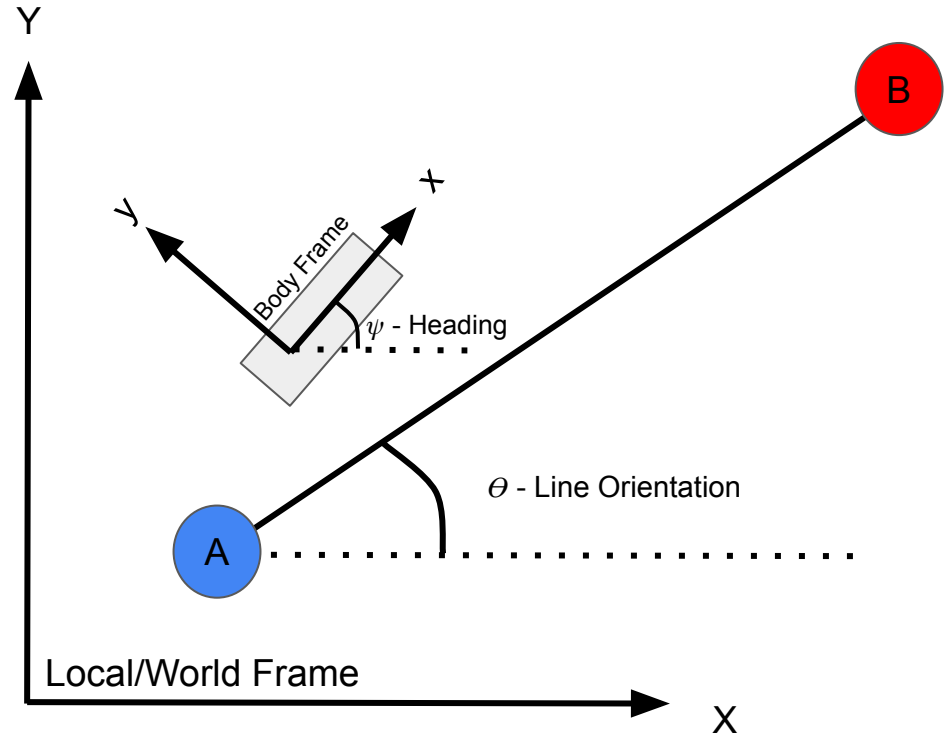


# Vehicle Path Guidance

Lee Thompson  
3/15/2022

# Goal

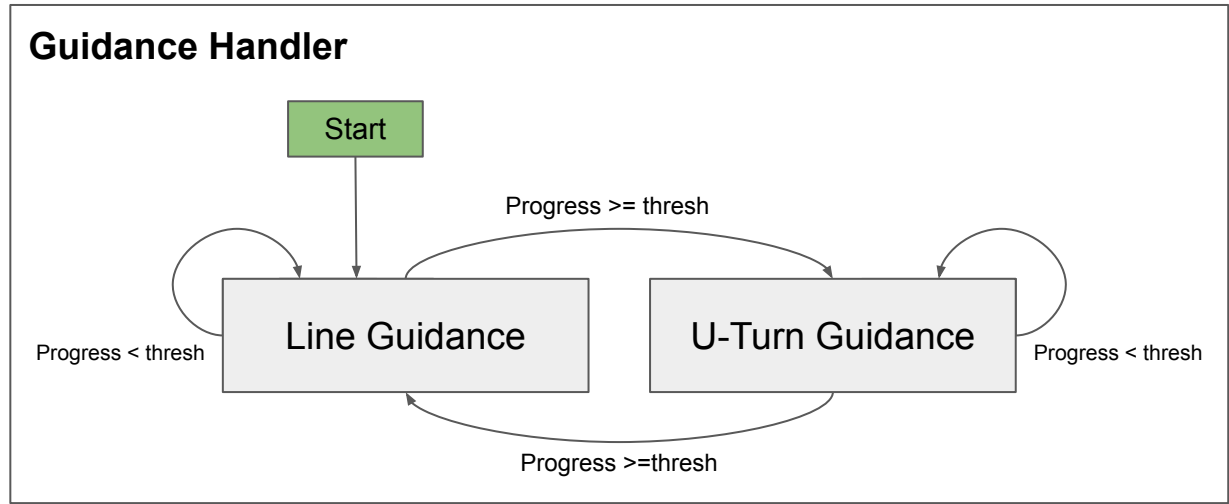
- Develop a steering solution that allows a vehicle to follow a straight line, perform a U-Turn, and then follow line again in opposite direction



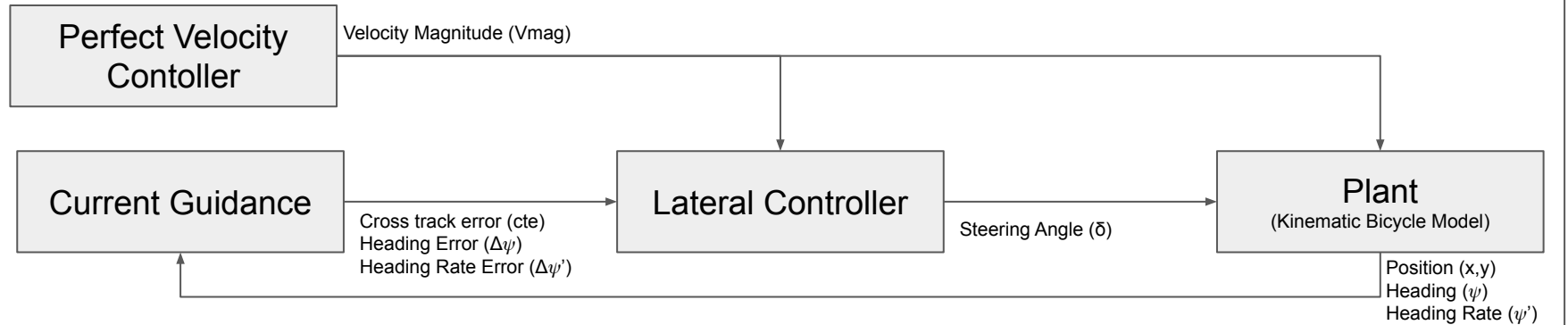
# Assumptions

- Kinematic Bicycle Model
  - Surface is perfectly planer/flat
  - Front and Rear wheels connected by rigid link of fixed length
  - Front wheels are steerable and act together
  - Pure rolling constraint
    - No slip or skidding
  - Non holonomic constraint
    - Move only along direction of heading, no lateral movements
- Longitudinal control
  - Perfect velocity control, only concerned with lateral control for this exercise
- Steering angle is only control input to plant
- No limitations on steering command rate of change
- No restriction how available space outside of line
- Parameters:
  - Wheel Base = 2.5 m
  - Max Steering Angle = 30 deg
  - Velocity Magnitude = 20 kph

