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C335

HW2.

1. $$\sum_{i=0}^{N-1} (r-1) * r^i = r^N - 1.$$

Base case

let $N=1$

$$\sum_{i=0}^{1-1} (r-1) * r^i = (r-1) * r^0 = r-1 \quad \checkmark$$

Inductive step.

let $N=k$

$$\sum_{i=0}^{k-1} (r-1) * r^i = r^{k-1}$$
$$= (r-1) + (r)(r-1) + \cdots + (r^{k-1})(r-1)$$

let $N=k+1$

$$\sum_{i=0}^{(k+1)-1} (r-1) * r^i = r^{k+1} - 1$$

$$\sum_{i=0}^{k} (r-1) * r^i = (r+1) + r(r-1) + \cdots + (r^{k-1})(r-1) + r^k(r-1)$$
$$= r^{k-1} - 1 + r^k(r-1)$$
$$= r^{k-1} + r^k - r - r^{k-1} - 1$$
$$= r^{k+1} - 1 \quad \checkmark$$

2.

let $p=6$
let $q=7$.
and both p and q are decimal.
we can represent p+q as

$$\begin{array}{r} \boxed{1\,1} \\ 6 \\ +\ 7 \\ \hline 13 \end{array}$$

let $p=1$
let $q=2$
and both p and q are decimal.
we can represent p+q as

$$\begin{array}{r} \boxed{1\,0} \\ 1 \\ +\ 2 \\ \hline 3 \end{array}$$

Since there are no p and q that can be added to result in greater than or equal to d_{i+2}, carry bits are always 0 or 1.
when we set p and q to be in the range of 0 to r, $(p+q)\%r$ is always 0 or 1.

3. given the formal definition,

$$-b_{N-1} * 2^{N-1} + \sum_{i=0}^{N-2} b_i * 2^i$$

for maximum,

$-b_{N-1} = 0$ therefore

Maximum two's complement number is

$\sum_{i=0}^{N-2} 2^i = 2^{N-1} - 1$ which is 011 ⋯ 11

For minimum,

$-b_{N-1} = 1$

that

$-b_{N-1} * 2^{N-1} = -2^{N-1}$ which is 100 ⋯ 00

is minimum.

4.

$$-(b_{N-1}\ldots b_0)_2 = (\overline{b_{N-1}\ldots b_0})_2 + 1$$

$$-(b_{N-1}\ldots b_0)_2 = -\left(-b_{N-1}2^{N-1} + \sum_{i=0}^{N-2} b_i 2^i\right)$$

$$= b_{N-1}2^{N-1} - \sum_{i=0}^{N-2} b_i 2^i$$

$$= b_{N-1}2^{N-1} - 2^{N-2} + 1$$

$$(\overline{b_{N-1}\ldots b_0})_2 + 1 = -\overline{b_{N-1}}2^{N-1} + (\overline{b_{N-2}\ldots b_0})_2 + 1$$

$$= -\overline{b_{N-1}}2^{N-1} + 2^{N-1} - \sum_{i=0}^{N-2} b_i 2^i$$

$$= -\overline{b_{N-1}}2^{N-1} + 2^{N-1} - 2^{N-2} + 1$$

$$\therefore\ b_{N-1}2^{N-1} = -\overline{b_{N-1}}2^{N-1} + 2^{N-1}$$

$$b_{N-1}2^{N-1} + \overline{b_{N-1}}2^{N-1} = 2^{N-1}$$

Therefore $-(b_{N-1}\ldots b_0)_2 = (\overline{b_{N-1}\ldots b_0})_2 + 1$

5. N_{th} digit $\rightarrow (b_{N-1}\, b_{N-2} \dots b_0)$

$$-b_{N-1}2^{N-1} + \sum_{i=0}^{N-2} b_i * 2^i$$

which is $-b_{N-1}2^{N-1} + b_{N-2}2^{N-2} + \cdots + b_0$

$N+k_{th}$ digit $\rightarrow (b_{N-1+k}\, b_{N-2+k} \cdots b_{N-1}\, b_{N-2} \cdots b_0)$

$$-b_{N-1}2^{N-1+k} + \sum_{i=0}^{N-2+k} b_i * 2^i$$

which is $-b_{N-1+k}2^{N-1+k} + b_{N-2+k}2^{N-2+k} + \cdots + b_{N-2}2^{N-2} + \cdots + b_0$

$$= -b_{N-1+k}2^{N-1+k} + b_{N-2}2^{N-2} + b_0$$

that N_{th} digit value is preserved in

$N+k_{th}$ Value.

