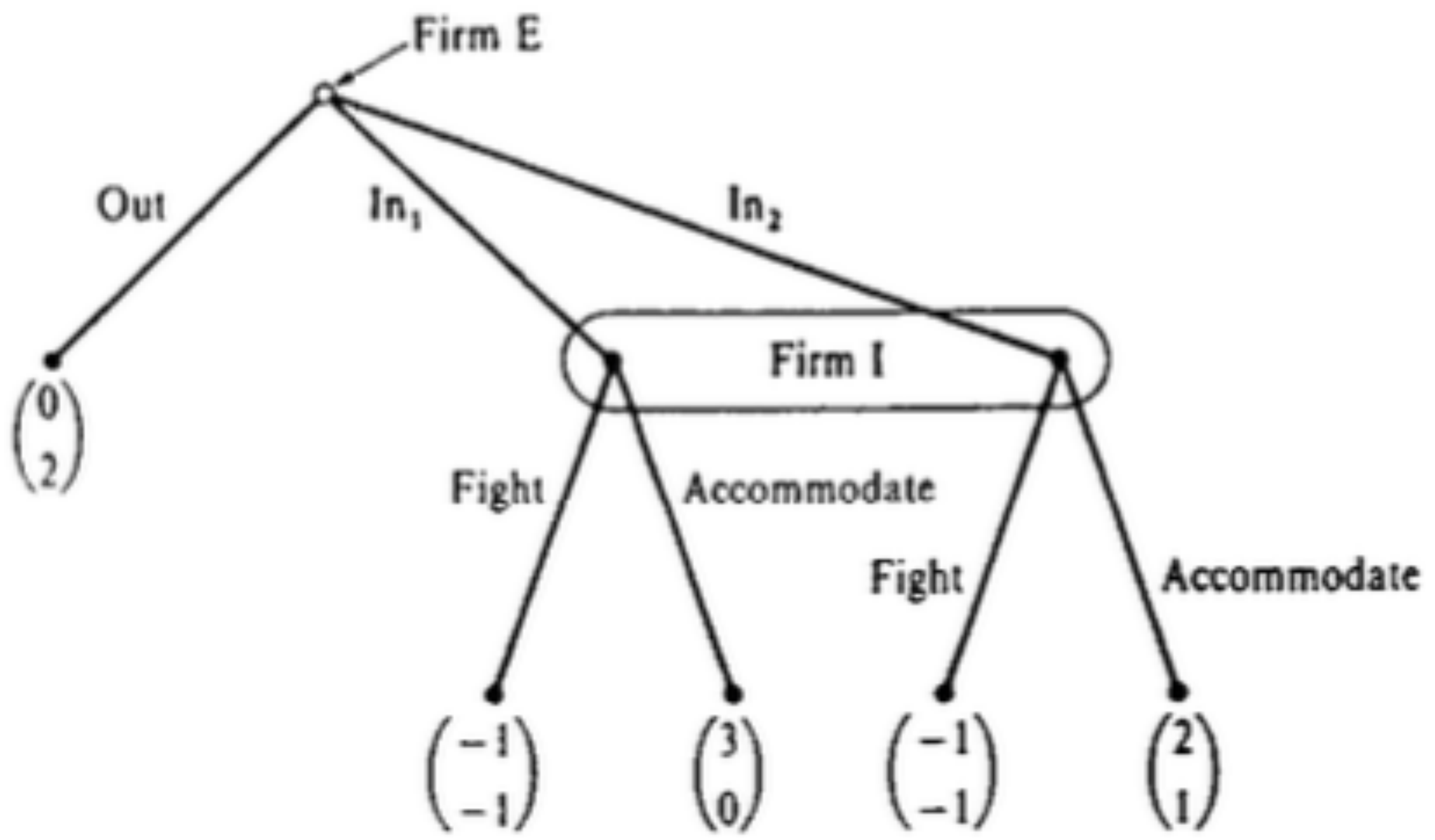
Beliefs and Sequential Rationality

- When SPNE concept fails to insure sequential rationality (Dalek game)



	f	a
out	0,2	0,2
In1	-1,-1	3,0
In2	-1,-1	2,1

N.E = {(out, f), (In1, a)}.

SPNE = {(out, f), (In1, a)}.

(out, f)는 SPNE이지만 뭔가 문제가 있어보임.

I의 information set H에서..

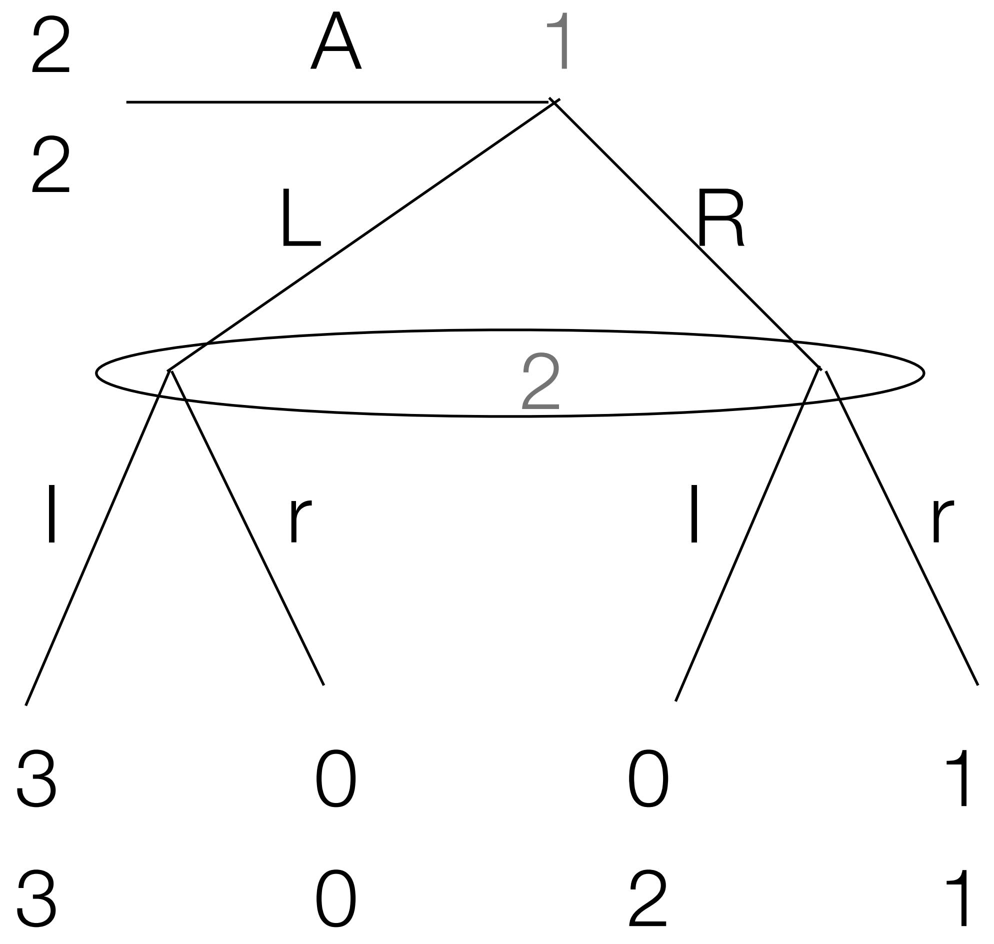
E가 In1을 선택하는 경우에도 I는 a를 선택하는게 낫고.

In2를 선택하는 경우에도 a를 선택하는게 낫기때문.

즉 I의 information set H에서는 f를 선택한다는 것이 비합리적. (In other words, at I's information set, f is strictly dominated by a for I).

다만, E가 out을 선택하는 상황이기 때문에 f를 선택하는 것이 문제라는게 드러나지 않은뿐임.

subgame perfection 절차로도 (out, f)를 배제할 수 없음. 이게임은 전체가 하나의 subgame이므로.



	l	r
A	2,2	2,2
L	3,3	0,0
R	0,2	1,1

NE = {(A, r), (L, l)}.

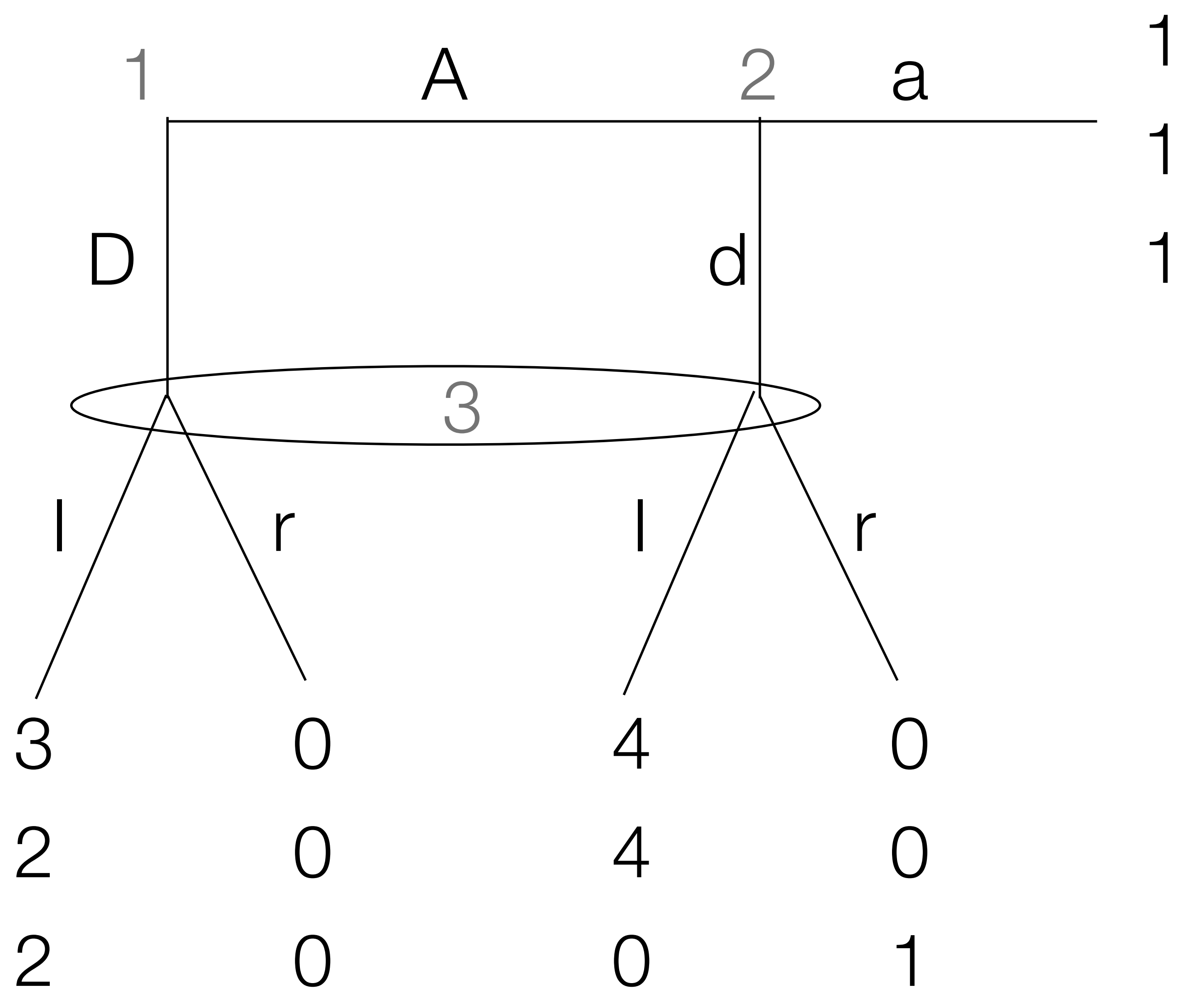
SPNE = {(A, r), (L, l)}.

여기서도 (A, r)이 같은 이유에서 비합리적.

Dalek robot in Doctor Who

Beliefs and Sequential Rationality

- When SPNE concept fails to insure sequential rationality (Zelten's horse)



3이 l을 선택했을때.

	a	d
A	1, 1, 1	4, 4, 0
D	3, 3, 3	3, 2, 2

3이 r을 선택했을 때

	a	d
A	2, 2, 1	0, 0, 1
D	0, 0, 0	0, 0, 0

(D, a, l) N.E.
(A, d, l) X

(A, a, r) N.E.
(D, d, r) X

So NE = {(D, a, l), (A, a, r)}
SPNE = {(D, a, l), (A, a, r)}.

이중 (D, a, l)은 비합리적.

(D, a, l)에서 경기자 2의 정보집합은 도달불가능한 정보집합이므로 a는 (D, l)에 대해 best response.
하지만 경기자 1과 3이 각각 (D, l)을 할때,
경기자 2의 정보집합에서는 (만일 도달가능하게 된다면) a를 해서 2를 얻는 것보다는 d를 해서 4를 얻는게 더 낫다. 따라서 (D, a, l)에서 a는 뭔가 문제 있는 대응.

