ECE5022: Digital Signal Processing

Lecture 11: Homomorphic Signal Processing

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Announcements

- Term Project open book No computer / tablet / phone
  - Due: 6/14
- Final Exam
  - 4:30PM – 5:30PM (60 Minutes) on Tuesday, 6/7
  - Place: Hi-Tech Center Room 228

(FFT 포함)
중간이후 전범위

- Grading
  - Exams: 30% (Midterm) + 30% (Final)
  - Project: 20%, Attendance: 10%, Homework: 10%

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Outline

- Homomorphic Signal Processing
- Cepstrum Definitions (Analysis
  - Real Cepstrum
  - Complex Cepstrum
- Homomorphic Deconvolution
- Speech Processing Applications

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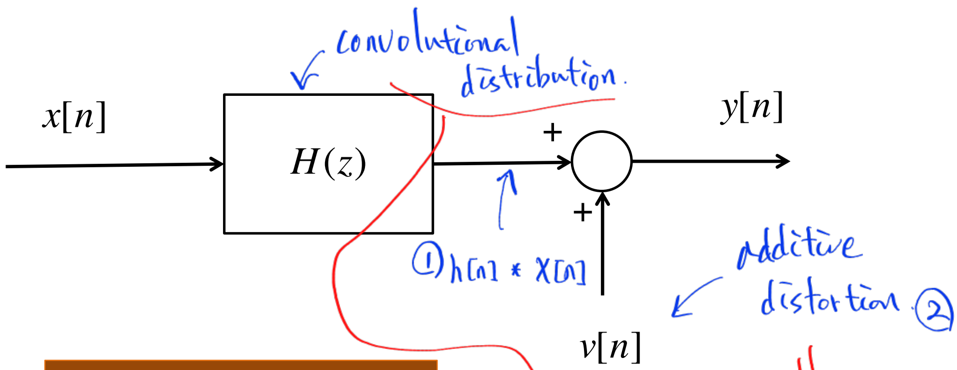
Signal Distortion Model



Convolutive Distortion

$$y[n] = \underbrace{h[n]*x[n]}_{①} + \underbrace{v[n]}_{②} \Rightarrow Y(e^{j\omega}) = H(e^{j\omega})X(e^{j\omega}) + V(e^{j\omega})$$

Additive Distortion

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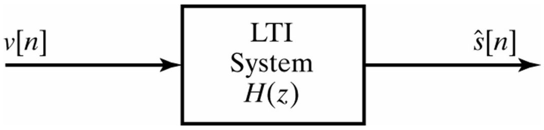
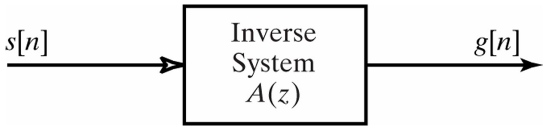
Two Types of Signal Distortion

- Additive Noise
  - Distortion caused by a noise signal ***added*** to the input
  - Example: Wiener Filtering for reducing additive noise
- Convolutive Noise
  - Distortion caused by ***convolving (filtering)*** the input with a noise signal

Removing Convolutive Noise

- Convolutive Noise
  - Need to find an inverse filter to remove it
  - Example: All-pole modeling gives an FIR inverse filter





$$H(z) = \frac{1}{1 - \sum_{k=1}^{p} a_k z^{-k}}$$

$$A(z) = \frac{1}{H(z)} = 1 - \sum_{k=1}^{p} a_k z^{-k}$$

Homomorphic Signal Processing

- Parametric signal modeling provides a simple inverse filter as far as the model is good enough
- Nonparametric approach may be required if the model is not good enough
- Convert the problem to an easier ~~solution~~ problem to solve.
  - Transform convolutive noise to additive noise

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Superposition Property

- Given a linear operator $L[\bullet]$

$$L\left[a_1x_1[n]+a_2x_2[n]\right]=a_1L\left[x_1[n]\right]+a_2L\left[x_2[n]\right]$$

z 변환

- Equivalently

$$L\left[a_1X_1(z)+a_2X_2(z)\right]=a_1L\left[X_1(z)\right]+a_2L\left[X_2(z)\right]$$

- Wiener filter can be used to remove the additive noise

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Generalized Superposition

- Given a nonlinear operator Φ that satisfies

$$\Phi[X_1(z)X_2(z)] = \Phi[X_1(z)] + \Phi[X_2(z)]$$

- If such a nonlinear operation exists, we can treat the convolutive noise as additive noise

problem

convolutive noise → additive noise

same shape

- Homomorphic signal processing
  - Convert the problem to an easier problem and solve
  - Transform the output signal back to the original domain

homomorphic signal process.

