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Statistical Arbitrage using High Frequency Pairs Trading

An algorithmic trading strategy in the US equity market

Abstract

Using three-month minute-by-minute data from S&P 500 constituents we examine and report the return of a high frequency trading strategy by creating an algorithm based on pairs trading. The algorithm is created based on two different approaches and is implemented using a broad set of portfolio configurations to compare the effect on trading profits. We show that high frequency pairs trading generates large positive excess returns before accounting for transaction costs and find that the returns express no significant traditional risk factors. Furthermore, testing of certain portfolios to estimate break-even points in respect of transaction cost and return is conducted. Finally, we implement an approach based on implied volatility to test pairs trading under different market conditions.

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1. Introduction

Pairs trading is a quantitative trading strategy where the goal is to achieve statistical arbitrage profits by quantifying, measuring, and predicting the relationship between pairs and trading on the relative mispricing between securities. The strategy identifies and match a pair of assets whose co-movement can be observed in their historical price. If prices diverge, the relative overvalued (undervalued) security is short sold (bought) with an expectation of convergence of the price relationship. The profitability of pairs trading using daily data is well documented by Gatev, Goetzmann and Rouwenhorst. (1999; 2006). The development of available data and processing tools has led to high frequency pairs trading. Since Bowen, Hutchinson, and O'Sullivan (2010) examined pairs trading on an intraday basis this has evolved to the new norm in this area. Research provided by Gatev et al. (1999; 2006) and Do and Faff (2010) suggest a declining trend in the profitability using pairs trading strategy to achieve risk adjusted returns. Moreover, Do and Faff (2012) suggest that pairs trading using daily data is highly unprofitable after 2002 when controlling for transaction cost.

In this study, we investigate the profitability from a pairs trading strategy when implementing pair formation restrictions and trading thresholds based on implied market volatility. Furthermore, we examine our top performing portfolio returns robustness to transaction costs. Back-testing high frequency pairs trading strategies using minute-by-minute data from the S&P 500 constituents. We show that transaction costs on a general basis eradicate portfolio returns confirming results from Stübinger and Bredthauer (2017). On the other hand, we report annualized returns of up to 17.33% and obtaining a Sharpe ratio of 3.33, after controlling for medium transaction cost.

Finally, we examine a specific portfolio return to traditional risk exposure factors using regression, and find no significant exposure to traditional risks with low explanatory power. Additionally, we test top six performing portfolios to find an estimated break-even point for additional transaction costs and find the sustainable transaction costs to vary from 1.096 basis-points to 7.015 basis-points per trade depending on the number of transactions. To optimize our algorithm, we test the best performing portfolio with an additional market volatility measure and restrict the algorithm to trade under certain market conditions finding high average return with low standard deviations.

This thesis is structured as follows. First, we introduce existing literature on pairs trading in section 2. Section 3 discuss a thorough review of pairs trading and the two most cited approaches related to pairs trading. Section 4 presents the dataset, the predefined formation period, the trading rules in the algorithm, and we discuss the implication of transaction costs and return computation. Section 5 provides the empirical results generated from back-testing our dataset. Moreover, we review specific portfolio risk and measures, demonstrate the break-even point for the top performing portfolios and introduce a volatility measure. Section 6 concludes with key findings and provide suggestions for future research.

2. Literature review

In this section we provide a brief overview of selected academic research papers related to pairs trading. The first two articles deal with pairs trading using daily return data, in contrast to the rest of the articles that back-test data of higher frequency.

Viewed as the pioneers of academic pairs trading research, Gatev et al. (1999; 2006) published the first paper that back-tested the strategy using daily data from the S&P 500 over the period from 1962 to 2002. To match pairs, they applied the minimum-distance criterion to measure co-movement with normalized historical prices. The study implements a two-stage methodology where the formation period over 12 months are followed by a trading period of 6 months. This methodology of two-to-one relationship for formation and trading is viewed as the standard of matching and trading pairs, although this number is chosen arbitrarily. They apply a simple trading rule where a position is opened when the price diverges more than two standard deviations in the formation period. Their portfolio of pairs yields average annualized returns up to 11%. The authors argue that their return is due to an unidentified market risk factor and back this up with high correlation between the return of portfolios consisting of non-overlapping pairs. The correlation is still present after controlling for Fama-French-Momentum-Reversal factors. Gatev et al. (1999; 2006) further discuss the decline in profitability using the pairs trading strategy in recent times. Moreover, the article dismisses that decline in profit is due to competition in the hedge fund sector. Alternatively, they argue that abnormal returns are a compensation to arbitrageurs for enforcing the “Law of One Price”.