# System Overview

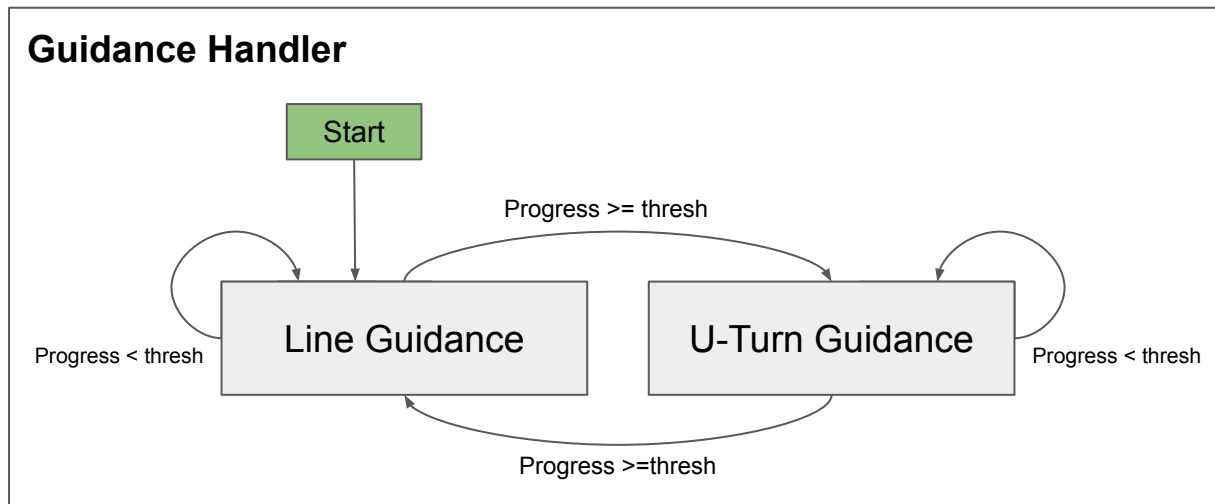


## Control Loop



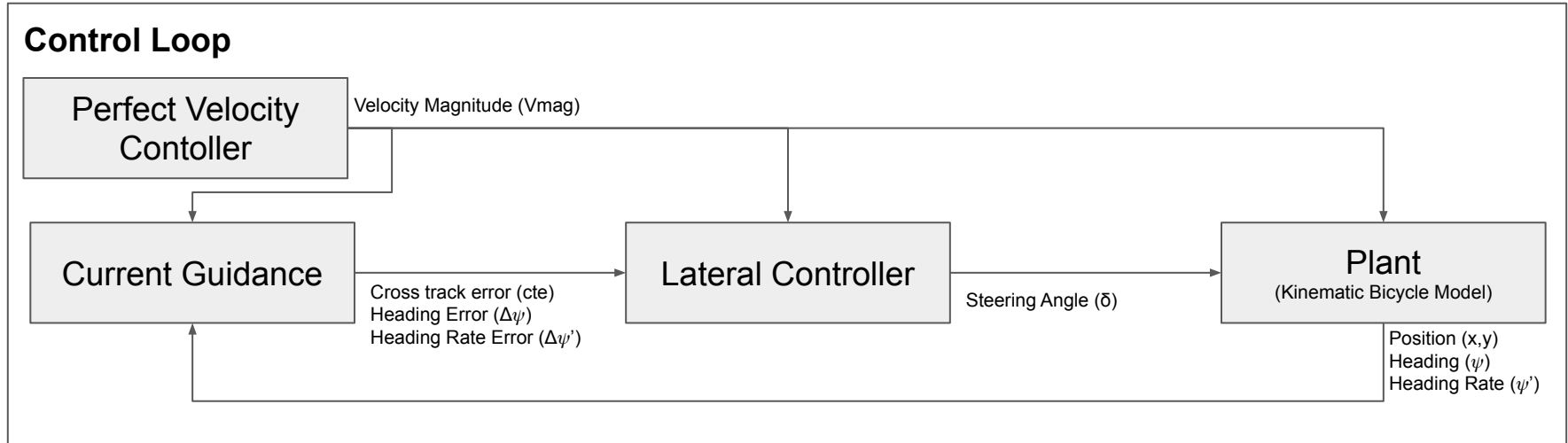
# Guidance Handler

- State machine to handle different operation modes
  - Line & U-Turn guidance modes
- Always starts in Line Mode
- Transition to next mode when progress criteria is met
  - “Progress” variable defined by guidance object, ranges 0-1



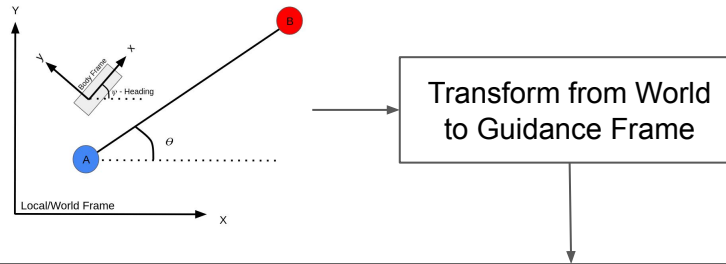
# Main Control Loop

- Current Guidance object determined by Guidance Handler
- Assuming perfect velocity control, holding a constant velocity

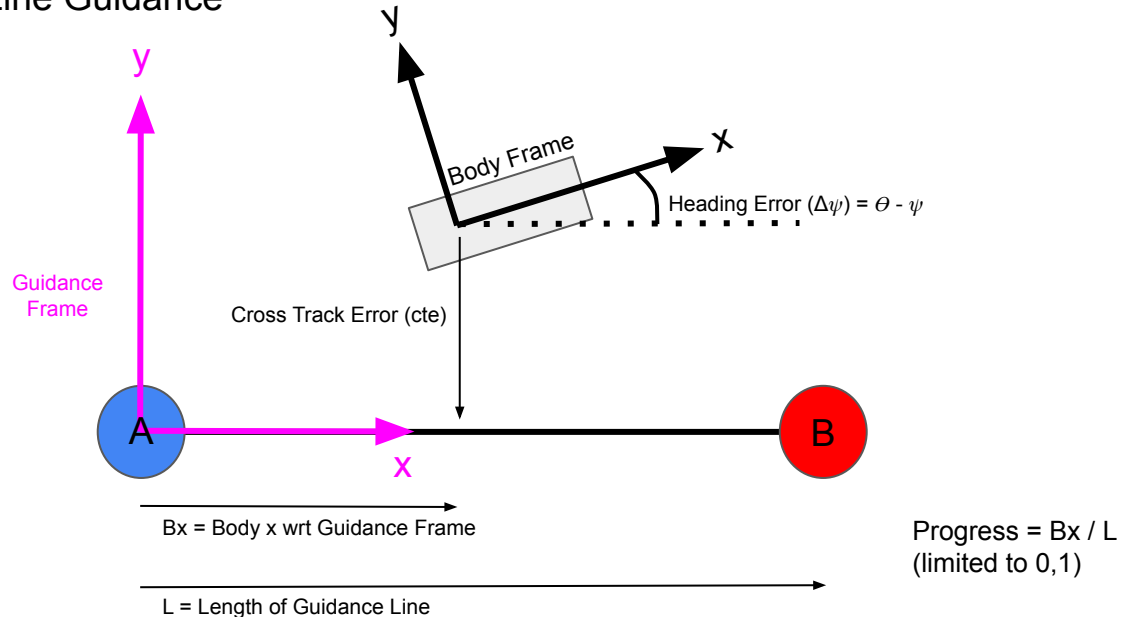


# Line Guidance

- Describe Body frame orientation and position with respect to Guidance Frame
- Output CTE, Heading Error, Heading Rate error, and Progress
- Fully defined by points A and B