Complex Logarithm I/II

- We can easily see that

$$\log_e[X_1(z)X_2(z)] = \log_e[X_1(z)] + \log_e[X_2(z)]$$

- For the DTFT (complex valued)

$X(e^{jw}) = |X(e^{jw})| e^{j\angle(e^{jw})}$

$$\log_e[X(e^{j\omega})] = \log_e\left[|X(e^{j\omega})| e^{j\angle X(e^{j\omega})}\right]$$
$$= \log_e\left[|X(e^{j\omega})|\right] + \log_e\left[e^{j\angle X(e^{j\omega})}\right]$$
$$= \log_e\left[|X(e^{j\omega})|\right] + j\angle X(e^{j\omega})$$

log of the magnitude of j times phase?

Complex Logarithm II/II

- Suppose

$$X(e^{j\omega}) = X_1(e^{j\omega})X_2(e^{j\omega})$$

- Then

실수(+짝수) even 홀수(odd) magnitude phase

$$\log\left[X(e^{j\omega})\right] = \log\left[\left(\left|X_1(e^{j\omega})\right| e^{j\angle X_1(e^{j\omega})}\right)\left(\left|X_2(e^{j\omega})\right| e^{j\angle X_2(e^{j\omega})}\right)\right]$$

$$= \log\left[\left|X_1(e^{j\omega})\right|\left|X_2(e^{j\omega})\right| e^{j\left(\angle X_1(e^{j\omega}) + \angle X_2(e^{j\omega})\right)}\right]$$

$$= \log\left|X_1(e^{j\omega})\right| + \log\left|X_2(e^{j\omega})\right| + j\left(\angle X_1(e^{j\omega}) + \angle X_2(e^{j\omega})\right)$$

Outline

- Homomorphic Signal Processing
- Cepstrum Definitions 켑스트럼.
  - Real Cepstrum
  - Complex Cepstrum
- Homomorphic Deconvolution
- Speech Processing Applications

Real Cepstrum

- For a real sequence $x[n]$
  - $|X(e^{j\omega})|$ is ***even*** and $\angle X(e^{j\omega})$ is ***odd*** and both are ***real***
  - $\log|X(e^{j\omega})|$ is ***real*** and ***even*** and $j\angle X(e^{j\omega})$ is ***imaginary*** and ***odd***
- If we define

$$C_x(e^{j\omega}) = \log\left|X(e^{j\omega})\right|$$

whose inverse DTFT is defined as the Real Cepstrum

$$c_x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} C_x(e^{j\omega})e^{j\omega n}\,d\omega = \frac{1}{2\pi}\int_{-\pi}^{\pi} \log\left|X(e^{j\omega})\right| e^{j\omega n}\,d\omega$$

Real Cepstrum

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되돌릴수있음!

각스텝은 invertable 함.

Complex Cepstrum

- If we define

convolutive noise in time domain

$X[n] \xrightarrow{①} \boxed{DTFT} \rightarrow \boxed{\log} \xrightarrow{③} \boxed{IDTFT} \rightarrow \hat{x}[n]$

$\boxed{IDTFT} \leftarrow \boxed{\exp} \leftarrow \boxed{DTFT}$

②

additive noise in freq. domain

$$\hat{X}(e^{j\omega}) = \log\left[X(e^{j\omega})\right]$$

- The complex cepstrum is defined as

항상 real

inverse dft 결과

$$\hat{x}[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} \hat{X}(e^{j\omega})e^{j\omega n}\,d\omega = \frac{1}{2\pi}\int_{-\pi}^{\pi} \log\left[X(e^{j\omega})\right]e^{j\omega n}\,d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi}\left[\log\left|X(e^{j\omega})\right| + j\angle X(e^{j\omega})\right]e^{j\omega n}\,d\omega$$

Even — real part

imaginary part — Odd

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Invertibility of Complex Cepstrum

- The inverse DTFT is invertible, i.e., DTFT
- The inverse of logarithm is invertible, i.e., exponential
- The complex cepstrum is therefore invertible
- The cepstrum of $X(e^{j\omega}) = X_1(e^{j\omega})X_2(e^{j\omega})$

$$\hat{X}(e^{j\omega}) = \hat{X}_1(e^{j\omega}) + \hat{X}_2(e^{j\omega})$$
$$= \log\left|X(e^{j\omega})\right| + j\angle X(e^{j\omega})$$

- The cepstrum is guaranteed to be real for real $x[n]$

Some Terminologies

- Quefrency
  - The time index n for the cepstrum

$x[n] \rightarrow \square \rightarrow \square \rightarrow \square \rightarrow \hat{x}[n]$

time index

~~time~~ ~~freq~~

quefrency domain

$$\hat{x}[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} \log\left[X(e^{j\omega})\right] e^{j\omega n}\, d\omega$$

  - Reverse of frequency -> quefrency 큐프런시.

