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Bitcoin price–volume: A multifractal cross-correlation approach

Marwane El Alaoui[a,*], Elie Bouri[b], David Roubaud[c]

[a] Faculty of Legal, Economic and Social Sciences Agdal, Mohammed V University of Rabat, Morocco
[b] USEK Business School, Holy Spirit University of Kaslik, Jounieh, Lebanon
[c] Montpellier Business School, Montpellier, France

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ABSTRACT

We study the price–volume cross-correlation in the Bitcoin market from July 17, 2010, to May 2, 2018, via the multifractal detrended cross-correlations analysis (MF-DCCA). Results show that Bitcoin prices changes and changes in trading volume mutually interact in a nonlinear way. Furthermore, multifractality is present and significant. By bringing fractal market and nonlinear theories into the analysis of Bitcoin price-volume behavior, we characterize the underlying mechanisms (i.e., nonlinear dependency and multifractality) that govern Bitcoin market dynamics. This deepens our insights into the effectiveness of technical trading strategies in the complex market of Bitcoin that seems to lack efficiency.

1. Introduction

The application of unconventional and sophisticated methods has been shown to appropriately account for the complex features and dynamic fluctuations of asset price series, making the prediction of prices more realistic and the design of risk management models more thorough. The complex features[1] have properties similar to those found in physical mechanics, such as multi-scaling properties and multifractal behavior (e.g., Zhou, 2008). Interestingly, the controversial market of Bitcoin exhibits multifractal behaviors given evidence on its price clustering (Urquhart, 2017), price explosivity (Fry and Cheah, 2016; Corbet et al., 2018), large price volatility (Bariviera et al., 2017; Alvarez-Ramirez et al., 2018), long-range dependence and persistence, and fat tails (Bariviera et al., 2017; Takaishi, 2018; Al-Yahyaee et al., 2018; Bouri et al., 2018a). Accordingly, many scholars have applied unconventional and sophisticated methods and found evidence of fractals and multifractality in the Bitcoin market (Bariviera et al., 2017; Alvarez-Ramirez et al., 2018; Lahmiri and Bekiros, 2018; Takaishi, 2018; Al-Yahyaee et al., 2018), as well as significant long-range cross-correlations between Bitcoin and several conventional assets (Gajardo et al., 2018). However, it is still unclear whether the cross-correlation between Bitcoin prices and trading volume is also subject to multifractality.

Therefore, the aim of this paper is to examine the multifractal cross-correlation between price changes and volume changes via the application of multifractal detrended cross-correlation analysis (MF-DCCA) of Zhou (2008), which has the power to capture the underlying scaling structure present in various systems.[2] Such an examination not only adds to Balcilar et al. (2017) and Bouri et al. (2018b), who apply conventional methods (i.e. causality methods) to the Bitcoin price–volume nexus and find mixed evidence, but it

* Corresponding author.
E-mail addresses: marwane.elalaoui@um5s.net.ma (M.E. Alaoui), eliebouri@usek.edu.lb (E. Bouri), d.roubaud@montpellier-bs.com (D. Roubaud).

[1] They are the result of many complex factors emanating from nonlinear interactions among heterogeneous agents with different time horizons, and complex events occurring in the external environment.

[2] The MF-DCCA approach was recently applied to the equity price-volume nexus (e.g., El Alaoui, 217).

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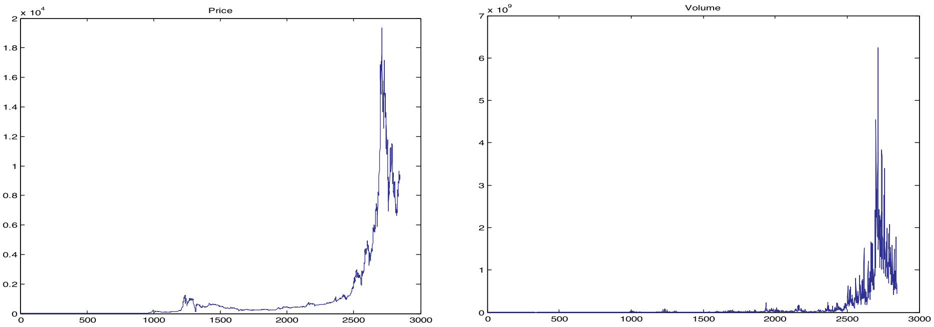

