ECE5022: Digital Signal Processing

Discrete Cosine Transform

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1

Finite-Length Transform

- Given a finite-length sequence $x[n]$ length N seq. $n=0,1,\cdots,N-1$
- A general form of finite-length transform pair

DFT: $X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi k}{N}n}$

$$\begin{cases} A[k] = \sum_{n=0}^{N-1} x[n]\phi_k^*[n], & 0 \le k \le N-1 \\ x[n] = \frac{1}{N}\sum_{k=0}^{N-1} A[k]\phi_k[n], & 0 \le n \le N-1 \end{cases}$$

$\phi_k^*[n]$ $*$: conjugate

$\phi_k[n] = e^{j\frac{2\pi k}{N}n}$

IDFT $= x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k] e^{j\frac{2\pi k}{N}n}$

2

Alternative Transform Pair

- A symmetric transform pair

$$
\begin{cases}
A[k] = \dfrac{1}{\sqrt{N}} \displaystyle\sum_{n=0}^{N-1} x[n]\phi_k^*[n], & 0 \le k \le N-1 \\
x[n] = \dfrac{1}{\sqrt{N}} \displaystyle\sum_{k=0}^{N-1} A[k]\phi_k[n], & 0 \le n \le N-1
\end{cases}
$$

Basis Sequence

- Basis sequence: $\phi_k[n]$
- Orthogonality of the basis sequences

$$
\frac{1}{N}\sum_{n=0}^{N-1} \phi_k[n]\phi_{\cancel{k}\,m}^*[n] = \begin{cases} 1, & m = k \\ 0, & m \ne k \end{cases}
$$

orthogonal basis functions

$\phi_k[n] = e^{j\frac{2\pi k}{n}n}$

- For DFT

$$
\phi_k[n] = W_N^{kn} = e^{-j(2\pi/N)kn}
$$

Discrete Cosine Transform (DCT)

- Discrete Cosine Transform uses cosine as the basis sequence $\phi_k[n]$
- Cosine basis sequence is always real (DFT is complex)
- Cosine is periodic (DFT is also periodic)
- Cosine has even symmetry
- Implicit assumption: periodic and even symmetric

Periodic with period $2N-2=6$

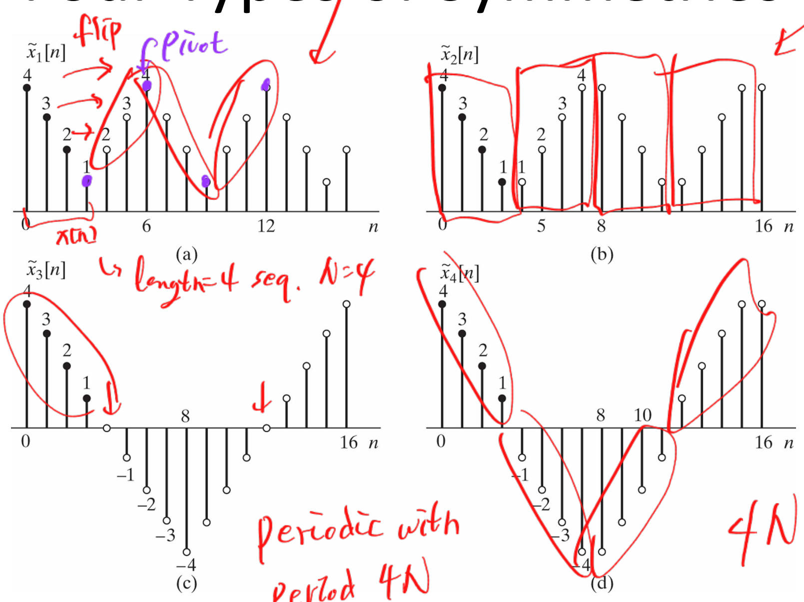
Four Types of Symmetries

periodic with period $2N$

Type-I

Type-II

DCT



Type-III

Type-IV

periodic with period 4N

4N

DCT−1

- DCT−1 uses the Type−1 Symmetry
- Finite sequence with length N is converted to a periodic sequence with length 2N−2 by ($x_{\alpha}[n] = \alpha[n]x[n]$)

$$\tilde{x}[n] = x_{\alpha}[(n)_{\mathrm{mod}\,2N-2}] + x_{\alpha}[(-n)_{\mathrm{mod}\,2N-2}]$$

$$\alpha[n] = \begin{cases} \frac{1}{2}, & \text{n=0}, N-1 \\ 1, & 1 \le n \le N-2 \end{cases}$$

