A Pairs Trading Strategy based on Mixed Copulas

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Abstract

We propose an alternative pairs trading strategy based on computing a mispricing index in a novel way via a mixed copula model, or more specifically via an optimal linear combination of copulas. We evaluate the statistical and economic performances of our proposed approach by analyzing S&P 500 daily stock returns between 1990 and 2015. Empirical results are obtained not only from the full sample analysis but also from subperiods analyses. These subperiods are chosen in two different ways: i) fixed time length; and ii) bull/bear market dependent. The fixed time length analysis is able to capture possible dynamics changes over time whereas the bull/bear analysis makes it possible for one to have distinct pairs trading strategies depending on the market state. Our results suggest that two-component mixed copulas perform very well, especially during bear market times when it regards to mean returns.

Keywords: Pairs Trading; Copula; Distance; Quantitative Trading Strategies; Short; Long; S&P 500; Statistical Arbitrage.

JEL Codes: C51, G10, G14.

1 Introduction

Pairs trading is a statistical arbitrage strategy based solely on past stock prices and simple contrarian principles. An investor has to find two stocks whose prices have strong historical co-movements. When their prices are abnormally apart from the equilibrium, i.e., when the spread between them widens up, the investor should, simultaneously, take a short position on the asset with the higher price and a long position on the other one. Thus, the strategy bets on price spreads convergence to the equilibrium. The performance of the strategy has attracted considerable recent interest in numerous sub-fields of finance and business, since they have potential to generate positive, low-volatility returns that are uncorrelated with market returns. The strategy has a long history on Wall Street. In the mid-1980s, the onetime astrophysicist Nunzio Tartaglia assembled a group of physicists, computer scientists and mathematicians to develop market neutral or long/short strategies using the most sophisticated statistical models and automated trading systems. However, it became popular through the study carried out by Gatev *et al.* (2006), who proposed the distance method as the main operational strategy for pairs trading.

The traditional distance method has been widely studied and tested throughout the pairs trading literature. The approach can be seen as a distribution-free method and thus does not subject the stock prices to be generated by any particular distribution. However, this approach only captures well the dependencies between stock prices in the case of elliptically distributed random variables. This assumption is generally not met in practice, motivating the use of copula-based models to address the stylized facts of multivariate financial stock

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price returns. Nevertheless, the use of copulas in this context is somehow recent and needs more comprehensive and profound studies.

The performance of the distance method has been measured thoroughly using different data sets and financial markets (Gatev *et al.*, 2006; Perlin, 2009; Do & Faff, 2010, 2012; Broussard & Vaihekoski, 2012; Caldeira & Moura, 2013; Rad *et al.*, 2016). In an efficient market, strategies based on mean-reversion concepts should not generate consistent profits. However, Gatev *et al.* (2006) find that pairs trading generates consistent statistical arbitrage profits in the U.S. equity market during 1962-2002 using CSRP data, although the profitability declines over the period. They obtain a mean excess return above 11% a year during the reported period. The authors attribute the atypical returns to a non-identified systematic risk factor. They support their view showing that there is a high degree of correlation between the excess returns of no overlapping top pairs even after accounting for risk factors from an augmented version of Fama & French (1993)'s three factors. Do & Faff (2010) extend their work expanding the data sample and also find a declining trend - 33 basis points (bps) mean excess return per month for 2003-09 versus 124 basis points mean excess return per month for 1962-88. Do & Faff (2012) show that the distance method is unprofitable after 2002 when trading costs are considered. Broussard & Vaihekoski (2012) test the profitability of pairs trading under different weighting structures and trade starting conditions using data from the Finnish stock market. They also find that their proposed strategy is profitable even after initiating the positions one day after the signal. Rad *et al.* (2016) evaluate distance, cointegration and copula methods using a long-term comprehensive data set spanning over five decades. They find that the copula method has a weaker performance than the distance and cointegration methods in terms of excess returns and various risk-adjusted metrics.

The distance strategy (Gatev *et al.*, 2006) uses the distance between normalized stock prices to capture the degree of mispricing between stocks. According to Xie *et al.* (2016) the distance method has a multivariate normal nature since it assumes a symmetric distribution of the spread between the normalized prices of the stocks within a pair and it uses a single distance measure, which can be seen as an alternative measurement of the linear association, to describe the relationship between two stocks. It is well known that if the random variables have joint gaussian distribution, then the linear correlation fully describes their dependence structure (Embrechts *et al.*, 2001). However, the dependence between two stock prices is rarely linear and thus the traditional hypothesis of (multivariate) gaussianity is inadequate (Campbell *et al.*, 1997; Artzner *et al.*, 1999; Cont, 2001; Ane & Kharoubi, 2003; Szegö, 2005; McNeil *et al.*, 2015). Therefore, a single distance measure may fail to catch the dynamics of the spread between a pair of stock prices, and thus initiate and close the trades at non-optimal positions. Throughout the years, different approaches have been devised to capture the mean-reverting properties of the spread in order to improve the profitability of the distance method (see, among others, Elliott *et al.* (2005); Do *et al.* (2006); Mudchanatongsuk *et al.* (2008); Montana *et al.* (2009); Avellaneda & Lee (2010); Triantafyllopoulos & Montana (2011); Bogomolov (2013); Murota & Inoue (2015); de Moura *et al.* (2016); Focardi *et al.* (2016); Stübinger *et al.* (2016); Krauss & Stübinger (2017); Liu *et al.* (2017); Ramos-Requena *et al.* (2017); Stübinger & Endres (2018); Wen *et al.* (2018)).

Due to the complex dependence patterns present in financial asset returns, a choice of a more complex multivariate distribution is usually more adequate than assuming multivariate normal returns. Given its flexibility, Copula-ARMA-GARCH models are capable to more adequately fit multivariate returns, in which case the role of copulas is clearly to allow possible nonlinear dependencies between returns, such as tail dependence. Besides nonlinear dependence, these empirically verified regularities, known as stylized facts in the financial literature, are often described in a simpler way within the univariate context. Nevertheless one should notice that non-standard features in the univariate distributions are passed on, in a more complex version, to their multivariate counterpart: (1) asymmetric conditional variance with higher volatility for negative returns than for positive returns (Hafner, 1998); (2) conditional skewness (Ait-Sahalia & Brandt, 2001; Chen *et al.*, 2001; Patton, 2006);

(3) leptokurdicity (Tauchen, 2001; Andreou *et al.*, 2001); and (4) nonlinear temporal dependence (Cont, 2001; Campbell *et al.*, 1997).

To capture more closely the stylized facts cited above and allow for a more close-to-reality description of the multivariate distribution, Liew & Wu (2013) propose a pairs trading strategy based on two-dimensional copulas. The authors evaluate its performance using only three pre-selected pairs over a period of less than three years. Xie *et al.* (2016) employ a similar methodology over a ten-year period with 89 stocks. Both studies show that the copula approach is a powerful alternative compared to the distance method. Rad *et al.* (2016) use a more comprehensive data set consisting of all stocks in the US market from 1962 to 2014. However, they find opposite results. Particularly, the distance, cointegration and Copula-GARCH strategies show an average monthly excess return of 36, 33, and 5 bps, respectively, after transaction costs and 88, 83, and 43 basis points before transaction costs. Thus, a copula-based approach may sound plausible but it may not lead to a viable standalone trading quantitative strategy due to overfitting issues.

Regarding the academic research related to pairs trading statistical arbitrage, the study by Krauss (2017b) carried out a survey of the related literature and listed more than 90 papers. The research by Yu & Lu (2017) analyzed the construction of cointegrated market-neutral portfolios in Hong Kong stock market, Mikkelsen (2018) compared the distance and cointegration methods for each high-frequency and daily data to verify whether it is profitable for Norwegian seafood companies, Clegg & Krauss (2018) used partial cointegration for the identification of pairs. More recently, Chen *et al.* (2019) considered Pearson correlation as as selection criteria and examine its application with the same data sample considered by Gatev *et al.* (2006), Kim & Kim (2019) evaluated the potential of applying Deep Reinforcement Learning in a Pairs Trading strategy, and obtained interesting results in comparison with more traditional technics, Sarmento & Horta (2020) considered machine learning approach to construct pairs trading strategy applied to a dataset with ETFs, and Caneo & Kristjanpoller (2020) analysed the profitability of pairs trading strategy in Latin American stock markets using a PCA method with a multi-factorial model and applied the methodology to six country with a dataset of 338 stocks. Furthermore, the literature analyzing pairs trading strategies also includes research that consider non-equity markets, such as Kanamura *et al.* (2010) in energy spreads, Göncü & Akyıldırım (2016) in commodity futures, Temnov (2017) in currencies, among others.

In this paper, we extend on Xie *et al.* (2016) and Rad *et al.* (2016) strategies on the highly liquid stocks of the S&P 500. Rad *et al.* (2016) suggests a relatively poor performance of the copula method. However, they estimate only copulas that model either lower or upper tail dependence, but not both. Our main contribution to the literature is three-fold. First, we make a methodological contribution to improve the dependence modeling in practice. We propose a mixed copula based model to capture linear and nonlinear associations and at the same time cover a wider range of possible dependence structures. In particular, our approach provides a flexible environment for the search of dependence measures that are better suited for capturing extreme co-movement asymmetries. Second, we show that this sophisticated strategy works well even in a highly efficient market, considering several economic performance measures. Third, following our subperiods analysis based on crisis (bear) and non-crisis (bull) market states, we can see that our approach is quite suitable to be used during crisis (bear) periods. In this way practitioners can have a tentative rule of using the mixed copula strategy during bear periods, whereas during bull periods other characteristics should be considered.

We compare the out-of-sample performances of the strategies using a variety of criteria, all of which are computed using a rolling period procedure similar to that used by Gatev *et al.* (2006) with the exception that the time horizon of formation and trading periods are rolled forward by six months as in Broussard & Vaihekoski (2012). The main criteria we focus on are: (1) mean and cumulative excess return, (2) risk-adjusted metrics as Sharpe and Sortino ratios, (3) percentage of negative trades, (4) t-values for various risk factors, and (5) maximum drawdown between two consecutive days and between two days within a maximum period of six months. Additionally we perform two subperiods analyses, one based on fixed time length intervals and the other based on the market state (bull/bear periods). For a glance at the results, our approach generates higher average and risk-adjusted excess returns (Sharpe Ratio) for all our choices of numbers of top pairs, considering or not cost transaction, for the fully invested capital strategy. Still on these performance measures, for the committed capital strategy our approach presents better results for the top 5 pairs, whereas the distance method is superior for top 20 and top 35 pairs. In terms of drawdown risk our approach is also consistently superior to the distance strategy.

Furthermore, to evaluate if pairs trading profitability is associated to exposure to different systematic risk factors[1], we regress daily excess returns on seven factors: daily Fama & French (2015)'s five research factors [2] plus momentum and long-term reversal. We find that the intercept is statistically greater than zero for all regressions at 1% level when considering the mixed copula strategy, showing that our results are robust to the augmented Fama & French (2015)'s risk adjustment factors. In addition, the share of observations with negative excess returns is smaller for the mixed copula than for the distance strategy.

The remainder of the paper is organized as follows. In Section 2, we present a brief review of the distance and copula methodologies for pairs trading. Additionally we propose our mixed copula based strategies. Section 3 summarizes the data and empirical results, followed by our conclusions in Section 4. Additional results are reported in the Appendix.

2 Methodology

In this Section we describe the strategies employed in our paper. The distance approach is described in Section 2.1, whereas the copula method is outlined in Section 2.2. We generalize the existing copula method by employing a mixture copula model. We want to evaluate if we can improve the profitability of pairs trading by capturing a wider variety of dependence structures.

2.1 Distance Framework

Our implementation of the distance strategy is similar to Broussard & Vaihekoski (2012) and follows the steps below:

1. Calculate the sum of Euclidean squared distances[3] of all $n(n+1)/2$ combinations of pairs during 12 months, named formation period (adjusting them by dividends, stock splits and other corporate actions):

$$\sum_{t=1}^{T_F} (p_{i,t} - p_{j,t})^2 \ ,$$

   where T_F is the number of observations during the formation period and $p_{i,t}$ is the price of asset i at time t. Specifically, the pairs are formed using data from January to December or from July to June. Prices are scaled to \$1 at the beginning of each formation period and then evolve using the return series[4].

2. We then select the top 5, 20 and 35 of those pairs with minimum distance. These pairs are then traded in the next six months (trading period).

[1] The single-factor capital asset pricing model (CAPM) of Sharpe (1964) and Lintner (1965), as well as its consumption based version (Breeden, 1979), among other extensions, has been empirically tested and rejected by numerous studies, which show that the cross-sectional variation in expected equity returns cannot be explained by the market beta alone, providing evidence that investors demand compensation for not being able to diversify firm-specific characteristics.

[2] Fama & French (2015) found evidences that the three factor model was not sufficient to explain a lot of the variation in average returns related to profitability and investment.

[3] Spread between the normalized daily closing prices.

[4] Missing values have been interpolated.

3. In Gatev *et al.* (2006), when the spread diverges by two or more standard deviations (which is calculated in the formation period) from the mean, the stocks are assumed to be mispriced in terms of their relative value to each other and thus one opens a short position in the outperforming stock and a long in the underperforming one.

4. The price divergence is expected to be temporary, i.e., the prices are expected to converge to their long-term mean value (mean-reverting behavior). Hence, the positions are closed once the normalized prices cross. The pair is then monitored for another divergence and thus a pair can complete multiple round-trip trades. Nonconvergent trades can result in a loss if they are still open at the end of the trading period when they are automatically closed. This results in fat left tails. Therefore, since the conditional variance is empirically higher for large negative returns and smaller for positive returns, it may be inappropriate to use constant trigger points because the volatility differs at different price levels.