Fig. 1. Price and volume values of Bitcoin from July 17, 2010 to May 2, 2018.

explains some of the underlying physical mechanisms that govern Bitcoin market dynamics. By doing so, we bring the fractal market theory and other nonlinear theory into the analysis of the Bitcoin price–volume nexus.

Our main results show that Bitcoin price and trading volume mutually interact in a nonlinear way and are subject to multifractality, which would help Bitcoin investors and traders make investment decisions that involve trading volume. Our empirical results are important, given incessant quest from scholars and practitioners to explain the puzzling price dynamics of Bitcoin, which so far have been found to be related to Google trend (see, e.g. Dastgir et al., 2018), economic policy uncertainty (Demir et al., 2018), macroeconomic news surprises (Al-Khazali et al., 2018), price behaviors of other large cryptocurrencies (Bouri et al., 2018c), and gold prices (Dyhrberg, 2016).

2. Data

Daily volume and price index data for Bitcoin against the US dollar are from July 17, 2010 to May 2, 2018. After being extracted from www.cryptocompare.com, the empirical investigation is conducted with 2845 logarithmic daily price changes and daily volume changes. Fig. 1 presents price and volume level series, whereas Fig. 2 depicts price changes and volume changes for the whole sample period.

Table 1 presents the summary statistics of Bitcoin price changes and trading volume changes for the full sample period. In both cases, the values of skewness and kurtosis indicate fat tails and non-normality.

3. Methods

3.1. Cross-correlation test

The cross-correlation test is a qualitative measure that analyzes the presence of cross-correlations between series. According to Podobnik et al. (2009), the cross-correlation function is

$$C_i = \frac{\sum_{k=i+1}^{N} x_k y_{k-i}}{\sqrt{\sum_{k=1}^{N} x_k^2 \sum_{k=1}^{N} y_k^2}} \tag{1}$$

x_t and y_t are two-time series with the same length N.

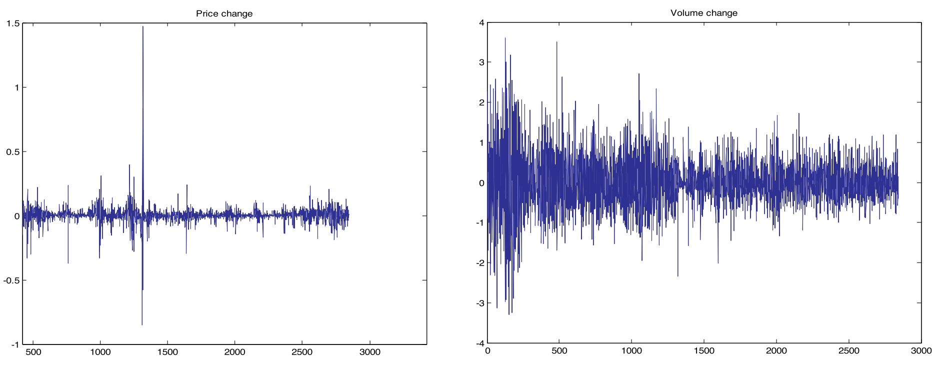

Fig. 2. Price changes and volume changes of Bitcoin from July 17, 2010 to May 2, 2018.

Table 1
Summary statistics.