Beliefs and Sequential Rationality

- **Definition**. *A system of belief* μ in extensive form game Γ_E is a specification of a probability $\mu(x) \in [0,1]$ for each decision node x in Γ_E such that $\Sigma_{x \in H} \mu(x) = 1$ for all information sets H.
- Let $E[u_i \mid H, \mu, \sigma_i, \sigma_{-i}]$ denote player i's expected utility starting at her information set H if her beliefs regarding the conditional probabilities of being at the various nodes in H are given by μ, if she follows strategy σ_i, and if her rivals use strategies σ_{-i}.
- **Definition**. A strategy profile $\sigma = (\sigma_1, \ldots, \sigma_I)$ in extensive form game Γ_E is *sequentially rational at information set* H *given a system of beliefs* μ if, denoting by $\iota(H)$ the player who moves at information set H, we have

$$E[u_{\iota(H)} \mid H, \mu, \sigma_{\iota(H)}, \sigma_{-\iota(H)}] \geq E[u_{\iota(H)} \mid H, \mu, \bar{\sigma}_{\iota(H)}, \sigma_{-\iota(H)}]$$

  for all $\bar{\sigma}_{\iota(H)} \in \Delta(S_{\iota(H)})$. If strategy profile σ satisfies this condition for *all* information sets H, then we say that σ is *sequentially rational given belief system* μ.
- In words, a strategy profile $\sigma = (\sigma_1, \ldots, \sigma_I)$ is sequentially rational if no player finds it worthwhile, once one of her information sets has been reached, to revise her strategy given her beliefs about what has already occurred (as embodied in μ) and her rivals' strategies.

Beliefs and Sequential Rationality

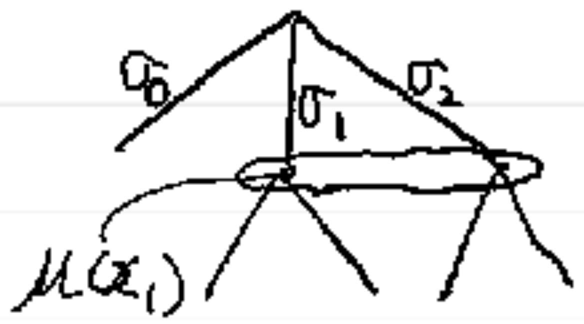
- **Definition**. A profile of strategies and system of beliefs (σ, μ) is a *weak perfect Bayesian equilibrium* (weak PBE) in extensive form game Γ_E if it has the following properties:

  (i) The strategy profile σ is sequentially rational given belief system μ.

  (ii) The system of beliefs μ is derived from strategy profile σ through Bayes' rule whenever possible. That is for any information set H such that $\text{Prob}(H \mid \sigma) > 0$ (read as "the probability of reaching information set H is positive under strategies σ"), we must have

$$\mu(x) = \frac{\text{Prob}(x \mid \sigma)}{\text{Prob}(H \mid \sigma)} \text{ for all } x \in H.$$

- First, strategies must be sequentially rational given beliefs. Second, whenever possible, beliefs must be consistent with the strategies.



$\mu(x_1) = \dfrac{\sigma_1}{\sigma_1 + \sigma_2}$ 이어야 Bayes 일관성을 가짐.

- $(\sigma_0, \sigma_1, \sigma_2) = (\frac{1}{2}, \frac{1}{6}, \frac{2}{6})$ 라면,

  $\mu(x_1) = \dfrac{\frac{1}{6}}{\frac{1}{6} + \frac{2}{6}} = \dfrac{1}{3}$ 이어야 함.

- $(\sigma_0, \sigma_1, \sigma_2) = (0, \frac{1}{4}, \frac{3}{4})$ 라면.

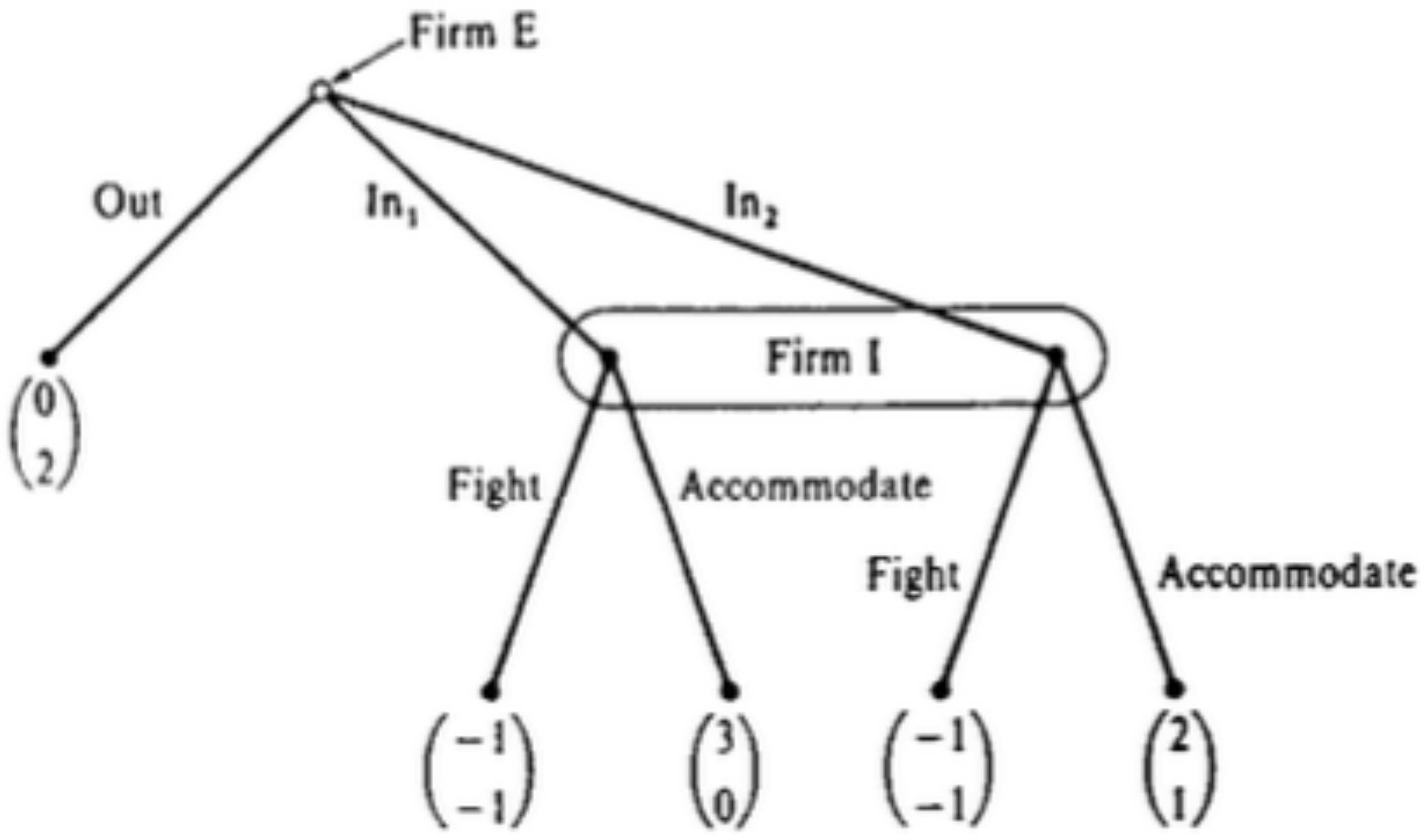
  $\mu(x_1) = \dfrac{\frac{1}{4}}{\frac{1}{4} + \frac{3}{4}} = \dfrac{1}{4}$ 이어야 함.

- $(\sigma_0, \sigma_1, \sigma_2) = (1, 0, 0)$. 라면,

  $\mu(x_1) = \dfrac{0}{0}$ 이므로 Bayes 일관성 확인불가.

  어떤 μ 체계도 okay.

Beliefs and Sequential Rationality



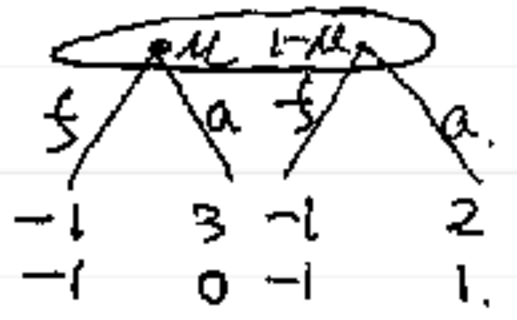
	f	a
out	0, 2	0, 2
In1	−1, −1	3, 0
In2.	−1, −1	2, 1.

N.E = {(out, f), (In1, a)}.

SPNE = {(out, f), (In1, a)}.

(out, f)는 SPNE이지만 뭔가 문제가 있어보임.

① (out, f)는 sequential rationality를 충족하지 않는다.



$E[u_I \mid H, \mu, out, f] = \mu(-1) + (1-\mu)(-1) = -1.$

$E[u_I \mid H, \mu, out, a] = 0 \cdot \mu + (1-\mu)1 = 1-\mu$

$\therefore E[u_I \mid H, \mu, out, f] < E[u_I \mid H, \mu, out, a]$ for any $\mu \in [0,1]$.

즉, H에서 f가 best가 되는 μ가 존재하지 않는다. (f를 합리화해줄 μ가 존재하지 않는다)

$\therefore$ (out, f) is not a wPBE.

① (In1, a)는 sequential rationality를 충족한다.

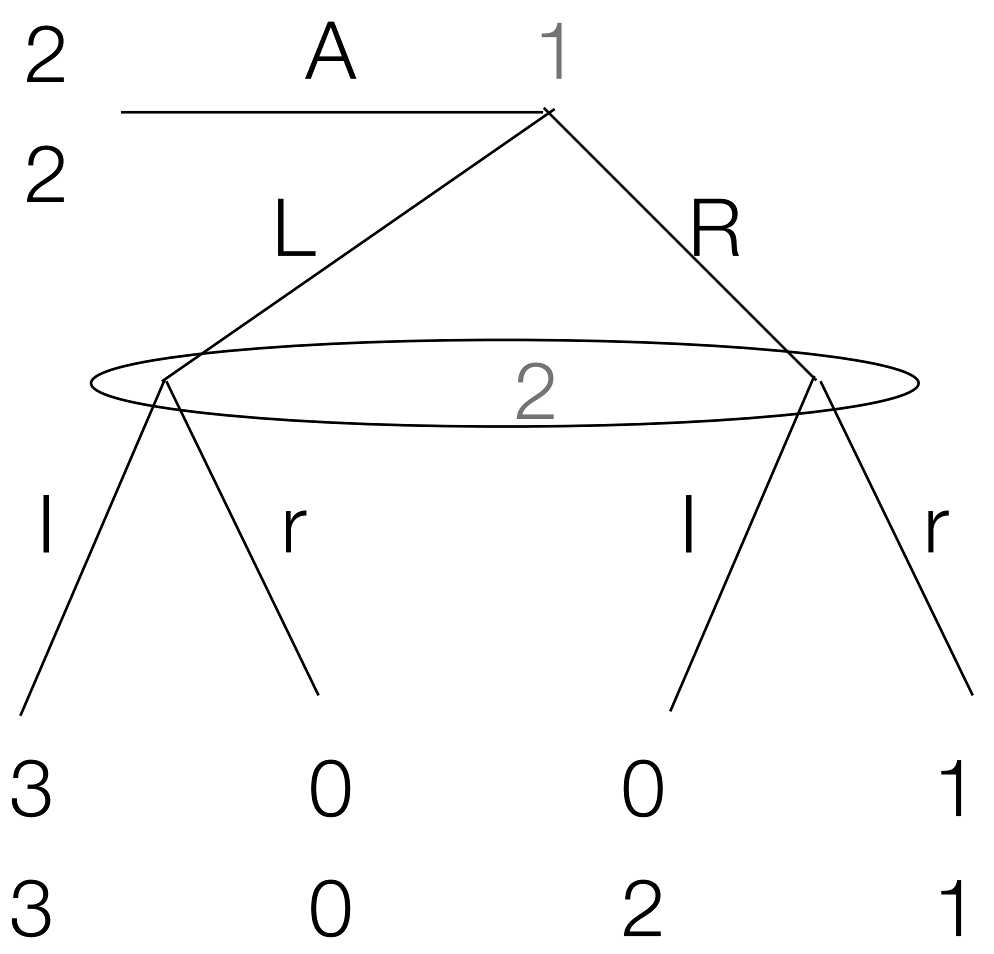
$E[u_I \mid H, \mu, out, f] < E[u_I \mid H, \mu, out, a]$ for any $\mu \in [0,1]$ 이므로.

②

$$\mu(x_1) = \frac{Prob(x_1 \mid (In1, a))}{Prob(H \mid (In1, a))} = \frac{1}{1} = 1.$$

$\therefore$ (In1, a, μ=1) is a wPBE.

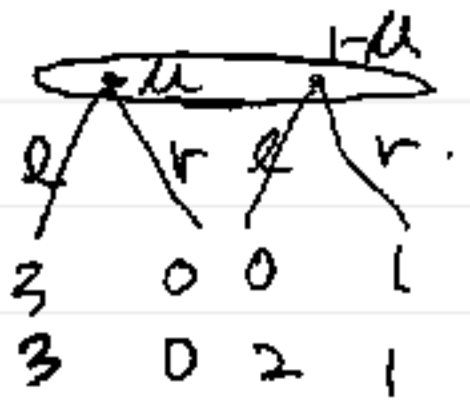
Beliefs and Sequential Rationality



	l	r
A	2,2	2,2
L	3,3	0,0
R	0,2	1,1

NE = {(A, r), (L, l)}.
SPNE = {(A, r), (L, l)}.

① (A, r) is not sequentially rational.



