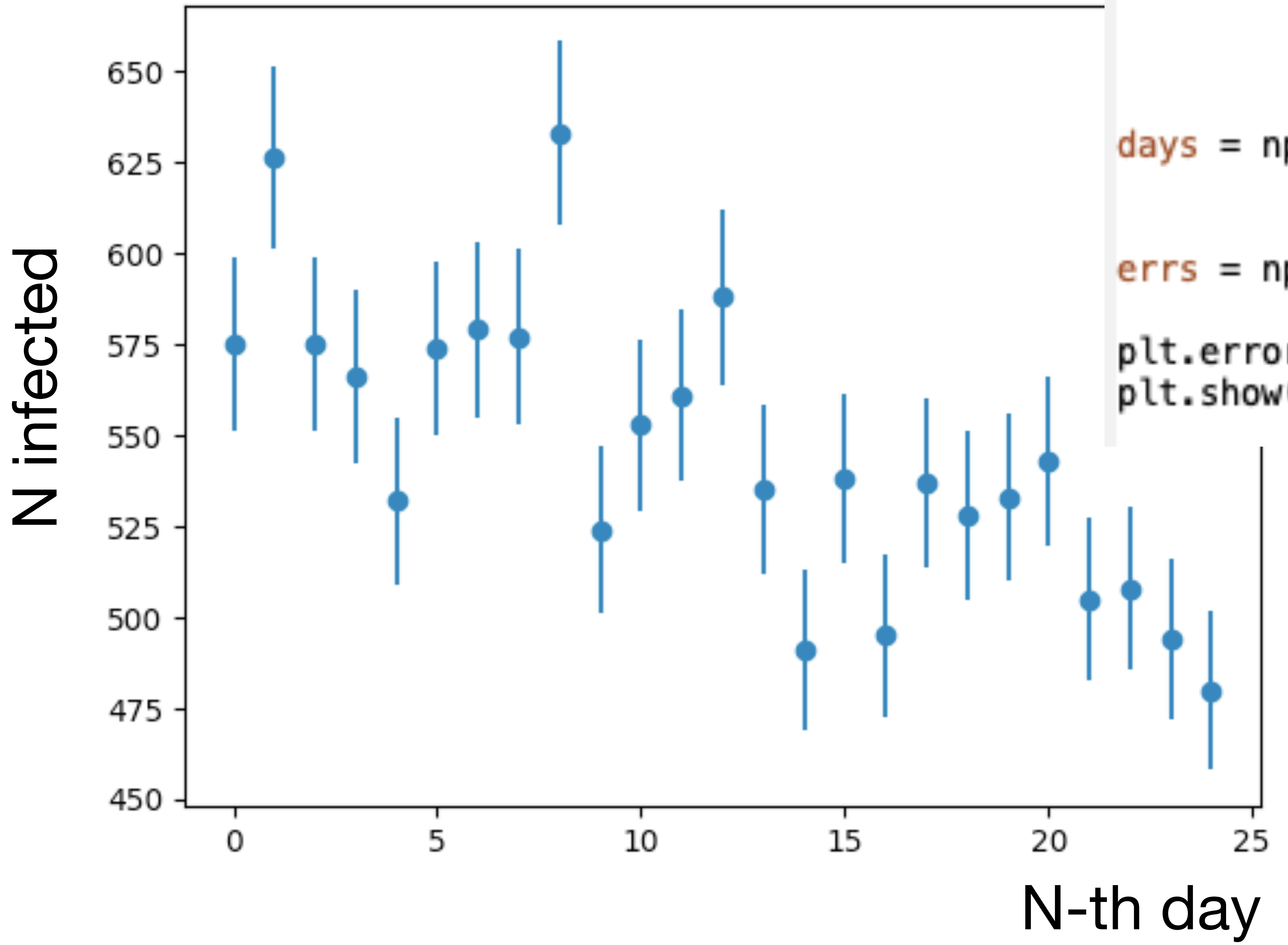
Computational Physics

Final Exam Solutions

Department of Physics, Chung-Ang University
Chang Hyon Ha

Problem I

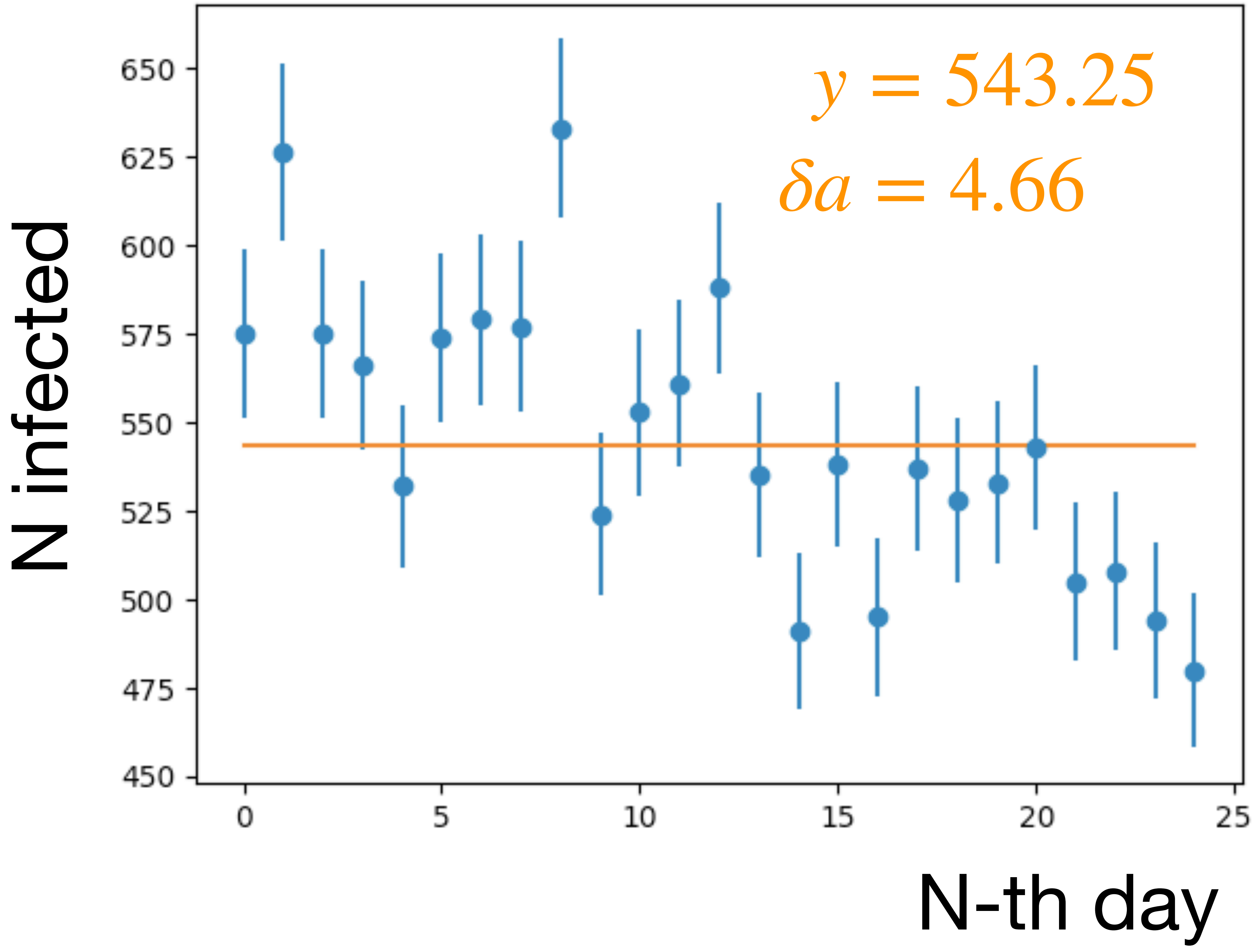
1)




```python
import numpy as np
import matplotlib.pyplot as plt

nums = np.array([575, 626, 575, 566, 532,
                 574, 579, 577, 633, 524,
                 553, 561, 588, 535, 491,
                 538, 495, 537, 528, 533,
                 543, 505, 508, 494, 480])

days = np.array([ 0,  1,  2,  3,  4,  5,  6,  7,  8,  9,
                 10, 11, 12, 13, 14, 15, 16, 17, 18, 19,
                 20, 21, 22, 23, 24])
errs = np.sqrt(nums)

plt.errorbar(days,nums,yerr=errs,fmt='o')
plt.show()
```