	Mean	Max.	Min.	Std. Dev.	Skewness	Kurtosis	Jarque–Bera	Prob.
Bitcoin	0.0040	1.4743	−0.8487	0.0698	2.8304	93.0937	965987.5	0.000
Volume	0.0067	3.6075	−3.2992	0.6818	0.2050	5.6445	846.9	0.000

Notes: This table presents key descriptive statistics of Bitcoin price changes and trading volume changes from July 17, 2010 to May 2, 2018

Whereas the cross-correlations statistic is

$$Q_{CC}(m) \equiv N^2 \sum_{i=1}^{m} \frac{C_i^2}{N-i} \tag{2}$$

which is approximately $\chi^2(m)$ distributed with m degrees of freedom. There is no cross-correlation when the cross-correlations test agrees well with $\chi^2(m)$ distribution. However, when the test exceeds the critical values, there is a significant cross-correlation according to a specific significance level.

3.2. The MF-DCCA method

The MF-X-DFA algorithm is an MF-DCCA method based on DFA. It consists of many steps.

Let $x(k)$ and $y(k)$ be time series of length N representing logarithmic returns. We suppose that these time series are of compact support, i.e. $(k) = 0$, $y(k) = 0$ for an insignificant fraction of values only. When $x(k) = y(k)$, the MF-X-DFA reduces to the MF-DFA algorithm.

Step 1: We determine the accumulated profile $X(i)$ and $Y(i)$ of the time series $x(k)$ and $y(k)$ for $i = 1, \ldots, N$

$$X(i) = \textstyle\sum_{k=1}^{i} [x(k) - \bar{x}], \quad Y(i) = \sum_{k=1}^{i} [y(k) - \bar{y}] \tag{3}$$

where $\bar{x}$ and $\bar{y}$ denote, respectively, the mean of the time series $x(k)$ and $y(k)$. We can easily verify that $X(N) = 0$ and $Y(N) = 0$.

Step 2: For a given time scale s, we divide the profiles $X(i)$ and $Y(i)$ into $N_s = int(N/s)$ non-overlapping segments of equal length s, where *int*() denotes the function, which gives the integer part of a real number. Since, in general, N is not often a multiple of s, a short part of the end of the profile may be disregarded. To incorporate this ignored part of the series, we repeat the same procedure starting from the end of the profile. We thus obtain $2N_s$ segments.

Step 3: We estimate for each of the $2N_s$ segments a local trend by fitting a polynomial to the data. We then calculate the variances by two formulas depending on the segment v:

– for each segment $v = 1, \ldots, N_s$:

$$F^2(v, s) = \frac{1}{s} \sum_{i=1}^{s} |X((v-1)s + i) - p_v^n(i)| \cdot |Y((v-1)s + i) - p_v^n(i)| \tag{4}$$

– for each segment $v = N_s + 1, \ldots, 2N_s$:

$$F^2(v, s) = \frac{1}{s} \sum_{i=1}^{s} |X(N - (v - N_s)s + i) - p_v^n(i)| \cdot |Y(N - (v - N_s)s + i) - p_v^n(i)| \tag{5}$$

where $p_v^n(i)$ is the n-th order fitting polynomial in the segment v. We can use linear DCCA1, quadratic DCCA2, cubic DCCA3 or higher order polynomials DCCAn for $n > 3$.

When the local detrending function is the moving averages, the algorithm returns to the MF-X-DMA (Kristoufek, 2011).

Step 4: We average the variances over all segments to obtain the q-th order fluctuation function:

– for $q \neq 0$:

$$F_q(s) = \left[\frac{1}{2N_s} \sum_{v=1}^{2N_s} [F^2(v, s)]^{q/2} \right]^{1/q} \tag{6}$$

– for $q = 0$:

$$F_0(s) = \exp\left[\frac{1}{4N_s} \sum_{v=1}^{2N_s} \ln[F^2(v, s)] \right] \tag{7}$$

The MF-DCCA procedure can be used to determine the behavior of the q-dependent fluctuation functions $F_q(s)$ with regard to the time scale s, for various values of q. When $q = 2$, it corresponds to the standard DCCA procedure. Hence, steps 2 through 4 must be repeated for different values of time scales s leading us to a final step.