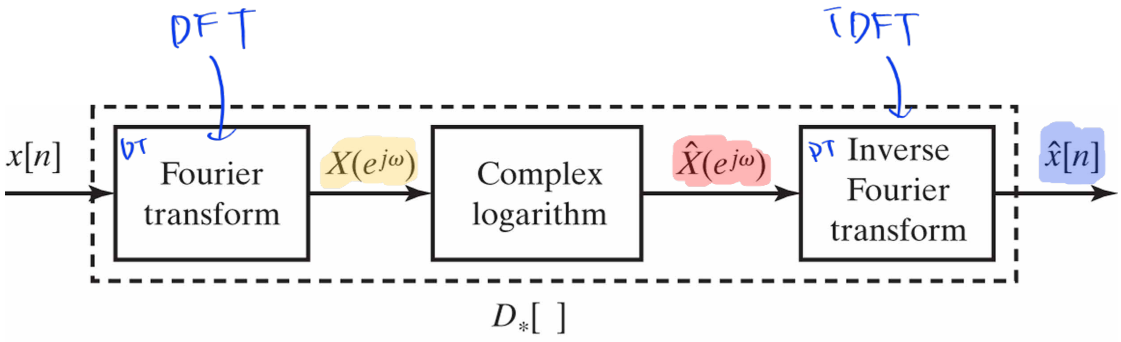
filtering

- Liftering 리프터링
  - Filtering in the cepstrum domain
  - Reverse of filtering -> liftering

시간축

↳ 새로운 도메인으로 봐야!!

Complex Cepstrum Diagram



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Computation of Complex Cepstrum

- The computation is represented as

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$ **DTFT**

$$\hat{X}(e^{j\omega}) = \log\left[X(e^{j\omega})\right]$$ **Complex Logarithm**

$$\hat{x}[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} \hat{X}(e^{j\omega})e^{j\omega n}\,d\omega$$ **Inverse DTFT**

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Complex Cepstrum in DFT

- Sampling the DTFT

$$X[k] = X(e^{j\omega})\Big|_{\omega=\frac{2\pi k}{N}} = \sum_{n=-\infty}^{\infty} x[n]e^{-j\frac{2\pi k}{N}n}$$

DFT

FFT 누구는 FFT해도 됨.

유사해서 쓰긴함.

$$\hat{X}[k] = \log\left[X(e^{j\omega})\right]\Big|_{\omega=\frac{2\pi k}{N}}$$

Complex Logarithm

$$\hat{x}_p[n] = \frac{1}{N}\sum_{k=0}^{N-1}\hat{X}[k]e^{-j\frac{2\pi n}{N}k}$$

Inverse DFT

FFT

incorrect!!

엄밀히는 틀림.

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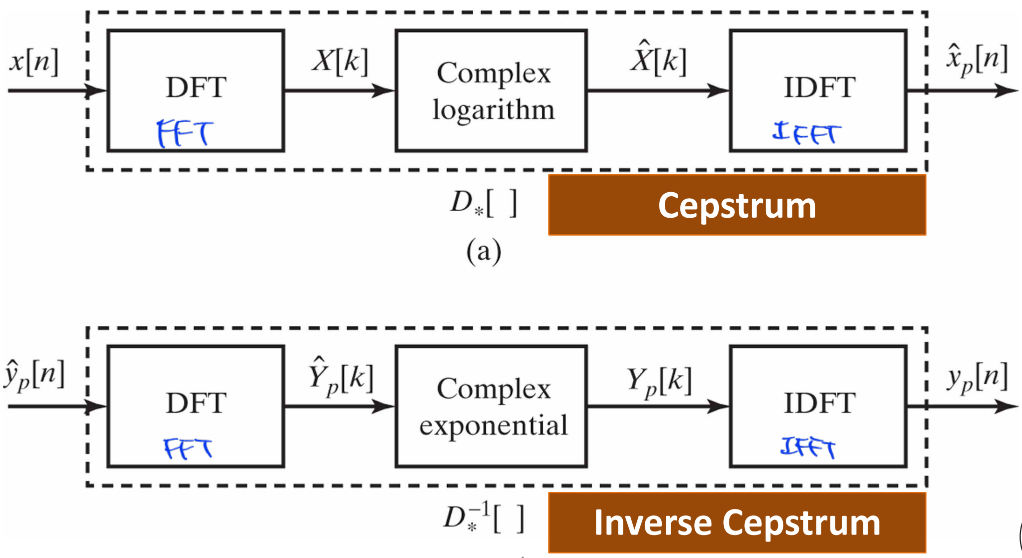
Cepstrum and Inverse Cepstrum in DFT



