, as $sin \theta$ ugh. Ugh. If you forget both backslashes, as $sin \theta$ ugh. Ouch. Don't forget backslashes! $sin \theta = s\sin \theta$.

Example 5.2. The congruence relation in modular arithmetic is expressed using \bmod, not \mod. Compare $18 \equiv 8 \mod 5$ and $18 \equiv 8 \mod 5$. There is too much space between the 8 and the "mod" in the second one (bmod is short for "binary mod," as in "mod as a binary relation").

Things get negated by putting \not in front of them, leading to \neq , $\not\equiv$, $\not\approx$, and so on.

6. Phancy Phonts

7. Lists

Here is the same two-item list in three forms, using \begin{itemize}, \end{itemize} for the first two and \begin{enumerate}, \end{enumerate} for the third. Look at the file to see how the (default) bullets are turned into letters in the second version.

- $e^{i\pi} = -1$ $65 = 1^2 + 8^2 = 4^2 + 7^2$
- (a) $e^{i\pi} = -1$
- (b) $65 = 1^2 + 8^2 = 4^2 + 7^2$
- $(1) e^{i\pi} = -1$
- (2) $65 = 1^2 + 8^2 = 4^2 + 7^2$

8. Set-Theoretic/Algebraic expressions

To write that x belongs to \mathbf{R} , use \setminus in: $x \in \mathbf{R}$. (If it's not, then $x \notin \mathbf{R}$, using \setminus not \setminus in.) A set is indicated with curly braces, which are typeset as \setminus {,} \setminus . For instance,

$${x \in \mathbf{R} : \sin x = 0} = {k\pi : k \in \mathbf{Z}}.$$

It is *essential* that you use the backslash with those curly braces or they are not recognized as set braces and they don't show up. You'd get

$$x \in \mathbf{R} : \sin x = 0 = k\pi : k \in \mathbf{Z},$$

which looks weird.

Unions and intersections are denoted with $\backslash \text{cup}$ and $\backslash \text{cap}$: $A \cup B$, $A \cap B$. The empty set is $\backslash \text{emptyset}$: \emptyset .

Square roots: $\sqrt{10} = \sqrt{10}$. For *n*-th roots, $\sqrt{10} = \sqrt[n]{10} = \sqrt[n]{10}$. To indicate positive and negative (plus/minus) together, $\sqrt{2} = \pm 2$.

Fractions are produced using \frac , e.g., $\frac{x}{3x-1}$ is $\frac{x}{3x-1}$. A fraction that is hard to read on a line of text can always be put on its own line using double dollar signs.

Binomial coefficients are $\binom{x}{n} = \dim\{x\}\{n\}$.

To indicate collected terms with braces from above, using \overbrace:

$$\underbrace{x + \dots + x}_{n \text{ times}} = nx.$$

If the word "times" was not placed in text mode, it comes out in italics, which looks bad:

$$\underbrace{x + \dots + x}_{n \text{ times}} = nx.$$

Use \underbrace for braces from below. To generate the · · · between the plus signs, use \cdots (center dots); don't just type three dots or you get ..., which is too low and cramped. We'll meet more kinds of dots when we see a big matrix in Section 10.

Here is a factored polynomial:

$$c_n x^n + c_{n-1} x^{n-1} + \dots + c_1 + c_0 = c_n (x - r_1)(x - r_2) \dots (x - r_n).$$

Greek letters are entered by writing the name of the letter preceded by a backslash, so α is α , π is π , and Π (capital pi) is π . For instance, the addition formula for the sine function comes out as

$$\sin(\alpha + \beta) = (\sin \alpha)(\cos \beta) + (\cos \alpha)(\sin \beta).$$

There are two versions of epsilon and phi: ε and ϵ , φ and ϕ . The first choices look better. They are \varepsilon and \varphi. If you want sums, don't use $\Sigma = \beta$ Instead use \sum, which will adapt the size of the Sigma to the location, which could be in text like $\sum_{n=1}^{N} n^2 = N(N+1)(2N+1)/6$ or displayed like

$$\sum_{n=1}^{N} n^2 = \frac{N(N+1)(2N+1)}{6}.$$

Similarly, products (if you need them) are \prod , not \prod , e.g.,

$$n! = \prod_{i=1}^{n} j.$$

9. Some calculus

An integral from a to b is $\int a^b$:

$$\int_0^\infty e^{-x} dx = -e^{-x} \Big|_0^\infty$$
$$= \lim_{b \to \infty} (-e^{-b} + 1)$$
$$= 1.$$

The vertical line with the bounds of integration would usually be only as tall as the nearby e^{-x} term. To get it as tall as the integral sign, a *hidden* extra integral sign was inserted right after that e^{-x} using $\phi \in \mathbb{N}$ and then several backspace commands $\psi \in \mathbb{N}$ brought the vertical line back up to the e^{-x} . Clever!

A double integral is \iint:

$$\iint_{\mathbf{R}^2} e^{-x^2 - y^2} \, \mathrm{d}x \, \mathrm{d}y = \int_0^\infty \int_0^{2\pi} e^{-r^2} r \, \mathrm{d}r \, \mathrm{d}\theta = \pi.$$

Triple integrals are \iiint: \iiint .