Step 5: We analyze the multi-scaling behavior of the fluctuation functions $F_q(s)$ by estimating the slope of log-log plots of $F_q(s)$ versus s for different values of q. If the analyzed times series $X(i)$ and $Y(i)$ present a long-range power law correlation as fractal proprieties, then the fluctuation function $F_q(s)$ will behave, for large sufficiently values of s, as the following power–law scaling

$$F_q(s) \sim s^{H_{XY}(q)} \tag{8}$$

In order to estimate the values of $H_{XY}(q)$ for various values of q (we chose 54 value[3] of q from –30 to 30), we regress the time series $H_{XY}(q)$ on the time series $F_q(s)$.

Generally, the exponent $H_{XY}(q)$ is a function depending on the variable q. In the presence of a stationary time series, we obtain the exponent $H_{XY}(2)$ that is identically equal to the standard Hurst exponent H. If the bivariate scaling exponent $H_{XY}(2) > 0.5$, the cross-correlations between X and Y are long-range persistent. If $H_{XY}(2) < 0.5$, then the cross-correlations between X and Y are anti-persistent. If $H_{XY}(2) = 0.5$, there is no cross-correlations between the two times series. The exponent $H_{XY}(q)$, which generalizes the Hurst exponent H, is commonly called the generalized Hurst exponent. We distinguish between monofractal and multifractal time series as follows. If $H_{XY}(q) = H$ (constant) for all values of q, then the time series under study is monofractal; otherwise $H_{XY}(q)$ is a monotonously decreasing function of q and the corresponding time series is multifractal. From Eqs. (3) and (4), we can infer that for positive values of q, the averaging fluctuation function $F_q(s)$ is dominated by the segments v holding large variances $F^2(v,s)$. Thus, for positive values of q, $H_{XY}(q)$ describes the scaling properties of large fluctuations. Conversely, for negative values of q, $H_{XY}(q)$ describes the scaling properties of small fluctuations.

The generalized Hurst exponent $H_{XY}(q)$ defined by the MF-X-DFA method is directly related to the multifractal scaling exponent $\tau_{XY}(q)$ commonly known as the Rényi exponent

$$\tau_{XY}(q) = qH_{XY}(q) - 1 \tag{9}$$

Clearly, the monofractal time series is characterized by a linear form for the Rényi exponent:

$$\tau_{XY}(q) = q \times H_{XY} - 1 \tag{10}$$

where, H_{XY} is the Hurst exponent.

Another interesting way to characterize the multifractality is to use the singularity spectrum $f_{XY}(\alpha)$ of the Hölder exponent α_{XY}. Notably, the singularity spectrum $f_{XY}(\alpha)$ is related to the Rényi exponent $\tau_{XY}(q)$ by the Legendre transform:

$$\alpha_{XY} = \tau'_{XY}(q) \quad \text{and} \quad f_{XY}(\alpha) = q\alpha_{XY} - \tau_{XY}(q) \tag{11}$$

The Hölder exponent α_{XY} characterizes the strength of the singularity, and the singularity spectrum $f_{XY}(\alpha)$ represents the Hausdorff dimension of the fractal subset with the exponent α_{XY}. The richness of multifractality can be determined by the spectrum width $\Delta\alpha_{XY} = \alpha_{XYmax} - \alpha_{XYmin}$. Thus, the wider the spectrum, the richer the multifractality behavior of the analyzed time series.

We can deduct from Eq. (7) the relation between the generalized Hurst exponent $H_{XY}(q)$ and the singularity spectrum $f_{XY}(\alpha)$:

$$\alpha_{XY} = H_{XY}(q) + qH_{XY}'(q) \quad \text{and} \quad f_{XY}(\alpha) = q[\alpha_{XY} - H_{XY}(q)] + 1 \tag{12}$$

3.3. DCCA coefficient

The DCCA cross-correlations coefficient is used to quantify level of cross-correlations. It is defined as the ratio between the detrended covariance and two detrended F^2_{DCCA} of Eq. (4) and the detrended variance function F_{DFA} (Podobnik et al., 2011).

$$\rho_{DCCA} \equiv \frac{F^2_{DCCA}}{F_{DFA}\{x(k)\}} F_{DFA}\{y(k)\}. \tag{13}$$

where $-1 \le \rho_{DCCA} \le 1$. If $\rho_{DCCA} = 0$, then there is no cross-correlation; if $\rho_{DCCA} = 1$, there is a perfect cross-correlation; if $\rho_{DCCA} = -1$, then it exists a perfect anti cross-correlation.