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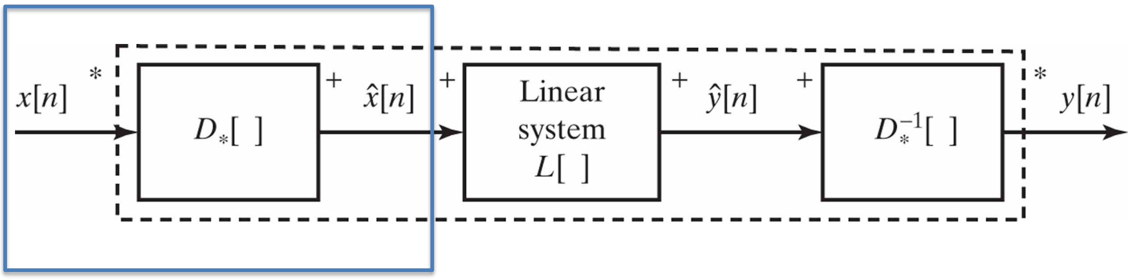
Homomorphic Signal Processing I/II

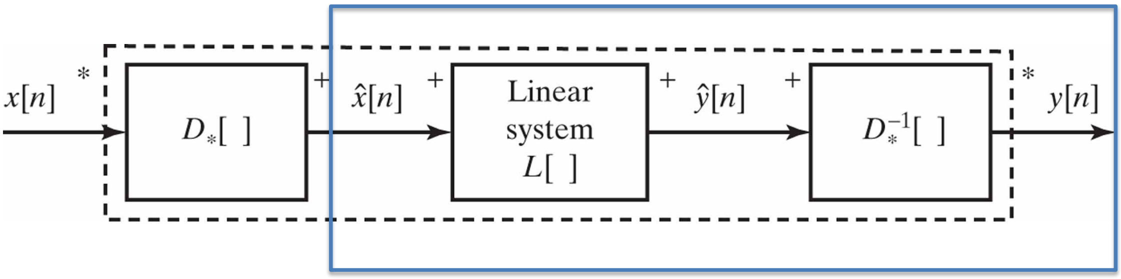


$$x[n] = x_1[n] * x_2[n] \Rightarrow X(z) = X_1(z) \cdot X_2(z)$$

$$\hat{X}(z) = \log\left[X(z)\right] = \log\left[X_1(z)\right] + \log\left[X_2(z)\right] = \hat{X}_1(z) + \hat{X}_2(z)$$

$$\hat{x}[n] = D_*\left[x_1[n] * x_2[n]\right] = \hat{x}_1[n] + \hat{x}_2[n]$$

Homomorphic Signal Processing II/II



$$\hat{x}[n] = \hat{x}_1[n] + \hat{x}_2[n]$$

$$\hat{y}[n] = L\left[\hat{x}_1[n] + \hat{x}_2[n]\right] = L\left[\hat{x}_1[n]\right] + L\left[\hat{x}_2[n]\right] = \hat{y}_1[n] + \hat{y}_2[n]$$

$$\hat{y}[n] = D_*^{-1}\left[\hat{y}_1[n] + \hat{y}_2[n]\right] = y_1[n] * y_2[n]$$

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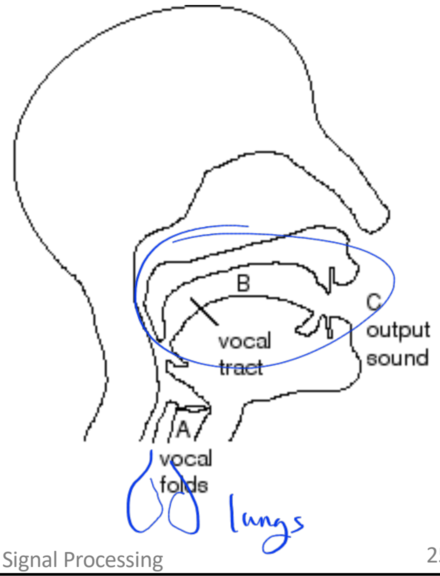
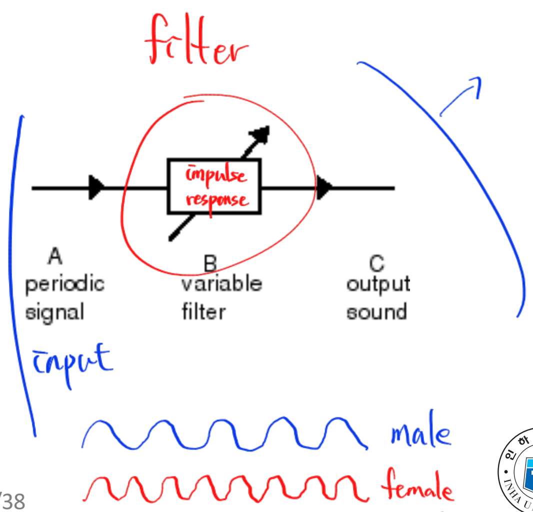
Source-Filter Model for Speech Production





음성인식

Cepstrum 사용

deconvolution

핵심

고주파!