2), 3),4)

Const. Model Best Fit



$$\chi^2_{min} = 68.74 \quad NDF = 24$$

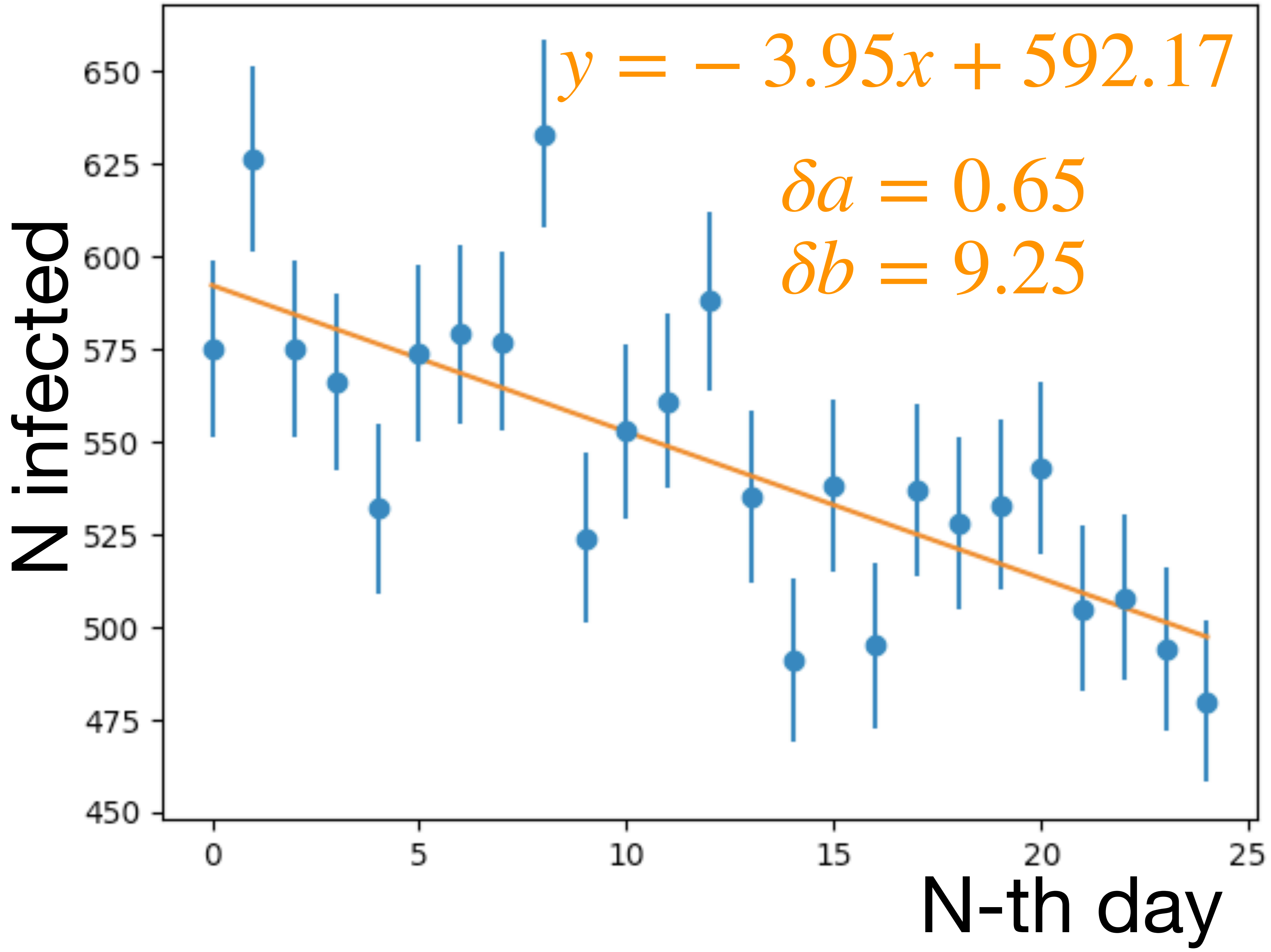
$$P - value = 3.4 \times 10^{-6}$$

```
from math import *
import numpy as np
import matplotlib.pyplot as plt
from scipy.optimize import curve_fit

def f(days, a):
    return np.zeros(N) + a

a = -1
N = 25

nums = np.array([575, 626, 575, 566, 532, 574, 579, 577, 633, 524, 553, 561, 588, 535, 491,
                 538, 495, 537, 528, 533, 543, 505, 508, 494, 480])
days = np.array([0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19,
                 20, 21, 22, 23, 24])
errs = np.sqrt(nums)

A = f(days, a) # constant function
start = (10) # starting position

popt,pcov = curve_fit(f,days,nums,sigma = errs,p0 = start,absolute_sigma=True)
print(popt)
perr = np.sqrt(np.diag(pcov))
print("Error = ", perr)

# Compute chi square
Nexp = f(days, *popt)
r = nums - Nexp
chisq = np.sum((r/errs)**2)
df = N - 1
print("chisq =",chisq,"ndf =",df)

plt.errorbar(days,nums,yerr=errs,fmt='o')
plt.plot(days, Nexp)
plt.show()
```

```
from scipy.integrate import quad
from math import *

def func (x,nu) :
    norm  = pow(2,nu/2.0) * gamma(nu/2.0)
    powe = pow(x,nu/2.0-1)
    expo = exp(-x/2.0)
    return (1./norm) * powe * expo

nu=24
I = quad(func ,68.74,inf,args=(nu))
print ("I = ", I)
```