4. Empirical results

4.1. Cross-correlation test

The cross-correlation test gives us a bird's eyes about the nature of relation between price–volume changes of Bitcoin series. Fig. 3 plots cross-correlations test statistics of price changes and volume changes of Bitcoin as well as critical value of $\chi^2(m)$ distribution at the 5% level of significance for the degrees of freedom, varying from 1 to $N - 1$.

[3] q = [−30:5:−1, −2.1:0.1:−0.1, 0.1:0.1:2.1, 5:5:30]

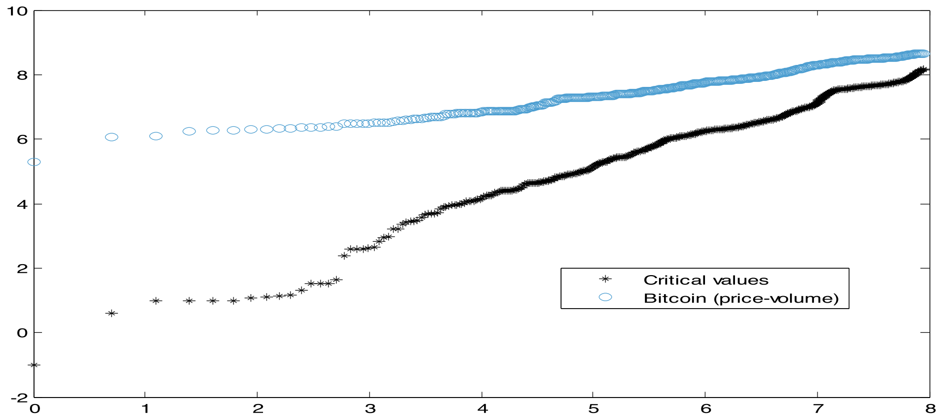

Fig. 3. Cross-correlation statistics for price changes and volume change of Bitcoin.

The Q-statistic $Q_{CC}(m)$ shows values that exceed the critical values of $\chi^2(m)$, suggesting the presence of significant long-range cross-correlations.

We next use the MF-DCCA approach to quantitatively analyze the cross-correlation.

4.2. MF-DCCA

Using the MF-X-DFA algorithm, which is an MF-DCCA method, the cross-correlations of price changes and volume changes are examined. Fig. 4 shows a long-range power law of price–volume multifractal cross-correlation of Bitcoin. In fact, $F_q(s)$ increases for large values of s, as a power law.

Using MF-X-DFA and MF-DFA algorithms, we calculate the generalized Hurst exponent, then we deduce the Rényi exponent and singularity spectrum. Fig. 5 plots the $H_{XY}(q)$ of price changes and volume changes. The Hurst exponent values of Bitcoin volume change series are less than 0.5, implying anti-persistence. However, Bitcoin price changes shows a strong persistence, as indicated by the values of generalized Hurst exponent, especially for negative values of q. The values of the generalized Hurst exponent of price–volume analysis range from around 0.2 to 0.6 for the different values of q.

In Fig. 6, the Rényi exponent presents a nonlinear curve, which confirms the existence of multifractal cross-correlations. Using the MF-DFA, we find that both volume change series and price change series have a nonlinear curve.

To further explore the multifractal cross-correlations features, we present in Fig. 7 the spectrum of singularity to deduce price–volume multifractal cross-correlations.