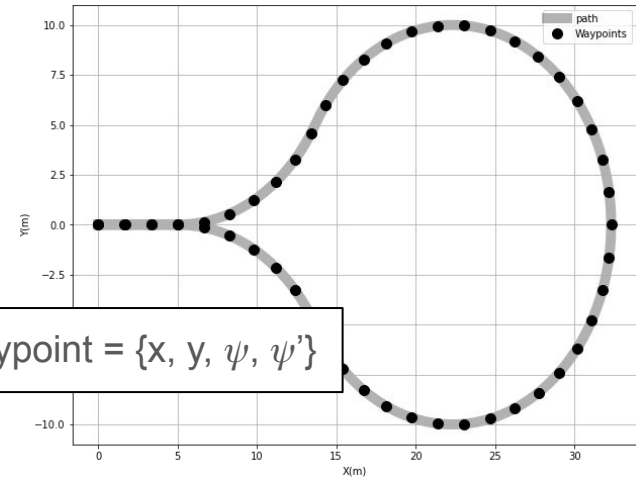
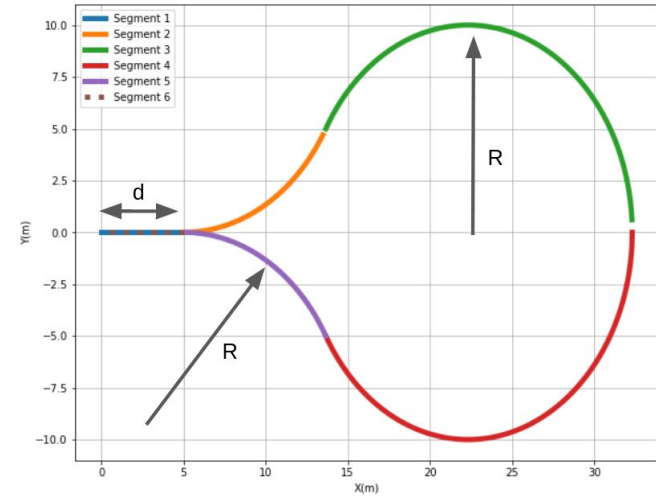


## Line Guidance



# U-Turn Guidance: Path Definition

- U-Turn Path made up of 6 line segments
  - Segment 1,6: Straight Lines
  - Segment 2-5: Constant Radius Curves
- Two Parameters define U-Turn shape
  - $d$ : Length of straight segments 1 & 6
  - $R$ : Radius of curved sections
- Transform U-Turn Path to World Frame
  - Based on end point and orientation of guidance line and
- Fit X and Y of path using Cubic Splines
  - Independent variable ( $t$ ) ranges from 0-1
  - $(x,y)$  for each waypoint
- Use derivative of Cubic Spline path to calculate heading at each waypoint
  - Heading for each waypoint
  - Heading =  $\arctan2(dy,dx)$
- Waypoint heading rate for constant radius turn
  - $\psi' = v_{Mag} / R$
  - Left turn (+), Right turn (-)



Waypoint =  $\{x, y, \psi, \psi'\}$



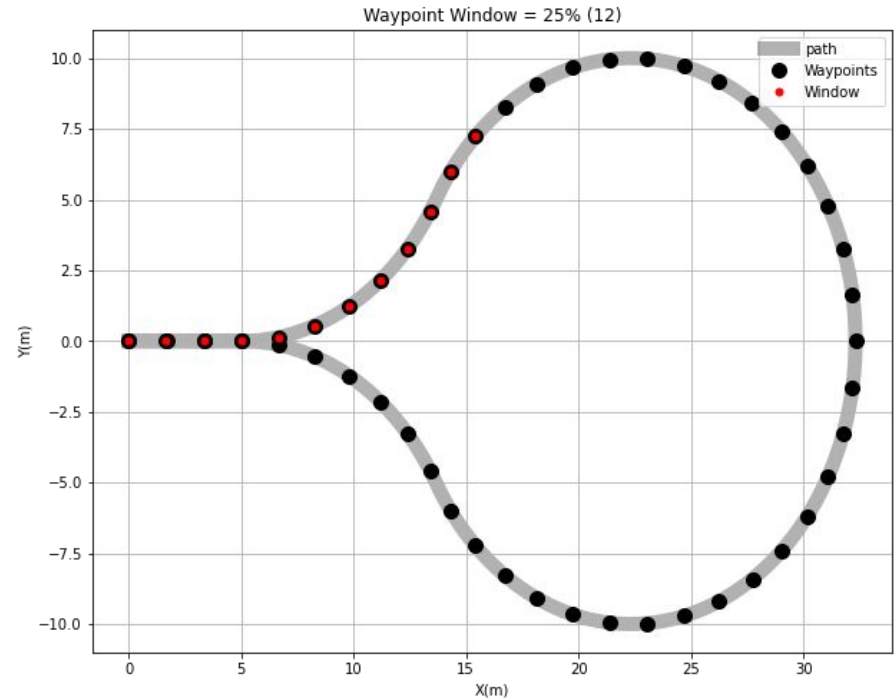
# U-Turn Guidance: Error Calculation

Two Steps:

1. Find the closest waypoint
2. Compute error

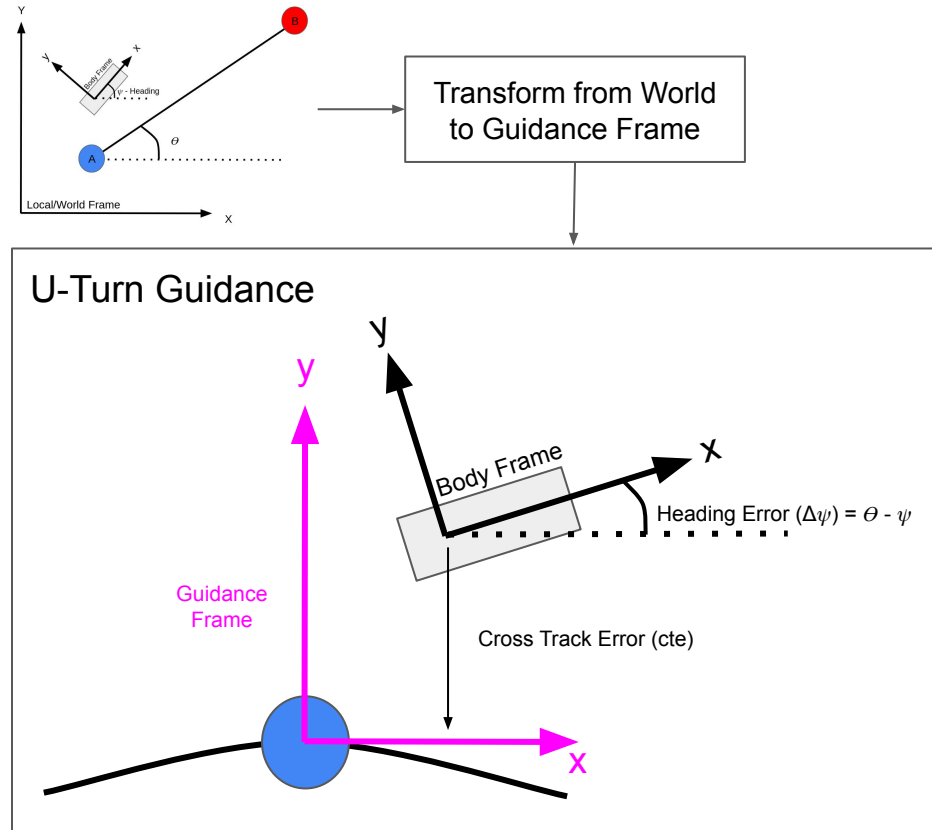
# U-Turn Guidance: Finding Closest Waypoint

- Look at window of waypoints around previous waypoint
  - Size of window is tuneable but based on percentage
- Choose closest waypoint (x,y) without ever choosing a previous waypoint
- Purpose of windowing is to not accidentally skip ahead
  - Window size = 25% is good, main trade off is computation speed



# U-Turn Guidance: Error Calculation

- Same as linear guidance but transform applied based on current waypoint position and heading
- Output CTE, Heading Error, Heading Rate error, and Progress
- Progress defined by current waypoint
- No interpolation between waypoints
  - If distance between waypoints is too large, causes undesirable behavior
  - Set point not smooth