To calculate the daily percentage returns for a pair, we compute

$$r_{pt} = w_{1t} r_t^L - w_{2t} r_t^S, \tag{1}$$

where L and S stands for long and short, respectively. Returns r_{pt} can be interpreted as excess returns since in (1) the riskless rate is canceled out when one calculates the long and short excess returns. The weights w_{1t} and w_{2t} are initially assumed to be one. After that, they change according to the changes in the value of the stocks, *i.e.*, $w_{it} = w_{it-1}(1 + r_{it-1})$.

The advantages of this methodology are relatively clear. First, functionally misspecified models and misestimations are avoided since the approach uses a non-parametric model. Second, the procedure is straightforward and easy to implement. However, the choice of Euclidean squared distance for identifying pairs is analytically suboptimal (Krauss, 2017b; Ramos-Requena *et al.*, 2017).

2.2 Copulas and Mispricing Index

Formally, we can define a copula function C as follows.

Definition 1. *An d-dimensional copula (or simply d-copula) is a function C with domain $[0,1]^d$, such that:*

1. C is grounded and d-increasing;

2. C has marginal distributions C_k, $k = 1, \ldots, d$, where $C_k(\mathbf{u}) = u$ for every $\mathbf{u} = (u_1, \ldots, u_d) \in [0,1]^d$.

A representation theorem due to Sklar (1959) has granted the motivation of copula modeling to dependence analysis.

Theorem 1. *(Sklar's Theorem) Let $X_1, \ldots, X_d$ be random variables with distribution functions $F_1, \ldots, F_d$, respectively. Then, there exists an d-copula C such that,*

$$F(x_1, ..., x_d) = C(F_1(x_1), \ldots, F_d(x_d)), \tag{2}$$

for all $\mathbf{x} = (x_1, ..., x_d) \in \mathbb{R}^d$. If $F_1, ..., F_d$ are all continuous, then the function C is unique; otherwise C is determined only on $\operatorname{Im} F_1 \times ... \times \operatorname{Im} F_d$.

Using Sklar's theorem we can derive an important relation between the marginal distributions and a copula. Let f be a joint density function (derived from the $d-$dimensional distribution function F) and $f_1, \ldots, f_d$ univariate density functions of the margins $F_1, \ldots, F_d$. Assuming that $F(\cdot)$ and $C(\cdot)$ are differentiable, by (2) we have

$$\frac{\partial^d F(x_1,\ldots,x_d)}{\partial x_1\ldots\partial x_d} \equiv f(x_1,\ldots,x_d) = \frac{\partial^d C(F_1(x_1),\ldots,F_d(x_d))}{\partial x_1\ldots\partial x_d} \tag{3}$$

$$= c(u_1,\ldots,u_d)\prod_{i=1}^{d} f_i(x_i), \tag{4}$$

where $u_i = F_i(x_i)$, $i = 1,\ldots,d$.

Thus, one can clearly see that copulas characterize the dependence structure among the variables. From a modeling perspective, Sklar's Theorem laid the theoretical foundation that allows us to estimate the multivariate distribution in two stages: (i) modeling stochastic marginal distributions; (ii) modeling dependency structure between the filtered data from (i). The choice of the copula function is also not dependent on the marginal distributions. Thus, by using copulas, different dependence structures can be modeled to allow for any non-linear dependences in a much finer way and thus identify more reliable trading opportunities[5].

A further important property of copulas concerns the partial derivatives of a copula with respect to its variables. Let now H be a bivariate function with marginal distribution functions F and G. According to Sklar (1959) there exists a copula $C : [0,1]^2 \to [0,1]$ such that $H(x_1,x_2) = C(F(x_1),G(x_2))$ for all $x_1,x_2 \in \mathbb{R}^2$. If F and G are continuous, then C is unique; otherwise, C is uniquely determined in $\operatorname{Im} F \times \operatorname{Im} G$. Conversely, if C is a copula and F and G are distribution functions, then the function H is a joint distribution function with marginals F and G and we can write

$$C(u_1,u_2) = H(F^{-1}(u_1),G^{-1}(u_2)), \tag{5}$$

where $u_1 = F(x_1) \Rightarrow x_1 = F^{-1}(u_1)$, $u_2 = G(x_2)) \Rightarrow x_2 = G^{-1}(u_2)$ and F^{-1} and G^{-1} are the quasi-inverses of F and G, respectively. For any copula C, $\frac{\partial C(u_1,u_2)}{\partial u_1}$ and $\frac{\partial C(u_1,u_2)}{\partial u_2}$ exist almost everywhere. The proposition below states that the partial derivatives of a copula function correspond to the conditional probabilities of the random variables (see Cherubini *et al.*, 2004; Nelsen, 2006).

Proposition 1. *Let U_1 and U_2 be two random variables with distribution $U(0,1)$. Then,*

$$P(U_1 \le u_1 \,|U_2 = u_2) = \frac{\partial C(u_1,u_2)}{\partial u_2} = P(X_1 \le x_1 \,|X_2 = x_2),$$

$$P(U_2 \le u_2 \,|U_2 = u_1) = \frac{\partial C(u_1,u_2)}{\partial u_1} = P(X_2 \le x_2 \,|X_1 = x_1).$$

By using the fact that the partial derivative of the copula function gives the conditional distribution function, Xie *et al.* (2016) define a measure to denote the degree of mispricing:

Definition 2. *(Mispricing Index) Let R_t^X and R_t^Y represent the random daily returns of stocks X and Y at time t, and r_t^X and r_t^Y represent the realizations of those returns at time t. Then define*

$$\begin{aligned} MI_t^{X|Y} = \frac{\partial C(u_1,u_2)}{\partial u_2} &= P(R_t^X < r_t^X \mid R_t^Y = r_t^Y) \\ &\textit{and} \\ MI_t^{Y|X} = \frac{\partial C(u_1,u_2)}{\partial u_1} &= P(R_t^X < r_t^X \mid R_t^Y = r_t^Y). \end{aligned} \tag{6}$$

[5]Copulas measures lower and upper tail dependencies and nonlinear and linear relationships in a rich set for describing dependencies between pairs. Copula is also invariant under strictly monotonic transformations (Cherubini *et al.*, 2004; Nelsen, 2006) and hence the same copula is obtained if we use price or return series, for example.

where $u_1 = F_X\left(r_t^X\right)$ *and* $u_2 = F_Y\left(r_t^Y\right)$.

Therefore, the conditional probabilities $MI_t^{X|Y}$ and $MI_t^{Y|X}$ indicate whether the return of X is considered high or low at time t, given the information on the return of Y at time t and the historical relation between the two stocks' returns, and vice-versa[6]. When we observe a value of $MI_t^{X|Y}$ equal to 0.5, r_t^X is neither too high nor too low given r_t^Y and their historical relation, i.e., we can say that this reflects no mispricing. In other words, the historical data indicates that, on average, there is an equal number of observations of the return of X being larger or smaller than r_t^X if the return of stock Y is equal to r_t^Y and therefore, a conditional value of 0.5 means that the stock X is fairly priced relative to stock Y at day t.

Note that the conditional probabilities, $MI_t^{X|Y}$ and $MI_t^{Y|X}$, only measure the degrees of relative mispricing for a single day. Following Xie *et al.* (2016) and Rad *et al.* (2016), we determine an overall degree of relative mispricing. Initially, let $m_{1,t}$ and $m_{2,t}$ be the corrected mispricing indexes of stocks X and Y, defined by $\left(MI_t^{X|Y} - 0.5\right)$ and $\left(MI_t^{Y|X} - 0.5\right)$, respectively. Then, at the beginning of each trading period two cumulative mispricing indexes $M_{1,t}$ and $M_{2,t}$ are set to zero and evolve for each day via

$$\begin{aligned} M_{1,t} &= M_{1,t-1} + m_{1,t}, \\ M_{2,t} &= M_{2,t-1} + m_{2,t} \end{aligned}$$

for $t = 1, \ldots, T$. By construction $M_{1,t}$ and $M_{2,t}$ are non-stationary time series. The properties of $M_{i,t}$ depend on the correlation between $m_{i,t}$ and $M_{i,t-1}$. If the correlation is zero, $M_{i,t}$ follows a pure random walk and (statistical) arbitrage opportunities should be absent. If the correlation is positive $M_{i,t}$ has a tendency to diverge, which generally results in a loss. Finally, if the correlation is negative $M_{i,t}$ has a tendency to converge when it moves away from zero significantly, which generally results in profits. Empirically, $M_{i,t}$ alternates across the three mechanisms and as long as the convergent mechanism dominates we can profit from our strategy.

This approach is attractive as it reflects the mispricings over multiple periods, thus reflecting how farther away the prices are out from their equilibrium. In contrast to the mispricing indices, it can lead to optimal trading strategies, since it results in a more stable strategy. Positive (negative) $M_{1,t}$ and negative (positive) $M_{2,t}$ are interpreted as stock 1 (stock 2) being overvalued relative to stock 2 (stock 1).

We perform a sensitivity analysis to open a long-short position once one of the cumulative indexes is above $0.05, 0.10, \ldots, 0.95$ and the other one is below $-0.05, -0.10, \ldots, -0.95$ at the same time for Top $5, 20$ and 35 pairs. The positions are closed when both cumulative mispricing indexes return to zero. The pairs are then monitored for other possible trades throughout the remainder of the trading period. Similar to Rad *et al.* (2016), we propose the following steps to obtain $M_{1,t}$ and $M_{2,t}$ using copula-based models:

1. First, we calculate daily returns for each stock during the formation period and estimate the marginal distributions of these returns separately by fitting an appropriate ARMA(p,q)-GARCH(r,s) model[7] to each univariate time series by obtaining the estimates $\widehat{\mu}_i$ and $\widehat{\sigma}_i$ of the conditional mean and standard deviation of these processes, respectively. Moreover, using the estimated parametric models, we construct the standardized residuals vectors given, for each $i = 1, \ldots, N$, $t = 1, \ldots, T$, where N is the number of assets and T is the time series length, by

$$\widehat{\varepsilon}_{i,t} = \frac{r_{i,t} - \widehat{\mu}_{i,t}}{\widehat{\sigma}_{i,t}}\,, \tag{7}$$

[6]Variables $MI_t^{X|Y}$ and $MI_t^{Y|X}$ are $U(0;1)$ distributed by the Rosenblatt probability integral transformation and are commonly used in multivariate goodness-of-fit tests or density forecast evaluations (see Diebold *et al.*, 1999).

[7]We look for the best ARMA(p,q)-GARCH(r,s) model up to order (2,2) for (p,q) and fixed (1,1) for (r,s), considering a t-asymmetric distribution as the most complex description of the error term.

where $r_{i,t}$ is the t-th return of asset i. The estimated standardized residuals vectors are then converted to the pseudo-observations $u_{i,t} = \frac{T}{T+1} F_{i,t}\left(\widehat{\varepsilon}_{i,t}\right)$, where $F_{i,t}$ is estimated by using their empirical distribution function[8], $\hat{F}_{i,t}$;

2. After obtaining the estimated marginal distributions from the previous step, we estimate the two-dimensional copula model to data that has been transformed to $[0, 1]$ margins to connect the joint distributions with the marginals $\hat{F}_{i,t}$ and $\hat{F}_{j,t}$, i.e.,

$$\hat{H}\left(r_{i,t}, r_{j,t}\right) = C\left(\hat{F}_{i,t}\left(\hat{\varepsilon}_{i,t}\right), \hat{F}_{j,t}\left(\hat{\varepsilon}_{j,t}\right)\right),$$

where $\hat{H}$ is the estimated joint cumulative distribution function and C is the copula. Copulas that are tested in this step are Gaussian/Normal (N) and Student t (T) (elliptical), Clayton (C), Symmetrized Joe-Clayton (SJC), Frank (F) and Gumbel (G) (Archimedean[9]). Here our contribution is to consider a mixture of d copulas, which is assumed to belong to a set $\mathcal{C}_M$, i.e.,

$$C\left(\cdot, \cdot\right) \in \mathcal{C}_M \equiv \left\{\sum_{i=1}^{d} \pi_i C^i\left(\cdot, \cdot\right) : \sum_{i=1}^{d} \pi_i = 1,\ \pi_i \geq 0,\ i = 1, \ldots, d\right\}, \tag{8}$$

where $C^i\left(\cdot, \cdot\right)$ denotes the j-th copula function. Hence we are able to build flexible mixed copula models with let us say two, three or four components (the number of copulas in the mixture is d). According to our empirical analysis, the two-component mixtures produce the best out-of-sample results. Among those, which totalize 15 possibilities, $\binom{6}{2}$, we use the following four best two-component performers: Clayton and Student (CT), Frank and Student (FT), Normal and Student (NT), Student and Gumbel (TG). Hence we can write these two-component mixed copulas in a general expression as

$$\mathcal{C}_{\theta}^{AB}\left(u_1, u_2\right) = \pi_1 \mathcal{C}_{\alpha}^{A}\left(u_1, u_2\right) + \left(1 - \pi_1\right) \mathcal{C}_{\beta}^{B}\left(u_1, u_2\right), \tag{9}$$

where A and B represent two of our chosen copulas, $\theta = (\alpha, \beta)'$ are the copula (dependence) parameters of A and B respectively and $\pi_1 \in [0, 1]$. The estimates are obtained by the minimization of the negative log-likelihood consisting of the weighted densities of the copulas;

3. Take the first derivative of the copula function to compute conditional probabilities and measure mispricing degrees $MI_{X|Y}$ and $MI_{Y|X}$ for each day in the trading period using the copula and estimated parameters;

4. Build long and short positions of Y and X on the days that $M_{1,t} > \Delta_1$ and $M_{2,t} < \Delta_2$ if there are no positions in X or Y. Conversely, build positions long/short of X and Y on the day that $M_{1,t} < \Delta_2$ and $M_{2,t} > \Delta_1$ if there are no positions in X or Y;

5. All positions are closed if $M_{1,t}$ reaches Δ_3 or $M_{2,t}$ reaches Δ_4, where $\Delta_1, \Delta_2, \Delta_3$ and Δ_4 are predetermined thresholds or are automatically closed out on the last day of the trading period if they do not reach the thresholds. Practitioners are free to choose the threshold trigger values. After an extensive data learning process we use $\Delta_1 = 0.40, \Delta_2 = -0.40$ and $\Delta_3 = \Delta_4 = 0$ for committed capital and $\Delta_1 = 0.55, \Delta_2 = -0.55$

[8]The asymptotically negligible scaling factor, $\frac{T}{T+1}$, is used to force the variates to fall inside the open unit hypercube to avoid, for example, problems with density evaluation at the boundaries.