The study from Gatev et al. (1999; 2006) is replicated by Do and Faff (2010) with near equal results and confirm a downward trend in the profitability of the trading strategy. Results from the study suggest that after controlling for commission, market impact and short selling fees, pairs trading proves profitable at a modest level before 2002. By extending the dataset from Gatev et al. (1999; 2006), they find that pairs trading performs better under periods of turbulence such as the financial crisis in 2007-2008. In line with the replicated article Do and Faff (2010) claims that hedge fund competition is only a part of the decline in profitability of the pairs trading strategy. Their study finds that worsening arbitrage risk accounts for 70 percent of the decline, while the rest is due to increased efficiency.

Bowen et al. (2010) is one of the first studies conducted on pairs trading using high frequency data. The authors examine returns from pairs trading strategy using the FTSE 100 constituent stocks intraday data from January to December 2007. They find that profits from pairs trading are highly sensitive to both transaction costs and the speed of execution. When implementing a wait-one-period method, excess returns are eliminated. Interestingly, the majority of profits occurred in the first hour of trading. Bowen et al. (2010), also finds that the return made from their trades are not related to traditional risk factors but stating that risk is related to market and reversal risk factors. This illustrates that the spread diverges further after entering a position in a pair.

Gundersen (2014) use high frequency intraday data from OBX in the first quarter of 2014. The study concludes that using unrestricted pairs, i.e. not including any fundamental factor restrictions, is a highly unprofitable strategy. On the other hand, when restricting the possible formation of pairs, allowing only assets from the same industry sector to be paired, significant returns are reported. The abnormal returns are also robust to moderate transaction costs and are uncorrelated with market returns.

Kishore (2012) examines two oil stocks using a co-integration approach. He finds that the preferable entry thresholds is dynamic when using high frequency data. Moreover, Kishore (2012) suggest that the open and closing thresholds should be unequal as opposed to the most cited methods. Similar to recent research, Kishore (2012) provides data which expresses high sensitivity to transaction costs. Finally, the article concludes that the algorithm is unable to take advantage of co-integration.

Stübinger and Bredthauer (2017) study pairs trading with high frequency strategy using minute-by-minute data of the S&P 500 constituent stocks from 1998 to 2015. Their best performing trading approach yields a significant return of 50.5% per annum after transaction costs. The study also confirms the declining trend in profitability using the pairs trading strategy. Interestingly, they find that the threshold formerly used on daily data is too aggressive when applied to minute-by-minute data. They conclude that the trading thresholds should not only be broader, but also suggest the use of dynamic trading signals.

3. Pairs Trading

The concept of pairs trading is credited a group of unconventional Wall Street traders consisting of mathematicians, physicists and computer scientists. Among other strategies the group developed the quantitative arbitrage strategy with advanced algorithms in the mid 1980's. Since its establishment it has become a popular trading strategy for hedge funds, proprietary trading desks and institutional investors (Vidyamurthy, 2004).

In pairs trading the investor exploit relative mispricing between two securities who share similarities in their return pattern. The basic idea of pairs trading is to identify and match pairs of securities that have moved together historically. When the relative prices diverge, the investor buy the relative undervalued stock, and simultaneously the relatively overvalued stock is sold short. It is important to note that the actual prices of the stocks in a formed pair are irrelevant. The price of one stock relative to the other is what we want to study. If the spread between the prices of a pair expands it indicates a mispricing between the assets, thus giving a buy- and sell signal. Accordingly, if the pairs follow an equilibrium relationship, the deviation from this equilibrium will correct itself and profits are made upon the convergence of relative prices.

While the basic idea of pairs trading is straightforward, an important question is how to identify pairs of securities that have an equilibrium relationship. Due to the strategy's proprietary nature, hedge funds and other investors practicing pairs trading are typically reluctant to share their approach. However, the years after Gatev et al.'s (1999) we have seen extensive research on pairs trading. This has resulted in various approaches for the formation of pairs, where the distance approach and co-integration approach are notably the most cited in empiric research.

The availability of technology and research has increased due to the popularity of statistical arbitrage and pairs trading in particular. It is a reasonable theory that the profitability decline is due to the competition amongst arbitrageurs. This is what Do and Faff (2010) identify as the "market efficiency effect". However, their findings indicate that the primary reason for the decline in profitability is due to a higher arbitrage risk for investors and not because of more competitive traders and hedge funds. While Do and Faff's (2010) study investigate pairs trading using daily data, articles in recent years have found pairs trading using high frequency data to be profitable. In these papers it is presented that arbitrageurs are inclined to encounter arbitrage risks as fundamental risk, noise-trader risk and synchronization risk. First, fundamental risk is due to a possible disruptive event causing the equilibrium relationship amidst a pair to vanish. Second, noise-trader risk is explained by traders who act irrationally in the market, making the spread between two securities to diverge further. Finally, synchronization risk refers to the timing of exploitation relative to other arbitrageurs which in effect close the difference between two similar assets at a different time, leading to exiting positions on time rather than convergence (Engleberg, Gao, Jagannathan, 2009).

