1. Which of the following are propositions? Give truth values for each proposition.

   (a) Jack Nicholson was born in Honolulu.

   Solution. This is a proposition. False.

   (b) What time is the train arriving?

   Solution. This is not a proposition. There is no truth value.

   (c) $2 + 2 = 4$

   Solution. This is a proposition. True.

   (d) $2 + 2 = \mathrm{x}$

   Solution. This is not a proposition.

2. For each pair of statements, determine whether the conjunction P $\wedge$ Q and the disjunction P $\vee$ Q are true

   (a) $P : \pi < 4, Q : e < 2$

   Solution.

P	Q	P $\wedge$ Q	P $\vee$ Q
T	F	F	T

   (b) P: Germany is the capital of Sweden, Q: The prime divsors of 15 are 2 and 5.

   Solution.

P	Q	P $\wedge$ Q	P $\vee$ Q
F	F	F	F

   (c) P: Mexico is south of Alaska, Q: 1738 is a multiple of 11

   Solution.

P	Q	P $\wedge$ Q	P $\vee$ Q
T	T	T	T

3. Make a truth table for each of the following propositional forms.

(a) $(P \vee Q) \wedge \neg R$

Solution.

P	Q	R	$\neg R$	$P \vee Q$	$(P \vee Q) \wedge \neg R$
T	T	T	F	T	F
T	T	F	T	T	T
T	F	T	F	T	F
T	F	F	T	T	T
F	T	T	F	T	F
F	T	F	T	T	T
F	F	T	F	F	F
F	F	F	T	F	F

(b) $Q \vee (P \vee \neg Q)$

Solution.

P	Q	$\neg Q$	$P \vee \neg Q$	$Q \vee (P \vee \neg Q)$
T	T	F	T	T
T	F	T	T	T
F	T	F	F	T
F	F	T	T	T

(c) $P \Rightarrow (Q \wedge P)$

Solution.

P	Q	$Q \wedge P$	$P \Rightarrow (Q \wedge P)$
T	T	F	F
T	F	T	T
F	T	F	T
F	F	T	T

(d) $(P \vee Q) \Rightarrow (P \wedge Q)$

Solution.

P	Q	$P \vee Q$	$P \wedge Q$	$(P \vee Q) \Rightarrow (P \wedge Q)$
T	T	T	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	F	T

4. Suppose that P is equivalent to Q and R is equivalent to S. Explain why it must be the case that $P \vee S$ is equivalent to $Q \vee R$.

   Solution. If P is equivalent to Q and S is equivalent to R then, you can substitute the proposition P with Q and S with R and vice versa. For example with, $P \vee S$, you can substitute P with Q and as P is equivalent to Q, the proposition $Q \vee S$ will still hold the same truth table, therefore equivalent

5. Identify the antecedent and consequent for each of the following conditional sentences. Assume that a, b, and f represent some fixed sequence, integer, or function, respectively

   (a) If the moon is made of cheese, then 8 is an irrational number.

   Solution.
   Antecedent: the moon is made of cheese
   Consequent: 8 is an irrational number

   (b) b divides 25 only if b divides 100.

   Solution.
   Antecedent: b divides 100
   Consequent: b divides 25

   (c) f is differentiable if f is continuous.

   Solution.
   Antecedent: f is continuous
   Consequent: f is differentiable

   (d) The fish bite whenever the moon is full.

   Solution.
   Antecedent: The moon is full
   Consequent: The fish will bite

6. Which of the following propositions are true?

   (a) A parallelogram is 5 sided iff 14 is prime.

   Solution. True

   (b) $\forall x, y, z \in \mathbb{Z}$, $xy = xz$ only if $y = z$

   Solution. True

   (c) $\forall x \in \mathbb{R}, x^2 \geq 0$ if $x > 0$.

   Solution. True.

7. Give, if possible, an example of a true conditional sentence for which

(a) the converse is false.

Solution. If the United States is in Europe, then Europe is a continent

(b) the contrapositive is true.

Solution. If $1 \in \mathbb{N}$ then $2 \in \mathbb{N}$

8. Give, if possible, an example of a false conditional sentence for which

(a) the converse is false.

Solution. Not possible, as the only way a condition is false is if the antecedent is true and the consequent is false. By taking the converse, that would result in the antecedent being false and the consequent being true which makes the conditional true.

(b) the contrapositive is true.

Solution. If $1 \in \mathbb{N}$ then $2.4 \in \mathbb{N}$