$E[u_2 | H, \mu, A, r] = 0 \cdot \mu + 1(1-\mu).$

$E[u_2 | H, \mu, A, l] = 3\mu + 2(1-\mu) = \mu + 2$

∴ $E[u_2 | H, \mu, A, r] \geq E[u_2 | H, \mu, A, l]$ 인 μ는 존재하지 않는다.

① (L, l) is sequentially rational.

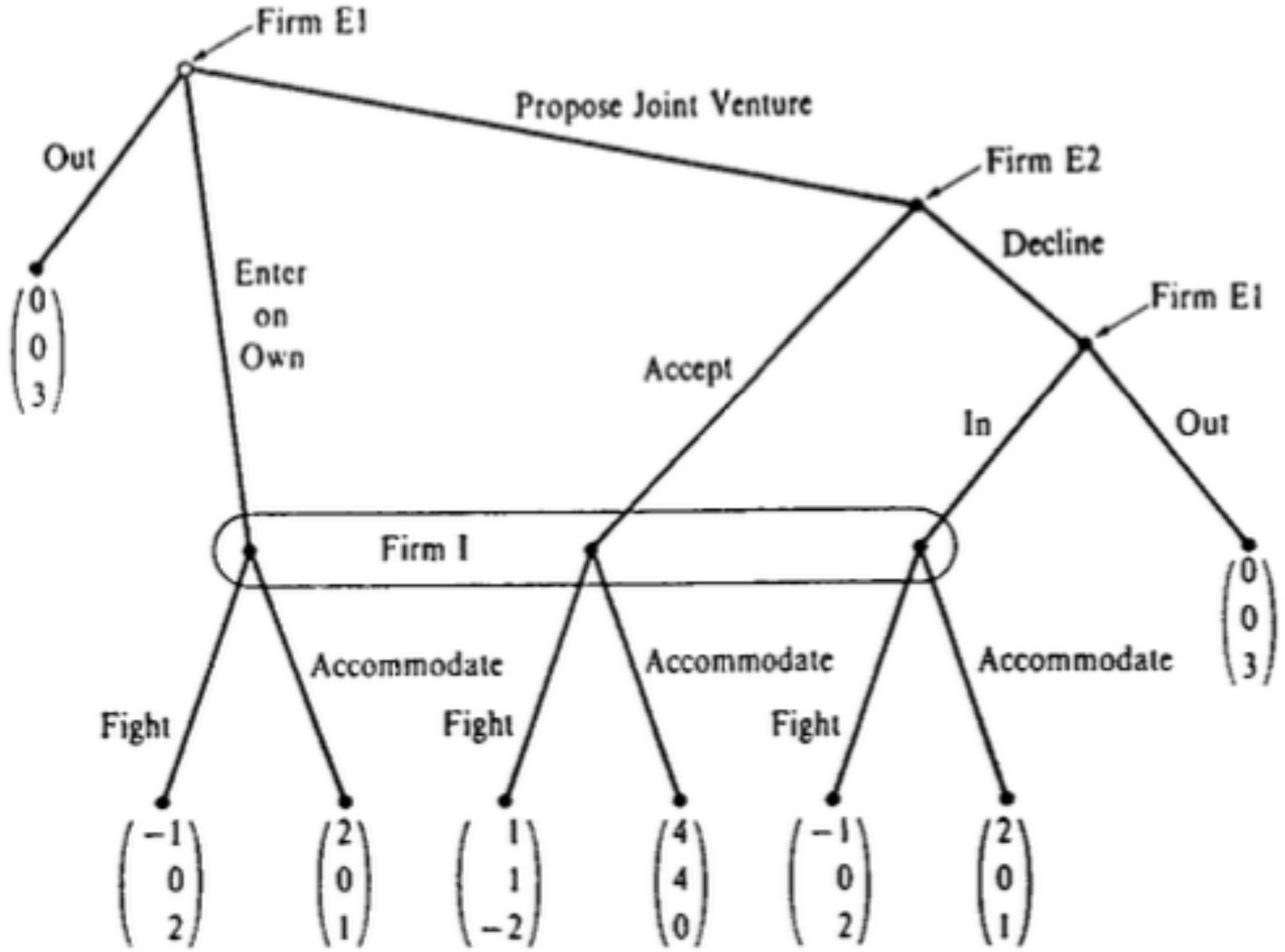
$E[u_2 | H, \mu, L, l] > E[u_2 | H, \mu, L, r]$ for any $\mu \in [0,1]$.

②

$$\mu(x_1) = \frac{Prob(x_1 | (L, l))}{Prob(H | (L, l))} = \frac{1}{1} = 1.$$

∴ $(L, l, \mu = 1)$ is a wPBE.

Beliefs and Sequential Rationality



S_{E1} = { out/In, Out/out, Enter on own/In, Enter on own/out, propose/In, propose/out }

S_{E2} = { accept, decline }

S_I = { fight, accommodate }.

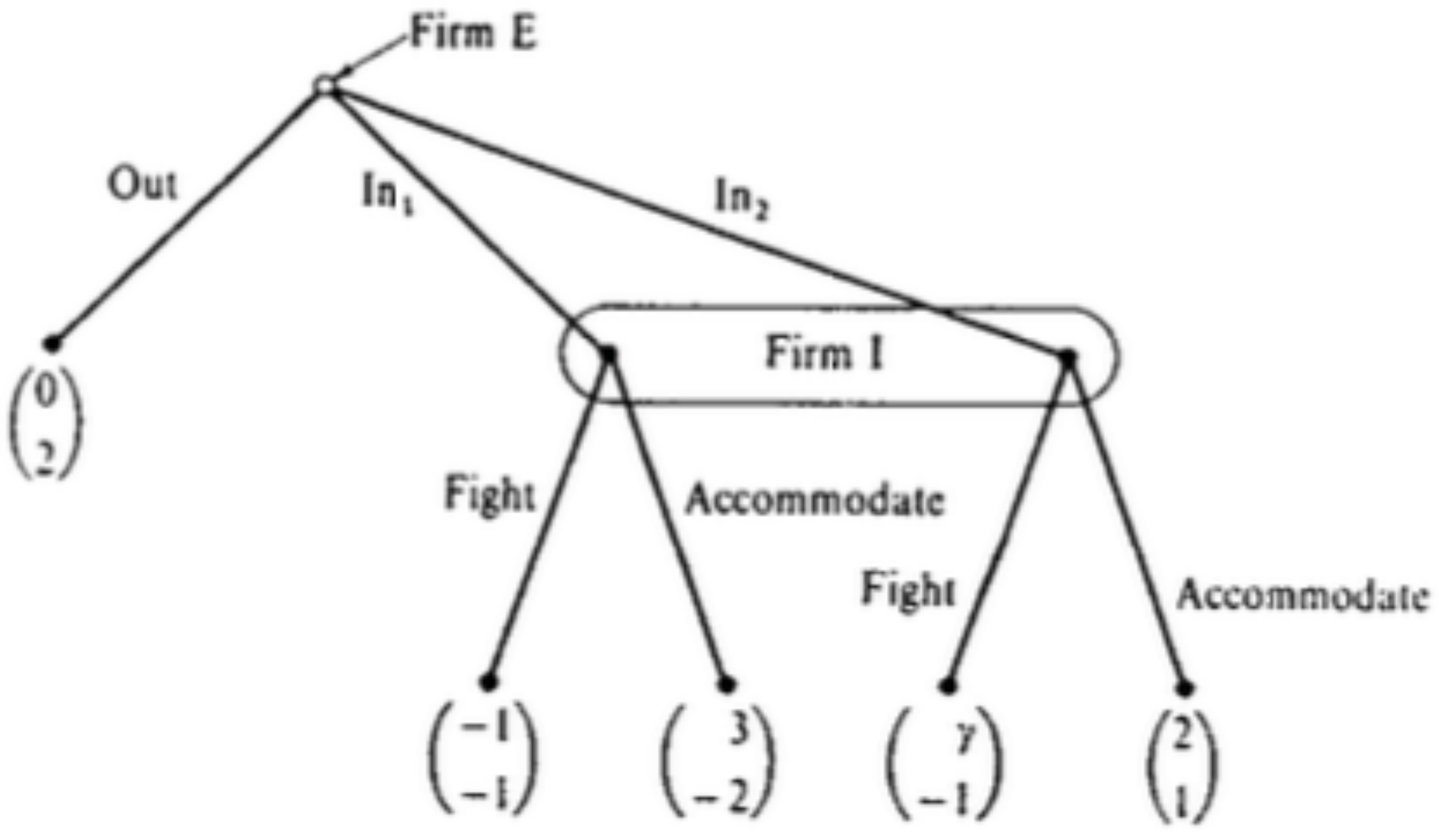
sequentially rational.

(propose/In, accept, accommodate)

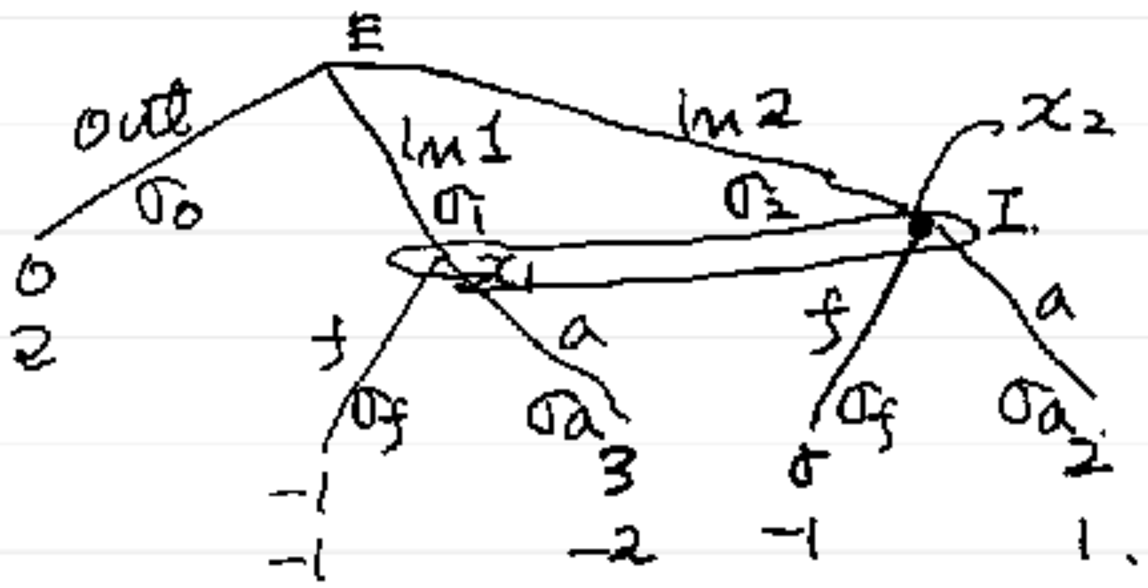
(propose/out, accept, accommodate).

not sequentially rational.

Beliefs and Sequential Rationality



$\gamma > -1$



	f	a
Out	0, 2	0, 2
In1	−1, −1	3, −2
In2	γ, −1	2, 1

$(\gamma > 0)$.

no pure strategy NE.

① I will play f with probability 1 if

$E[u_I \mid H, \mu, \sigma_E, f] > E[u_I \mid H, \mu, \sigma_E, a]$

$-1\cdot\mu + (-1)(1-\mu) = -1$. $\quad -2\mu + 1(1-\mu) = 1-3\mu$

$\therefore \mu > \frac{2}{3}$.

그런데 I가 f를 prob. 1로 하면, E는 In2를 prob 1로 할것. 따라서 $\mu(x_1) = \frac{0}{1} = 0$.

$\therefore \mu > \frac{2}{3}$는 f를 합리화하는 믿음이지만. $(\cdot, f)$ 전략쌍 하에서는 not consistent.

② $\mu < \frac{2}{3}$은 a를 합리화. 하지만 이때 E는 In1을 prob 1로 할 것이므로 $\mu < \frac{2}{3}$은 not consistent

③ 남아 있는 가능성은 $\mu = \frac{2}{3}$.

- $\mu = \frac{2}{3}$ 이면 $(\sigma_F, 1-\sigma_F) \in \Delta$ 인 모든 σ_F가 best response. 즉 $\mu = \frac{2}{3}$은 $\sigma_F \in [0,1]$인 모든 σ_F를 합리화. (sequentially).
- 경기자 E는

  Out을 선택하면 0을 얻고.

  In 1을 선택하면 $-1\sigma_F + 3(1-\sigma_F) = 3 - 4\sigma_F$.

  In 2를 선택하면 $\gamma\sigma_F + 2(1-\sigma_F) = 2 + (\gamma-2)\sigma_F$

  즉 $\sigma_F = \frac{1}{\gamma+2}$ 이면, In1을 할때 $\frac{3\gamma+2}{\gamma+2}$를 얻고, In2를 할때도 $\frac{3\gamma+2}{\gamma+2}$를 얻음. $\gamma > 0$ 이면 $\frac{3\gamma+2}{\gamma+2} > 0$. 즉, out 해서 얻게될 보수보다 크므로 In1, In2 만을 사용해서 randomize.