[9]An Archimedean copula has the form

$$C\left(u_1, \ldots, u_n\right) = \psi\left(\psi^{-1}\left(u_1\right) + \psi^{-1}\left(u_2\right) + \ldots + \psi^{-1}\left(u_n\right)\right),$$

for an appropriate generator $\psi\left(\cdot\right)$, where $\psi : [0, \infty] \to [0, 1]$ and satisfies (i) $\psi(0) = 1$ and $\psi(\infty) = 0$; (ii) ψ is d-monotone, i.e., ψ has continuous derivatives on $(0, \infty)$ up to the order $d - 2$, $(-1)^k \psi^{(k)}(x) \geq 0$ for any $k \in \{1, \ldots, d - 2\}$ and $(-1)^{d-2} \psi^{(d-2)}$ is non-negative, non-increasing and convex on $(0, \infty)$. The most known copulas from this class of copulas have a closed form expression. Moreover, each member has a single parameter that controls the degree of dependence, which allows modeling dependence in arbitrarily high dimensions with only one parameter.

and $\Delta_3 = \Delta_4 = 0$ for fully invested.

Following Gatev *et al.* (2006), two measures of excess returns for each portfolio are computed: the return on committed capital (CC) and on fully invested capital (FI). The former commits[10] equal amounts of capital to each one of the pairs even if the pair has not been traded[11], whereas the latter divides all capital among the pairs that are open during the trading period. For example, if, in the Top 20 pairs trading portfolio, only ten pairs are open based on the historical two standard deviation trigger or cumulative mispricing indexes criteria, then the FI portfolio returns are scaled by 10. Hence, CC portfolio returns are more conservative.

3 Data and Empirical Results

Our data set consists of daily data of adjusted closing prices of all stocks that belongs to the S&P500 market index from July 1990 to December 2015, a period that covers several market upturns and downturns, as well as relatively calm periods. We obtain adjusted closing prices from Bloomberg, whereas returns on the Fama and French factors are pulled from French's website[12]. The data set spans 6,426 trading days and includes a total of 1,100 stock price series that appeared at least once as constituents of the S&P500. To remove survivorship bias only stocks that are listed during the formation period are included in the analysis, *i.e.*, around the leading 500 companies of the US stock market in each trading period. We assume that all trades occur at the closing price of that day.

Using data from the Center for Research in Security Prices (CRSP) from 1980 to 2006, French (2008) estimates that the cost of active investing, including total commissions, bid-ask spreads, and other cost investors pay for trading services, has dropped from 146 basis points in 1980 to 11 basis points in 2006. Do & Faff (2012) estimate institucional one-way commissions to be 7-9 basis points for 2007-2009. Avellaneda & Lee (2010); Stübinger *et al.* (2016); Liu *et al.* (2017); Stübinger & Endres (2018) assume transaction costs of 5 basis points per share half-turn, thus 10 basis points for the round-trip transaction cost. Given that our sample extends until 2015 and trading costs have become even lower with the rise of dark pools and rebates to supply liquidity, we follow these studies and assume trading costs of 0.20% (20 basis points) as proxy for the S&P 500 constituents per round-trip pair trade.

3.1 Profitability of the Strategies

In this section, we describe the results obtained from the following eleven cases: six individual copulas, four two-component mixed copulas and distance. Concerning the portfolio construction, we impose that all pairs are equally weighted in the portfolio under committed capital and fully invested capital schemes. In addition, we consider that the capital resulting from the short position is immediately applied in the long position. This approach eliminates net equity market exposure, so the returns provided should not be affected by the market's direction. Finally, all the results are presented for both without and with transaction costs. We consider transaction costs of 0.20% 20 bps, including commissions, market impact and short-selling constraints (Do & Faff, 2012).

Table 1 reports, for top 5 pairs, annualized mean excess returns, annualized Sharpe and Sortino ratios (see Newey & West (1987)), adjusted t-statistics, share of negative excess returns, the maximum drawdown in terms of maximum percentage drop between two consecutive days (MDD1) and between two days within a period of maximum six months (MDD2), annualized standard deviation, skewness, kurtosis, minimum and maximum daily returns, without and with transaction costs, for committed capital and fully invested strategies for the

[10] We assume zero return for non-open pairs, although in practice one could earn returns on idle capital.

[11] The commited capital is considered more realistic as it takes into account the opportunity cost of the capital that has been allocated for trading.

[12] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/det_st_rev_factor_daily.html

full sample period from July 1991 to December 2015 (6,173 observations). By analyzing this table, it is possible to observe that there is always one combination of the mixed copula that presents the best results in any of the performance measures. It might not be always the same combination, nevertheless there is always one regardless whether there are transaction costs or not and also whether the strategy is committed capital or fully invested. For a glance at the results, our approach generates higher average and risk-adjusted excess returns (Sharpe Ratio) as well as lower drawdown risk and standard deviation measures.

Tables 2 and 3 display exactly the same information as Table 1, but for top 20 and top 35 pairs, respectively. Now the dominance of our approach is not uniform over all measures and strategies. For top 20 pairs in Table 2 most of the results favor our approach. The exceptions are the standard deviation and one of the maximum drawdown measures for the fully invested strategy, either with or without transaction costs, and mean returns for the committed capital strategy, also either with or without considering transaction costs. In summary, we can still state that in general our approach is superior for top 20 pairs if one considers many performance measures. Jumping to Table 3, we look at the results for top 35 pairs, which, in summary, follow very closely to those for top 20 pairs.

Figure 1 shows (after costs) cumulative excess returns of the best individual copulas (elliptical and archimedean), the best mixed copula and the distance method through the full dataset for both investment strategies (committed and fully invested capital) for top 5 (top), top 20 (center) and top 35 (bottom) pairs. The left panels display cumulative returns on committed capital, whereas the right panels on fully invested capital. It should be noted that the mixed copula strategy shows a superior out-of-sample performance relative to the distance approach after the subprime mortgage crisis for all choices of number of pairs under the fully invested strategy. For the committed capital strategy it happens only for top 5 pairs, whereas the distance methods provides dramatically best results for top 20 and top 35 pairs.

Table 4 presents some relevant trading statistics concerning the number of pairs found for each one of the different pair's search techniques being considered. There are two main blocks in this table, the top one for committed capital and the bottom one for fully invested capital. Each block has 3 panels, one for each number of top pairs, 5, 20 and 35. Each line inside a panel represents a trading statistic. The columns, as in the previous tables, represent the following eleven cases: distance, six individual copulas and four two-component mixed copulas. The average price deviation trigger for opening pairs is listed in the first row of each panel. We can observe that, in average, we initiate the positions before when using the SJC copula. For instance, the positions are initiated when prices have diverged by 3.9%, 4.8%, and 5.0% for top 5, top 20 and top 35 pairs (committed capital), respectively. Similar to Gatev *et al.* (2006), the trigger spread increases, for both investment strategies, with the number of pairs for all approaches[13]. The table reveals that the average number of pairs traded per six-month period is only similar among the strategies for top 5 pairs. For top 20 and top 35 pairs, the distance approach has a much higher average number. This suggests that a two standard deviation trigger as opening criterion (Gatev *et al.*, 2006) is less conservative than the opening threshold suggested by Rad *et al.* (2016) using the cumulative mispricing indexes $M_{1,t}$ and $M_{2,t}$. Thus, the distance approach will be able to identify more trading opportunities to profit making the comparison less meaningful, although in practice the benefits are partly offset by the trading costs. Finally, notice for instance that each pair is held open, for fully invested capital, top 35 pairs, in average, for 52.62 trading days under the distance approach against many fewer days in all but one copula approach, the SJC, with an average of 105.8 trading days. In other words, all but one copula strategy perform better than (lower average number of open trading days) the distance method in this evaluation.

[13]Gatev *et al.* (2006) explains that the standard deviation of the prices increases as the proximity of the securities in price space decreases, thus increasing the trigger spreads.

Table 1: Excess returns of pairs trading strategies on portfolios of Top 5 pairs before and after costs

Note: This table summarizes statistics of the annualized excess returns, standard devations, Sharpe and Sortino ratios on portfolios of top 5 pairs between July 1991 and December 2015 (6,173 observations). Pairs are formed based on the smallest sum of squared deviations. The t-statistics are computed using Newey-West standard errors with a six-lag correction. The columns labeled MDD1 and MDD2 compute the largest drawdown in terms of maximum percentage drop between two consecutive days and between two days within a period of maximum six months, respectively. **Bold** entries indicate better performance between individual models and between mixture.

Committed Capital (No Costs)	Distance	Individual Copula						Mixed Copula			
		Clayton	Frank	Gaussian	Gumbel	SJC	Student	CT	FT	NT	TG
Mean Return	2.800	3.300	3.100	2.200	**4.300**	−0.800	2.300	3.200	1.400	2.700	**5.100**
Sharpe Ratio	0.334	0.532	0.495	0.359	**0.798**	−0.087	0.418	0.538	0.789	0.498	**0.876**
Sortino Ratio	0.625	0.899	0.849	0.607	1.356	−0.069	0.700	0.892	1.278	0.829	1.557
t-statistic	1.983	2.735	2.537	2.051	4.340	−0.223	2.312	2.823	4.131	2.790	4.703
% of negative excess returns	47.10	39.32	39.09	40.88	37.56	48.15	39.24	37.67	37.38	39.24	37.90
MDD1	6.74	6.790	6.790	4.650	3.470	8.320	4.640	6.780	1.260	5.230	3.510
MDD2	19.62	15.08	18.51	15.91	7.870	24.32	12.46	14.23	2.980	13.48	12.62
Annualized STD(%)	0.083	0.060	0.062	0.054	0.052	0.089	0.053	0.057	0.017	0.053	0.057
Skewness	0.345	−0.183	0.247	−0.267	0.156	0.104	−0.243	−0.379	0.128	−0.248	0.819
Kurtosis	8.570	17.06	15.31	12.89	10.06	10.10	10.67	19.31	1.130	15.47	12.98
Minimum Daily Ret	−4.500	−4.700	−4.7	−4.400	−3.500	−6.000	−3.700	−4.700	−1.000	−4.700	−3.000
Maximum Daily Ret	5.500	3.400	3.900	3.300	3.300	6.100	3.200	3.500	1.100	3.300	3.400
Fully Invested Capital (No Costs)											
Mean Return	4.300	10.10	**14.20**	1.110	12.90	−3.500	12.50	11.10	**17.20**	15.00	13.00
Sharpe Ratio	0.297	0.680	**0.953**	0.681	0.875	−0.253	0.860	0.754	**1.199**	1.007	0.905
Sortino Ratio	0.616	1.296	1.754	1.206	1.607	−0.299	1.586	1.411	2.213	1.863	1.687
t-statistic	1.941	3.754	5.208	3.937	4.794	−0.959	4.757	4.208	6.478	5.461	4.945
% of negative excess returns	47.10	38.67	36.54	38.11	37.70	47.62	37.30	36.31	36.26	37.56	36.68
MDD1	8.710	7.920	7.700	23.92	8.020	8.330	8.180	7.140	7.550	7.83	7.470
MDD2	29.57	26.57	30.41	30.19	26.10	38.13	29.92	30.03	20.17	28.47	30.08
Annualized STD(%)	0.142	0.141	0.139	0.154	0.138	0.138	0.137	0.139	0.133	0.139	0.135
Skewness	0.482	0.930	0.419	−0.864	0.422	0.572	0.533	1.070	0.684	0.508	0.660
Kurtosis	10.71	14.28	7.040	65.29	9.810	13.60	10.42	16.21	9.940	9.040	8.790
Minimum Daily Ret	−6.500	−7.900	−7.700	−23.10	−8.000	−6.000	−8.100	−7.100	−7.500	−7.400	−7.300
Maximum Daily Ret	10.10	12.70	7.300	10.20	9.700	12.70	9.500	12.70	9.700	8.700	9.700
Committed Capital (Costs = 20bps)											
Mean Return	2.200	2.800	2.700	1.300	**3.800**	−1.100	1.700	2.700	1.100	2.100	**4.600**
Sharpe Ratio	0.263	0.461	0.421	0.233	**0.711**	−0.122	0.311	0.459	0.650	0.392	**0.793**
Sortino Ratio	0.505	0.783	0.727	0.406	1.206	−0.127	0.528	0.764	1.042	0.657	1.407
t-statistic	1.611	2.395	2.185	1.387	3.890	−0.411	1.757	2.432	3.413	2.233	4.280
% of negative excess returns	47.24	39.47	39.26	41.17	37.81	48.30	39.48	37.91	37.73	39.52	38.16
MDD1	6.730	6.790	6.790	4.650	3.470	8.320	4.640	6.790	1.260	5.230	3.500
MDD2	19.64	15.08	18.61	15.98	7.940	24.40	12.48	14.23	2.990	13.58	12.65
Annualized STD(%)	0.083	0.060	0.062	0.054	0.052	0.089	0.053	0.057	0.017	0.053	0.057
Skewness	0.326	−0.194	0.242	−0.292	0.134	0.106	−0.256	−0.401	0.065	−0.274	0.805
Kurtosis	8.510	17.13	15.41	13.08	10.15	10.11	10.78	19.51	12.95	15.60	1.130
Minimum Daily Ret	−4.500	−4.700	−4.700	−4.400	−3.500	−6.000	−3.700	−4.700	−1.000	−4.700	−3.000
Maximum Daily Ret	5.400	3.400	3.900	3.300	3.300	6.100	3.200	3.500	1.100	3.300	3.400
Fully Invested Capital (Costs = 20bps)											
Mean Return	3.400	9.200	**13.10**	9.400	11.90	−3.900	11.20	10.00	**16.00**	13.60	11.90
Sharpe Ratio	0.233	0.624	**0.887**	0.586	0.813	−0.282	0.779	0.687	**1.123**	0.923	0.837
Sortino Ratio	0.506	1.196	1.635	1.049	1.496	−0.347	1.439	1.291	2.071	1.708	1.563
t-statistic	1.603	3.480	4.882	3.460	4.488	−0.116	4.354	3.876	6.102	5.053	4.613
% of negative excess returns	47.24	38.84	36.71	38.28	37.86	47.70	37.49	36.50	36.50	37.81	36.86
MDD1	8.710	7.920	7.700	23.92	8.020	8.330	8.180	7.140	7.550	7.830	7.470
MDD2	29.57	26.80	30.60	30.40	26.39	38.25	30.13	30.24	20.49	28.75	30.29
Annualized STD(%)	0.141	0.141	0.139	0.153	0.138	0.138	0.136	0.138	0.132	0.139	0.135
Skewness	0.475	0.930	0.417	−.894	0.412	0.571	0.518	1.071	0.684	0.505	0.656
Kurtosis	10.72	14.36	7.090	66.23	9.880	13.60	10.46	16.37	10.04	9.170	8.860
Minimum Daily Ret	−6.500	−7.900	−7.700	−23.10	−8.000	−6.000	−8.100	−7.100	−7.500	−7.400	−7.300
Maximum Daily Ret	10.100	12.70	7.300	10.20	9.700	12.70	9.500	12.70	9.700	8.700	9.700