# Setting U-Turn Guidance Parameters

Parameter	Description	Notes
d	Sets length of straight segment that connects with Line Guidance	Want this to be long enough to allow CTE to be minimized before start of Guidance Line
R	Sets turning radius	If too small, will be difficult to track at speed. Will either need to decrease vMag or increase R
spline_dt	Sets step size for spline independent parameter	U-Turn Guidance does not interpolate between waypoints. If spline_dt is too large this will introduce undesirable behavior
Window size	Sets number of waypoints to look at when searching for nearest waypoint. Window size defined as % of number of waypoints	25% should be good for most cases unless using a small number of waypoints

# Lateral Control

Based on Stanley Controller

- Can be shown to be globally asymptotically stable at zero CTE<sup>[1]</sup>

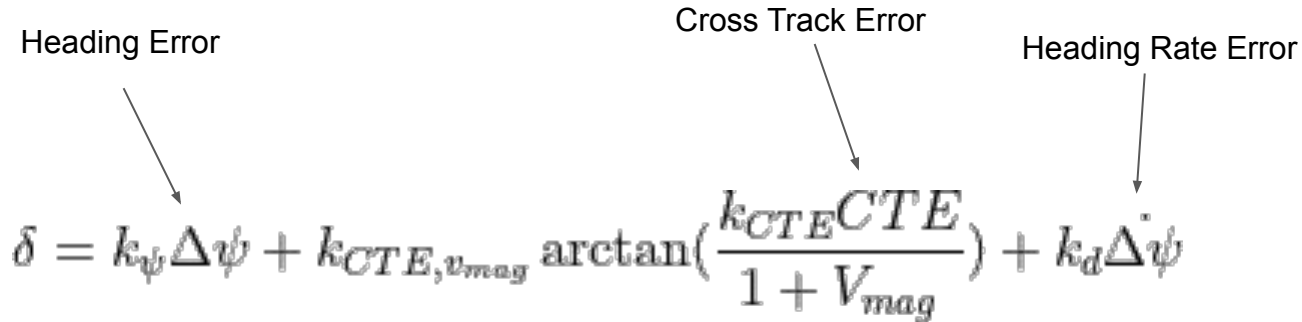


Diagram illustrating the Stanley lateral control equation, showing the relationship between the steering angle  $\delta$  and the error signals:

Heading Error  $\rightarrow k_{\psi} \Delta\psi$

Cross Track Error  $\rightarrow k_{CTE, v_{mag}} \arctan(\frac{k_{CTE} CTE}{1 + V_{mag}})$

Heading Rate Error  $\rightarrow k_d \dot{\Delta\psi}$

$$\delta = k_{\psi} \Delta\psi + k_{CTE, v_{mag}} \arctan\left(\frac{k_{CTE} CTE}{1 + V_{mag}}\right) + k_d \dot{\Delta\psi}$$

# Recommended Tuning Strategies

Gain Parameter	Good Starting Point	Notes
$k_\psi$	1	Used used to decrease CTE during turns and reduce path overshoot
$k_{CTE, v_{mag}}$	1	Use to enable/disable CTE error component while debugging.
$k_{CTE}$	1	Used to null CTE
$k_d$	0	Used to dampen response, more important at high speeds

Heading Error                      Cross Track Error                      Heading Rate Error

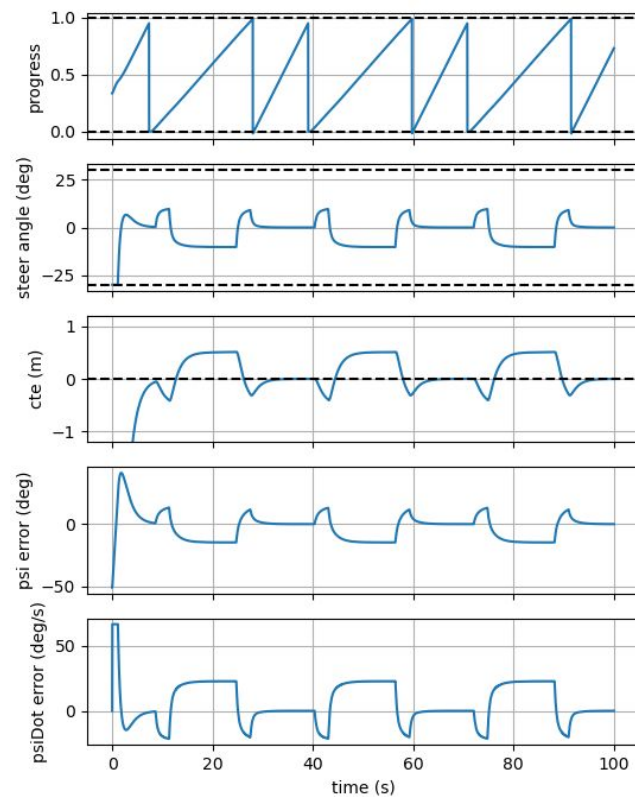
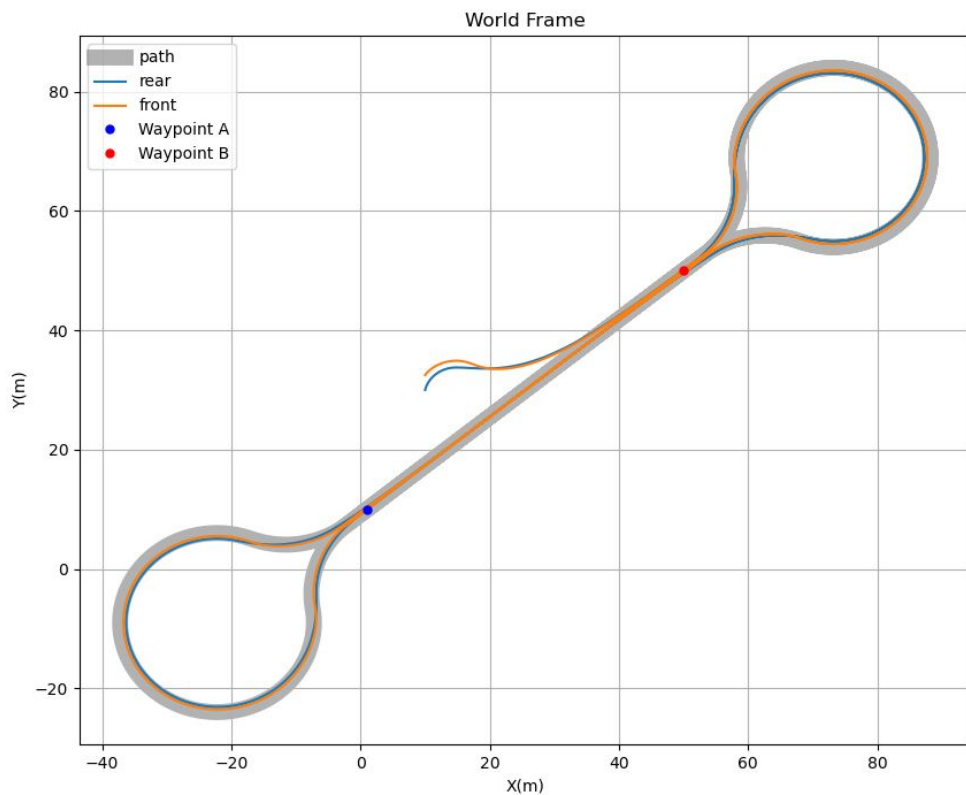
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$$\delta = k_\psi \Delta\psi + k_{CTE, v_{mag}} \arctan\left(\frac{k_{CTE} CTE}{1 + V_{mag}}\right) + k_d \dot{\Delta\psi}$$