Table 2 shows that the value of spectrum width Δα is 0.767 for price changes series, whereas it is equal to 0.349 for volume change series and attains 0.450 for price–volume cross-correlations series. Taken together, multifractal cross-correlations are important in the Bitcoin series, especially for the price change series that shows the highest value of spectrum width. Given that the widths of multifractal spectra for price changes, volume changes, and price–volume cross-correlation are significantly nonzero, it follows that there are clear departures from random walk process for the three cases.

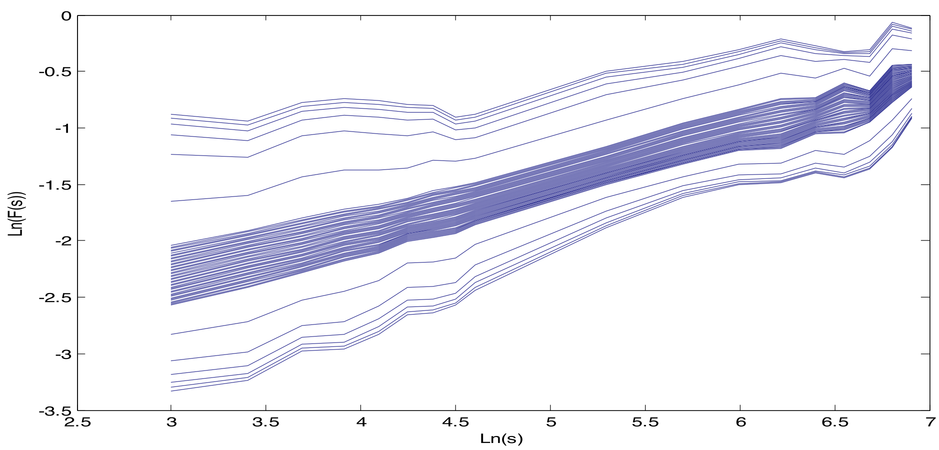

Fig. 4. Cross-correlations fluctuation functions of price changes and volume changes of Bitcoin.

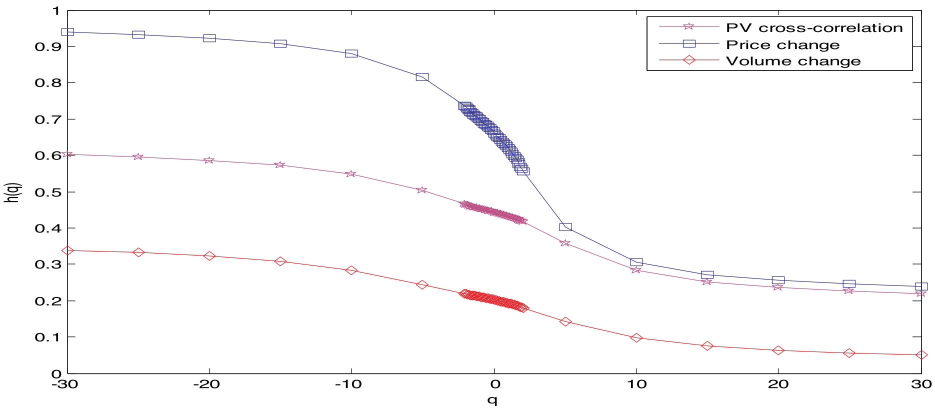

Fig. 5. Generalized Hurst exponent for different values of q for price–volume cross-correlations (MF-DCCA), price change (MF-DFA) and volume change (MF-DFA).

