# Numerical Analysis Test 2:

**Name:**________________________________________

**1.** [**Orthogonal Projections**] (a) Suppose that you have a subset of orthonormal vectors $\mathcal{O} = \{o_1, o_2, o_3, ...o_k\}$. How would you construct the projection of a vector $f$ which lives in the same space onto $\mathcal{O}$, or $Proj_{\mathcal{O}} f$?

(b) Suppose you have a bunch of vectors $V = \{v_1, v_2, v_3, ..., v_k\}$. How would you construct an orthonormal set of vectors which span the same space? What is this process called?

**2. [Least Squares]** (a) Explain the least squares method and the basic difference between Least Squares and Lagrange Interpolation.

(b) Derive the Normal Equations for solving the least squares problem.

**3. [Numerical Integration]** Assume that you are given $\{f(x_k)\}$ where $\{x_k\} = a + hk$, with $k = 0...N$ and $h = (b-a)/N$. The goal is to estimate $\int_a^b f(x)dx$.

(a) State the Left and Right Riemann Integration formulas.

(b) Show how to combine the left and right Riemann Integration formulas to get the Trapezoid rule.

(c) Show that the local error formula for the Trapezoid rule is $O(h^3)$.

**4. [Simpson's Rule]** (a) Explain the method behind Simpson's Rule and state the rule for a single 3 point segment.

(b) Show that Simpson's Rule is exact for third order ( cubic ) polynomials.

(c) Given that Simpson's Rule is exact for cubics, derive an local error estimate which is $O(h^5)$.

**5. [Numerical Differences]** Assume that you are given $\{f(x_k)\}$ where $\{x_k\} = a + hk$, with $k = 0...N$ and $h = (b-a)/N$. The goal is to estimate $\{f'(x_k)\}$ at each point.

(a) Use Taylor's Formula to derive the Backward and Forward Difference Formulas, and their error formulas.

(b) Show that the Symmetric Difference can easily be obtained from the Forward and Backward Difference formulas.

(c) Use Taylor's Formula to Derive the Symmetric Difference and it's error formula, which is $O(h^2)$.

**6. [Second Difference]** (a) Use Taylor's formula to derive a formula for the 2nd derivative $f''(x_k)$.

(b) Use Taylor's Formula to show the Second Difference formula is exact for cubics, and derive its error formula.