# “Starting” Gains

$$\delta = k_{\psi} \Delta\psi + k_{CTE, v_{mag}} \arctan\left(\frac{k_{CTE} CTE}{1 + V_{mag}}\right) + k_d \dot{\Delta\psi}$$

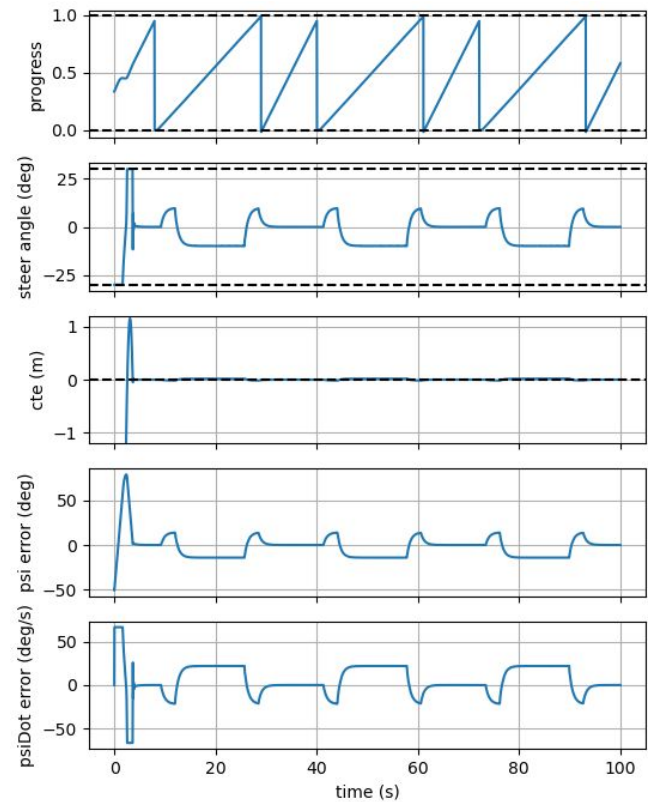
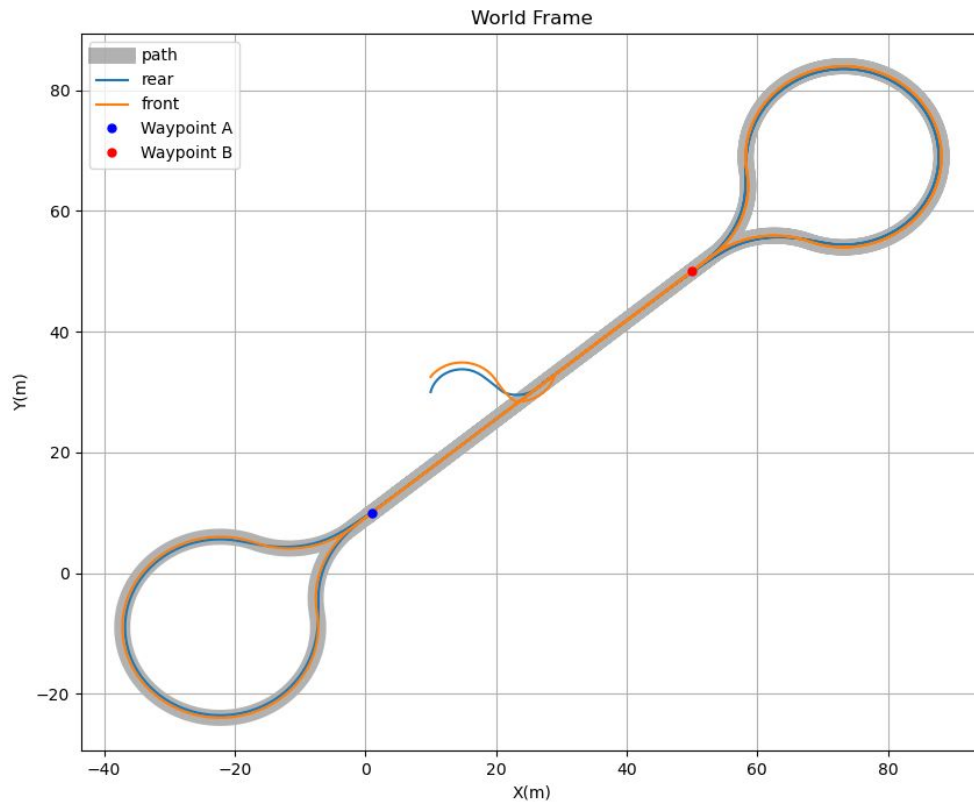
Velocity Magnitude = 20 kph



# Increase K\_cte

$$\delta = k_{\psi} \Delta\psi + k_{CTE, v_{mag}} \arctan\left(\frac{k_{CTE} CTE}{1 + V_{mag}}\right) + k_d \dot{\Delta\psi}$$

Velocity Magnitude = 20 kph

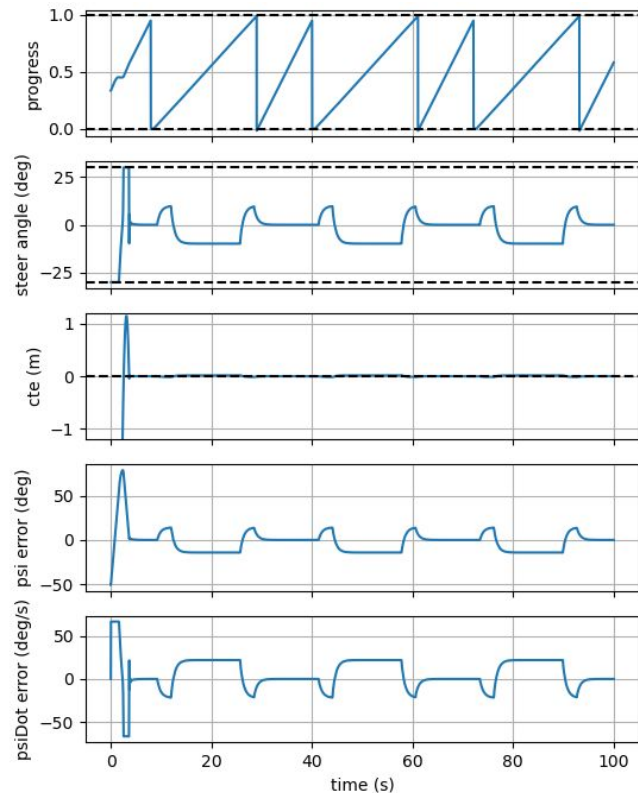
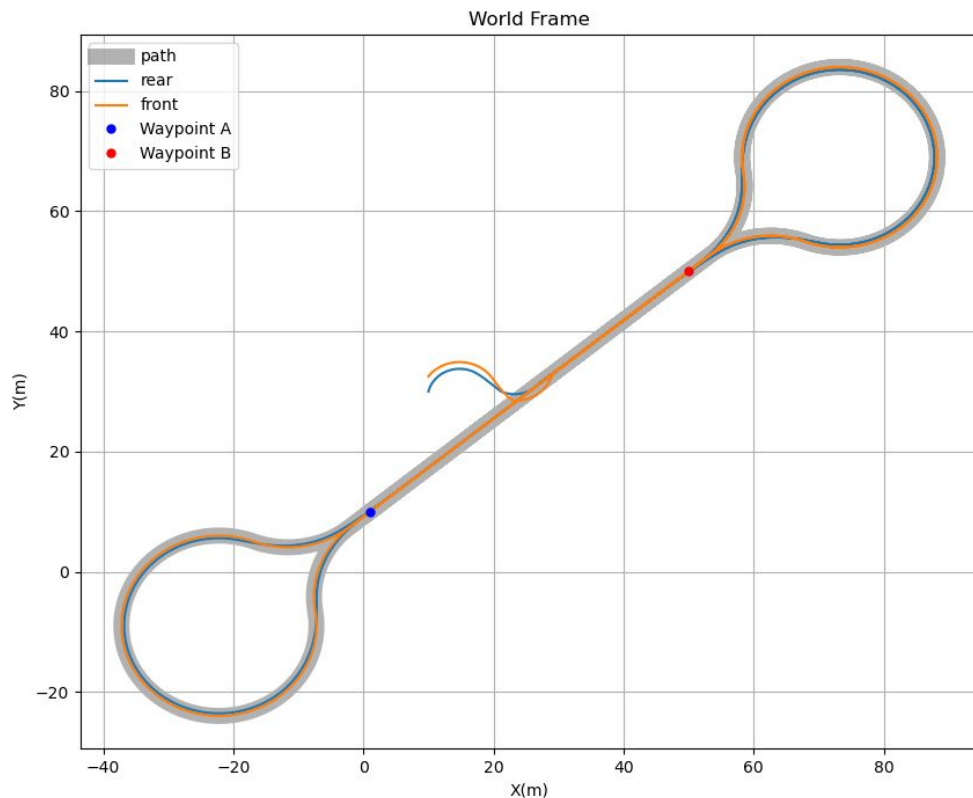