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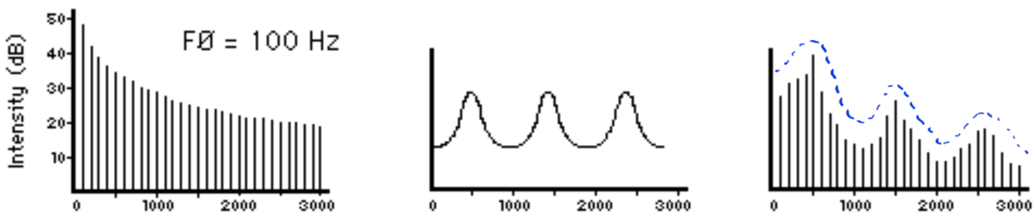
Source-Filter Model for Voiced Speech



Excitation	Vocal Tract	Output
$E(e^{j\omega})$	$V(e^{j\omega})$	$X(e^{j\omega}) = E(e^{j\omega})V(e^{j\omega})$

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Logarithm of the Fourier Transform

- The output signal $x[n]$ is the convolution of
  - The excitation signal $e[n]$ and the vocal tract filter $v[n]$
- In the frequency domain

$$X(e^{j\omega}) = E(e^{j\omega})V(e^{j\omega})$$

- If we take the magnitude, then

$$|X(e^{j\omega})| = |E(e^{j\omega})||V(e^{j\omega})|$$

- If we take the logarithm, then

$$\log|X(e^{j\omega})| = \log|E(e^{j\omega})| + \log|V(e^{j\omega})|$$

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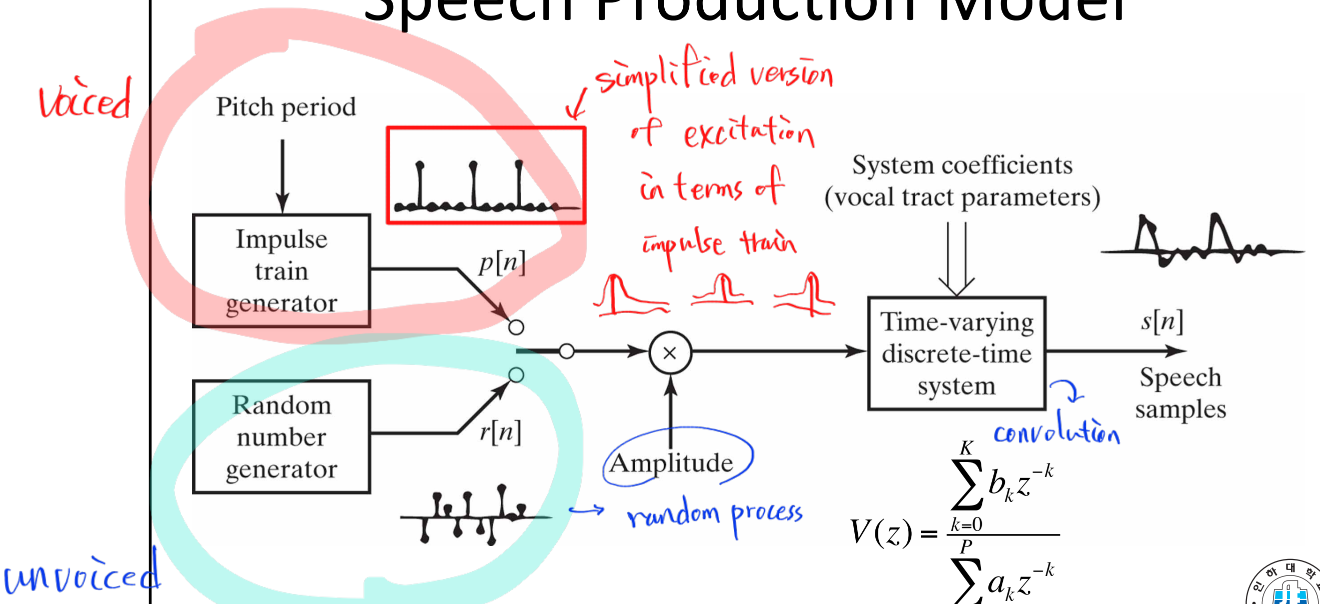
Speech Production Model



$$V(z) = \frac{\sum_{k=0}^{K} b_k z^{-k}}{\sum_{k=0}^{P} a_k z^{-k}}$$

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* noise ? => unwanted signal.

즉, 목적에 따라 noise는 달라짐.