Evaluating a derivative:

$$\frac{\mathrm{d}}{\mathrm{d}x}\Big|_{x=2} \left(\frac{x}{1+x^2} \right) = \frac{1-x^2}{(1+x^2)^2} \Big|_{x=2} = -\frac{3}{25}.$$

The command $\left(\right)$ is used around $\frac{x}{1+x^2}$ to get properly sized left and right parentheses in the displayed equation. If you just use plain (,) then you get something silly:

$$\left(\frac{x}{1+x^2}\right)$$
.

An ordinary and partial differential equation:

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + (\sin x) \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 + (\cos x)y = 0, \quad \frac{\partial^2 f}{\partial x^2} = \frac{\partial f}{\partial t}.$$

10. Some linear algebra

In text, a 2×2 matrix is produced using (\begin{smallmatrix}, \end{smallmatrix}). For instance, look at this: $\binom{a \ b}{c \ d}$. If you forget to include the parentheses, then you get a poor floating array $\binom{a \ b}{c \ d}$ which very much wants its parentheses!

Here are matrices multiplied by vectors

$$\begin{pmatrix} 1 & 2 & 0 \\ 4 & 9 & 1 \\ 3 & 2 & 0 \\ 9 & 1 & 3 \end{pmatrix} \begin{pmatrix} 5 \\ 4 \\ 3 \end{pmatrix}, \qquad \begin{bmatrix} 1 & 2 & 0 \\ 4 & 9 & 1 \\ 3 & 2 & 0 \\ 9 & 1 & 3 \end{bmatrix} \begin{bmatrix} 5 \\ 4 \\ 3 \end{bmatrix}.$$

They are produced using \begin{array},\end{array}. The different borders are produced using \left(,\right) in the code for the first matrix and \left[,\right] in the code for the second matrix. The command {ccc} which is in the LATEX code for the matrices after \begin{array} tells LATEX that there are 3 columns (so 3 c's). The column vectors (which

are really 1-column matrices) are produced using {c} after \begin{array}. A monster matrix can be done too:

$$\begin{pmatrix} \lambda & 1 & \cdots & 0 & 0 \\ 0 & \lambda & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \mu & 4 \\ 0 & 0 & \cdots & 1 & \nu \end{pmatrix}.$$

Here the command {cccc} is used since this matrix is built out of 5 columns. Notice the effect of \cdots (center dots), \vdots (vertical dots), and \ddots (diagonal dots) in the code which generated this nice matrix.

Instead of surrounding a matrix with parentheses, we can use other delimiters such as \left|,\right| and it looks like a determinant:

$$\det A = \left| \begin{array}{cc} a & b \\ c & d \end{array} \right| = ad - bc.$$

Perpendicularity is indicated with \perp and the transpose operation with \top:

$$\mathbf{v} \perp \mathbf{w}, \quad V^{\perp}, \quad A^{\top}.$$

The length of a vector can be indicated with double parallel lines, which are produced with \Vert: || (the capital V is important, since \vert outputs |, a single vertical line), so the Cauchy-Schwarz inequality says

$$|\mathbf{v} \cdot \mathbf{w}| \le ||\mathbf{v}|| ||\mathbf{w}||.$$

If you literally typed in parallel lines as ||, then the inequality looks less nice:

$$|\mathbf{v} \cdot \mathbf{w}| \le ||\mathbf{v}|| ||\mathbf{w}||.$$

A common notation for inner products is $\langle \mathbf{v}, \mathbf{w} \rangle$. These brackets are obtained using \langle,\rangle. If you use ordinary less than and greater than signs, you get the uglier $\langle \mathbf{v}, \mathbf{w} \rangle$.

11. Tables

Table 1 gives a two-column table without borders. In Table 2 there are boundaries added all around and a double line down the middle. The code for tables can be found in the file. The first table has the columns centered using {c|c}, while the second table has its first column left justified and its second column right justified using {|l||r|}. The vertical lines appearing in these commands tell LaTeX where to place vertical lines in the tables. Appropriately placed \hline commands tell LaTeX where to put horizontal lines.

| Integers | Real polynomials | |
|---------------------------|-------------------|--|
| Positive | Monic | |
| Prime | Irreducible | |
| Composite | Reducible | |
| ± 1 | Non-zero constant | |
| ?? | Derivative | |
| Table 1. Useful analogies | | |

| Integers | Real polynomials |
|-----------|-------------------|
| Positive | Monic |
| Prime | Irreducible |
| Composite | Reducible |
| ±1 | Non-zero constant |
| ?? | Derivative |

Table 2. Useful analogies

12. Bibliography

Some sample books, web pages, and articles (from journals and conference proceedings) appear below in the reference section, which is generated using \begin{thebibliography}{N}, \end{thebibliography}, where N is the number of references. (To be honest, N can be any number, for instance in the file it is actually 5 while there are 6 references; one was added at the last minute. Just some number belongs in the N position.) If you want to cite a reference use \cite{name} (not \ref{name}), like this: \cite{irros} is [2]. If you want to cite a page or specific result from a reference, you could do [2, p. 340], [2, pp. 340–342], or [2, Theorem 20.1.1]. Place a tilde in the space between p. or pp. and the page numbers to prevent a line break in that space. Take a look at the file to see how the citations were produced just above to know what this is about.

References

- [1] B. H. Gross and J. T. Tate, Commentary on algebra, pp. 335–336 in: "A Century of Mathematics in America, Part II," Amer. Math. Soc., Providence, 1989.
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