$\therefore \gamma > 0$이면 $\sigma_E \equiv (\sigma_0, \sigma_1, \sigma_2) = (0, \frac{2}{3}, \frac{1}{3})$,

$\sigma_I \equiv (\sigma_F, 1-\sigma_F) = (\frac{1}{\gamma+2}, \frac{\gamma+1}{\gamma+2})$일 때,

$(\sigma_E, \sigma_I, \mu = \frac{2}{3})$은 WPBE

Beliefs and Sequential Rationality

- **Proposition**. A strategy profile σ is a Nash equilibrium of extensive form game Γ_E if and only if there exists a system of beliefs μ such that

  (i) The strategy profile σ is sequentially rational given belief system μ at all information sets H such that $\text{Prob}(H \mid \sigma) > 0$.

  (ii) The system of beliefs μ is derived from strategy profile a through Bayes' rule whenever possible.

$\sigma = (\sigma_1, \dots, \sigma_I)$ is a Nash equilibrium of game $\Gamma_N = [I, \{\Delta(S_i)\}, \{u_i(\cdot)\}]$ if for $\forall i \in I$.
$u_i(\sigma_i, \sigma_{-i}) \geq u_i(\sigma_i', \sigma_{-i})$ for all $\sigma_i' \in \Delta_i$

① 모든 i가 상대방 전략상 σ_{-i}에 대해 최적대응을 하고 있다 (즉, rationality 충족)
$\Leftrightarrow$ for $\forall i \in I$.
$u_i(\sigma_i, \sigma_{-i}) \geq u_i(\sigma_i', \sigma_{-i})$ for all $\sigma_i' \in \Delta_i$

② 서로의 최적대응으로 이루어진 전략상.

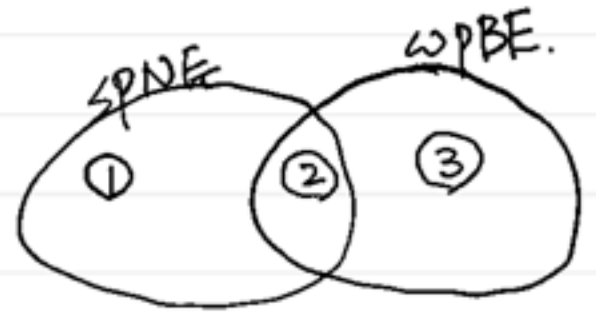
$\Downarrow$

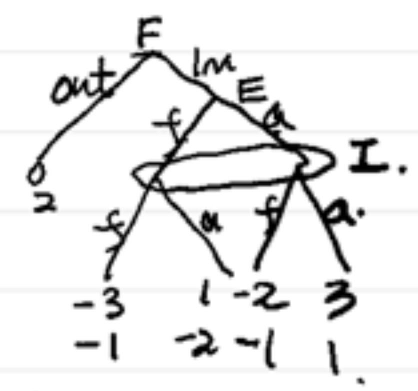
①' 모든 i가 상대방의 행동을 예측하고, 그 믿음下에서 최적 대응을 하고 있다.
(즉 어떤 믿음에 대해 최적 대응 전략을 선택한다는 의미에서 sequentially rational).

②' 그 믿음은 실제 상대방들의 행동과 일관성을 갖고 있어야 함 (i.e., Bayes 일관성).

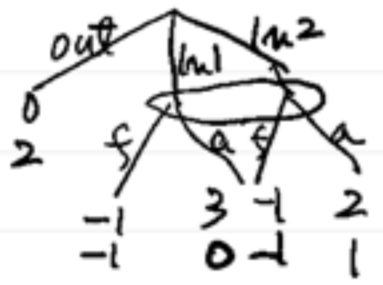
$\Rightarrow$ 위의 proposition.

	sequential rationality		belief consistency	
	on the equilibrium path	off the equilibrium path	on the equilibrium path	off the equilibrium path
NE	O		O	
SPNE	O	O	O	
wPBE	O	O	O	
PBE	O	O	O	O
SE	O	O	O	O



③ 

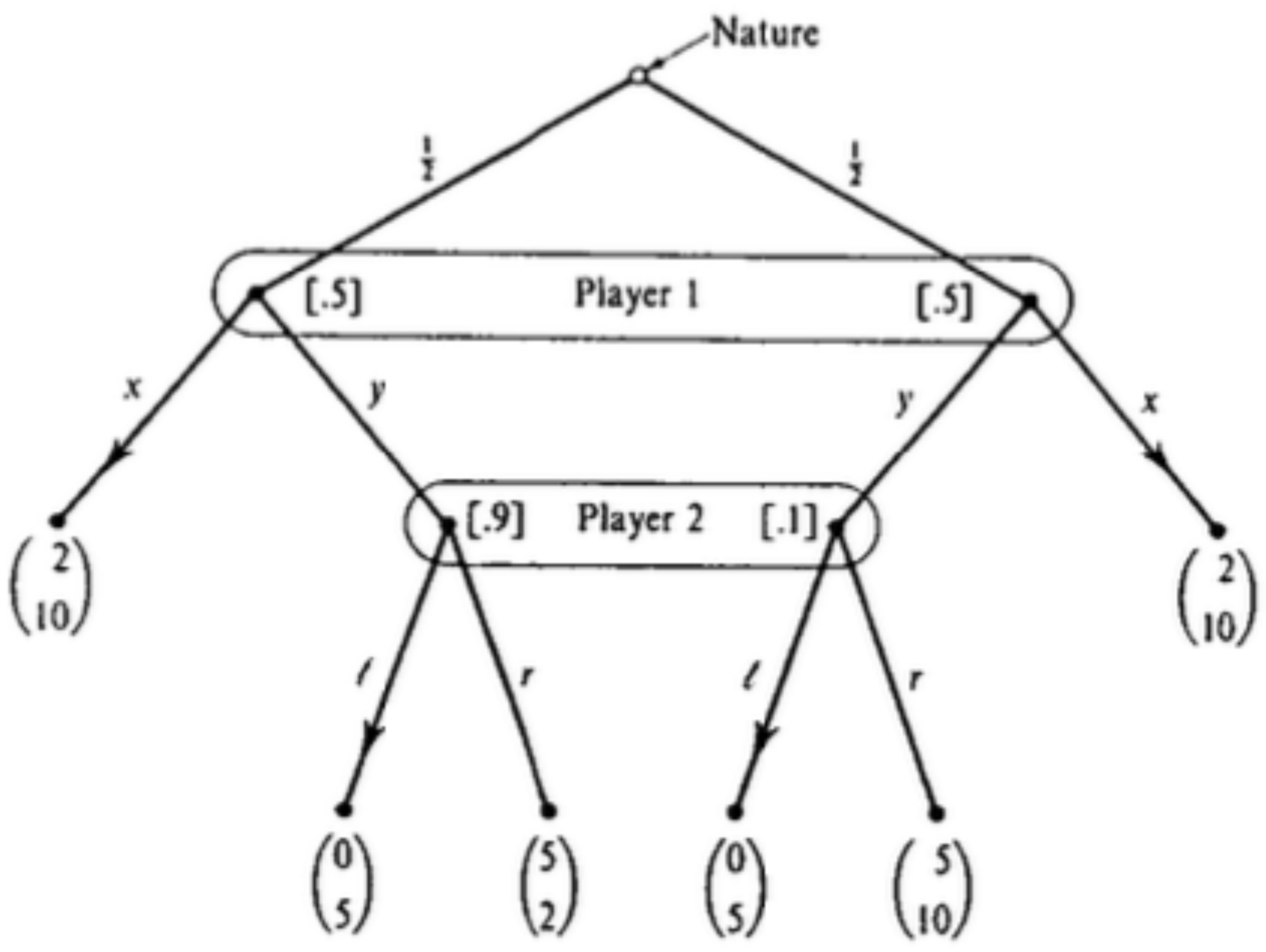
(out/f, a) is a wPBE but not a SPNE.

① 

(out, f) is a SPNE but not a wPBE

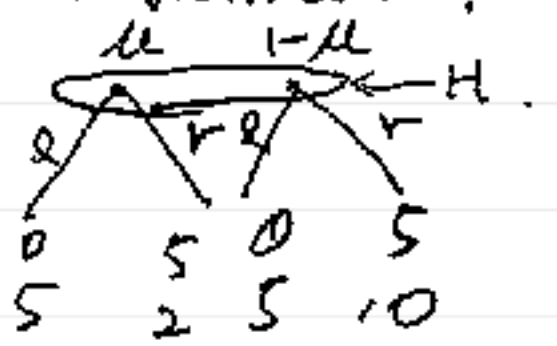
Beliefs and Sequential Rationality

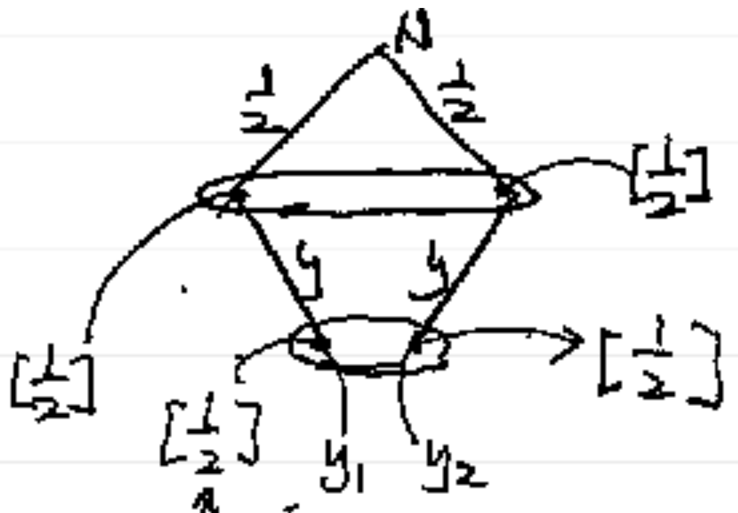
- Beliefs in a weak PBE may not be structurally consistent.



Both (x, l) and (y, r) are wPBE.

① (x, l)은 sequentially rational at H 인가?





$l \to 5$

$r \to 2\mu + 10(1-\mu)$

$5 > 10 - 8\mu$ if $\mu > \frac{5}{8}$

$$\mu(y_1) = \frac{\text{Prob}[\cdot \mid (y, r)]}{\text{Prob}[H \mid (y, r)]} = \frac{\frac{1}{2}}{\frac{1}{2} + \frac{1}{2}} = \frac{1}{2}$$

즉 y_1에 있을 확률이 $\frac{5}{8}$보다 크다는 믿음下에서 l은 sequentially rational at H.

② (x, l)은 Bayes consistency를 만족하는가?

- (x, l)下에서 H는 도달불가능한 정보집합이므로 Bayes consistency의 위배는 확인할수없다. 따라서 wPBE.

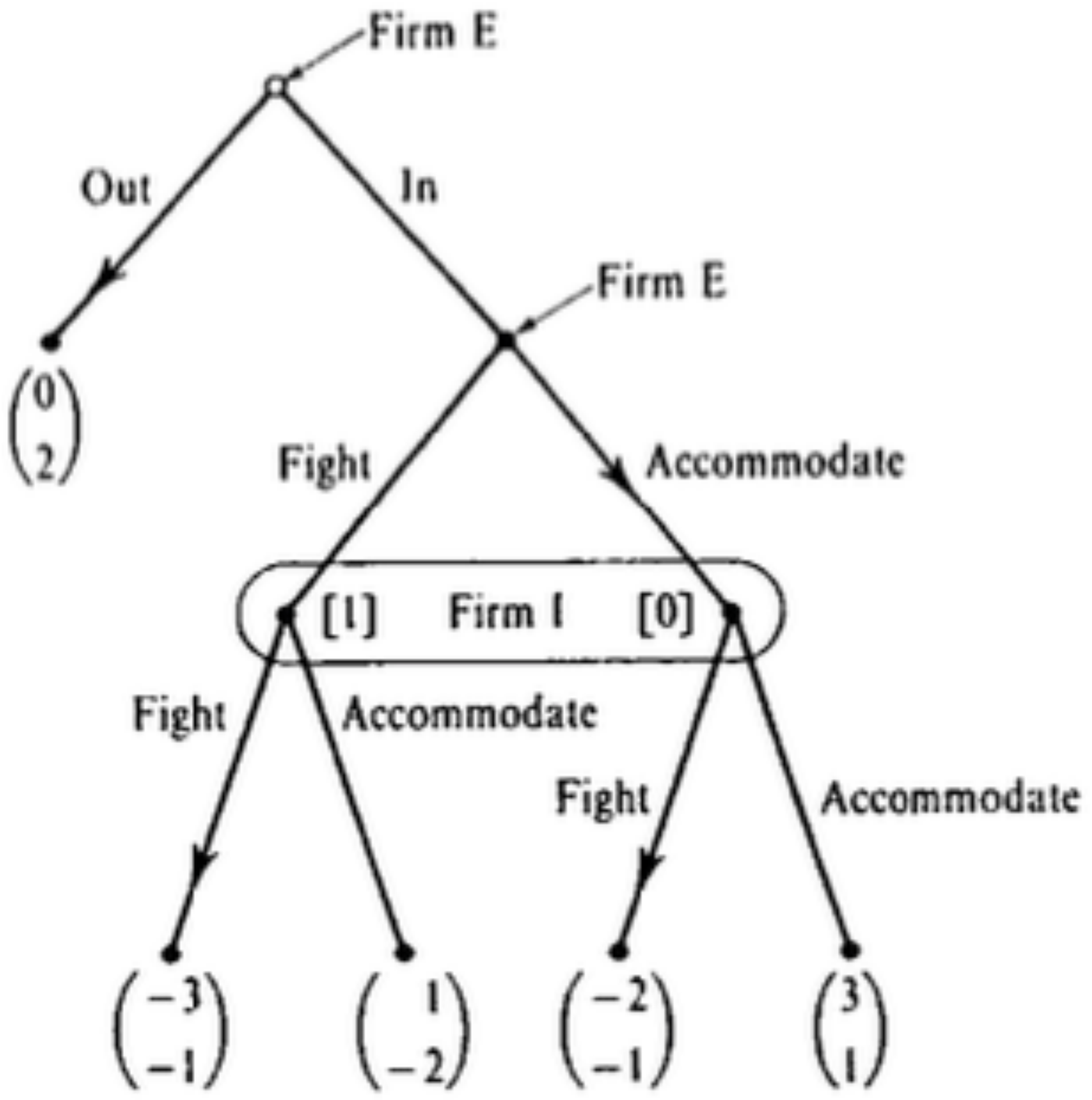
③ 그럼에도 불구하고. 게임의 구조상 H의 y_1노드에 있을 확률은 $\frac{1}{2}$일 수 밖에 없음.
따라서 $\mu > \frac{5}{8}$이라는 믿음下에서 sequentially rational 한 (x, l)은 Bayes consistency 조건은 충족하지만 구조적 비일관성을 갖는다.