6.

- 107

$11010 11_2$

2 | 107
2 | 53 ... 1
2 | 26 ... 1
2 | 13 ... 0
2 | 6 ... 1
2 | 3 ... 0
1 ... 1

- 2312

100100001000_2

2 | 2312
2 | 1156 ... 0
2 | 578 ... 0
2 | 289 ... 0
2 | 144 ... 1
2 | 72 ... 0
2 | 36 ... 0
2 | 18 ... 0
2 | 9 ... 0
2 | 4 ... 1
2 | 2 ... 0
1 ... 0

• 31333

111101001100101_2

$2\,|\,31333$
$2\,|\,15666 \cdots 1$
$2\,|\,7833 \cdots 0$
$2\,|\,3916 \cdots 1$
$2\,|\,1958 \cdots 0$
$2\,|\,979 \cdots 0$
$2\,|\,489 \cdots 1$
$2\,|\,244 \cdots 1$
$2\,|\,122 \cdots 0$
$2\,|\,61 \cdots 0$
$2\,|\,30 \cdots 1$
$2\,|\,15 \cdots 0$
$2\,|\,7 \cdots 1$
$2\,|\,3 \cdots 1$
$1 \cdots 1$

• 97

1100001_2

$2\,|\,97$
$2\,|\,48 \cdots 1$
$2\,|\,24 \cdots 0$
$2\,|\,12 \cdots 0$
$2\,|\,6 \cdots 0$
$2\,|\,3 \cdots 0$
$1 \cdots 1$

7.

- $103 - 92$

  $1000 - 92 = 908$

  $103 + 908 = 1011$

  $\therefore\ 103 - 92 = 011$

- $1027 - 11$

  $10000 - 11 = 9989$

  $1027 + 9989 = 11016$

  $\therefore\ 1027 - 11 = 1016$

- $129 - 33$

  $1000 - 33 = 967$

  $129 + 967 = 1096$

  $\therefore\ 129 - 33 = 096$

- $2222 - 222$

  $10000 - 222 = 9778$

  $2222 + 9778 = 12000$

  $\therefore\ 2222 - 222 = 2000$

8.

- −91

$$\begin{array}{r|l} 2 & 91 \quad \cdots \\ 2 & 45 \quad \cdots 1 \\ 2 & 22 \quad \cdots 1 \\ 2 & 11 \quad \cdots 0 \\ 2 & 5 \quad \cdots 1 \\ 2 & 2 \quad \cdots 1 \\ & 1 \quad \cdots 0 \end{array}$$

1011011

→ 0100100 add 1

→ 0100101

∴ 10100101_2

- −96

$$\begin{array}{r|l} 2 & 96 \\ 2 & 48 \quad \cdots 0 \\ 2 & 24 \quad \cdots 0 \\ 2 & 12 \quad \cdots 0 \\ 2 & 6 \quad \cdots 0 \\ 2 & 3 \quad \cdots 0 \\ & 1 \quad \cdots 1 \end{array}$$

1100000

→ 0011111 add 1

→ 0100000

∴ 10100000_2

• −126

2 | 126
2 | 63 ... 0
2 | 31 ... 1
2 | 15 ... 1
2 | 7 ... 1
2 | 3 ... 1
1 ... 1

1111110
→ 0000001 add 1
→ 0000010
→ 10000010_2

• 101

2 | 101
2 | 50 ... 1
2 | 25 ... 0
2 | 12 ... 1
2 | 6 ... 0
2 | 3 ... 0
1 ... 1

1100101
→ 01100101_2

• 76

2 | 78
2 | 39 ... 0
2 | 19 ... 1
2 | 9 ... 1
2 | 4 ... 1
2 | 2 ... 0
1 ... 0

1001110
→ 01001110_2