1. *Solution.*

   (a) Everyone has sent an email to someone

   (b) Everyone has sent an email to everybody

   (c) Every q has sent an email to some p and for every r, if every t has sent an email to r, then r is p

2. *Solution.* $\exists! p\ (\forall q S(q,p))$

3. *Solution.* Suppose x and y are even. Then, from the definition of even, we can rewrite x and y in the form $x = 2n$ and $y = 2m$ for some integers n and m. Then, plugging in $2n$ and $2m$ we are left with the new equation $5(2n) - 3(2m)^2 + 3$. Since $3 = 2 + 1$, we can rewrite the equation as $10n - 12m^2 + 2 + 1$. From here, we can factor out a 2 from the first three terms and we are left with an equation of the form $2(5n - 6m^2 + 1) + 1$. Let $s = (5n - 6m^2 + 1)$ Since n and m are integers, and since $\mathbb{Z}$ is closed under addition and multiplication, $s = (5n - 6m^2 + 1)$ is also an integer. Therefore, since $5x - 3y^2 + 3 = 2(5n - 6m^2 + 1) + 1 = 2s + 1$, we conclude $5x - 3y^2 + 3$ must be an odd integer if x and y are even.

4. *Solution.* Suppose a, b and c were integers. Assume a divides b and c divides d. In that case, if we factor out b from our number $2bd + b^2d^2$ we are left with $b(2d + bd^2)$. Since a divides b, we know this whole number is divisible by a. We can also factor out a d from our number: $d(2b + b^2d)$. Since c divides d, we also know our number can be divided by c. Therefore, if we factor out both b and d from our number, we can see that the number must be divisible by both a and c. Thus, the number $2bd + b^2d^2$ is divisible by ac

5. *Solution.* Suppose $x \in \mathbb{R}$ and x is arbitrary. Assume $3 - x \leq 0$. Since $3 - x \leq 0$ we can add x from both sides to get $3 \leq x$. Squaring both sides of the inequality, $9 \leq x^2$ and cubing both sides of the assumed inequality, $27 \leq x^3$. Adding the two inequalities, $27 + 9 = 36 \leq x^3 + x^2$. Therefore, $1 < 36 \leq x^3 + x^2$. Thus, if $8 - x^3 - x^2 \geq 7$, then $3 - x > 0$.

6. *Solution.* Assume $x \in \mathbb{Z}$ and is an arbitrary integer. Assume x is odd. Then, by definition, x can be written in the form $x = 2(2k)+1 = 4k+1$ for some integer k. Then, by algebra, $x^2 - 1 = (4k+1)^2 - 1 = (16k^2+8k+1) - 1 = 8(2k^2 + k) + 1 - 1 = 8(2k^2 + k)$. Since $x^2 - 1 = 8(2k^2 + k)$ for some arbitrary integer k, and because of the closure properties of $\mathbb{Z}$, $x^2 - 1$ is divisible by 8. Thus, we conclude if 8 does not divide $x^2 - 1$ then x is even.

7. *Solution.*

   (a) Assume $(x, y) \in \mathbb{R}^2$ and $(x - 1)^2 + (y - 4)^2 \leq 9$. Then, this equation can be rewritten as the distance between the points $(1, 4)$ and (x, y) less than or equal to 3: $d((1, 4), (x, y)) \leq 3$. We also know the distance between the two centers of the circles is: $d((1, 4), (2, 3)) = \sqrt{2}$ Therefore, the distance between the two centers and the distance from $(1, 4)$ to any point (x, y) satisfying $d((x, y), (1, 4)) \leq 3$ is at most $\sqrt{2} + 3$, which is less than or equal to 5: $d((2, 3), (1, 4)) + d((1, 4), (x, y)) \leq \sqrt{2} + 3 \leq 5$. Therefore, if given any $(x, y) \in \mathbb{R}$ which satisfies $(x - 1)^2 + (y - 4)^2 \leq 9$, must also satisfy $(x - 2)^2 + (y - 3)^2 \leq 25$

   (b) With two circles of radius 3 and 5, we know that if we place the centers of the two circles at certain points, we find that the circle with a larger radius can completely cover the one with the samller radius. Therefore, if the two centers of the two circles are within the difference between the two radii, we find that every point within the circle of smaller radius must be contained within the circle of the larger radius.

8. *Solution.* Assume $|m| > |n|$ then, there are four cases: either both are positive, both are negative, m is negative and n is positive and m is positive and n is negative.

   (a) Case 1: m and n are both positive
   If m and n are both positive, then $|m| = m$ and $|n| = n$. Therefore, $m > n$. Squaring both sides: $m^2 > n^2$ and subtracting n^2 from both sides: $m^2 - n^2 > 0$

   (b) Case 2: m and n are both negative
   If m and n are both negative, then $|-m| = m$ and $|-n| = n$. Therefore, $m > n$. Squaring both sides: $m^2 > n^2$ and subtracting n^2 from both sides: $m^2 - n^2 > 0$

   (c) Case 3: m is negative, n is positive
   If m is negative and n is positive, then $|-m| = m$ and $|n| = n$. Therefore, $m > n$. Squaring both sides: $m^2 > n^2$ and subtracting n^2 from both sides: $m^2 - n^2 > 0$

   (d) Case 4: m is positive, n is negative
   If m is positive and n is negative, then $|m| = m$ and $|-n| = n$. Therefore, $m > n$. Squaring both sides: $m^2 > n^2$ and subtracting n^2 from both sides: $m^2 - n^2 > 0$

Since every case prodcues the same result, namely $m^2 - n^2 > 0$, we conclude if $|m| > |n|$, then $m^2 - n^2 > 0$