Table 2: Excess returns of pairs trading strategies on portfolios of Top 20 pairs before and after costs

Note: This table summarizes statistics of the annualized excess returns, standard devations, Sharpe and Sortino ratios on portfolios of top 20 pairs between July 1991 and December 2015 (6,173 observations). Pairs are formed based on the smallest sum of squared deviations. The t-statistics are computed using Newey-West standard errors with a six-lag correction. The columns labeled MDD1 and MDD2 compute the largest drawdown in terms of maximum percentage drop between two consecutive days and between two days within a period of maximum six months, respectively. **Bold** entries indicate better performance between individual models and between mixture.

Committed Capital (No Costs)	Distance	Individual Copula						Mixed Copula			
		Clayton	Frank	Gaussian	Gumbel	SJC	Student	CT	FT	NT	TG
Mean Return	3.600	**1.700**	1.100	0.900	1.600	0.500	0.700	1.300	1.400	1.100	**1.900**
Sharpe Ratio	0.703	**0.887**	0.559	0.576	0.966	0.115	0.454	0.764	0.789	0.685	**1.111**
Sortino Ratio	1.233	1.481	0.899	0.937	1.603	0.221	0.726	1.234	1.278	1.123	1.937
t-statistic	3.636	4.518	2.906	3.088	5.164	0.748	2.416	3.862	4.131	3.638	5.794
% of negative excess returns	47.58	38.98	38.96	40.55	37.17	48.36	39.13	37.49	37.38	39.09	37.59
MDD1	3.950	1.720	1.720	1.130	1.090	3.060	1.120	1.720	1.260	1.310	1.26
MDD2	11.73	4.220	4.360	3.840	2.620	11.31	2.600	3.740	2.98	3.190	2.970
Annualized STD(%)	0.050	0.019	0.019	0.015	0.017	0.042	0.015	0.017	0.017	0.015	0.017
Skewness	0.343	0.263	0.081	0.005	0.289	0.021	−0.114	0.011	0.128	0.110	0.857
Kurtosis	10.12	13.16	13.75	13.25	13.11	5.170	9.850	16.82	1.130	14.29	15.58
Minimum Daily Ret	−2.800	−1.200	−1.200	−1.100	−0.900	−1.900	−1.000	−1.200	−1.000	−1.200	−1.000
Maximum Daily Ret	2.900	1.100	1.100	0.900	1.100	2.500	0.900	1.100	1.100	0.900	1.200
Fully Invested Capital (No Costs)											
Mean Return	6.300	11.50	**16.40**	11.70	14.70	0.700	14.10	14.70	**17.20**	17.20	16.00
Sharpe Ratio	0.682	0.815	**1.129**	0.727	1.030	0.077	0.983	1.026	**1.199**	1.160	1.131
Sortino Ratio	1.226	1.522	2.066	1.279	1.885	0.186	1.800	1.908	2.213	2.141	2.106
t-statistic	3.670	4.434	6.059	4.171	5.478	0.634	5.361	5.560	6.478	6.238	6.083
% of negative excess returns	47.58	38.40	36.55	38.09	37.49	48.09	37.04	36.02	36.26	37.36	36.71
MDD1	4.710	7.920	7.700	23.92	8.020	7.330	8.180	7.140	7.55	7.830	7.47
MDD2	19.77	23.12	30.06	28.17	21.51	38.13	19.66	24.28	20.17	17.75	19.54
Annualized STD(%)	0.089	0.134	0.135	0.153	0.134	0.084	0.135	0.134	0.133	0.137	0.132
Skewness	0.039	1.002	0.439	−0.906	0.474	−0.162	0.548	1.227	0.684	0.531	0.758
Kurtosis	5.610	16.93	7.710	66.92	11.05	19.08	11.01	18.24	9.940	9.460	9.850
Minimum Daily Ret	−4.800	−7.900	−7.700	−23.10	−8.000	−5.900	−8.100	−7.100	−7.500	−7.400	−7.300
Maximum Daily Ret	3.600	12.70	7.300	10.20	9.700	7.400	9.500	12.70	9.700	8.700	9.700
Committed Capital (Costs = 20bps)											
Mean Return	3.300	**1.500**	0.800	0.600	1.400	0.200	0.500	1.100	1.100	0.800	**1.700**
Sharpe Ratio	0.591	0.761	0.422	0.382	**0.823**	0.045	0.285	0.623	0.650	0.518	**0.966**
Sortino Ratio	1.037	1.261	0.675	0.612	1.341	0.106	0.453	0.996	1.042	0.839	1.656
t-statistic	3.081	3.895	2.213	2.069	4.428	0.360	1.529	3.170	3.413	2.771	5.074
% of negative excess returns	4.480	39.19	39.11	40.88	37.49	48.62	39.29	37.86	37.73	39.35	37.85
MDD1	3.950	1.720	1.720	1.130	1.090	3.060	1.120	1.720	1.260	1.310	1.260
MDD2	11.90	4.400	4.440	3.860	2.630	11.46	2.630	3.740	2.990	3.210	2.980
Annualized STD(%)	0.050	0.019	0.019	0.015	0.017	0.042	0.015	0.017	0.017	0.014	0.017
Skewness	0.321	0.208	0.037	−0.175	0.111	0.007	−0.206	−0.086	0.065	0.005	0.762
Kurtosis	10.07	12.94	13.75	12.81	12.94	5.190	9.770	16.79	12.95	13.99	15.43
Minimum Daily Ret	−2.800	−1.200	−1.200	−1.100	−0.900	−1.900	−1.000	−1.200	−1.000	−1.200	−1.000
Maximum Daily Ret	2.900	1.100	1.100	0.900	1.100	2.500	0.900	1.100	1.100	0.900	1.200
Fully Invested Capital (Costs = 20bps)											
Mean Return	5.300	10.60	**15.20**	1.100	13.60	0.300	12.80	13.60	**16.00**	15.70	14.80
Sharpe Ratio	0.580	0.752	**1.056**	0.629	0.961	0.027	0.895	0.951	**1.123**	1.071	1.056
Sortino Ratio	1.049	1.409	1.933	1.118	1.759	0.108	1.642	1.771	2.071	1.975	1.968
t-statistic	3.157	4.126	5.703	3.681	5.141	0.368	4.932	5.195	6.102	5.808	5.719
% of negative excess returns	4.480	38.61	36.78	38.27	37.69	48.28	37.18	36.28	36.50	37.60	36.96
MDD1	4.710	7.920	7.700	23.92	8.020	7.330	8.180	7.140	7.550	7.830	7.470
MDD2	20.03	23.33	30.25	28.39	21.71	38.25	19.98	24.32	20.49	17.84	19.83
Annualized STD(%)	0.089	0.134	0.134	0.152	0.133	0.084	0.134	0.134	0.132	0.137	0.131
Skewness	0.027	1.003	0.438	−0.934	0.466	−0.163	0.536	1.230	0.684	0.529	0.756
Kurtosis	5.600	17.05	7.770	67.87	11.15	19.05	11.05	18.44	10.04	9.610	9.930
Minimum Daily Ret	−4.800	−7.900	−7.700	−23.10	−8.000	−5.900	−8.100	−7.100	−7.500	−7.400	−7.300
Maximum Daily Ret	3.600	12.70	7.300	10.20	9.700	7.400	9.500	12.70	9.700	8.700	9.700

Table 3: Excess returns of pairs trading strategies on portfolios of Top 35 pairs before and after costs

Note: This table summarizes statistics of the annualized excess returns, standard devations, Sharpe and Sortino ratios on portfolios of top 35 pairs between July 1991 and December 2015 (6,173 observations). Pairs are formed based on the smallest sum of squared deviations. The t-statistics are computed using Newey-West standard errors with a six-lag correction. The columns labeled MDD1 and MDD2 compute the largest drawdown in terms of maximum percentage drop between two consecutive days and between two days within a period of maximum six months, respectively. **Bold** entries indicate better performance between individual models and between mixture.