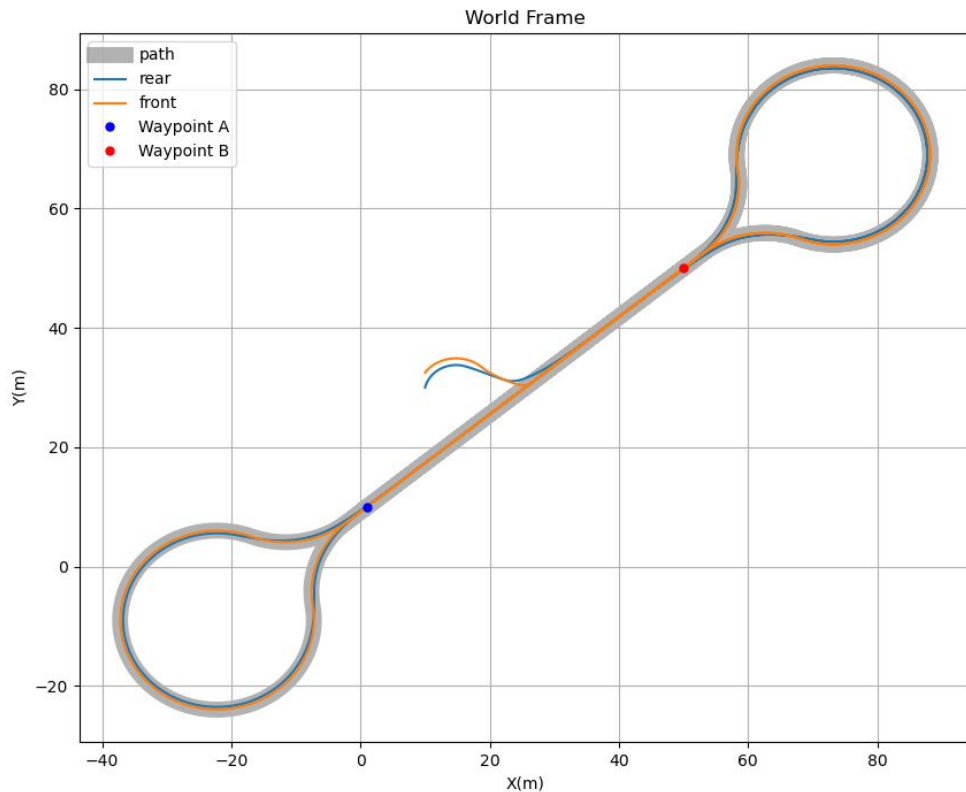
# Increase Kd to add rate damping

$$\delta = k_{\psi} \Delta\psi + k_{CTE, v_{mag}} \arctan\left(\frac{k_{CTE} CTE}{1 + V_{mag}}\right) + k_d \dot{\Delta\psi}$$

Velocity Magnitude = 20 kph

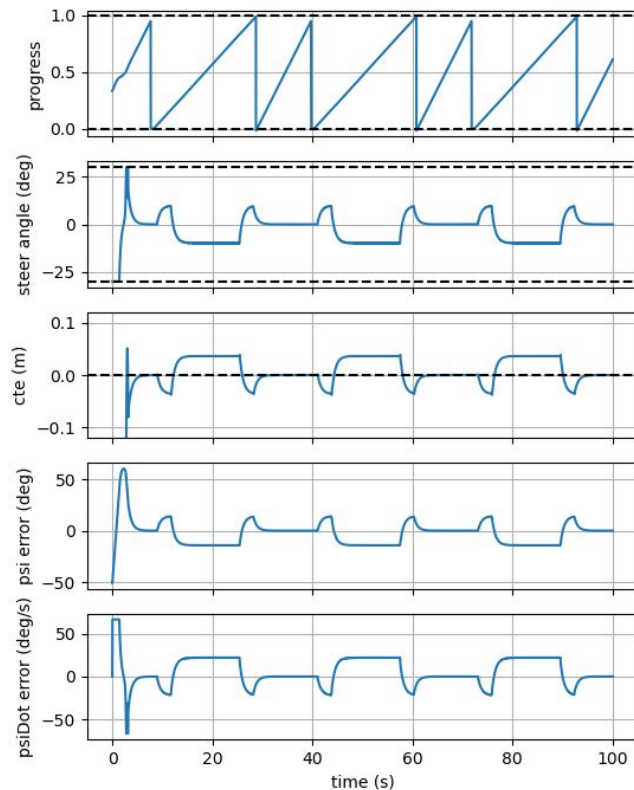


# Increase $K_{\psi}$ to further reduce overshoot on turns



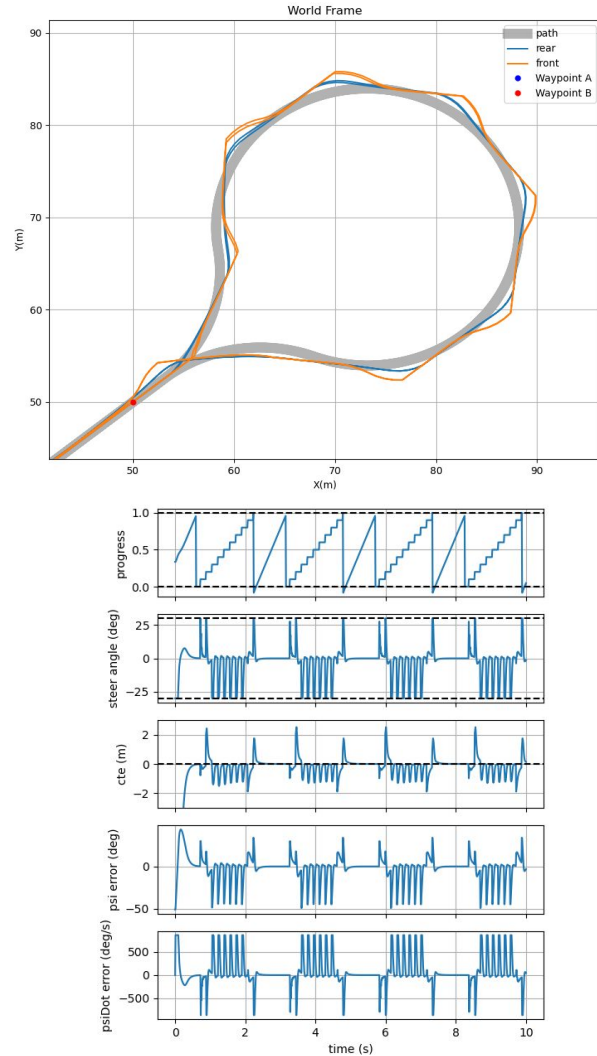
$$\delta = k_{\psi} \Delta\psi + k_{CTE, v_{mag}} \arctan\left(\frac{k_{CTE} CTE}{1 + V_{mag}}\right) + k_d \dot{\Delta\psi}$$