Speech Deconvolution

- For voiced speech

$$s[n] = v[n] * p[n]$$

- After windowing

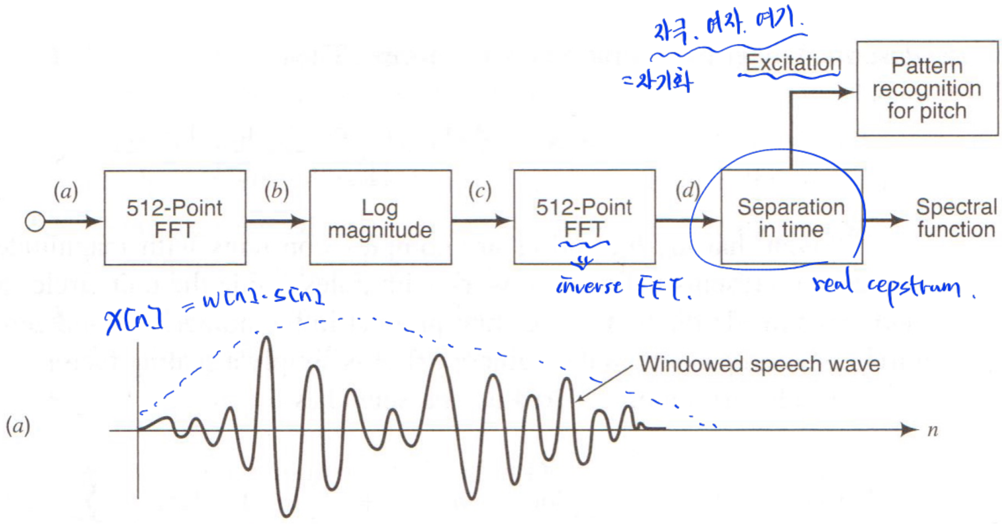
$$\underline{w[n]}s[n] = v[n] * \underline{p_w[n]}, \quad p_w[n] = w[n]p[n]$$

- The complex cepstrum

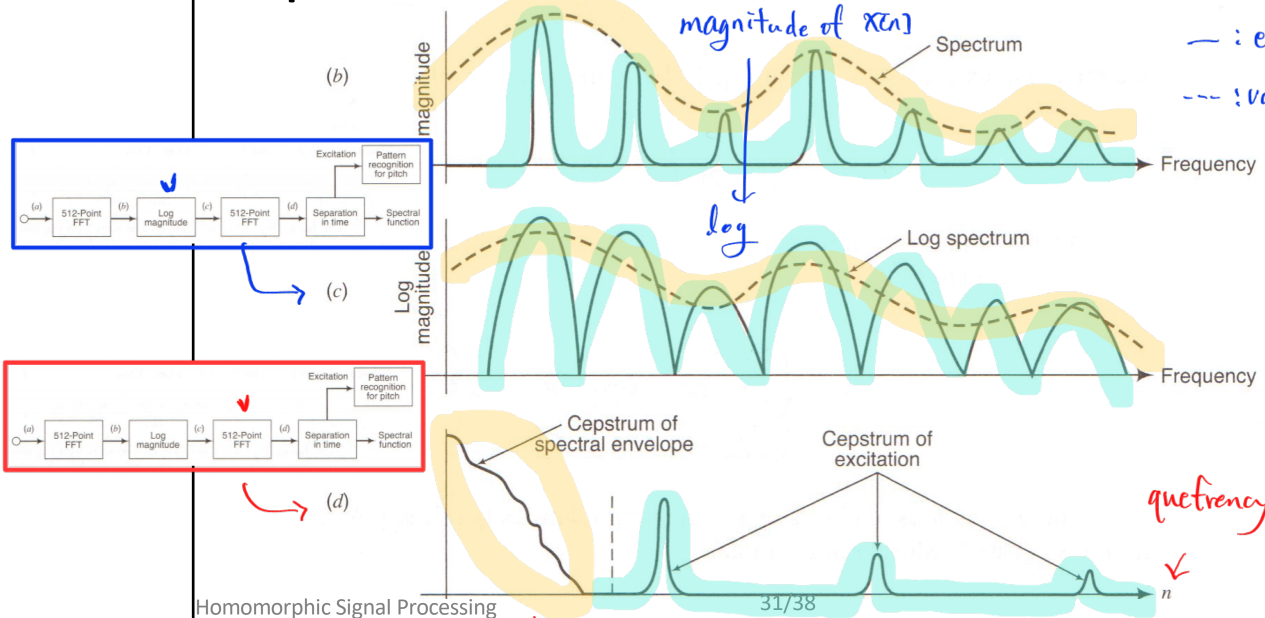
$$\hat{x}[n] = \hat{v}[n] + \hat{p}_w[n]$$

windowing. to preserve this part.

Speech Deconvolution Overview I/II



Speech Deconvolution Overview II/II



— : excitation (P[n])

--- : vocal tract (V[n])

excitation 고정

vocal 바꾸면

lower part 바뀜

반대로

excitation 바꾸면

higher part 바뀜.