2), 3),4)

Linear Model Best Fit

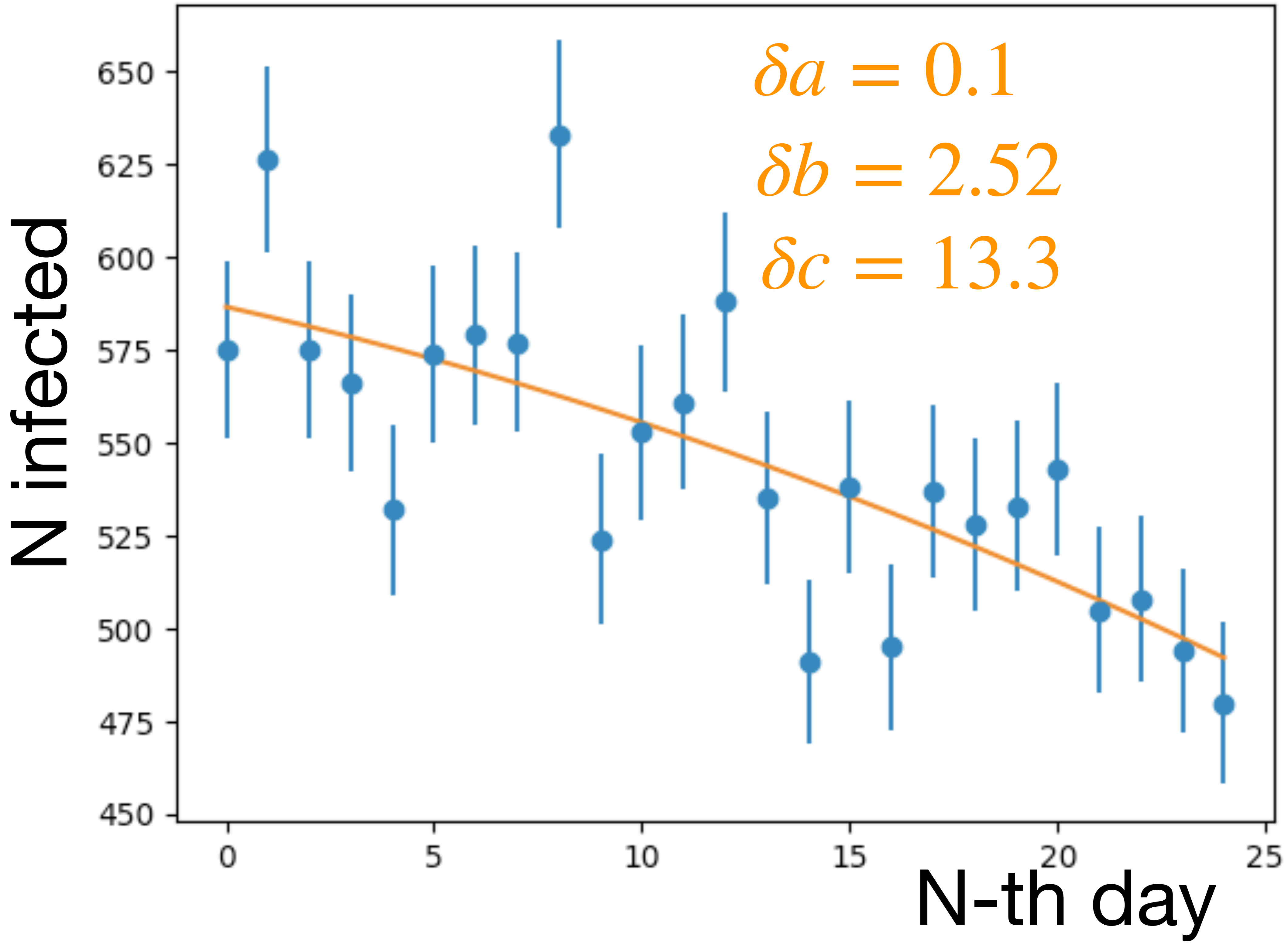


$\chi^2_{min} = 31.22 \quad NDF = 23$

$P - value = 0.12$

Quadratic Model Best Fit

$y = -0.058x^2 - 2.52x + 586.56$



$\chi^2_{min} = 30.88 \quad NDF = 22$

$P - value = 0.099$

5) The Constant Model is not valid for explaining this data because the p-value is too low $P - value = 3.4 \times 10^{-6}$

The Linear Model is valid for explaining this data (the p-value is not too low).