DCT−1 Analysis and Synthesis

- DCT−1 transform pair

$$\begin{cases} X^{c1}[k] = 2\sum_{n=0}^{N-1} \alpha[n]x[n]\cos\left(\frac{\pi kn}{N-1}\right), & 0 \le k \le N-1 \\ x[n] = \frac{1}{N-1}\sum_{k=0}^{N-1} \alpha[n]X^{c1}[k]\cos\left(\frac{\pi kn}{N-1}\right), & 0 \le n \le N-1 \end{cases}$$

DCT−2

- DCT−2 uses the Type−2 Symmetry
- Finite sequence with length N is converted to a periodic sequence with length 2N by

$$\tilde{x}[n] = x_\alpha[(n)_{\mathrm{mod}\,2N}] + x_\alpha[(-n-1)_{\mathrm{mod}\,2N}]$$

DCT−2 Analysis and Synthesis

- DCT−2 transform pair

DCT−2

$$\begin{cases} X^{c2}[k] = 2\sum_{n=0}^{N-1} x[n]\cos\left(\dfrac{\pi k(2n+1)}{2N}\right), & 0 \le k \le N-1 \\ x[n] = \dfrac{1}{N}\sum_{k=0}^{N-1} \beta[k] X^{c2}[k]\cos\left(\dfrac{\pi k(2n+1)}{2N}\right), & 0 \le n \le N-1 \end{cases}$$

$$\beta[k] = \begin{cases} \dfrac{1}{2}, & \mathrm{k{=}0} \\ 1, & 1 \le k \le N-1 \end{cases}$$

Symmetric DCT−2

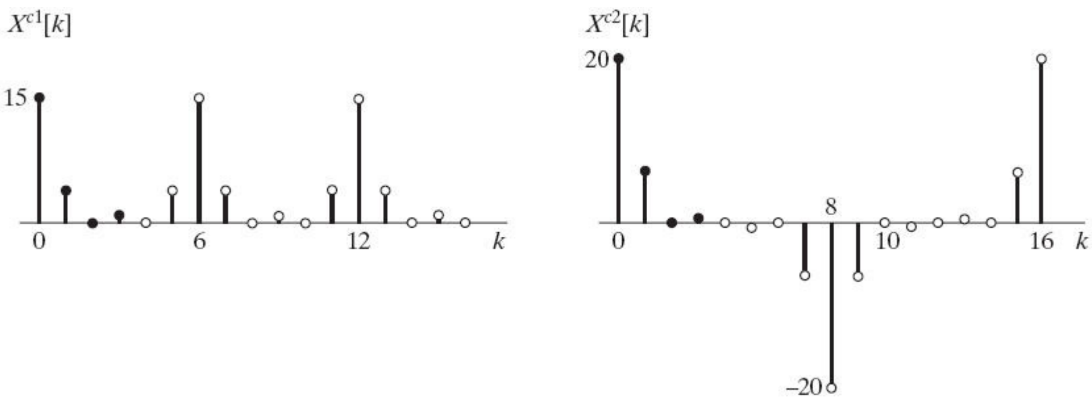
- Alternative DCT−2 transform pair

$$\begin{cases} \tilde{X}^{c2}[k] = \sqrt{\dfrac{2}{N}}\tilde{\beta}[k]\displaystyle\sum_{n=0}^{N-1} x[n]\cos\left(\dfrac{\pi k(2n+1)}{2N}\right), & 0 \le k \le N-1 \\ x[n] = \sqrt{\dfrac{2}{N}}\displaystyle\sum_{k=0}^{N-1} \tilde{\beta}[k]\tilde{X}^{c2}[k]\cos\left(\dfrac{\pi k(2n+1)}{2N}\right), & 0 \le n \le N-1 \end{cases}$$

$$\beta[k] = \begin{cases} \dfrac{1}{\sqrt{2}}, & \text{k=0} \\ 1, & 1 \le k \le N-1 \end{cases}$$