Velocity Magnitude = 20 kph



# Limitations

- U-Turn Guidance does not interpolate between waypoints
  - Need to “smooth” response by using many waypoints
- U-Turn Guidance results in step change in desired yaw rate
- Vehicle dynamics not modeled
  - Kinematic controllers are known to have poor tracking at high speeds where assumptions may no longer be valid

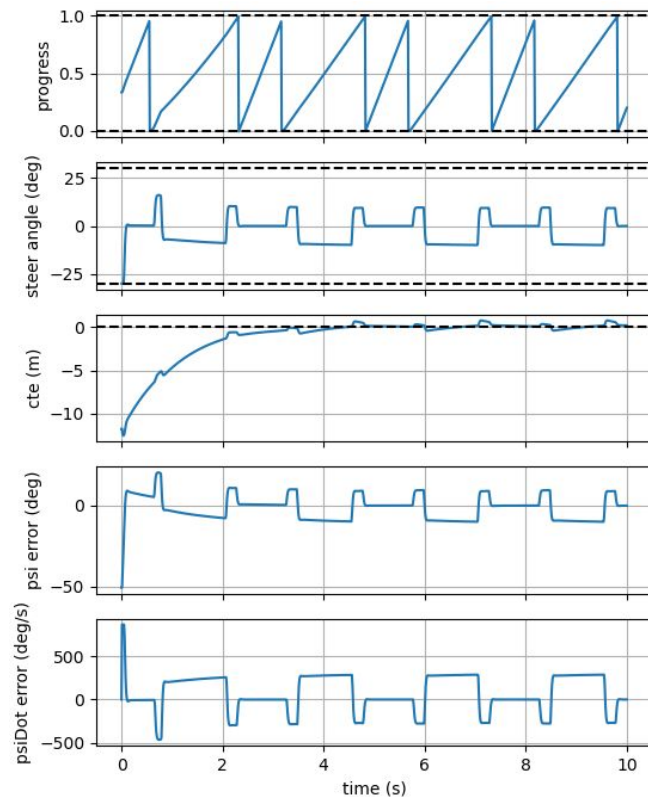
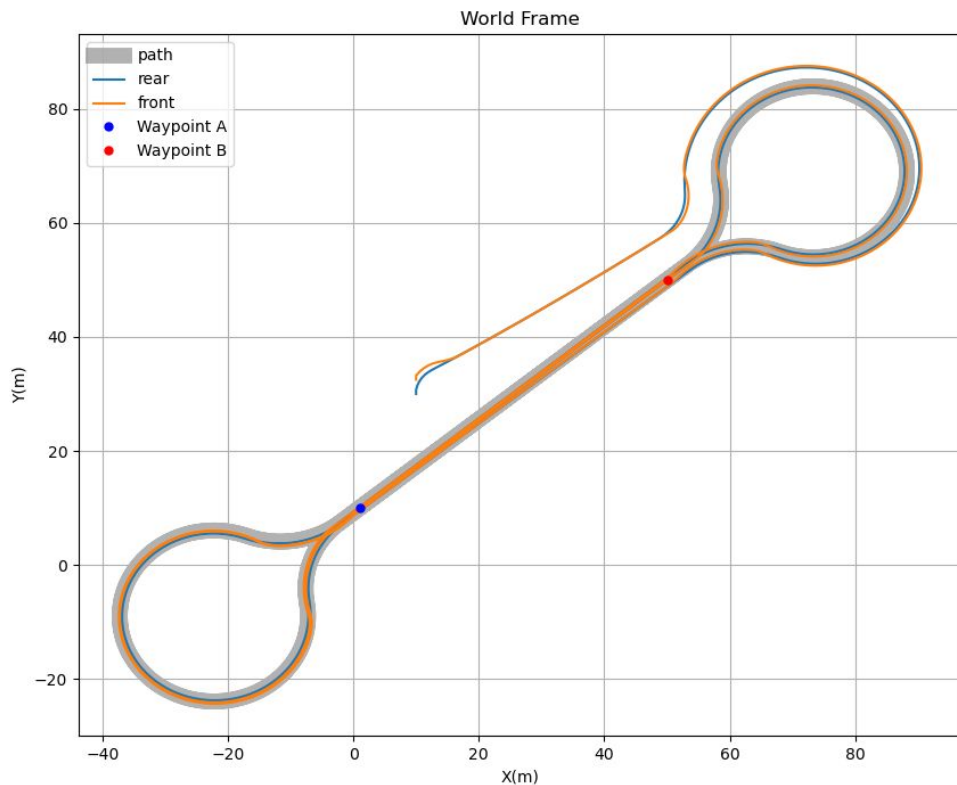


Gain tuning at 260 kph

# “Starting” Gains

$$\delta = k_{\psi}\Delta\psi + k_{CTE,v_{mag}} \arctan\left(\frac{k_{CTE}CTE}{1 + V_{mag}}\right) + k_d\dot{\Delta\psi}$$