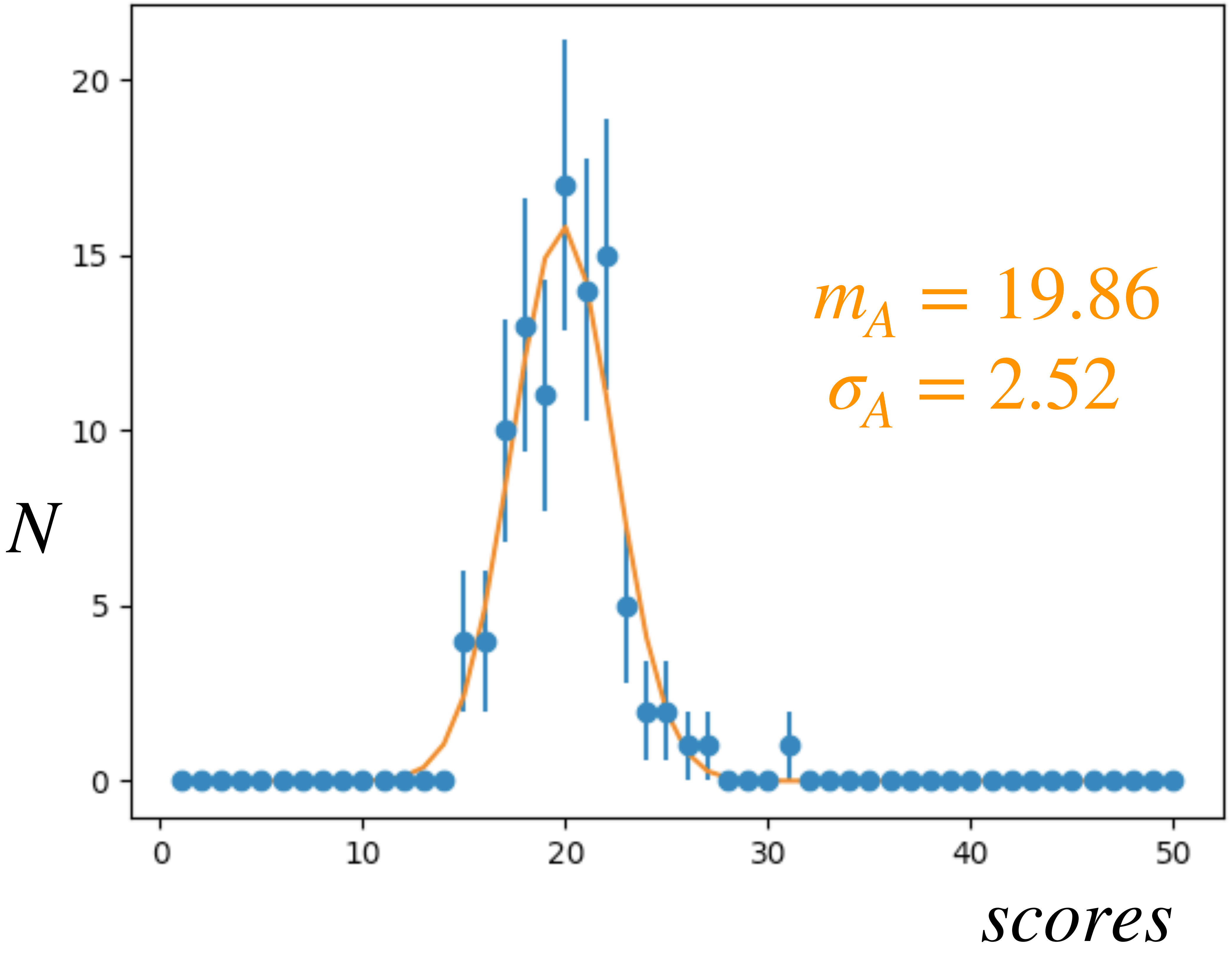
The Quadratic Model is also valid for explaining this data (the p-value is not too low)

6) The Constant Model should be rejected. If repeated, the probability of getting χ^2 value larger than 68.74 is roughly 3 out of a million chance. This is between 4 and 5 sigma, which is very rare. Therefore, the model is not good.