$\boxed{(y, r)\text{에서는}}$
player2의 H에서
y_1 노드에 있을 확률은 $\frac{1}{2}$이어야한다.
player 2의 belief는 이것과 같아야 함.
그런데 (y, r) is $\mu < \frac{5}{8}$은 $\mu(y_1) = \frac{1}{2}$가 일관됨

∴ (y, r) with $\mu = \frac{1}{2}$ is a wPBE.

Beliefs and Sequential Rationality

- A weak PBE may not even be subgame perfect



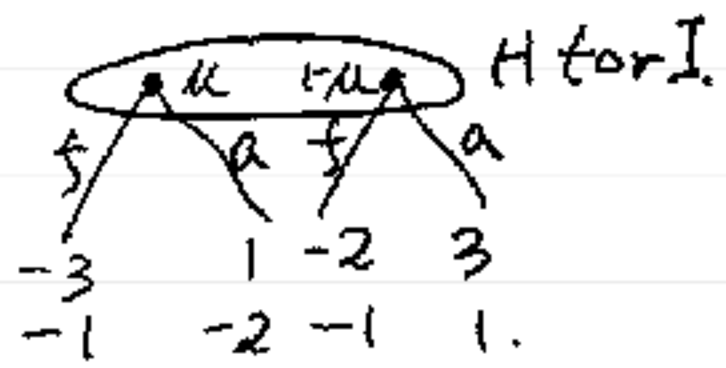
	f	a
out/f	0.2	0.2
out/a	0.2	0.2
In/f	−3, −1	1, −2
In/a	−2, −1	3, 1.

NE = {(out/f, f), (out/a, f)
(In/a, a)}.
SPNE = {(In/a, a)}.
wPBE = {(out/a, f)(In/a, a)}.

(out/a, f) is not an SPNE but a wPBE.

① is sequentially rational.

$E[u_I | H, \mu, \text{out}/a, f]$
$= -1(\mu) + (-1)(1-\mu) = -1.$
$E[u_I | H, \mu, \text{out}/a, a]$
$= -2(\mu) + 1(1-\mu) = 1 - 3\mu.$



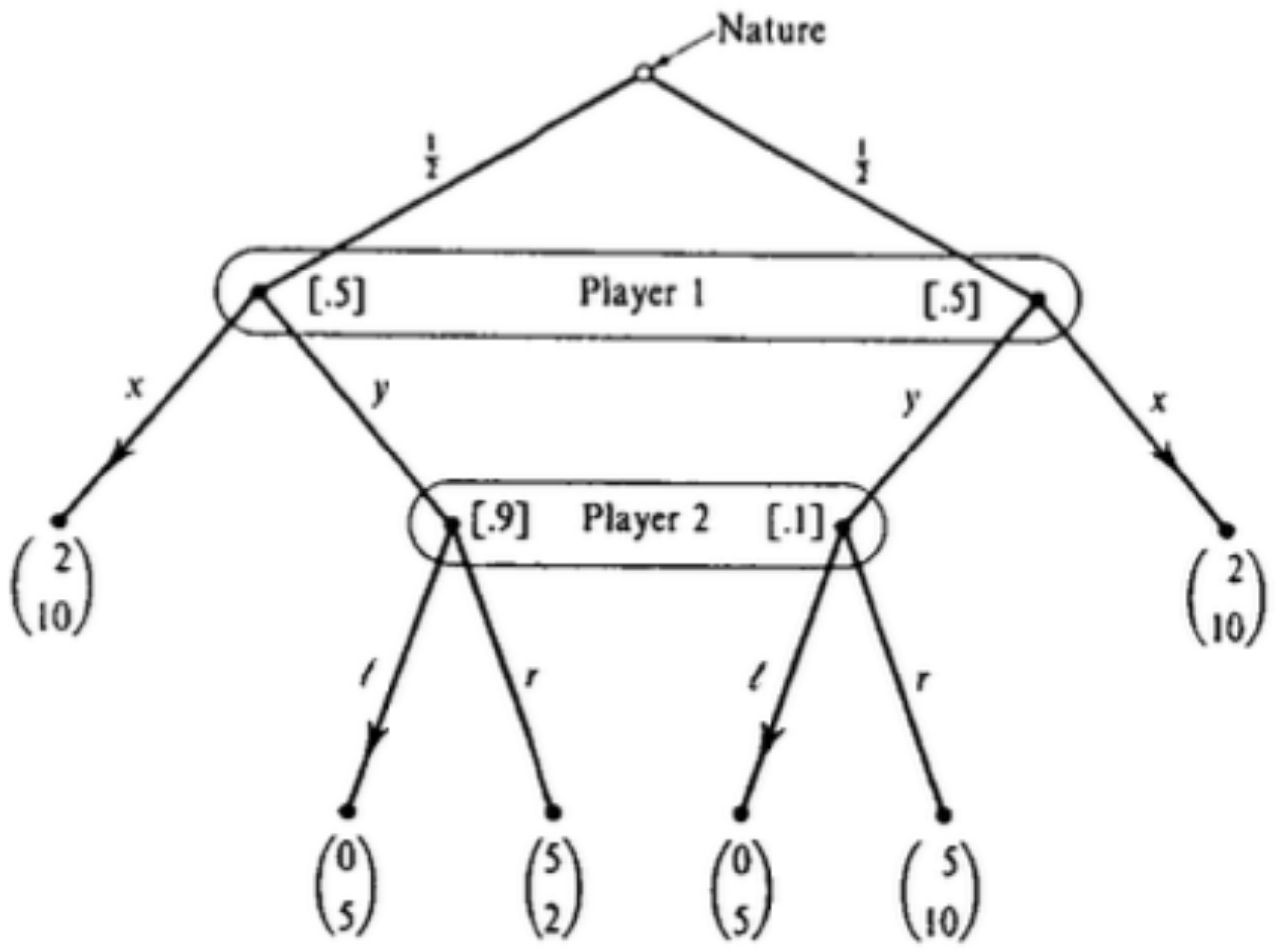
So (out/a, f) if $\mu > \frac{2}{3}$ is sequentially rational at H.

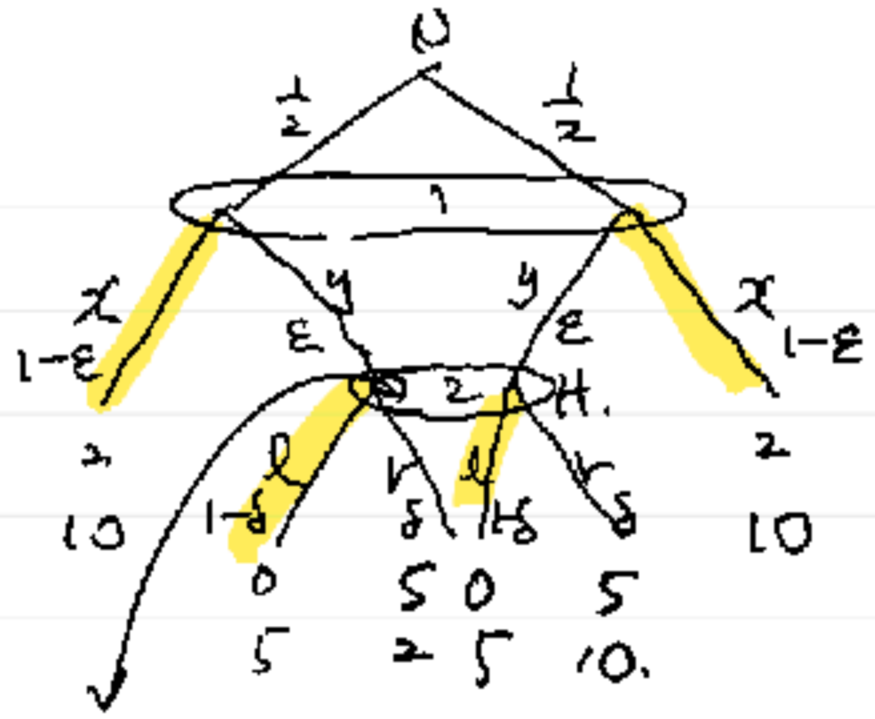
② Bayes consistency 위배 여부는 확인불가능.
(out/a, f) 下에서 H는 off the equilibrium path 상에 존재하기 때문 (즉 도달불가능한 H).

Beliefs and Sequential Rationality

- **Definition**. A strategy profile and systme of beliefs (σ, μ) is a *sequential equilibrium* of extensive form game Γ_E if it has the following properties:
- (i) Strategy profile σ is sequentially rational given belief system μ.
- (ii) There exists a sequence of completely mixed strategies $\{\sigma^K\}_{k=1}^{\infty}$, with $\lim_{k\to\infty}\sigma^k = \sigma$, such that $\mu = \lim_{k\to\infty}\mu^k$, where μ^k denotes the beliefs derived from strategy profile σ^k using Bayes' rule.

Beliefs and Sequential Rationality





$$\frac{\frac{1}{2}\varepsilon}{\frac{1}{2}\varepsilon+\frac{1}{2}\varepsilon}=\frac{1}{2}.$$

$\boxed{(x, l)\text{의 경우}}$

① (x, l) is sequentially rational if $\mu > \frac{5}{8}$

② 경기자 2의 H는 (x, l)下에서 도달불가. 따라서 어떤 μ를 가져도 Bayes consistency를 위반하지 않는다. → 아게 문제였음.

⇓

②′ 1의 정보집합에서 x 눈할 확률은 σ_x, 2의 정보 집합에서 l을 할 확률은 σ_l로 놓고 (x, l)을 (σ_x, σ_l)로 나타내면 $(\sigma_x^*, \sigma_l^*) = (1, 1)$.
$\sigma_x^k \equiv \sigma_x^* - \varepsilon^k = 1-\varepsilon^k$, $\sigma_l^k \equiv \sigma_l^* - \delta^k = 1-\delta^k$로 놓자. 단 $\lim_{k\to\infty}(\sigma_x^k, \sigma_l^k) = (\sigma_x^*, \sigma_l^*)$가 되도록 $\lim_{k\to\infty}\varepsilon^k = 0$, $\lim_{k\to\infty}\delta^k = 0$을 가정.

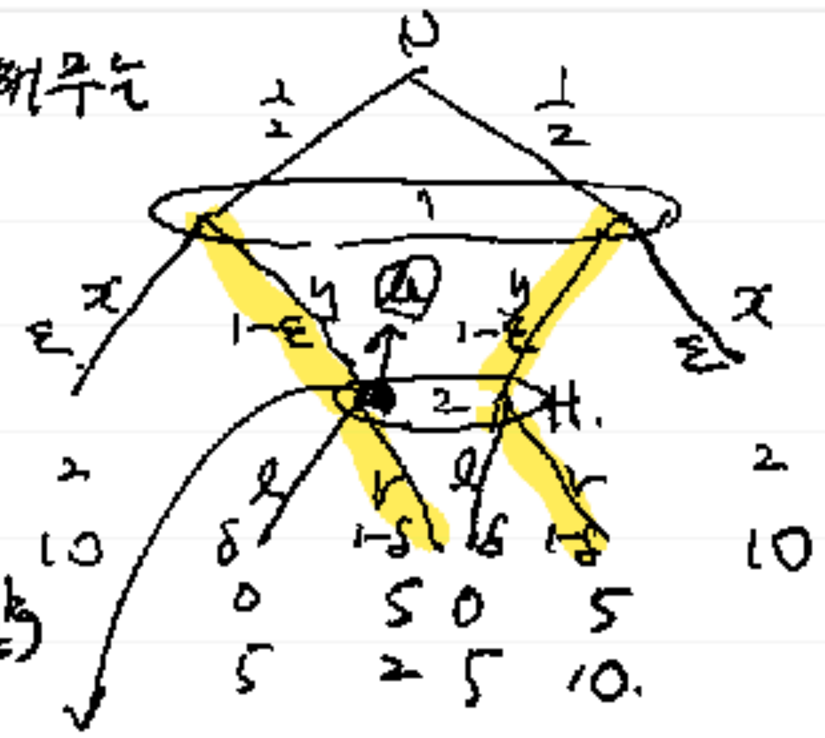
$$\frac{\frac{1}{2}\varepsilon^k}{\frac{1}{2}\varepsilon^k+\frac{1}{2}\varepsilon^k}=\frac{1}{2}.$$

$\lim_{k\to\infty}(\sigma_x^k, \sigma_l^k) = (\sigma_x^*, \sigma_l^*) = (1, 1)$이 되면서 $\lim_{k\to\infty}\mu^k = \frac{1}{2}$인데, (x, l)을 합리화해주는 $\mu > \frac{5}{8}$은 이 조건을 위반.