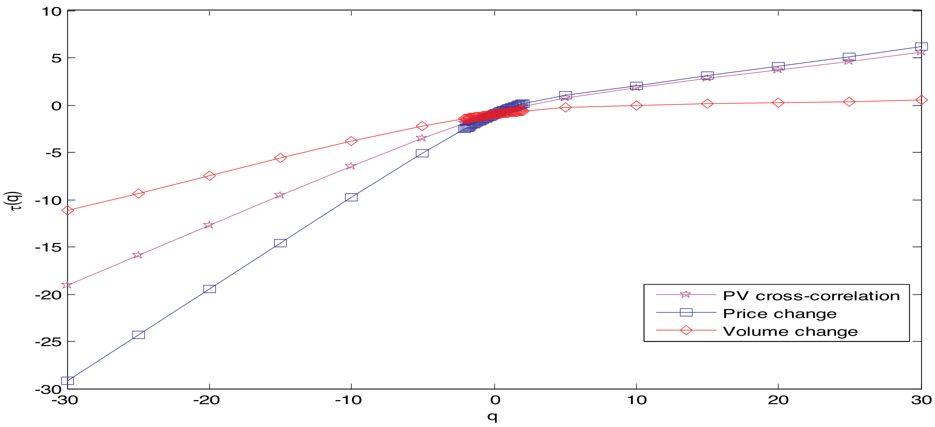

Fig. 6. Rényi exponent for different values of q for price–volume cross-correlations (MF-DCCA), price change (MF-DFA), and volume change (MF-DFA).

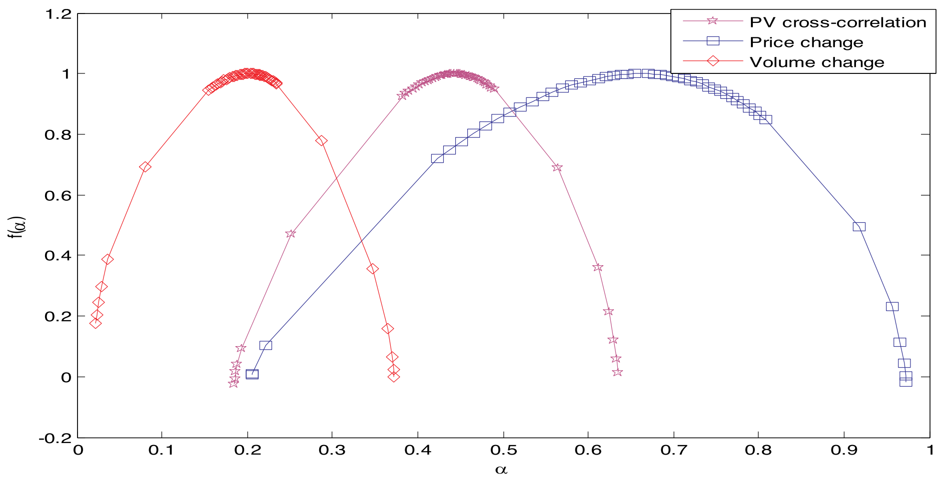

Fig. 7. Spectrum singularity for price–volume cross-correlations (MF-DCCA), price change (MF-DFA), and volume change (MF-DFA).

Table 2
Values α_{max}, α_{min} and $\Delta\alpha$.

	α_{max}	α_{min}	$\Delta\alpha = \alpha_{max} - \alpha_{min}$
Price change	0.973	0.2058	0.7671
Volume change	0.3722	0.0228	0.3494
Cross-correlation	0.6351	0.1851	0.4500

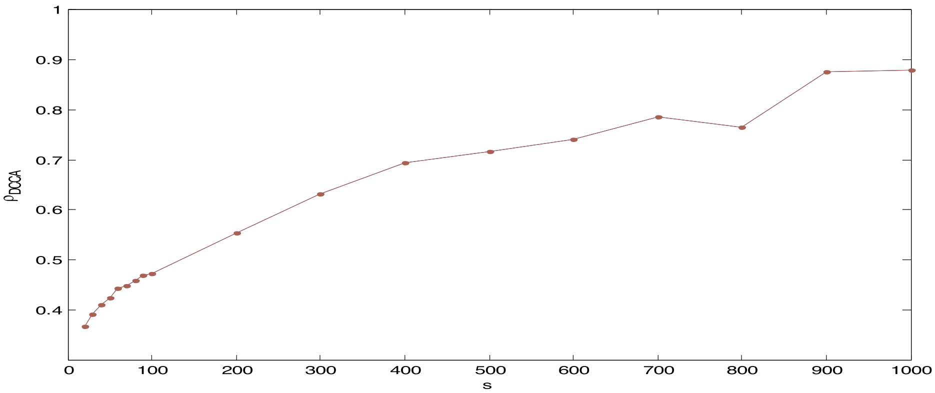

Fig. 8. DCCA cross-correlation coefficient between price change and volume change of Bitcoin.