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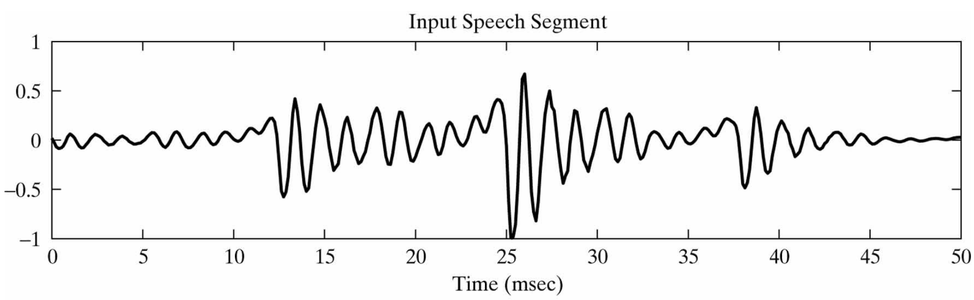
inverse cepstrum 하면

vocal part 정보 겟.

이쪽 inverse cepstrum 하면

excitation part 겟.

Speech Deconvolution Example I/V



- Input speech signal after windowing

$$x[n] = w[n]s[n] = v[n] * p_w[n], \quad p_w[n] = w[n]p[n]$$

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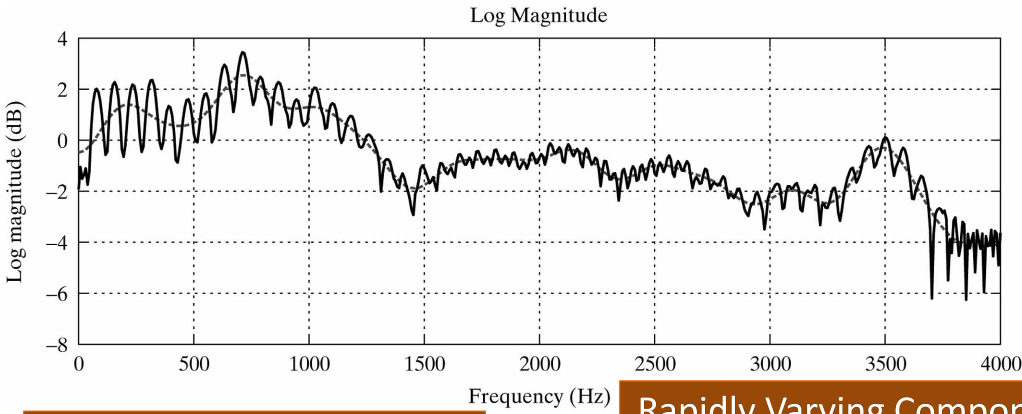
Speech Deconvolution Example II/V

