STA 3032 - Practice set modules 1&2

True or False

1. A quality engineer in a factory is interested in the proportion of all computer chips that her assembly line produces that meet a particular quality requirement. She selects a sample of 100 chips and finds that 85 pass the test. This means that $p = 0.85$ is the proportion of all chips meeting the requirement.

2. The (large-sample) sampling distribution for the sample mean is normally distributed ONLY if the underlying population of measurements is normal.

Open Ended Problems

1. A plant researcher has measured the amount of growth among five plants that received a growth formula. He finds that the amounts of growth were: 4, 8, 6, 7, 10 cm. Give the sample mean, median, standard deviation and 33rd percentile of the amounts of growth.

2. Among male birds of a species, 20% have a particular gene. Among females of the species, 10% have the gene. The males comprise 40% of all the birds of the species (thus, females comprise 60%).

   (a) What is the probability a randomly selected bird of this species has the gene?

   (b) What is the probability the bird is male, given it has the gene?

3. A sample of 5 animals of a particular species is selected at random from the population being managed in a wildlife refuge. If 5% (0.05) of the population have a particular trait, what is the probability that none of the 5 tested have the trait?

4. You are going to a foreign nation to conduct your research. On a weather website you see that the average high temperature during the period you will be there has been historically 20 degrees Celsius and variance 5 degrees^2. What is the mean and variance of the dataset in degrees Fahrenheit? Hint: $°\text{F} = 32 + (9/5)°\text{C}$

5. In a population of 100 watt light bulbs manufactured by a company, 80% (0.80 as a proportion) have lifetimes exceeding 800 hours. An inspector samples 10 bulbs at random. What is the probability at least 2 of the bulbs lifetimes exceed 800 hours?

6. Among students taking a standardized exam, scores are normally distributed with a mean of 550 and standard deviation 100.

   (a) What proportion of the students score above 700?

   (b) What is 15th percentile?

7. Based on the following contingency table, complete the following parts:

	Concussion	No Concussion	Total
Male	40	25596	25636
Female	60	27107	27167
Total	100	52703	52803

   (a) What is the probability of being female and have no concussion?

   (b) Among Males, what is the probability of concussion?

   (c) Among individuals with concussions, what is the probability of being male?

   (d) Among Females, what is the probability of concussion?

8. A random variable has p.d.f.

$$f(x) = \frac{1}{10}x^3 \qquad 1 < x < 41^{1/4}.$$

   Find the mean, variance, 85th percentile and $P(X \leq 2)$.

9. A discrete random variable has p.m.f.

x	1	5	7	8	9	10
$p(x)$	0.13	0.17	0.07	0.22	0.11	0.30

   Find the mean, variance, 65th percentile and $P(X \leq 2)$.

10. Five applicants for a job are ranked according to ability, with applicant 1 being the best, number 2 being the second best, and so on. These ranking are unknown to an employer, who simply hires two applicants (for two identical positions) at random. What is the probability that this employer hires exactly one of the two best applicants? Hint: First find the total number of applicant selection possibilities and then calculate how many possibilities result in exactly one of the two best being selected.

11. Let

$$f(x_1, x_2) = 3x_1, \quad 0 \leq x_2 \leq x_1 \leq 1$$

Find

(a) $P(0.2 < X_2 < 0.4)$

(b) $P(X_2 < 0.2 | X_1 = 0.5)$

(c) $\text{Cov}(X_1, X_2)$

12. For a certain manufacturing industry, the number of industrial accidents average 3 per (6 work day) week. Find the probability that two accidents will occur in a given day.