4.3. Cross-correlations coefficient

Before we quantify the level of cross-correlation via computing the cross-correlations (DCCA) (Podobnik et al., 2011), we indicate that the linear correlation coefficient between Bitcoin price changes and Bitcoin volume changes is very close to zero (-0.0057). This result presumes nonlinear cross-correlation between the series. The DCCA allows us to explore the level of cross-correlations while considering the scaling effect and other complex features of the series; we observe that the DCCA cross-correlations coefficient (Fig. 8) is positive and strong. It exceeds 0.800 when $s > 900$ and attains 0.879 when $s = 1000$.

From this result, we infer that there is a strong cross-correlation between volume change and price change, especially for long-range periods. Consequently, investors should take into consideration the analysis of such cross-correlations because it could be misleading to make investment inferences on linear measures, such as the Pearson's correlation coefficient, that detect almost zero correlation.

Taken together, our results indicate that nonlinear dependency (in the form of cross-correlation) and multifractality characterize the price and volume relationship in the Bitcoin market. Accordingly, several practical implications can emerge. First, evidence of nonlinear dependency implies that a trader or a technical analyst can extend her knowledge on Bitcoin market dynamics from the integrated standpoint that price and volume are the joint products of a single market mechanism. This suggests that Bitcoin volume may help predict the underlying dynamics of Bitcoin price changes. In other words, an inference based on Bitcoin price cannot be complete without a simultaneous inference of Bitcoin trading volume, and vice-versa. Given that both price and volume are characteristics of a Bitcoin market's trading activities, evidence of cross-correlation between price and volume may be used by market participants who seek to base their trading strategies on Bitcoin market dynamics. However, our related results on the mutual interaction in Bitcoin price–volume cannot simply mean causality, suggesting an inability to make inferences on whether price is used to predict volume or vice-versa.

Second, evidence of significant nonlinear dependency may be related to market (in)efficiency as academia and policy-makers often assess the degree of market efficiency via the cross-correlation exponents. In fact, we find that the cross-correlation exponent deviates from zero, which suggests that the market is by no means perfectly efficient, and thus it is possible to make inferences on the dynamics of trading volume (price) from that of the price (volume).

Third, our findings on Bitcoin price-volume may also concern market participants in the CME and CBOE Bitcoin futures markets. The fact that price changes affect the changes in trading volume in futures contracts may suggest that a close look at the underlying Bitcoin spot market can help market participants, such as speculators, gain profound insight on Bitcoin futures contracts.

5. Conclusion

This study adds another piece of empirical evidence on the Bitcoin market dynamics by revealing that both nonlinear dependency and multifractality exist in the Bitcoin price–volume series. The level of correlations is high, and the volume change series shows anti-persistence for both positive and negative values of moments "q". The clear departures from zero for the cross-correlation imply that the Bitcoin market is not efficient from the perspective of the price–volume relationship. This finding is somewhat consistent with earlier findings on the inefficiency of Bitcoin (e.g. Kristoufek, 2018). Results also confirm the presence of complex features such as memory and nonlinearity effects in this series, which concurs with prior studies (Bariviera et al., 2017; Takaishi, 2017; Al-Yahyaee et al., 2018; Bouri et al., 2018a). Nonlinear dependency in the form of cross-correlations implies that price and volume in the Bitcoin market mutually interact, which might assist market participants in the Bitcoin market in maximizing the profitability of technical trading strategies involving Bitcoin price and trading volume (Balcilar et al., 2017; Bouri et al., 2018b). The evidence of multi-fractality in the cross-correlations implies the presence of dynamic and turbulent features in Bitcoin price dynamics (Gajardo et al., 2018), which deserves further research involving the price dynamics of other leading cryptocurrencies as well as micro-structure factors.

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