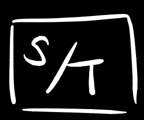
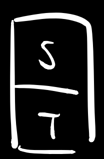
Given an n × m board, starting from the top left, you can only move right one square or down one square.
How many unique ways can you get to the bottom right square?

Diagram

let's start small:
n = 1, m = 1 ⟶ f(1,1) = 0



n = 1, m = 2



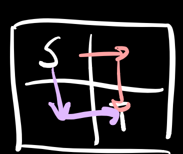
f(1,2) = 1

n = 2, m = 1

f(2,1) = 1

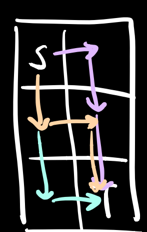
? If n ≠ m, do they produce different answers if flipped? (is it symmetric?)

n = 2, m = 2



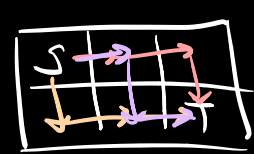
f(2,2) = 2

n = 2, m = 3



f(2,3) = 3

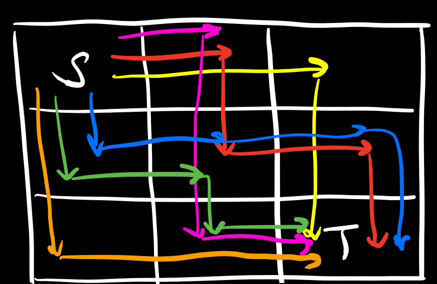
n = 3, n = 2



f(3,2) = 3

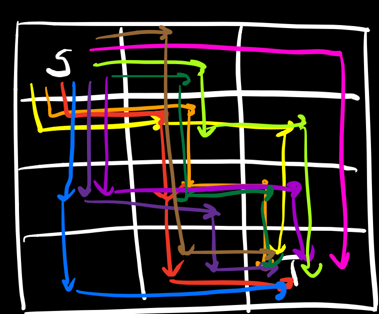
judging from this, it seems the answer is no?

n = 3, m = 3



f(3,3) = 6

n = 3, m = 4

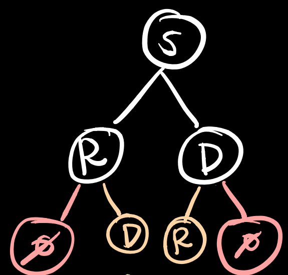


f(3,4) = 10

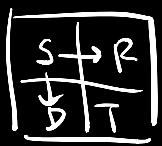
in comparison to (n = 3, m = 3), the addition of the 1 on one side increased the paths by 3 going down and 1 going right.

- This problem seems similar to the stairs problem, and may have a recurrence relation.

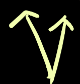
- can we model the problem with a tree?



this is a 2x2 board

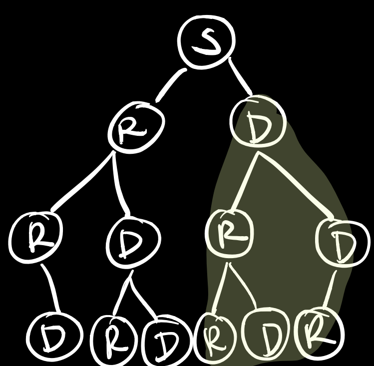


- each node can only be R or D (for right & down)
- the outermost nodes on the left & right branches should be null because they are not possible moves on the board

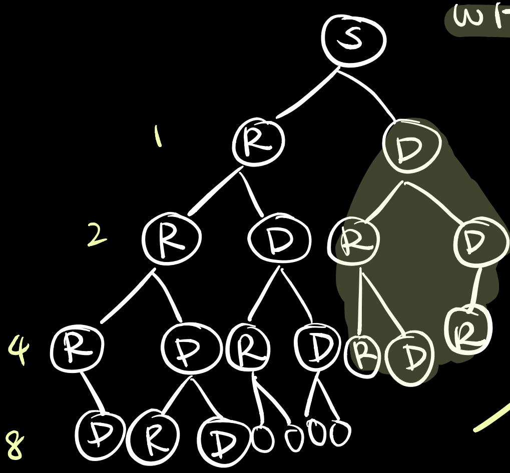


$f(2,2) = 2$

the answer would be the total number of paths, or nodes without children



3x3 board, 6 paths
$f(3,3) = 6$



4x3 board, 10 paths
$f(4,3) = 10$

the height of the subtrees must match the size of the corresponding side of the board

$Lh = m - 1$
$Rh = n - 1$

If we can model the grid by using the properties of a binary tree, the formula to give the total number of unique paths is:

$$2^{(m-1)} - 1 + 2^{(n-1)} - 1 = \text{answer}$$

ex) for 4 x 3 grid

null outermost node referenced above

$Lh = m - 1 = 4 - 1 = 3 \rightarrow 2^3 - 1 = 7$
$Rh = n - 1 = 3 - 1 = 2 \rightarrow 2^2 - 1 = 3$

$7 + 3 = \boxed{10}$