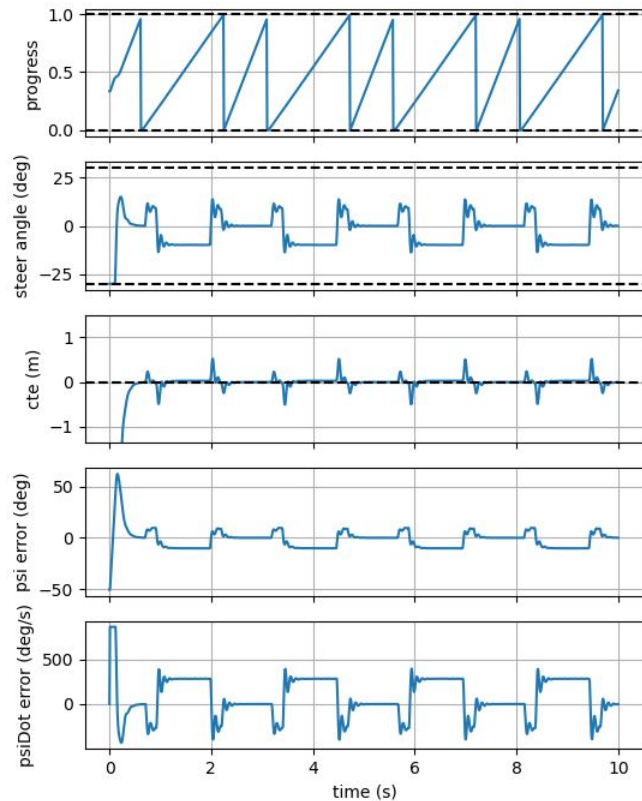
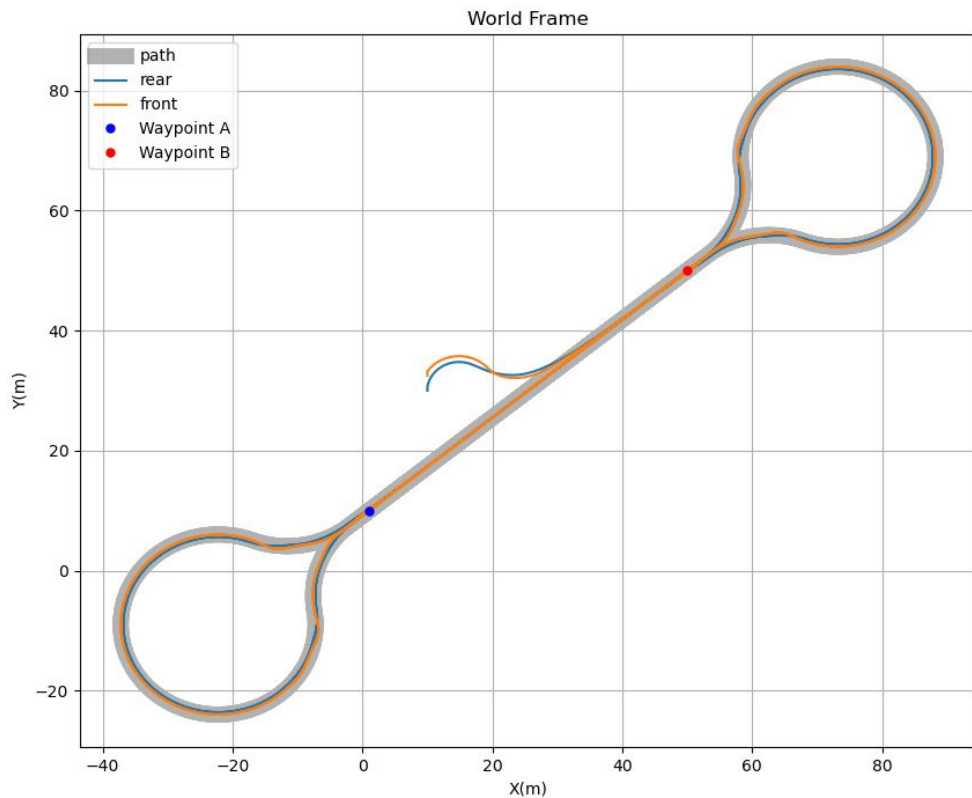
Velocity Magnitude = 260 kph



# Increase K\_cte

$$\delta = k_{\psi} \Delta\psi + k_{CTE, v_{mag}} \arctan\left(\frac{k_{CTE} CTE}{1 + V_{mag}}\right) + k_d \dot{\Delta\psi}$$

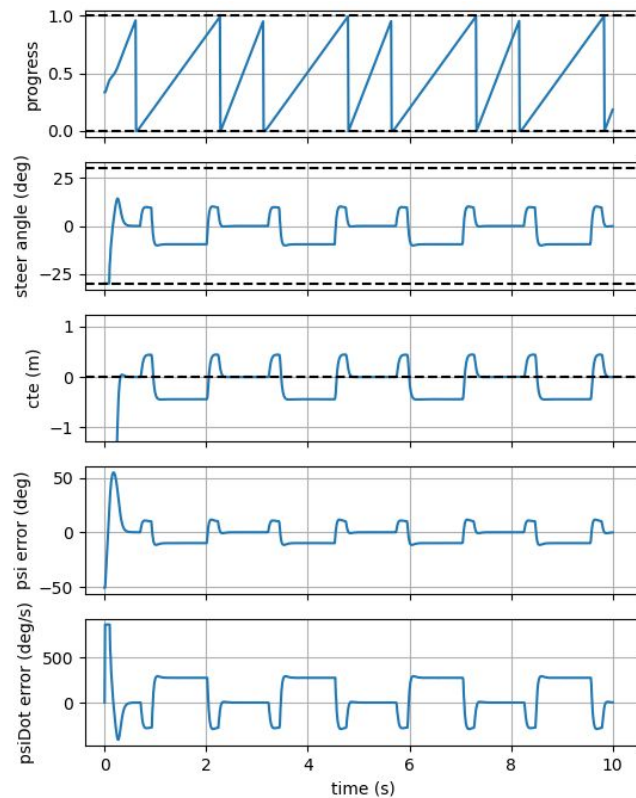
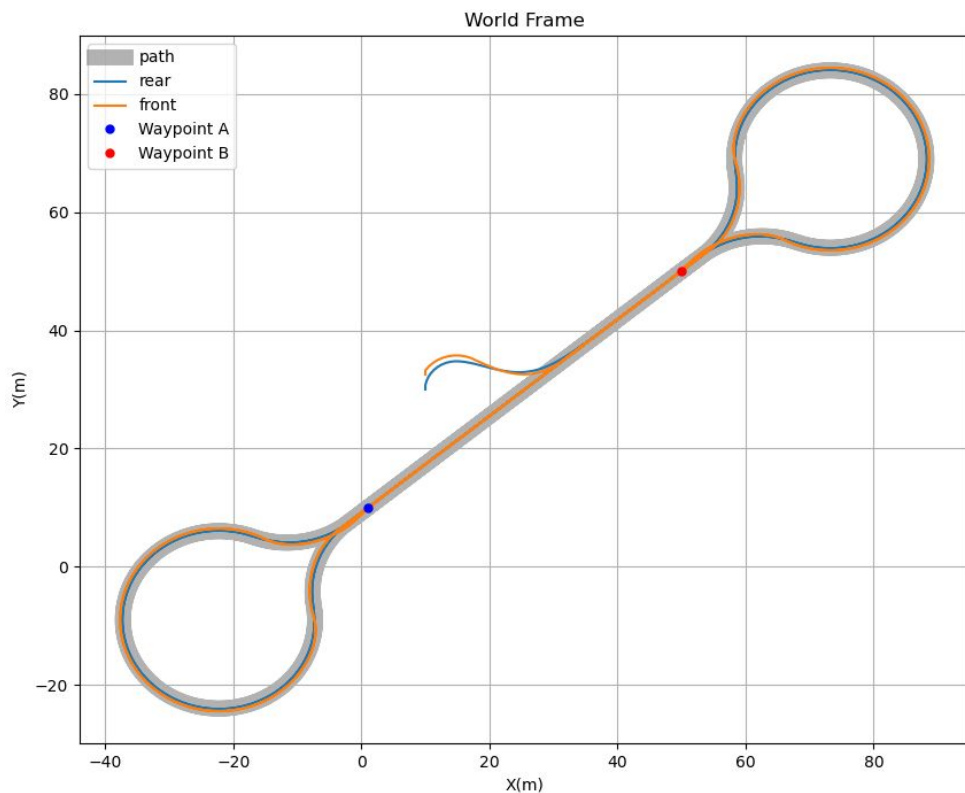
Velocity Magnitude = 260 kph



# Increase Kd to add rate damping

$$\delta = k_{\psi} \Delta\psi + k_{CTE, v_{mag}} \arctan\left(\frac{k_{CTE} CTE}{1 + V_{mag}}\right) + k_d \dot{\Delta\psi}$$

Velocity Magnitude = 260 kph



# Increase $K_{\psi}$ to further reduce CTE during turns

$$\delta = k_{\psi} \Delta\psi + k_{CTE, v_{mag}} \arctan\left(\frac{k_{CTE} CTE}{1 + V_{mag}}\right) + k_d \dot{\Delta\psi}$$

Velocity Magnitude = 260 kph