∴ (x, l) with $\mu > \frac{5}{8}$은 wPBE이지만 sequential equilibrium은 아니다.

$\boxed{(y, r)\text{의 경우}}$

②′ (y, r)을 합리화 해주는 믿음은 $\mu < \frac{5}{8}$.
$(\sigma_x^*, \sigma_l^*) = (0, 0)$.
ε^k와 δ^k가 0에 수렴할 때



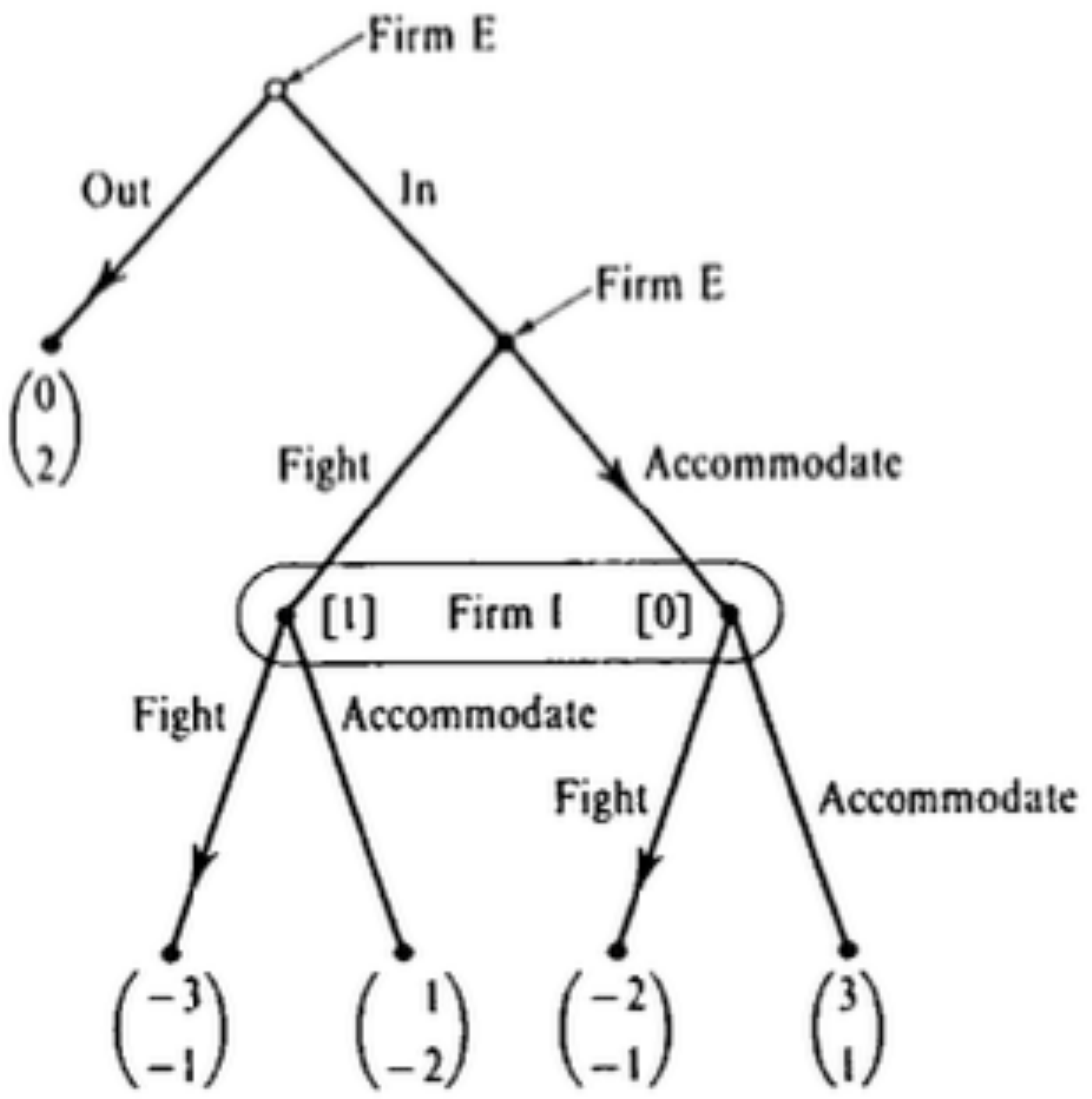
$$\lim_{k\to 0}\mu^k = \frac{\frac{1}{2}(1-\varepsilon^k)}{\frac{1}{2}(1-\varepsilon^k)+\frac{1}{2}(1-\varepsilon^k)} = \frac{1}{2}$$

$$\frac{\frac{1}{2}\varepsilon}{\frac{1}{2}\varepsilon+\frac{1}{2}\varepsilon}=\frac{1}{2}.$$

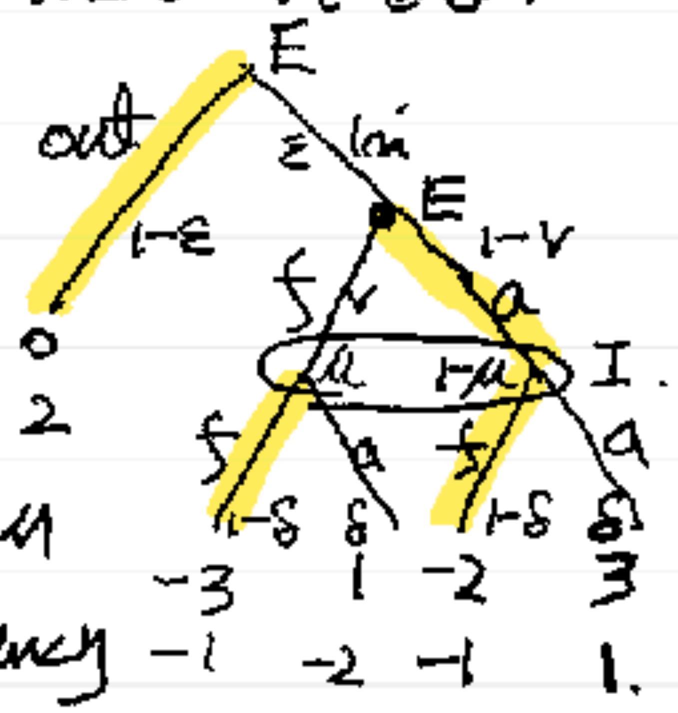
이 때 $\lim_{k\to 0}\mu^k = \frac{1}{2}$은 $\mu < \frac{5}{8}$과 일관된다.

∴ (y, r) with $\mu = \frac{1}{2}$은 wPBE이면서 sequential equilibrium.

Beliefs and Sequential Rationality



(out/a, f)는 wPBE 인데 SPNE는 아닌 균형.



① (out/a, f)는 경기자 I의 H에서 $\mu > \frac{2}{3}$ 이면 합리화됨. (sequentially rational)

② 경기자 I의 H는 (out/a, f) 下에서 도달불가능하므로 Bayes consistency 확인불가.

so (out/a, f) with $\mu > \frac{2}{3}$ 은 wPBE.

②′ $\lim_{k=\infty}(\varepsilon, v, \delta)^k = (0, 0, 0)$ 이 되는 $(\varepsilon, v, \delta)^k$의 sequence를 가정하여

$\lim_{k=\infty}(\text{out}/a, f)^k = (\text{out}/a, f) = (1, 0, 1)$ 인

σ_{out} σ_{in} and f σ_f

sequence를 가정

이때 $\lim_{k\to\infty}\mu^k = \lim_{k\to\infty}\frac{\varepsilon^k v^k}{\varepsilon^k v^k + \varepsilon^k(1-v^k)} = \lim_{k\to\infty} v^k = 0$

즉 (out/a, f)에서는 μ^k는 0에 수렴해야 함.

그런데 (out/a, f)를 합리화해주는 μ는 $\mu > \frac{2}{3}$ 이어야 하므로 (out/a, f)는 ②′조건을 만족하지 않는다.

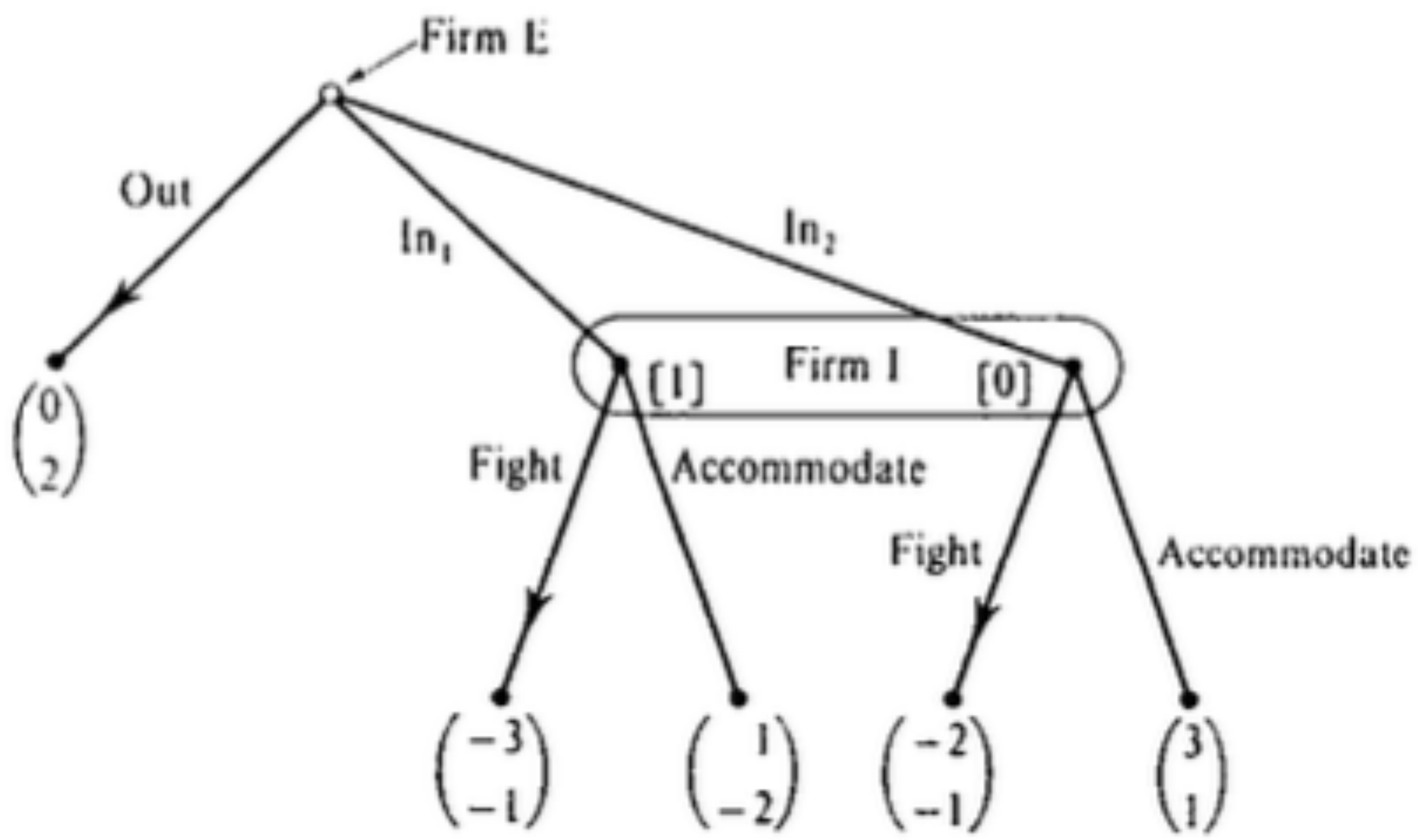
∴ (out/a, f) with $\mu > \frac{2}{3}$ 은 wPBE 이지만 sequential equilibrium은 아니다.

Beliefs and Sequential Rationality

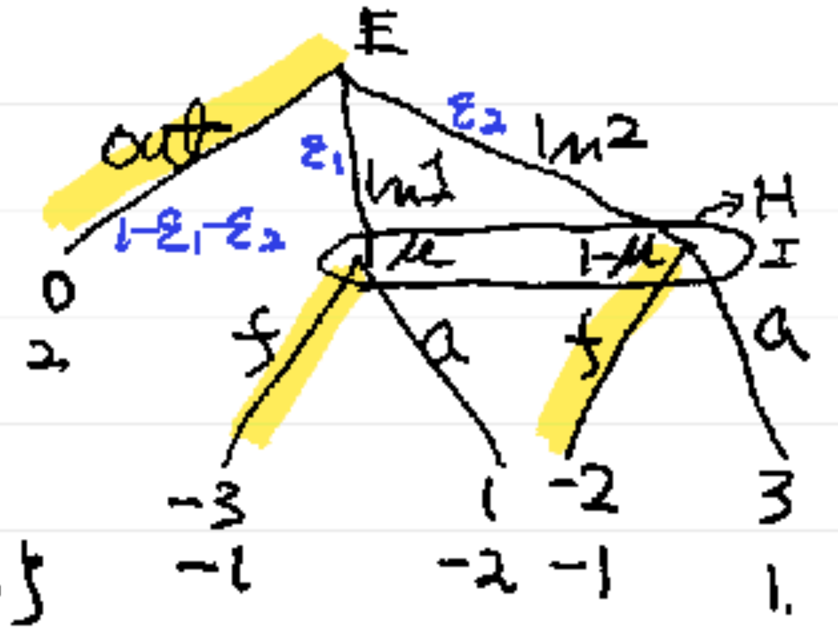
- **Proposition**. In every sequential equilibrium (σ, μ) of an extensive form game Γ_E, the equilibrium strategy profile σ constitutes a subgame perfect Nash equilibrium of Γ_E.