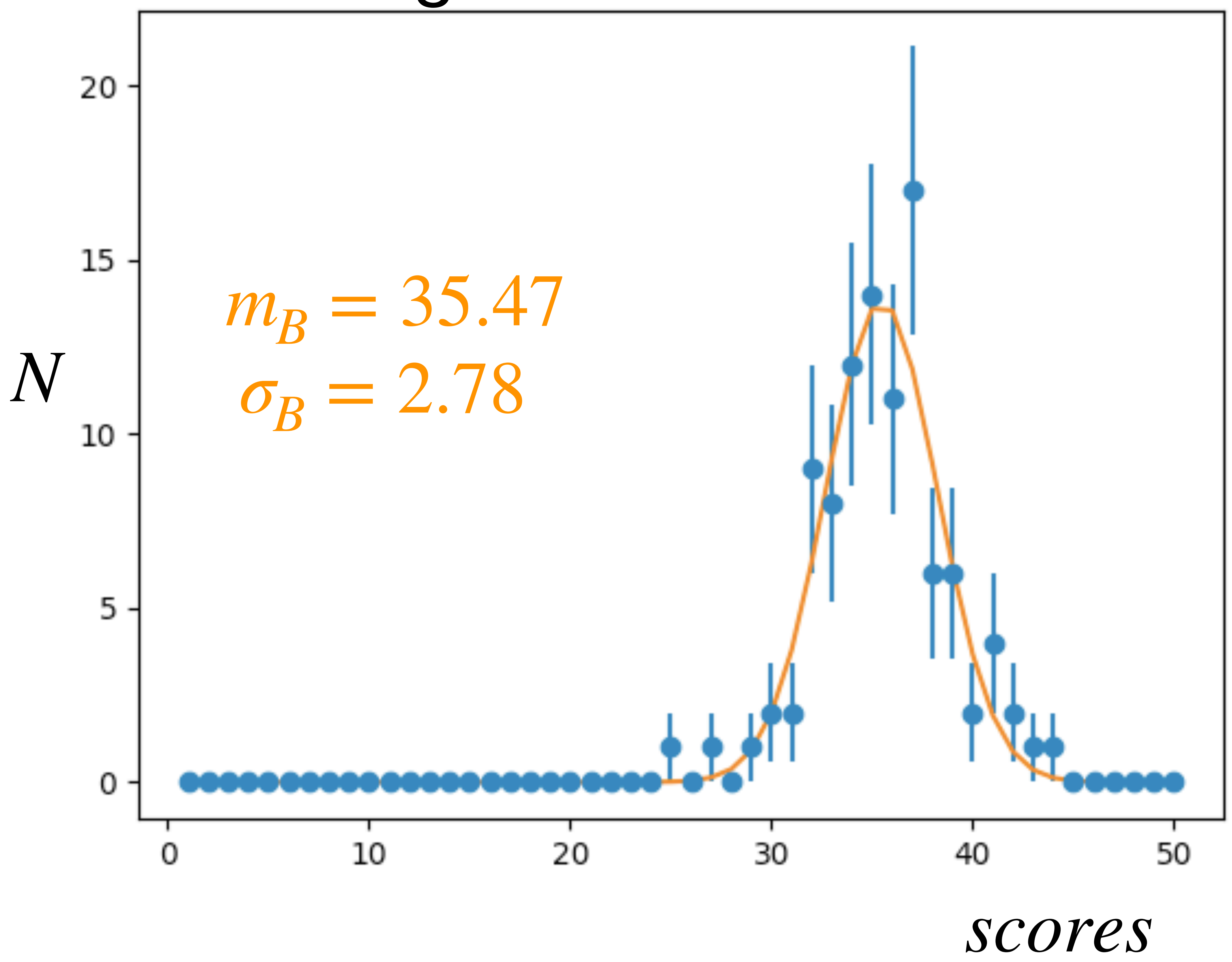
Other two models have p-values around 10% level. If repeated, 10% chance of getting worse fitting than the current ones. So, this is not rare. Since the Linear and Quadratic models have similar p-values, we can not say one is better than the other one. They are both reasonable models.

Problem II

“A” high school distribution



“B” high school distribution



$$\textit{Figure of Merit} = \frac{|m_A - m_B|}{\sqrt{\sigma_A^2 + \sigma_B^2}} = \frac{|19.86 - 35.47|}{\sqrt{2.52^2 + 2.78^2}} = 4.16$$

(FoM)

FoM=4.16 is significant. Two schools are different at 4.2 sigma level in terms of their exams. Therefore, “B” is better than “A”. Or, this is not by chance.