Committed Capital (No Costs)	Distance	Individual Copula						Mixed Copula			
		Clayton	Frank	Gaussian	Gumbel	SJC	Student	CT	FT	NT	TG
Mean Return	3.800	1.100	0.600	0.600	**1.100**	0.700	0.500	0.800	0.800	0.700	**1.200**
Sharpe Ratio	0.934	0.899	0.538	0.621	**1.070**	0.161	0.508	0.771	0.794	0.743	**1.199**
Sortino Ratio	1.677	1.478	0.849	1.024	1.843	0.292	0.809	1.244	1.289	1.234	2.101
t-statistic	4.798	4.624	2.719	3.325	5.729	0.990	2.673	3.827	4.015	3.896	6.150
% of negative excess returns	47.73	38.77	38.96	40.50	37.05	47.87	3.390	37.38	37.35	39.03	37.49
MDD1	2.430	0.990	1.240	0.650	0.630	3.060	0.640	0.990	0.870	0.750	1.020
MDD2	8.040	2.300	2.680	2.220	1.500	8.710	1.550	2.230	2.240	1.840	1.720
Annualized STD(%)	0.040	0.012	0.011	0.009	0.010	0.041	0.009	0.010	0.010	0.009	0.010
Skewness	0.388	0.275	−0.093	0.405	1.139	−0.093	−0.056	0.275	0.334	0.669	0.850
Kurtosis	4.850	1.170	17.39	15.71	22.12	4.600	9.980	19.84	14.98	22.06	15.09
Minimum Daily Ret	−1.600	−0.700	−0.800	−0.700	−0.600	−2.100	−0.600	−0.700	−0.600	−0.700	−0.600
Maximum Daily Ret	1.800	0.900	0.700	0.600	1.100	1.700	0.500	0.700	0.700	0.900	0.600
Fully Invested Capital (No Costs)											
Mean Return	6.600	1.120	**16.60**	12.10	15.10	2.200	14.70	14.80	17.30	**17.70**	16.40
Sharpe Ratio	0.864	0.847	**1.145**	0.750	1.059	0.241	1.017	1.031	**1.205**	1.188	1.155
Sortino Ratio	1.576	1.580	2.095	1.317	1.933	0.465	1.860	1.917	2.221	2.193	2.150
t-statistic	4.604	4.584	6.124	4.291	5.598	1.453	5.538	5.574	6.488	6.373	6.195
% of negative excess returns	47.73	38.40	36.45	37.90	37.44	48.04	36.91	36.11	36.13	37.22	36.57
MDD1	3.390	7.920	7.700	23.92	8.020	4.330	8.180	7.140	7.550	7.830	7.470
MDD2	14.91	23.14	30.10	28.17	19.99	21.70	19.78	24.28	20.04	17.75	19.27
Annualized STD(%)	0.073	0.133	0.134	0.152	0.133	0.080	0.135	0.134	0.132	0.137	0.131
Skewness	0.251	1.014	0.443	−0.914	0.461	0.234	0.549	1.233	0.686	0.538	0.758
Kurtosis	3.610	17.18	7.780	67.21	11.16	6.500	11.05	18.36	10.01	9.480	9.910
Minimum Daily Ret	−2.800	−7.900	−7.700	−23.10	−8.000	−3.600	−8.100	−7.100	−7.500	−7.400	−7.300
Maximum Daily Ret	3.200	12.70	7.300	10.20	9.700	4.400	9.500	12.70	9.700	8.700	9.700
Committed Capital (Costs = 20bps)											
Mean Return	3.300	0.900	0.500	0.400	**0.900**	0.400	0.300	0.600	0.700	0.500	**1.100**
Sharpe Ratio	0.799	0.749	0.372	0.398	**0.903**	0.095	0.303	0.600	0.627	0.551	**1.023**
Sortino Ratio	1.428	1.216	0.580	0.641	1.515	0.185	0.475	0.951	1.100	0.900	1.749
t-statistic	4.128	3.861	1.892	2.151	4.889	0.628	1.605	2.989	3.175	2.918	5.298
% of negative excess returns	48.02	39.05	39.16	40.88	37.43	48.10	39.22	37.78	37.69	39.34	37.75
MDD1	2.430	0.990	1.240	0.650	0.630	3.060	0.640	0.990	0.870	0.750	1.020
MDD2	8.230	2.430	2.750	2.230	1.550	8.890	1.690	2.360	2.240	1.860	1.720
Annualized STD(%)	0.040	0.012	0.011	0.009	0.010	0.041	0.009	0.010	0.010	0.009	0.010
Skewness	0.366	0.148	−0.184	0.105	0.828	−0.111	−0.204	0.066	0.181	0.419	0.669
Kurtosis	4.830	16.03	17.29	13.99	20.05	4.600	9.900	18.99	14.45	19.40	14.52
Minimum Daily Ret	−1.600	−0.700	−0.800	−0.700	−0.600	−2.100	−0.600	−0.700	−0.600	−0.700	−0.600
Maximum Daily Ret	1.800	0.900	0.700	0.500	0.900	1.700	0.500	0.700	0.700	0.800	0.600
Fully Invested Capital (Costs = 20bps)											
Mean Return	5.600	11.10	**15.40**	10.40	14.00	1.100	13.30	13.60	16.00	**16.20**	15.20
Sharpe Ratio	0.741	0.783	**1.072**	0.653	0.990	0.115	0.930	0.955	**1.128**	1.099	1.080
Sortino Ratio	1.354	1.465	1.961	1.155	1.806	0.248	1.702	1.779	2.079	2.027	2.010
t-statistic	3.985	4.273	5.767	3.803	5.260	0.813	5.108	5.208	6.110	5.945	5.830
% of negative excess returns	48.02	38.54	36.70	38.11	37.72	48.43	37.18	36.32	36.41	37.52	36.79
MDD1	3.390	7.920	7.700	23.92	8.020	7.560	8.180	7.140	7.550	7.830	7.470
MDD2	15.19	23.34	30.29	28.39	20.22	31.58	20.09	24.32	20.35	17.84	19.56
Annualized STD(%)	0.073	0.133	0.134	0.152	0.133	0.082	0.134	0.134	0.132	0.137	0.131
Skewness	0.241	1.016	0.443	−0.943	0.454	0.005	0.537	1.238	0.687	0.536	0.757
Kurtosis	3.580	17.31	7.840	68.23	11.27	15.13	11.10	18.56	10.12	9.630	9.990
Minimum Daily Ret	−2.800	−7.900	−7.700	−23.10	−8.000	−4.800	−8.100	−7.100	−7.500	−7.400	−7.30
Maximum Daily Ret	3.200	12.70	7.300	10.20	9.700	7.100	9.500	12.70	9.700	8.700	9.700

Figure 1: Cumulative excess returns of pairs trading strategies after costs

Note: This figure shows how an investment of $1 evolves from July 1991 to December 2015 for each of the strategies.

3.2 Regression on Fama-French asset pricing factors

In an attempt to understand the economic drivers behind our findings as well as to evaluate whether pairs trading profitability is a compensation for risk, we regress daily excess returns onto various risk factors: daily Fama & French (2015)'s five research factors[14], the excess return on a broad market portfolio, $(R_m - R_f)$, the difference between the return on a portfolio of small stocks and the return on a portfolio of large stocks (SMB, small minus big), the difference between the return on a portfolio of high book-to-market stocks and the return

[14] All the data used to fit the above regressions are described in and obtained from Kenneth French's data library http://http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

Table 4: Trading statistics

Note: Trading statistics for portfolio of top 5, 20 and 35 pairs between July 1991 and December 2015 (49 periods). Pairs are formed over a 12-month period according to a minimum-distance (sum of squared deviations) criterion and then traded over the subsequent 6-month period. Average price deviation trigger for opening a pair is calculated as the price difference divided by the average of the prices.

Committed Capital	Distance	Individual Copula						Mixed Copula			
Top 5 Pairs		Clayton	Frank	Gaussian	Gumbel	SJC	Student	CT	FT	NT	TG
Av Price Dev trigger for open pairs	0.060	0.069	0.069	0.060	0.072	0.039	0.065	0.067	0.069	0.062	0.064
Total number of pairs opened	355	256	275	407	276	193	344	275	283	341	286
Average number of pairs opened per six-month period	7.245	5.225	5.613	8.307	5.633	3.939	7.021	5.613	5.776	6.96	5.837
Average number of round-trip trades per pair in months	1.449	1.045	1.123	1.662	1.127	0.788	1.405	1.123	1.156	1.392	1.168
Standard Deviation	1.002	0.997	0.993	1.896	1.062	0.458	1.465	1.188	1.131	1.466	1.160
Average time pairs are open in days	51.33	44.22	39.68	24.45	35.70	108.8	28.24	37.34	36.04	28.87	35.67
Standard Deviation	39.85	36.88	35.92	31.99	35.09	24.97	35.39	36.01	34.60	33.23	35.13
Median time pairs are open in days	39.00	32.50	27.00	10.00	20.50	119.0	11.00	23.00	22.00	15.00	21.00
Top 20 Pairs											
Av Price Dev trigger for open pairs	0.070	0.086	0.088	0.073	0.086	0.048	0.080	0.088	0.089	0.079	0.086
Total number of pairs opened	1330	566	598	676	572	734	593	567	563	581	585
Average number of pairs opened per six-month period	27.143	11.55	12.21	13.80	11.67	14.98	12.10	11.57	11.49	11.86	11.94
Average number of round-trip trades per pair in months	1.358	0.578	0.611	00690	0.584	0.749	0.606	0.579	0.575	0.593	0.597
Standard Deviation	0.986	0.767	0.767	1.211	0.796	0.468	0.972	0.836	0.819	0.976	0.831
Average time pairs are open in days	52.09	28.70	23.73	16.28	22.61	106.6	18.61	22.59	22.46	18.98	21.99
Standard Deviation	39.84	31.83	30.73	26.99	29.59	26.16	29.68	30.43	28.97	28.31	29.76
Median time pairs are open in days	42.00	13.50	10.00	5.00	10.00	118.0	6.00	9.00	10.00	7.00	9.00
Top 35 Pairs											
Av Price Dev trigger for open pairs	0.074	0.091	0.091	0.080	0.090	0.050	0.086	0.094	0.096	0.086	0.093
Total number of pairs opened	2261	728	762	798	710	1252	728	741	700	696	740
Average number of pairs opened per six-month period	46.14	14.86	15.55	16.29	14.49	25.55	14.86	15.12	14.29	14.21	15.10
Average number of round-trip trades per pair in months	1.319	0.425	0.445	0.466	0.414	0.731	0, 425	0.433	0.409	0.406	0.432
Standard Deviation	1.017	0.669	0.671	0.984	0.684	0.470	0.806	0.712	0.698	0.805	0.711
Average time pairs are open in days	52.62	23.50	19.36	13.98	18.89	105.8	15.47	17.99	18.53	16.03	17.95
Standard Deviation	40.51	30.09	28.53	25.43	27.64	26.58	27.59	27.92	27.20	26.71	27.63
Median time pairs are open in days	42.00	10.00	7.00	4.00	7.00	117.0	4.00	6.00	7.00	5.00	6.00
Fully Invested Capital											
Top 5 Pairs	Distance	Clayton	Frank	Gaussian	Gumbel	SJC	Student	CT	FT	NT	TG
Av Price Dev trigger for open pairs	0.060	0.068	0.071	0.064	0.064	0.041	0.068	0.067	0.072	0.068	0.067
Total number of pairs opened	355	220	241	335	224	182	289	243	250	288	245
Average number of pairs opened per six-month period	7.245	4.490	4.919	6.837	4.572	3.715	5.898	4.960	5.103	5.878	5.000
Average number of round-trip trades per pair in months	1.449	0.898	0.984	1.368	0.915	0.743	1.180	0.992	1.021	1.176	1.000
Standard Deviation	1.002	0.826	0.963	1.636	0.853	0.448	1.160	0.963	1.006	1.309	0.937
Average time pairs are open in days	51.33	48.40	39.86	26.69	43.18	107.6	32.31	41.69	39.28	31.53	41.16
Standard Deviation	39.85	36.79	35.33	32.27	36.45	23.63	34.05	36.03	34.73	33.78	36.07
Median time pairs are open in days	39.00	42.50	29.00	12.00	31.00	117.0	16.0	29.00	29.50	17.00	29.00
Top 20 Pairs											
Av Price Dev trigger for open pairs	0.070	0.088	0.087	0.078	0.087	0.049	0.087	0.087	0.089	0.089	0.088
Total number of pairs opened	1330	481	511	541	538	697	545	506	539	539	506
Average number of pairs opened per six-month period	27.14	9.817	10.43	11.04	10.98	14.26	11.12	10.33	11.00	11.00	10.33
Average number of round-trip trades per pair in months	1.358	0.491	0.522	0.553	0.549	0.712	0.557	0.517	0.550	0.550	0.517
Standard Deviation	0.986	0.658	0.720	1.026	0.682	0.461	0.824	0.733	0.744	0.874	0.719
Average time pairs are open in days	52.09	32.93	25.75	18.12	27.75	106.5	21.51	26.69	24.57	20.26	26.60
Standard Deviation	39.84	32.67	30.34	27.84	30.65	23.70	28.58	31.92	29.66	28.08	31.50
Median time pairs are open in days	42.00	18.00	13.00	7.00	14.00	117.0	10.00	12.00	12.00	9.00	12.00
Top 35 Pairs											
Av Price Dev trigger for open pairs	0.074	0.091	0.091	0.080	0, .90	0.050	0.086	0.094	0.096	0.086	0.093
Total number of pairs opened	2261	728	762	798	710	1252	728	741	700	696	740
Average number of pairs opened per six-month period	46.14	14.86	15.55	16.29	14.49	25.55	14.86	15.12	14.29	14.21	15.10
Average number of round-trip trades per pair in months	1.319	0.425	0.445	0.466	0.414	0.731	0.425	0.433	0.409	0.406	0.432
Standard Deviation	1.017	0.669	0.671	0.984	0.684	0.470	0.806	0.712	0.698	0.805	0.711
Average time pairs are open in days	52.62	23.50	19.36	13.98	18.89	105.8	15.47	17.99	18.53	16.03	17.95
Standard Deviation	40.51	30.09	28.53	25.43	27.64	26.58	27.59	27.92	27.20	26.71	27.63
Median time pairs are open in days	42.00	10.00	7.00	4.00	7.00	117.0	4.00	6.00	7.00	5.00	6.00

on a portfolio of low book-to-market stocks (HML, high minus low), the difference between the return of the most profitable stocks and the return of the least profitable stocks (RMW, robust minus weak), the difference between the return of stocks that invest conservatively and the return of stocks that invest aggressively (CMA, conservative minus aggressive) plus momentum (Mom), and long-term reversal (LRev) factors, *i.e.*,

$$R_{i,t} - R_{f,t} = \alpha_i + \beta_i \left(R_{m,t} - R_{f,t}\right) + s_i \mathrm{SMB}_t + h_i \mathrm{HML}_t + r_i \mathrm{RMW}_t + c_i \mathrm{CMA}_t + m_i \mathrm{Mom}_t + l_i \mathrm{LRev}_t + \varepsilon_{i,t}, \tag{10}$$

with $E(\varepsilon_{i,t}) = 0$, $Var(\varepsilon_{i,t}) = \sigma^2_{\varepsilon_i}$ and $E(\varepsilon_{i,t}\varepsilon_{i,s}) = 0, t \neq s$, where i and t stand for portfolio and time indexes, respectively.

The main purpose of these regressions is to estimate the intercept alpha, the average excess return not explained after controlling for these factors, as a measure of risk-adjusted performance. The errors have been adjusted for heteroscedasticity and autocorrelation by using Newey-West adjustment with six lags.

Tables 5, 6 and 7 report the coefficients and corresponding Newey-West t-statistics of regressing daily portfolio return series against Fama & French (2015)'s five research factors plus momentum and long-term reversal factors for each of the strategies after transaction costs, for top 5, top 20 and top 35 pairs. Whereas tables 5 and 6 refer to the distance method and to individual copula models considering committed capital and fully invested capital schemes (after transaction costs), respectively, Table 7 reports the results for mixed copulas.

It is clear that the mixed copula approach produces higher adjusted alphas than the distance method for both weighting schemes, especially under fully invested capital (a maximum value of 6.3 daily basis points - approximately 17.20% per year - with a t-value of 6.13). It should also be noted that the risk-adjusted returns provided by mixed copula and distance strategies are positive and significant at least at 5% and 10%, respectively, after accounting for all the previously mentioned factors.