Reasonable Beliefs and Forward Induction

- Forward induction



	f	a
out	0,2	0,2
In1	-3,-1	1,-2
In2	-2,-1	3,1



NE = {(out, f), (In2, a)}

SPNE = {(out, f), (In2, a)}

wPBE = {(out, f, $\mu > \frac{2}{3}$), (In2, a, $\mu = 1$)}.

$\boxed{(\text{out}, f, \mu > \frac{2}{3})\text{의 경우}}$ H가 도달불가능하기에 Bayes consistency를 확인할 수 없다.

$\lim_{k\to\infty} (\varepsilon_1^k, \varepsilon_2^k) = 0$ 이 되도록 sequence를 만들어도

$\lim_{k\to\infty} \mu^k = \lim_{k\to\infty} \frac{\varepsilon_1^k}{\varepsilon_1^k + \varepsilon_2^k}$ 의 극한을 찾을 수 없기 때문에 sequential equilibrium 기준을 적용할 수도 없다.

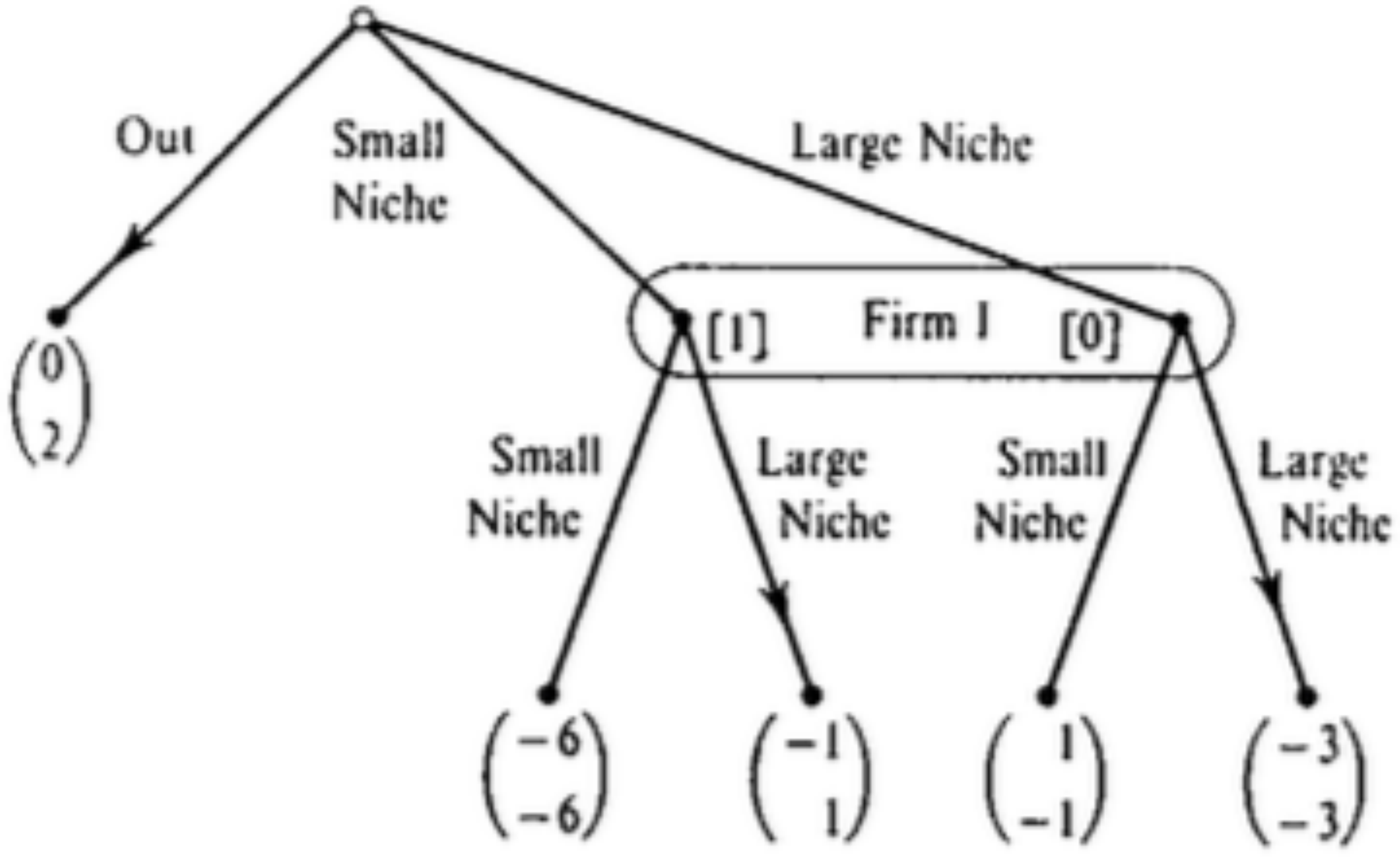
forward induction.

① 경기자 I가 보기에, 경기자 E가 "진입하기로 결정했다는 전제下에서"
In1은 In2에 의해 "strictly dominated".
따라서 $\mu = 0$일 것이라 생각하는 것이 타당.

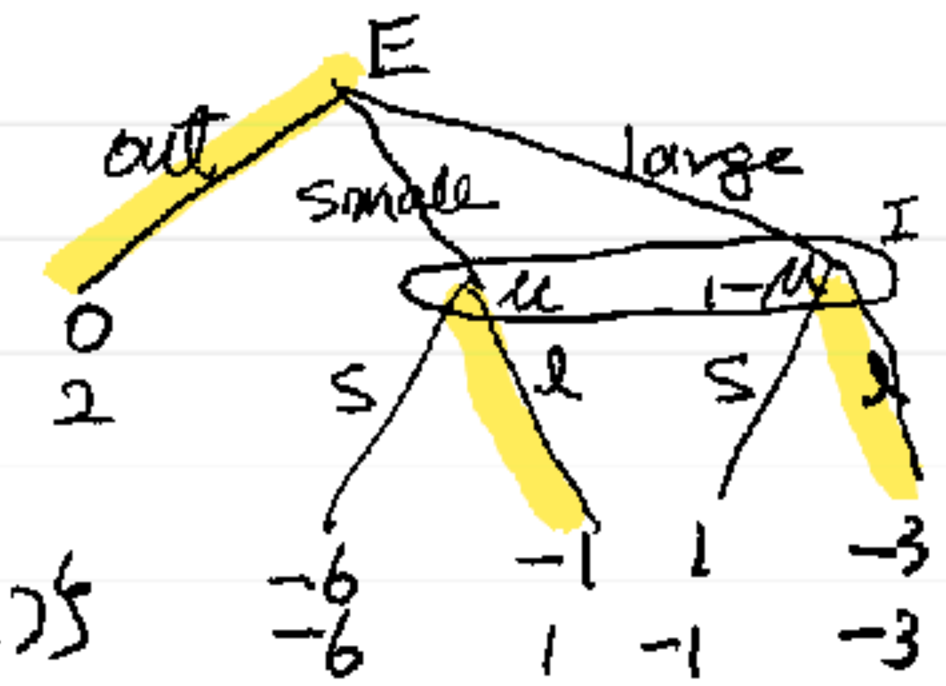
② 따라서 (out, f, $\mu > \frac{2}{3}$)은 forward induction에 의거 배제가능.

Reasonable Beliefs and Forward Induction

- Forward induction



	s	l
out	0, 2	0, 2
small	−6, −6	−1, 1
large	1, −1	−3, −3



$NE = \{(out, l), (large, s)\}$

$SPNE = \{(out, l), (large, s)\}$.

$wPBE = \{(out, l, \mu > \frac{4}{9}), (large, s, \mu = 0)\}$

$\boxed{(out, l, \mu > \frac{4}{9})\text{에 대해}}$

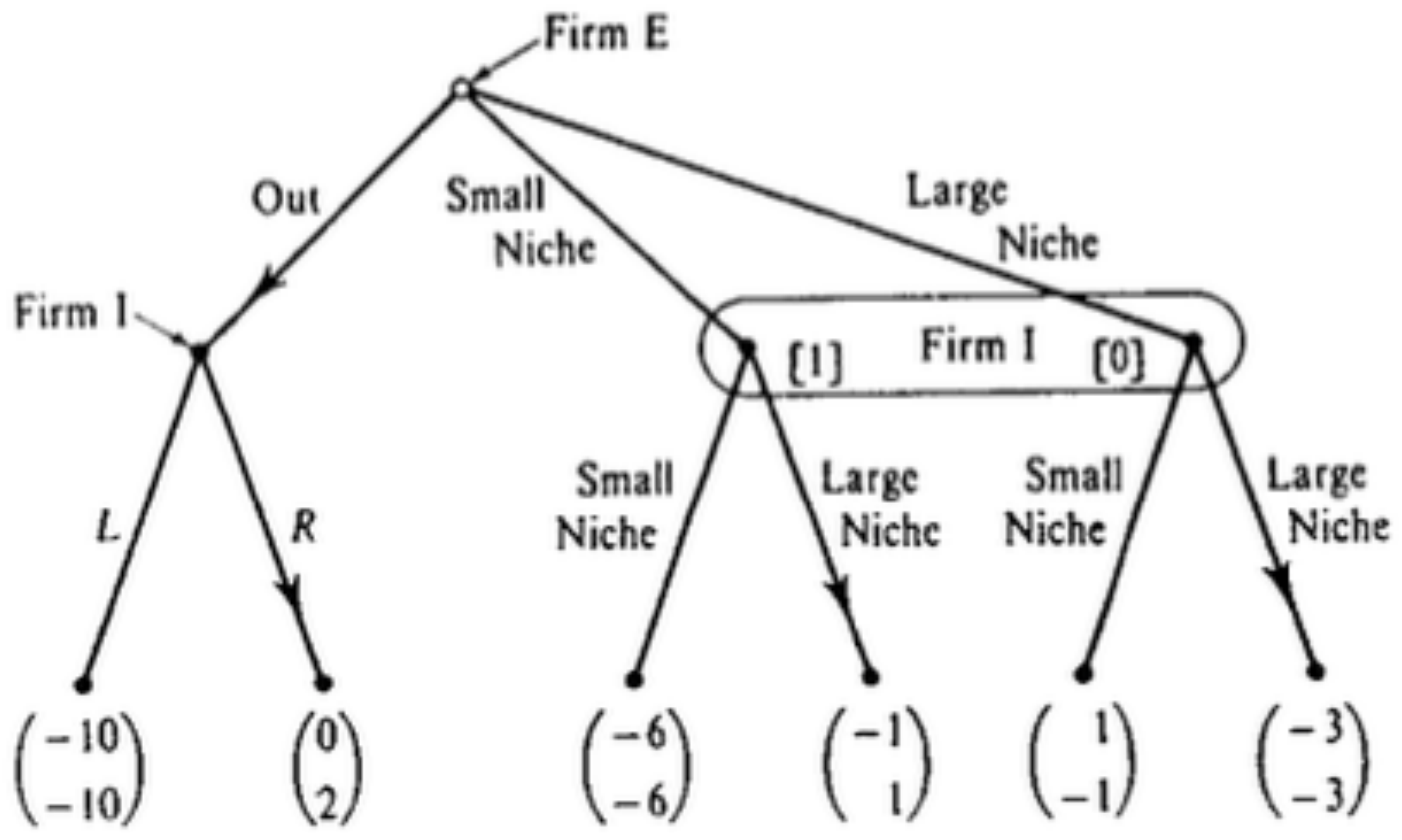
Forward Induction.

① 경기자 E에게 small은 out에 의해 strictly dominated.
즉 진입을 했다면 large를 택할 것이다.
small을 택하려면 차라리 out이 낫다는 것을
경기자 I가 추론함. 즉 μ should be 0.

② Forward Induction에 의해
$(out, l, \mu > \frac{4}{9})$는 배제.

Reasonable Beliefs and Forward Induction

- Equilibrium domination



	Ls	Ll	Rs	Rl
out	−10 −10	−10 −10	0 2	$\boxed{0\ 2}$
Small	−6 −6	$\boxed{-1\ 1}$	−6 −6	−1 1
large	1 −1	−3 −3	$\boxed{1\ -1}$	−3 −3

NE = { (out, Rl), (Small, Ll), (large, Ls)
(large, Rs) }.

SPNE = { (out, Rl), (large, Rs) }.