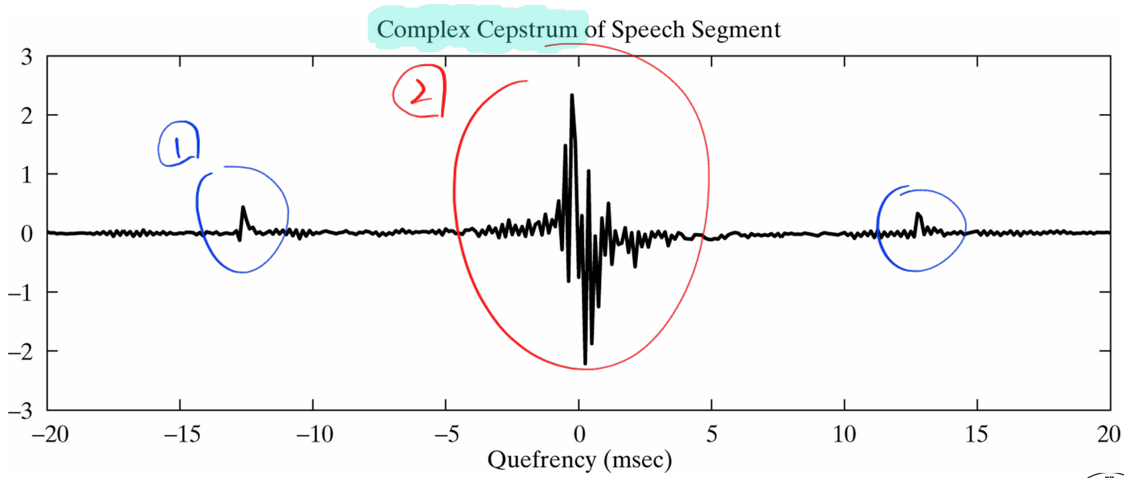


Slowly Varying Component

Rapidly Varying Component

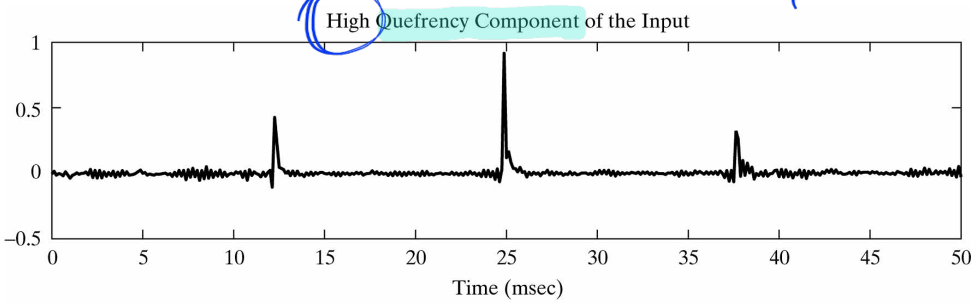
$$x[n] = v[n] * p_w[n] \Rightarrow V(e^{j\omega}) P_w(e^{j\omega})$$

Speech Deconvolution Example III/V



Speech Deconvolution Example IV/V

invers cepstrum of (1)

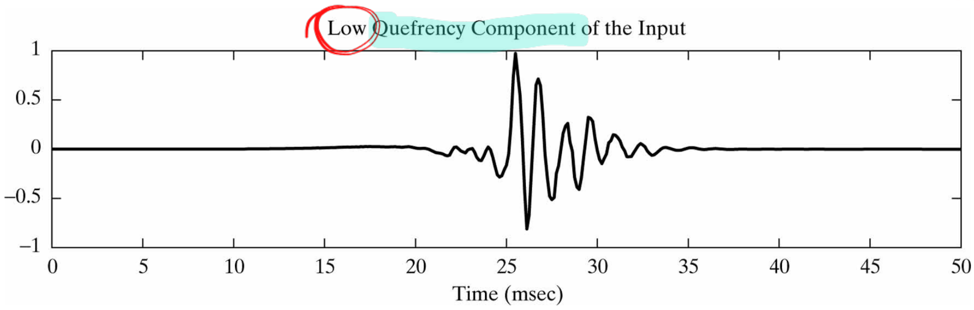


- High quefrency component

$$p_w[n] = w[n]p[n]$$

Speech Deconvolution Example V/V

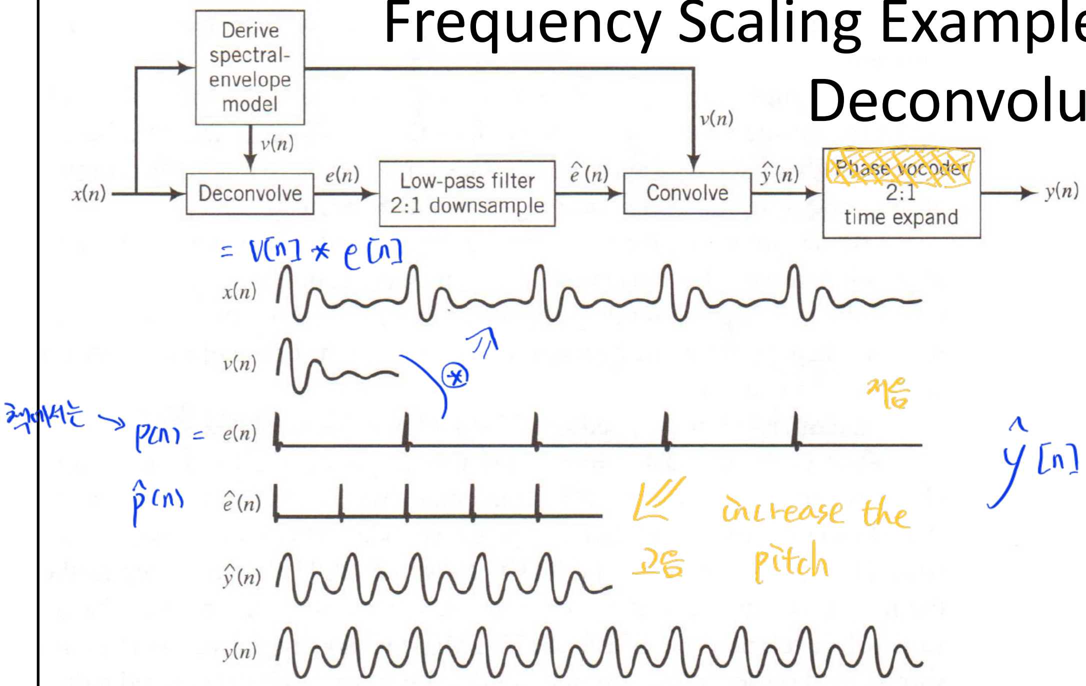
inverse cepstrum of (2)



- Low quefrency component

$$v[n]$$

Frequency Scaling Example via Deconvolution



$\hat{y}$[n] has higher pitch than x[n]

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feature extraction → knocker 가능.

Questions?

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