From these tables one could also observe that the magnitude of the loadings on the market factor are large and with high t-values (significant at 1%), under both investment strategies, for the distance method and the mixed copulas, thus contributing to the pairs trading profitability. Among the other factors, the loadings on the momentum factor are negative and significant at least at 5% under committed capital. In addition, the correlations between the excess returns and other traditional equity risk premia factors (RMW and HML) are typically near to zero. Finally, it should be noted that the results show that the exposures to the various sources of systematic profile risk provide a low explanation of the average excess returns for any strategy, with adjusted R^2 ranging from 0.2% to 3.4%, particularly for the copula-based pairs strategy, indicating that the method is nearly factor-neutral over the whole sample period.

Table 5: Daily risk profile of pairs 1991-2015: Fama and French (2016)'s five factors plus Momentum and Long-Term Reversal (Committed Capital)

Note: This table shows results of regressing daily portfolio return series onto Fama and French (2016)'s five factors factors plus momentum and long-term reversal over July 1991 and December 2015 (6173 observations). Section 1 shows the Return on Committed Capital and Section 2 on Fully Invested Capital after transaction costs. Pairs are formed based on the smallest sum of squared deviations. The *t*-statistics (shown in parentheses) are computed using Newey-West standard errors with six lags. Intercepts have been multiplied by 100 to aid interpretation. Newey-West *t*-statistics in parenthesis.

Method		Intercept	$R_m - R_f$	SMB	HML	RMW	CMA	MOM	LREV	R^2	R^2_{adj}
Distance	TOP 5	0.042	0.048	−0.020	0.046	0.002	−0.020	−0.051	−0.033	0.029	0.028
		(2.026)	(4.719)	(−0.923)	(1.889)	(0.045)	(−0.561)	(−4.623)	(−1.475)		
	TOP 20	0.015	0.029	−0.014	−0.006	−0.008	0.036	−0.038	−0.032	0.029	0.028
		(3.832)	(5.392)	(−0.981)	(−0.434)	(−0.383)	(2.066)	(−4.971)	(−2.266)		
	TOP 35	0.015	0.031	−0.004	−0.013	0.012	0.030	−0.032	−0.020	0.035	0.034
		(4.85)	(5.98)	(−0.464)	(−1.171)	(0.875)	(2.086)	(−5.342)	(−1.586)		
Clayton	TOP 5	0.014	0.030	−0.002	−0.003	0.038	−0.030	−0.025	−0.016	0.017	0.016
		(2.735)	(3.947)	(−0.109)	(−0.120)	(1.568)	(−1.029)	(−3.114)	(−0.870)		
	TOP 20	0.007	0.011	−0.001	−0.002	0.008	−0.004	−0.008	−0.004	0.018	0.017
		(4.442)	(5.38)	(−0.254)	(−0.280)	(1.190)	(−0.409)	(−3.060)	(−0.828)		
	TOP 35	0.004	0.007	−0.002	0.001	0.005	−0.005	−0.004	−0.002	0.018	0.017
		(4.565)	(5.278)	(−0.573)	(0.039)	(1.198)	(−0.981)	(−2.819)	(−0.579)		
Frank	TOP 5	0.011	0.037	−0.019	0.036	0.015	−0.043	−0.035	−0.004	0.030	0.029
		(2.219)	(5.512)	(−1.031)	(1.662)	(0.624)	(−1.504)	(−4.443)	(−0.239)		
	TOP 20	0.004	0.010	−0.006	0.007	0.006	−0.011	−0.009	0.001	0.021	0.020
		(2.17)	(4.633)	(−1.145)	(1.372)	(0.888)	(−1.393)	(−3.719)	(0.165)		
	TOP 35	0.003	0.006	−0.004	0.004	0.002	−0.006	−0.005	0.002	0.020	0.019
		(2.733)	(4.746)	(−1.492)	(1.268)	(0.480)	(−1.255)	(−3.843)	(0.409)		
Gaussian	TOP 5	0.006	0.018	−0.019	−0.004	0.020	0.013	−0.016	−0.023	0.009	0.008
		(1.346)	(2.704)	(−1.643)	(−0.17)	(0.968)	(0.511)	(−2.011)	(−1.311)		
	TOP 20	0.003	0.004	−0.004	−0.003	0.005	0.004	−0.005	−0.006	0.008	0.007
		(2.089)	(2.239)	(−1.202)	(−0.464)	(0.921)	(0.561)	(−2.421)	(−1.185)		
	TOP 35	0.003	0.003	−0.003	−0.002	0.004	0.002	−0.003	−0.003	0.008	0.007
		(3.311)	(2.384)	(−1.499)	(−0.460)	(1.136)	(0.473)	(−2.310)	(−0.845)		
Gumbel	TOP 5	0.015	0.020	−0.018	−0.019	0.032	0.007	−0.022	0.014	0.01	0.009
		(3.775)	(3.644)	(−1.599)	(−1.303)	(2.000)	(0.312)	(−3.158)	(0.960)		
	TOP 20	0.006	0.008	−0.004	−0.007	0.011	0.009	−0.008	0.004	0.012	0.011
		(4.226)	(4.392)	(−1.086)	(−1.610)	(2.213)	(1.426)	(−3.559)	(0.854)		
	TOP 35	0.004	0.005	−0.002	−0.004	0.007	0.004	−0.005	0.003	0.011	0.01
		(4.695)	(4.399)	(−0.852)	(−1.551)	(2.304)	(1.115)	(−3.474)	(1.116)		
SJC	TOP 5	−0.004	0.031	−0.027	0.057	−0.006	0.016	−0.008	−0.014	0.009	0.008
		(−0.59)	(3.256)	(−1.487)	(2.418)	(−0.155)	(0.433)	(−0.633)	(−0.569)		
	TOP 20	0.006	0.012	−0.014	0.009	0.008	0.001	0.001	0.011	0.003	0.002
		(1.507)	(2.613)	(−1.487)	(0.807)	(0.480)	(0.027)	(0.148)	(0.822)		
	TOP 35	0.001	0.012	0.008	0.003	0.009	0.020	−0.003	0.010	0.004	0.003
		(0.068)	(2.838)	(0.936)	(0.308)	(0.614)	(1.227)	(−0.446)	(0.921)		
Student	TOP 5	0.009	0.027	−0.011	0.003	0.025	−0.011	−0.005	−0.014	0.011	0.010
		(2.110)	(4.359)	(−0.956)	(0.169)	(1.554)	(−0.465)	(−0.587)	(−0.847)		
	TOP 20	0.002	0.008	−0.002	0.002	0.012	−0.001	−0.002	−0.003	0.011	0.010
		(1.216)	(4.579)	(−0.473)	(0.234)	(2.574)	(−0.078)	(−0.913)	(−0.701)		
	TOP 35	0.002	0.005	−0.001	0.001	0.007	−0.001	−0.002	−0.001	0.011	0.010
		(2.375)	(4.545)	(−0.430)	(0.046)	(2.563)	(−0.060)	(−1.235)	(−0.356)		

Table 6: Daily risk profile of pairs 1991-2015: Fama and French (2016)'s five factors plus Momentum and Long-Term Reversal (Fully Invested Capital)

Note: This table shows results of regressing daily portfolio return series onto Fama and French (2016)'s five factors factors plus momentum and long-term reversal over July 1991 and December 2015 (6173 observations). Section 1 shows the Return on Committed Capital and Section 2 on Fully Invested Capital after transaction costs. Pairs are formed based on the smallest sum of squared deviations. The *t*-statistics (shown in parentheses) are computed using Newey-West standard errors with six lags. Intercepts have been multiplied by 100 to aid interpretation. Newey-West *t*-statistics in parenthesis.

Method		Intercept	$R_m - R_f$	SMB	HML	RMW	CMA	MOM	LREV	R^2	R^2_{adj}
Distance	TOP 5	0.021	0.079	−0.006	0.073	0.001	−0.018	−0.079	−0.061	0.025	0.024
		(1.934)	(4.9)	(−0.171)	(2.056)	(0.004)	(−0.298)	(−4.426)	(−1.645)		
	TOP 20	0.027	0.051	−0.019	0.014	−0.020	0.060	−0.073	−0.057	0.033	0.032
		(3.829)	(5.409)	(−0.807)	(0.551)	(−0.606)	(1.932)	(−4.986)	(−2.246)		
	TOP 35	0.026	0.057	−0.016	−0.013	0.011	0.073	−0.059	−0.045	0.038	0.037
		(4.61)	(5.87)	(−0.979)	(−0.568)	(0.424)	(2.682)	(−4.986)	(−1.821)		
Clayton	TOP 5	0.042	0.053	−0.039	0.021	0.078	−0.084	−0.049	0.036	0.011	0.010
		(3.731)	(3.77)	(−1.400)	(0.551)	(1.622)	(−1.577)	(−2.570)	(1.025)		
	TOP 20	0.046	0.060	−0.027	0.005	0.075	−0.076	−0.052	0.043	0.013	0.012
		(4.397)	(4.389)	(−0.995)	(0.122)	(1.603)	(−1.452)	(−2.757)	(1.269)		
	TOP 35	0.048	0.061	−0.028	0.004	0.071	−0.079	−0.050	0.045	0.013	0.012
		(4.551)	(4.462)	(−1.012)	(0.084)	(1.526)	(−1.510)	(−2.654)	(1.310)		
Frank	TOP 5	0.052	0.068	−0.069	0.037	0.012	−0.012	−0.060	0.014	0.017	0.016
		(4.869)	(5.032)	(−2.180)	(0.886)	(0.263)	(−0.219)	(−3.583)	(0.411)		
	TOP 20	0.060	0.073	−0.072	0.035	0.001	−0.004	−0.056	−0.006	0.02	0.018
		(5.68)	(5.475)	(−2.329)	(0.880)	(0.003)	(−0.079)	(−3.441)	(−0.187)		
	TOP 35	0.065	0.072	−0.073	0.035	−0.005	−0.006	−0.057	−0.007	0.019	0.018
		(6.12)	(5.361)	(−2.351)	(0.879)	(−0.096)	(−0.112)	(−3.475)	(−0.218)		
Gaussian	TOP 5	0.042	0.029	−0.067	−0.050	−0.004	0.097	−0.036	−0.102	0.009	0.007
		(3.64)	(1.491)	(−1.953)	(−0.941)	(−0.036)	(1.282)	(−1.344)	(−1.867)		
	TOP 20	0.044	0.03	−0.061	−0.052	−0.001	0.103	−0.036	−0.104	0.009	0.007
		(3.851)	(1.583)	(−1.774)	(−0.985)	(−0.009)	(1.368)	(−1.345)	(−1.917)		
	TOP 35	0.051	0.033	−0.064	−0.053	−0.001	0.107	−0.037	−0.104	0.009	0.008
		(4.466)	(1.701)	(−1.896)	(−1.013)	(−0.001)	(1.413)	(−1.387)	(−1.922)		
Gumbel	TOP 5	0.047	0.062	−0.062	−0.039	0.023	0.068	−0.036	0.017	0.010	0.009
		(4.316)	(3.986)	(−2.249)	(−1.114)	(0.581)	(1.473)	(−2.012)	(0.447)		
	TOP 20	0.053	0.062	−0.061	−0.030	0.014	0.087	−0.033	−0.006	0.010	0.009
		(4.981)	(4.287)	(−2.262)	(−0.9)	(0.351)	(1.986)	(−1.970)	(−0.146)		
	TOP 35	0.054	0.063	−0.057	−0.032	0.018	0.084	−0.034	−0.006	0.010	0.009
		(5.099)	(4.336)	(−2.115)	(−0.985)	(0.449)	(1.934)	(−2.011)	(−0.160)		
SJC	TOP 5	−0.013	0.040	−0.041	0.149	−0.024	−0.027	−0.01	−0.045	0.012	0.011
		(−1.237)	(2.424)	(−1.367)	(3.897)	(−0.501)	(−0.488)	(−0.549)	(−0.967)		
	TOP 20	0.009	0.030	−0.035	0.081	−0.027	−0.031	−0.007	−0.036	0.011	0.010
		(1.295)	(2.322)	(−1.403)	(2.416)	(−0.748)	(−0.673)	(−0.506)	(−0.911)		
	TOP 35	0.001	0.044	0.004	0.092	−0.029	−0.005	−0.006	−0.040	0.019	0.018
		(0.146)	(3.516)	(0.142)	(2.886)	(−0.866)	(−0.095)	(−0.508)	(−1.051)		
Student	TOP 5	0.049	0.051	−0.017	−0.099	0.081	0.036	−0.009	0.025	0.008	0.007
		(4.557)	(3.183)	(−0.63)	(−2.977)	(2.143)	(0.785)	(−0.479)	(0.734)		
	TOP 20	0.049	0.054	−0.021	−0.096	0.073	0.049	−0.008	0.007	0.009	0.008
		(4.721)	(3.406)	(−0.772)	(−2.909)	(1.964)	(1.075)	(−0.417)	(0.203)		
	TOP 35	0.056	0.055	−0.023	−0.098	0.071	0.047	−0.007	0.010	0.009	0.008
		(5.327)	(3.471)	(−0.854)	(−2.966)	(1.912)	(1.035)	(−0.368)	(0.283)		

Table 7: Daily risk profile of mixture pairs 1991-2015: Fama and French (2016)'s five factors plus Momentum and Long-Term Reversal

Note: This table shows results of regressing daily portfolio return series onto Fama and French (2016)'s five factors factors plus momentum and long-term reversal over July 1991 and December 2015 (6173 observations). Section 1 shows the Return on Committed Capital and Section 2 on Fully Invested Capital after transaction costs. Pairs are formed based on the smallest sum of squared deviations. The t-statistics (shown in parentheses) are computed using Newey-West standard errors with six lags. Intercepts have been multiplied by 100 to aid interpretation. Newey-West t-statistics in parenthesis.