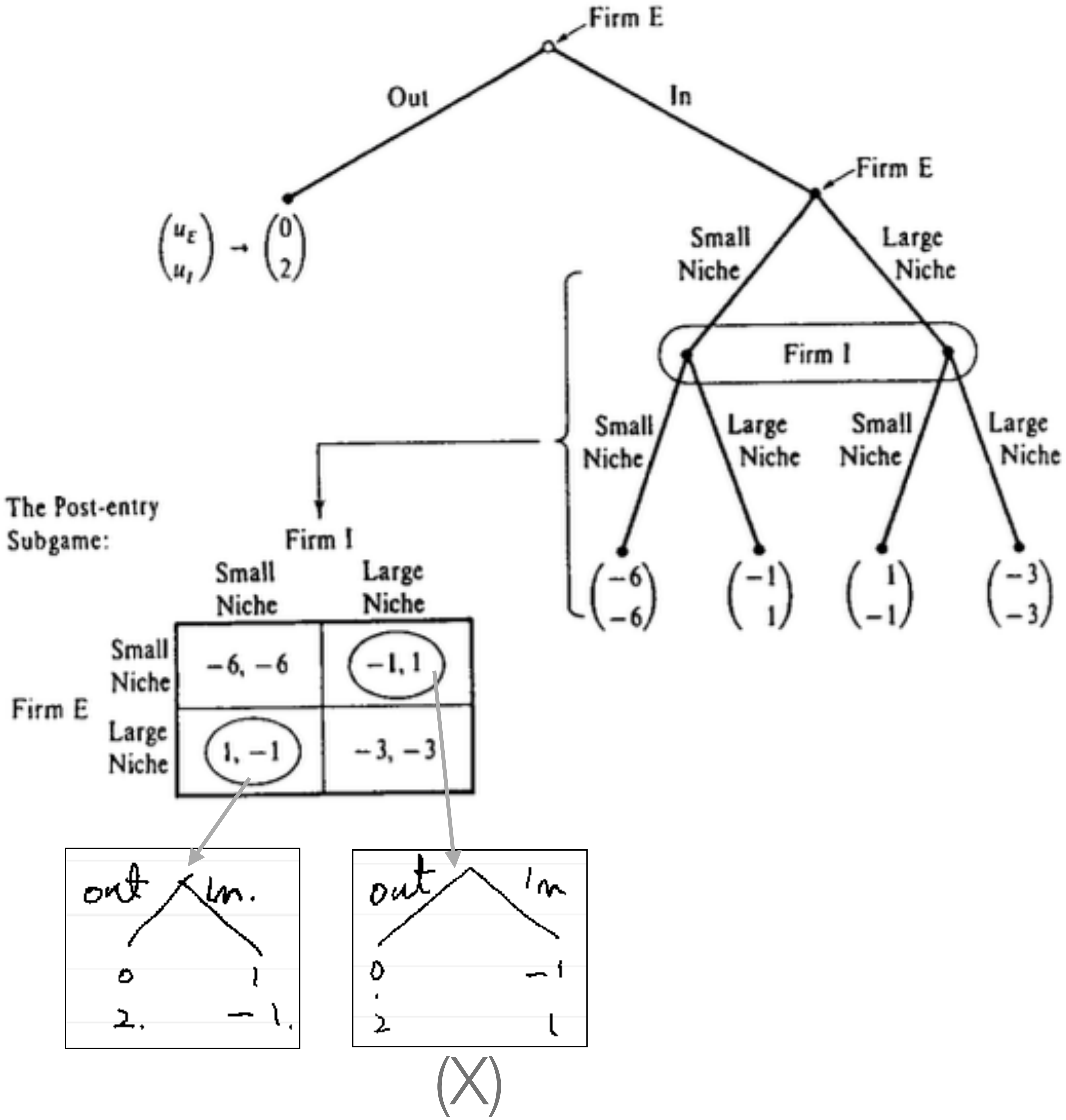
(out, Rl) is sequentially rational if
$\mu > \frac{4}{11}$.

① for E. out is not strictly dominated by small any more.
Therefore the same type of forward induction cannot apply.

② equilibrium domination.
for E, out is dominated by small if E treats its equilibrium payoff as something that it can achieve with certainty by following its equilibrium strategy.
i.e. by doing out (with I playing R)
E gets 0.
by doing small. it will get −6 or −1 which are greater than it gets by playing out.
∴ "out" is equilibrium dominated by "small" for firm E.

Reasonable Beliefs and Forward Induction

- Forward induction



	s	l
out/small	0, 2	0, 2
out/large	0, 2	0, 2
In/small	−6, −6	−1, 1
In/large	1, −1	−3, −3

← subgame.

NE = {(out/small, l), (out/large, l), (In/large, s)}

SPNE = {(out/small, l), (In/large, s)}.

① (out/small, l) is a SPNE. But...

for E, In/small is strictly dominated by "out".

I는 E가 In을 택한다면 large로 In할 것이라 추론. Small로 In 할거면 out이 더 나은선택일 것이므로.

따라서 forward induction에 의해 (out/small, l)은 배제가능.

Reasonable Beliefs and Forward Induction

- Forward induction: some potential problems
- For example, suppose that we are in a world where players make mistakes with some small probability. In such a world, are the forward induction arguments just given convincing? Perhaps not. To see why, suppose that firm E enters in the same shown in Figure 9.D.l(a) when it was supposed to play "out." Now firm I can explain the deviation to itself as being the result of a mistake on firm E's part, a mistake that might equally well have led firm E to pick "in$_1$" as “in$_2$". And firm E’s speech may not fall on very sympathetic ears: "Of course, firm E is telling me this,” reasons the incumbent, "it has made a mistake and now is trying to make the best of it by convincing me to accommodate."

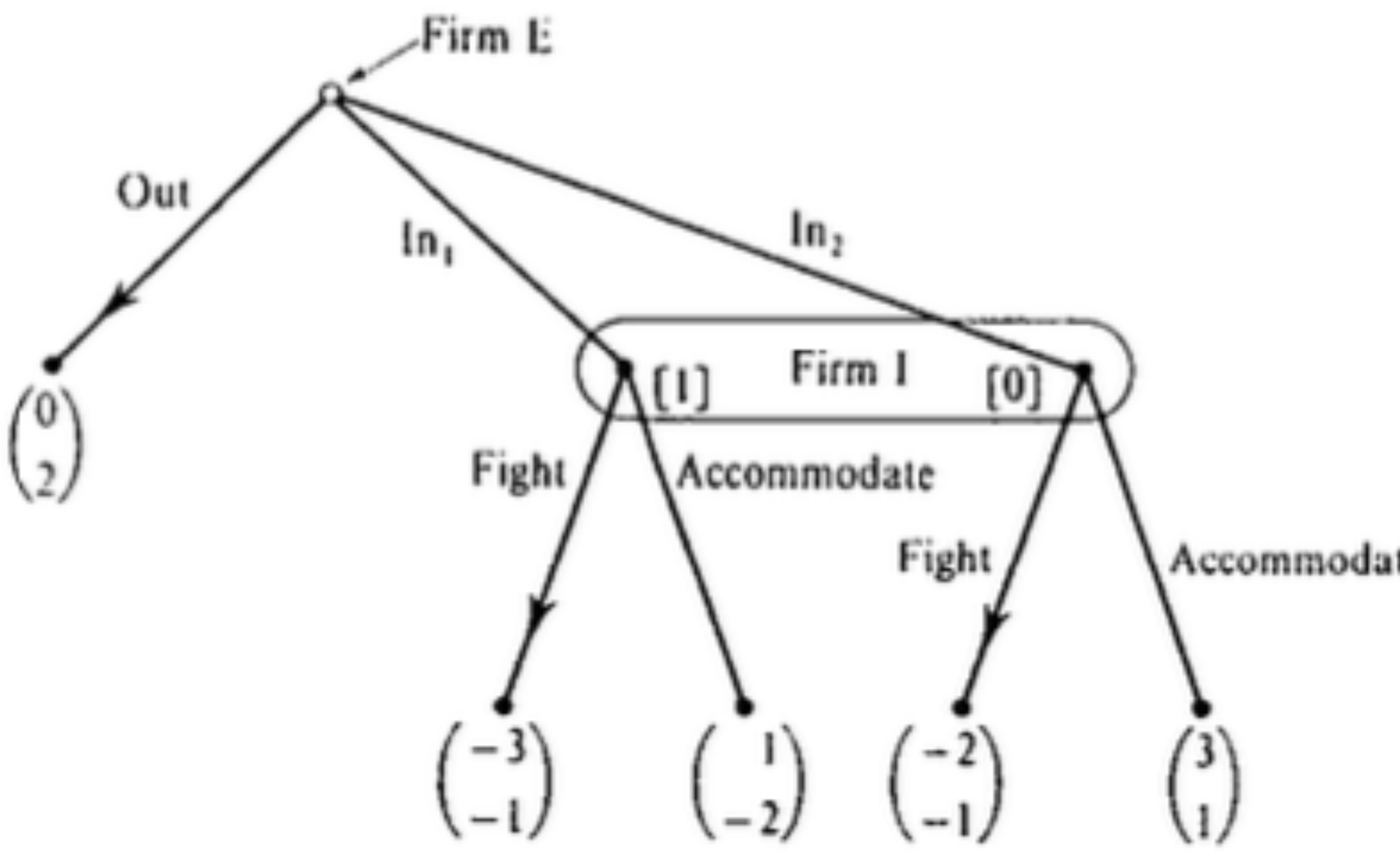


Figure 9.D.l(a)

Reasonable Beliefs and Forward Induction

- Forward induction: some potential problems
- To see this in an even more striking manner, consider the game in Figure 9.D.3.
- Now, after firm E has entered and the two firms are about to play the simultaneous-move post-entry game, firm E makes its speech. But the incumbent retorts: “Forget it! I think you just made a mistake—and even if you did not, I'm going to target the large niche!"

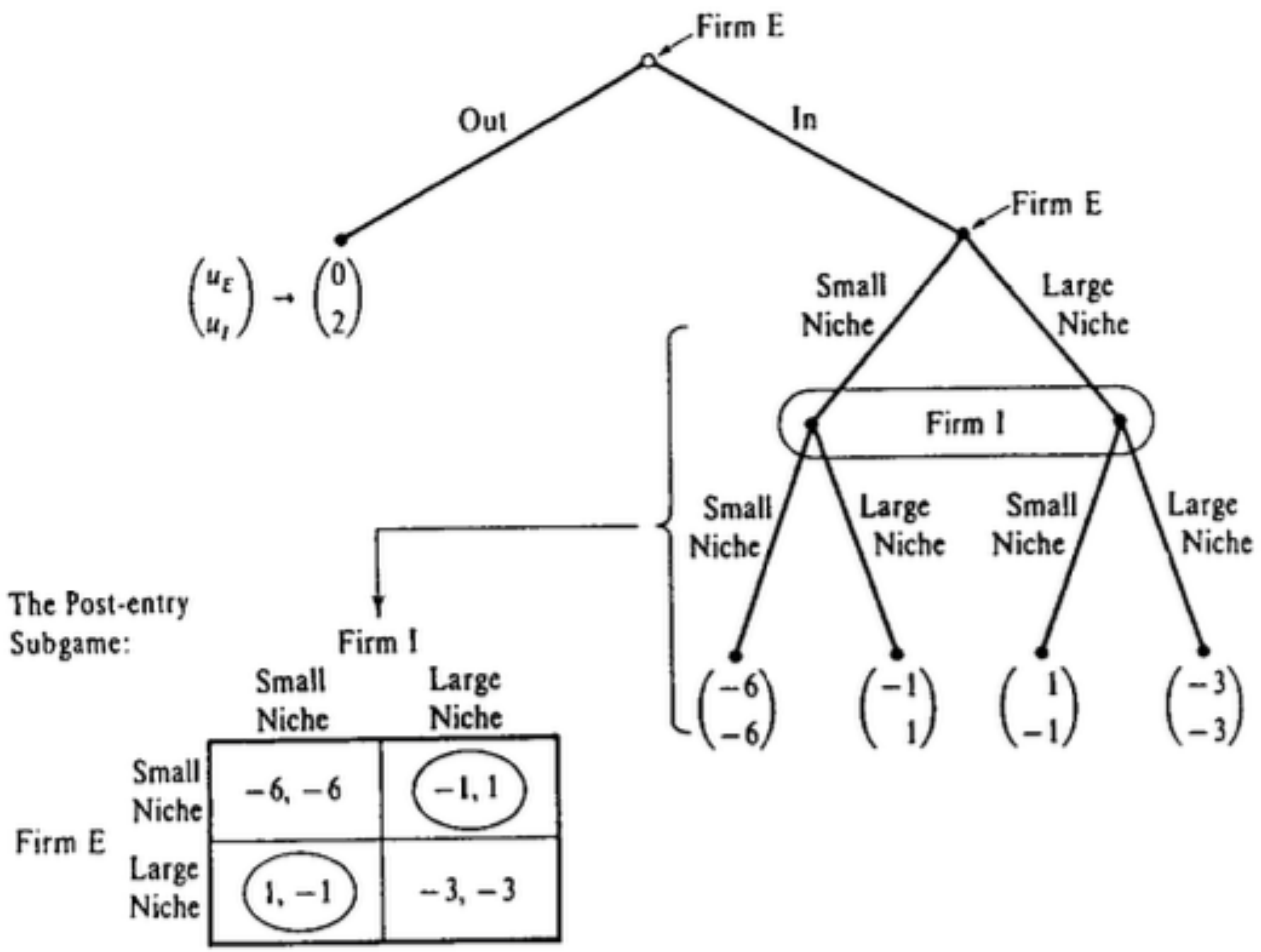


Figure 9.D.3