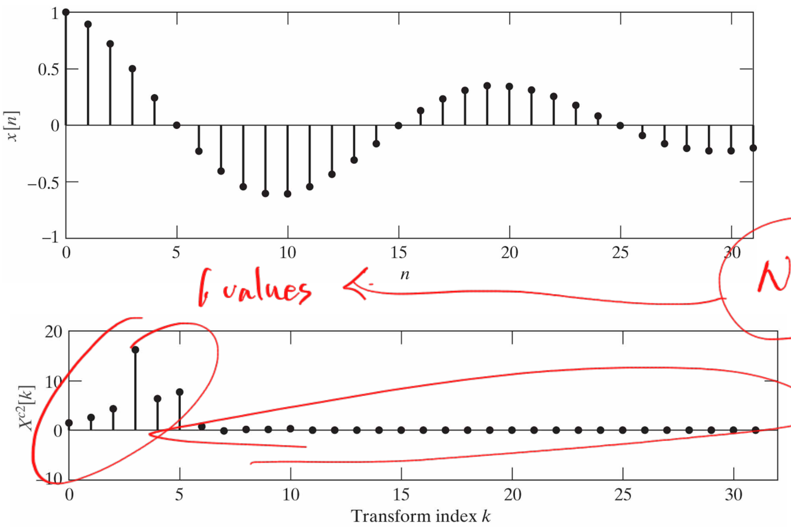
11

DCT−1 & DCT−2 Examples



12

Energy Compaction Property of DCT



6 values

$N=32$

DCT 쓰면 Error 비약적하게 줄일 수 있음.

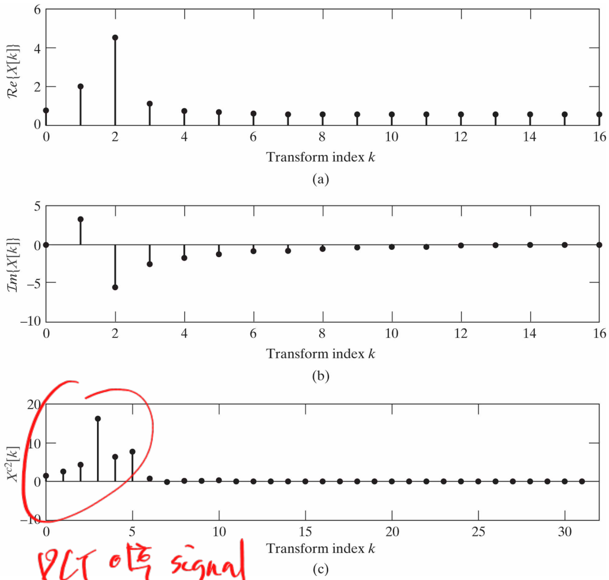
DFT는 불가!

Time-Domain Sequence

DCT-2 Coefficients

13

DFT *vs.* DCT



DCT 이득 signal

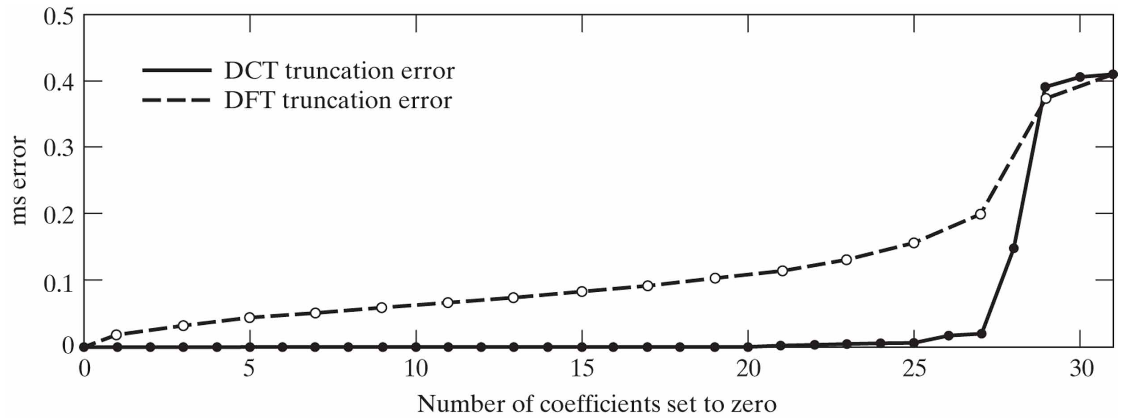
DFT Real Coefficients

DFT Imaginary Coefficients

DCT-2 Coefficients

14

DFT *vs.* DCT Compression



Discrete Cosine Transform 15/15

15