Method		Intercept	$R_m - R_f$	SMB	HML	RMW	CMA	MOM	LREV	R^2	R^2_{adj}
Committed Capital											
CT	TOP 5	0.013	0.024	−0.013	−0.03	0.037	0.015	−0.024	−0.015	0.013	0.012
		(2.823)	(3.311)	(−0.953)	(−2.057)	(1.538)	(0.552)	(−3.225)	(−0.873)		
	TOP 20	0.005	0.007	−0.004	−0.006	0.009	0.001	−0.005	−0.005	0.01	0.009
		(3.15)	(3.298)	(−0.931)	(−1.374)	(1.266)	(0.022)	(−2.361)	(−0.884)		
	TOP 35	0.004	0.005	−0.003	−0.004	0.005	−0.001	−0.003	−0.002	0.011	0.01
		(3.795)	(3.791)	(−1.186)	(−1.454)	(1.196)	(−0.108)	(−2.315)	(−0.477)		
FT	TOP 5	0.015	0.027	−0.015	−0.026	0.042	0.017	−0.021	−0.003	0.013	0.011
		(3.53)	(3.547)	(−1.11)	(−1.847)	(2.389)	(0.732)	(−2.907)	(−0.133)		
	TOP 20	0.005	0.008	−0.004	−0.007	0.01	0.004	−0.005	−0.001	0.01	0.009
		(3.251)	(3.819)	(−1.019)	(−1.722)	(1.923)	(0.535)	(−2.176)	(−0.129)		
	TOP 35	0.003	0.005	−0.002	−0.005	0.006	0.003	−0.003	0.001	0.01	0.008
		(3.043)	(3.937)	(−0.865)	(−1.983)	(1.662)	(0.648)	(−2.222)	(0.082)		
NT	TOP 5	0.009	0.025	−0.013	−0.021	0.021	0.021	−0.02	0.004	0.011	0.009
		(2.119)	(3.276)	(−1.081)	(−1.339)	(0.937)	(0.814)	(−2.716)	(0.191)		
	TOP 20	0.003	0.007	−0.004	−0.006	0.005	0.006	−0.006	0.004	0.011	0.01
		(2.639)	(3.567)	(−1.273)	(−1.489)	(0.877)	(0.884)	(−2.699)	(0.755)		
	TOP 35	0.002	0.004	−0.003	−0.004	0.004	0.004	−0.003	0.003	0.011	0.009
		(2.77)	(3.69)	(−1.545)	(−1.472)	(0.936)	(0.819)	(−2.503)	(0.807)		
TG	TOP 5	0.018	0.034	−0.02	0.004	0.029	0.004	−0.021	0.018	0.016	0.015
		(4.053)	(3.896)	(−1.606)	(0.219)	(1.788)	(0.181)	(−2.905)	(1.019)		
	TOP 20	0.007	0.01	−0.005	0.001	0.005	−0.004	−0.006	0.006	0.015	0.014
		(4.909)	(3.968)	(−1.185)	(0.178)	(0.899)	(−0.567)	(−2.521)	(1.046)		
	TOP 35	0.005	0.006	−0.003	−0.001	0.004	−0.004	−0.004	0.005	0.017	0.015
		(5.974)	(4.277)	(−1.005)	(−0.247)	(1.122)	(−0.906)	(−2.676)	(1.385)		
Fully Invested Capital											
CT	TOP 5	0.045	0.065	−0.037	0.006	0.048	−0.051	−0.072	0.034	0.016	0.015
		(4.259)	(4.641)	(−1.105)	(0.126)	(0.916)	(−1.005)	(−4.054)	(0.937)		
	TOP 20	0.054	0.063	−0.036	0.018	0.038	−0.042	−0.064	0.013	0.016	0.015
		(5.254)	(4.544)	(−1.07)	(0.414)	(0.736)	(−0.868)	(−3.815)	(0.388)		
	TOP 35	0.058	0.065	−0.035	0.018	0.041	−0.042	−0.063	0.013	0.016	0.015
		(5.624)	(4.691)	(−1.047)	(0.419)	(0.785)	(−0.87)	(−3.707)	(0.38)		
FT	TOP 5	0.055	0.059	−0.05	0.021	0.045	−0.013	−0.065	−0.033	0.016	0.015
		(5.215)	(3.297)	(−1.652)	(0.553)	(1.039)	(−0.231)	(−3.554)	(−0.829)		
	TOP 20	0.063	0.061	−0.043	0.011	0.038	−0.01	−0.066	−0.047	0.018	0.017
		(6.136)	(3.467)	(−1.454)	(0.29)	(0.904)	(−0.181)	(−3.655)	(−1.224)		
	TOP 35	0.063	0.062	−0.043	0.007	0.042	−0.011	−0.067	−0.04	0.018	0.017
		(6.136)	(3.512)	(−1.46)	(0.18)	(0.995)	(−0.195)	(−3.717)	(−1.038)		
NT	TOP 5	0.058	0.072	−0.032	0.025	0.027	0.024	−0.052	−0.031	0.015	0.014
		(5.373)	(4.227)	(−1.08)	(0.627)	(0.632)	(0.452)	(−2.86)	(−0.816)		
	TOP 20	0.061	0.078	−0.028	0.025	0.028	0.014	−0.051	−0.031	0.016	0.015
		(5.703)	(4.602)	(−0.939)	(0.644)	(0.699)	(0.262)	(−2.832)	(−0.824)		
	TOP 35	0.062	0.079	−0.028	0.023	0.027	0.018	−0.052	−0.03	0.017	0.016
		(5.839)	(4.682)	(−0.949)	(0.591)	(0.679)	(0.329)	(−2.904)	(−0.809)		
TG	TOP 5	0.048	0.069	−0.048	0.008	0.063	0.003	−0.067	−0.031	0.018	0.017
		(4.576)	(4.006)	(−1.579)	(0.193)	(1.463)	(0.052)	(−3.724)	(−0.798)		
	TOP 20	0.058	0.074	−0.042	0.008	0.054	0.003	−0.064	−0.045	0.02	0.019
		(5.661)	(4.27)	(−1.395)	(0.219)	(1.289)	(0.047)	(−3.591)	(−1.181)		
	TOP 35	0.063	0.074	−0.04	0.005	0.057	0.002	−0.066	−0.04	0.021	0.019
		(6.134)	(4.296)	(−1.325)	(0.117)	(1.365)	(0.027)	(−3.712)	(−1.052)		

3.3 Sub-period analysis

The existing literature on trading strategies provides evidence of the performance sensitivity according to different market conditions. Therefore, our empirical results are obtained not only from the full sample analysis but also from subperiods analyses. These subperiods are chosen in two different ways: i) fixed time length; and ii) bull/bear market (non-crisis/crisis) dependent. The fixed time length analysis is able to capture possible dynamics changes over time whereas the bull/bear analysis makes it possible for one to have distinct pairs trading strategies depending on the market state.

3.3.1 Fixed time length: Five-years sub-periods

We split the full sample period into five sub-periods: (1) July 1991 to December 1995, (2) January 1996 to December 2000, (3) January 2001 to December 2005, (4) January 2006 to December 2010, and (5) January 2011 to December 2015. The third sub-period is marked by strong events such as the dotcom crisis and the September 11th terrorist attack, whereas the fourth sub-period encompasses the subprime mortgage financial crisis period.

Figures 2 and 3 show the profitability and risk-adjusted patterns for top 5 (top), top 20 (center) and top 35 (bottom) pairs after costs, respectively, for each sub-period under committed (left) and fully invested capital (right). We only report results for the distance method, the best individual elliptical copula, the best individual archimedean copula and the best two-element mixed copula. The copula based strategies yield to a superior out-of-sample average excess returns relative to the distance approach in all subperiods and number of top pairs under fully invested capital. For committed capital this superiority is maintained only for top 5 pairs, with the exception of the first subperiod. In terms of Sharpe Ratio, there is a dominance of copula based methods almost everywhere.

3.3.2 Crisis (bear) versus non-crisis (bull) sub-periods

We also conduct a subperiod analysis considering crisis and non-crisis periods. For that, the following subperiods are classified as belonging to the crisis subgroup[15] over the sample: (1) 24 of March 2000 to 21 of September 2001, (2) 04 of January 2002 to 09 of October 2002, (3) 09 of October 2007 to 20 of November 2008 and (4) 06 of January 2009 to 09 of March 2009. This crisis group adds up to 894 trading days.

Figures 4 and 5 show the profitability and risk-adjusted patterns of the distance method, the best individual elliptical copula, the best individual archimedean copula and the best two-element mixed copula for top 5 (top), top 20 (center) and top 35 (bottom) pairs after costs, for crisis and non-crisis sub-periods under committed capital (left) and fully invested capital (right). Figure 4 shows that on average the returns of the copula based strategies have been typically much higher during the crisis subperiods. Specifically, the mixed copula Student-Gumbel presents the best results during crisis period under committed capital for all numbers of top pairs. In general, even during the non-crisis subperiod there are only two situations where the distance method outperforms the copula based ones: top 20 and top 35 pairs under committed capital. Finally if one looks at the Sharpe Ratios in 5, there is a uniform dominance of copula based methods. Particularly, the mixed copula approach dominates in all non-crisis scenarios as well as in all crisis scenarios under committed capital.

[15] https://www.hartfordfunds.com/dam/en/docs/pub/whitepapers/CCWP045.pdf

Figure 2: Average excess returns of pairs trading strategies after costs for each sub-period

Note: This figure shows how the 5-year rolling window Sharpe ratio densities evolve from July 1996 to December 2015 for each of the strategies.

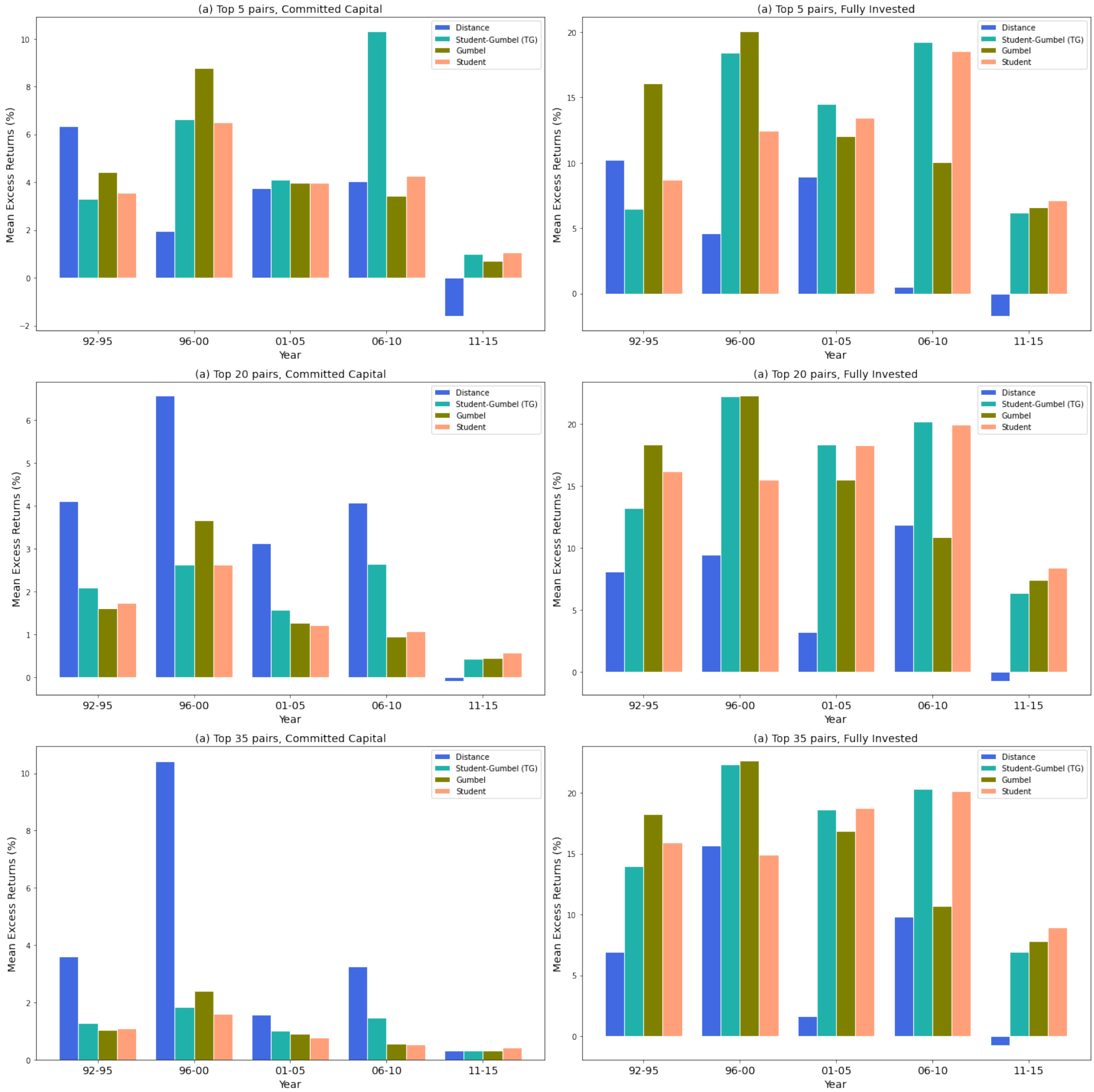


Figure 3: Sharpe Ratio of pairs trading strategies after costs for each sub-period

Note: This figure shows how the 5-year rolling window Sharpe ratio densities evolve from July 1996 to December 2015 for each of the strategies.

