Option Pricing Excercises

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Black Scholes Option Pricing Formula

Black and Scholes (1973) derive a formula that can be used to value a *European* option that does **not** pay dividends before the options expiration date:

Let

- S = Stock Price
- X = Strike/Exercise Price
- r = Risk Free Rate
- T = Time to expiration in years
- σ = Volatility of the relative price change of the underlying stock price
- $N(x)$ = The cumulative normal distribution function

Then

$$c = SN(d_1) - Xe^{-rT}N(d_2)$$
$$p = Xe^{-rT}N(-d_2) - SN(-d_1)$$

Where

$$d_1 = \frac{ln(\frac{S}{X}) + (\frac{r+\sigma^2}{2})T}{\sigma\sqrt{T}}$$
$$d_2 = \frac{ln(\frac{S}{X}) + (\frac{r-\sigma^2}{2})T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

Let's code it up as a function:

```
price_euro_option_ <- function(S, X, r, sigma, `T`) {

  d1 <- (log(S/X) + ((r + sigma^2)/2)*`T`) / sigma * sqrt(`T`)
  d2 <- d1 - sigma * sqrt(`T`)

}

# alternative naming convention that could make more sense
price_euro_option <- function(spot, strike, rf_rate, vol, ttmy, type = "c") {

  d1 <- (log(spot/strike) + (rf_rate + vol^2/2)*ttmy)/(vol*sqrt(ttmy))
  d2 <- d1 - vol*sqrt(ttmy)

  if (type == "c"){

    spot*pnorm(d1) - strike*exp(-rf_rate*ttmy)*pnorm(d2)
```

```
  } else if (type == "p"){

    strike*exp(-rf_rate*ttmy)*pnorm(-d2) - spot*pnorm(-d1)

  }

}
```

Let's consider the following scenario:

- European option
- 3 months to expiry, therfore $\frac{3}{12} = 0.25$ because we measure in years
- Stock(spot) price = R60.00
- Strike price = R65.00
- Risk free rate = 8% p.a.
- Volatility = 30%

This is calculated as follows:

```
price_euro_option(
  spot = 60,
  strike = 65,
  rf_rate = 0.08,
  ttmy = 0.25,
  vol = 0.3,
)
```

```
## [1] 2.133368
```