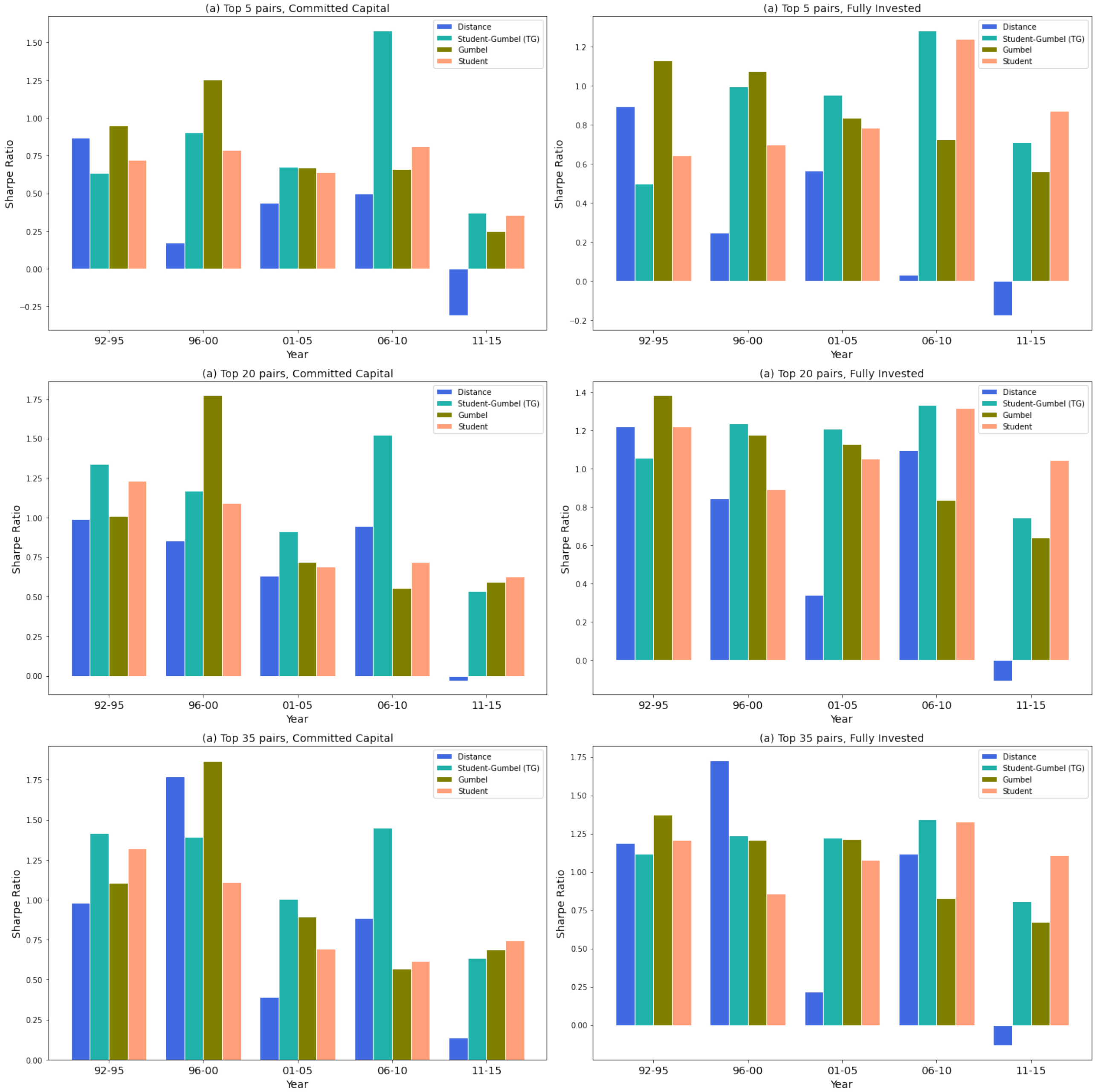


Figure 4: Average excess returns of pairs trading strategies after costs for each sub-period

Note: This figure shows how the 5-year rolling window Sharpe ratio densities evolve from July 1996 to December 2015 for each of the strategies.

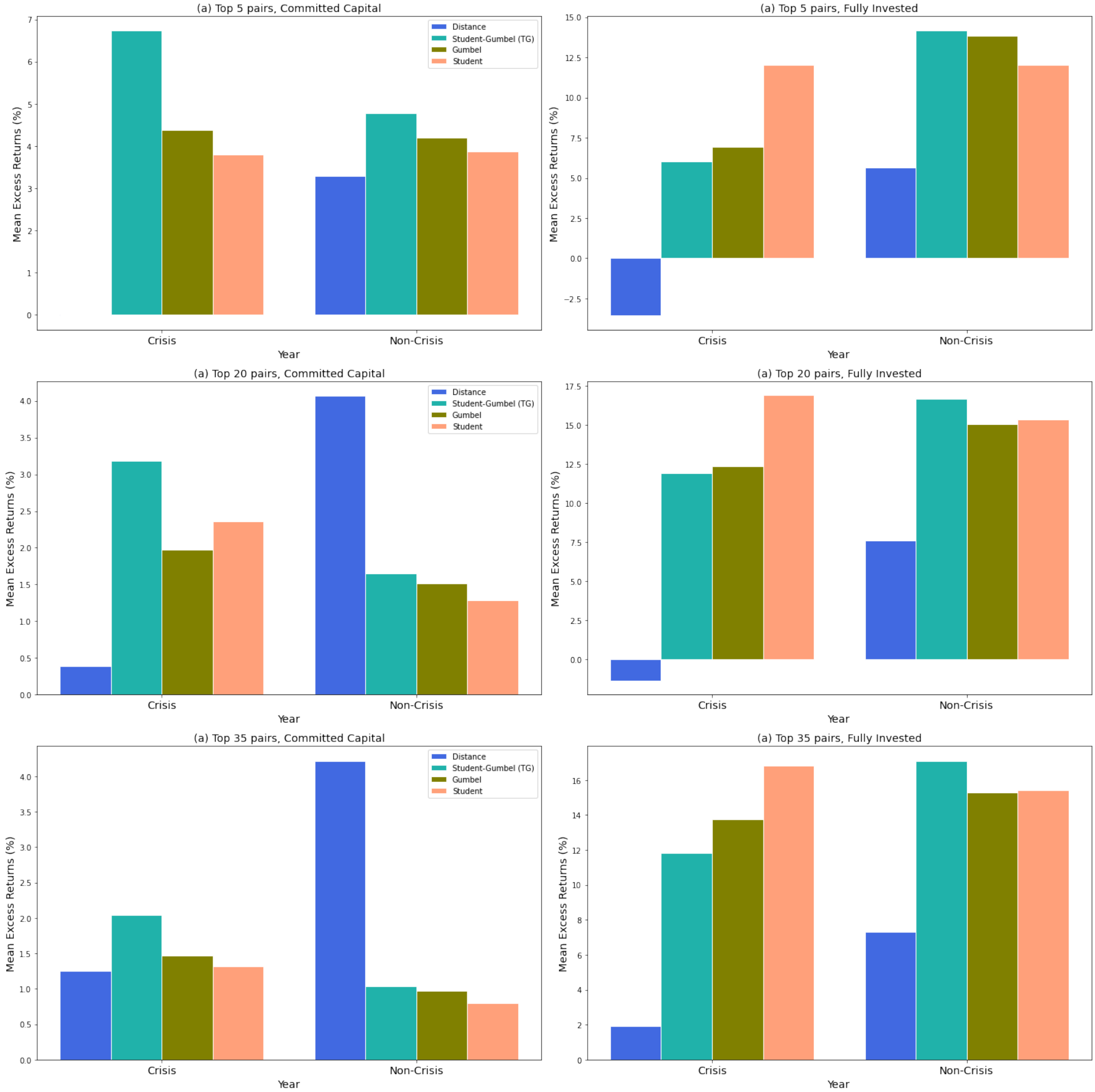
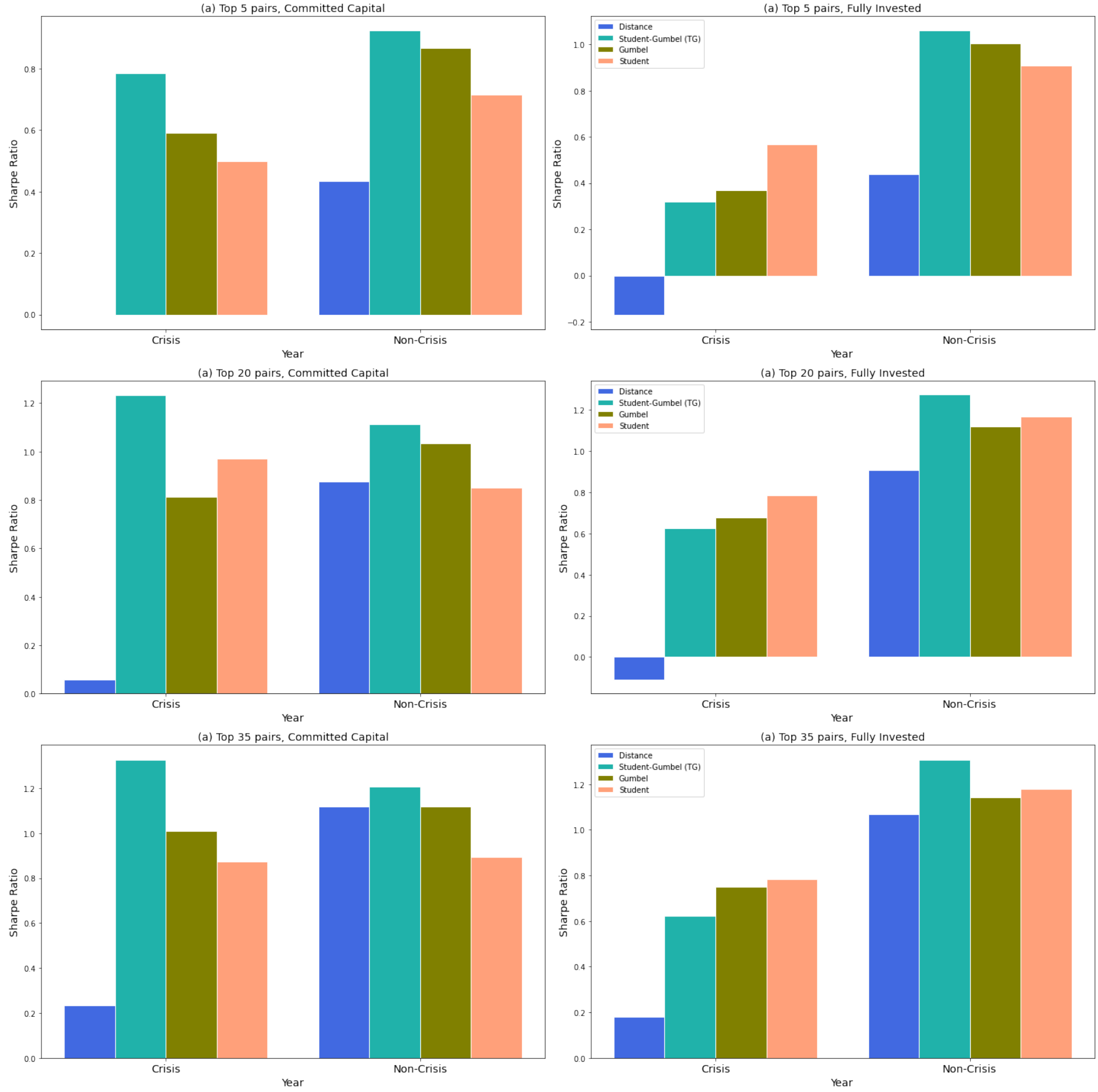


Figure 5: Sharpe Ratio of pairs trading strategies after costs for each sub-period

Note: This figure shows how the 5-year rolling window Sharpe ratio densities evolve from July 1996 to December 2015 for each of the strategies.



4 Concluding Remarks

The main goal of this paper is to verify if a strategy composed of a mixture of copulas is able to generate better and more robust economic performances, such as average returns and Sharpe ratios, than the distance methodology. Our analysis is comprehensive and contemplates subperiods analysis for dynamics detection and market state dependence modelling. We are also interested in understanding better the way that financial factors affect the profitability of pairs trading under different approaches. In this respect the mixed copula approach delivers economically larger alphas than the distance method for fully invested capital.

Using a long-term comprehensive data set spanning 25 years, our empirical analysis suggests that overall the mixed copula strategy has a superior performance than the distance approach especially during crisis periods, both in terms of average returns and risk adjusted average returns (Sharpe ratio). Nevertheless, no strategy consistently shows superiority over all subperiods, investment strategies and number of top pairs.

We propose here a very flexible way of performing pairs trading via a mixed copula dependence modeling, associated with an ARMA-GARCH structure for conditional mean/variance. This approach can cope with possible nonlinear dependencies, asymmetries, heavy tails, etc. Our empirical results indicate a robust performance of our proposal. As a conclusion, the results presented reinforce the use of the mixed copula modeling as an important tool in the quantitative management of funds. In practical terms, it provides an effective tool to monitor and hedge risks in the markets.

Data Availability Statement

The data that support the findings of this study are available upon request.

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