3.1 The distance approach

The Euclidean distance approach was first introduced by Gatev et al. (1999) who constructed this approach based on their interpretation and descriptions of pairs trading given by trading professionals. This approach has two stages, the formation period and the trading period. Primarily, an estimation to find securities that historically have moved together is carried out during the formation period. This co-movement is measured by estimating the Euclidean distance between two securities which is measured by a straight line in a Euclidean space. Thus, pairs are identified by the sum of squared differences between their normalized price series. The normalized price series is set from the start of the formation period where a cumulative total return index is constructed at the end of the formation period for each stock.

$$Normalized\ price\ series_t = NP_t = \prod_{t=1}^{T}(1 + r_t), \qquad where\ r_t = \frac{P_t - P_{t-1}}{P_{t-1}}$$

$$Distance = \sum_{t=1}^{T}\left(NP_t^{Stock\ A} - NP_t^{Stock\ B}\right)^2$$

Second, the pairs are ranked based on the distance with a smaller distance being preferable. Gatev et al. (1999) chose to study the top 5 and 20 pairs, and although this was chosen arbitrarily it has become the standard procedure when using the distance approach. The distance between the pairs are monitored and positions are opened when the distance reaches a pre-determined threshold. Gatev et al. (1999; 2006) use a two-standard deviation metric to trigger their trades. However, this threshold is proven too aggressive when using high frequency data according to Stübinger and Bredthauer (2017), who also suggest using a dynamic time-varying threshold. When the spread deviates from its mean and reaches this threshold as estimated during the formation period, a long- and short-position are opened in the pair. This position is maintained until a stop-loss threshold is triggered in case of further divergence, a given trading period is reached, one company is delisted, or when the prices converges to its mean.

The long and short trading signals are given when the spread reaches the trading threshold computed as simply subtracting the normalized price of one stock by the other.

$$i) \quad NP_t^{AB} > \mu^{AB} + k \times \sigma^{AB} \rightarrow Short\ stock\ A, buy\ stock\ B$$

$$ii) \quad NP_t^{AB} < \mu^{AB} - k \times \sigma^{AB} \rightarrow Buy\ stock\ A, short\ stock\ B$$

Where $NP_t^{AB} = NP_t^A - NP_t^B$ is the normalized spread between the paired stocks, μ^{AB} and $\sigma^{AB}$is the respective mean and standard deviation of the spread from the formation period. The parameter k determines the aggressiveness of the entry thresholds, with Gatev et al. (1999; 2006) applying a k of 2.

The distance approach has been a popular in the literature of pairs trading. There are two main reasons for this, the first being its simplicity and the second being because empiric results have shown pairs trading to be profitable across several markets, asset classes and periods of time. Krauss (2017) explains that basing pairing choices on the squared distance between securities is suboptimal for the profit maximizing investor. A profit maximizing investor using pairs trading would want to achieve the highest profit she can per pair, which can be calculated as profit per trade multiplied with trades per pair. Therefore, a profit maximizing investor would want their pairs to exhibit high spread variance with prices that reverts to the mean of the pair. However, a perfect pair with the distance approach would have the distance of zero. Thus, they generate no profits even though it is expected that the lowest ranking pair would be the most profitable.

Therefore, the approach of forming pairs based on the distance-criterion between them may be of disadvantage given that low distance leads to lower spread variance. Do, Faff and Hamza (2006) argues that a fundamental problem with the distance-criterion in pairs trading is the assumption of static relationship throughout the time period, or that the return of pairs of securities are in parity. This may be the case for short periods of time, but most likely to be so for pairs whose risk-return profiles are close to identical. However, the authors also argue that the approach has an advantage given it is economic model-free and are therefore not prone to misestimations and misspecifications. On the other hand, being non-parametric means that the strategy lacks forecasting abilities regarding the convergence time or expected holding period.

3.2 The co-integration approach

The co-integration approach was first introduced by Vidyamurthy (2004), who created this approach based on the framework provided by Engle and Granger (1987). Along with the distance approach of Gatev et al. (1999; 2006), the co-integration approach is the most widely applied in the pairs trading literature. Important academic research applying this approach includes Lin, McCrae, and Gulati (2006), Kishore (2012), Rad, Low, and Faff (2015), as well as Do et al. (2006) and Krauss (2017). Before explaining how co-integration is achieved, we find it necessary to introduce the terms of stationarity and nonstationarity. In conclusion, we elaborate cointegration in a pairs trading perspective.

For a times series to be weakly stationary[1] the first two moments must be time-invariant. In other words, the mean and variance of a given time series, x_t, is constant over time as well as the autocovariance (Tsay, 2010). This can be denoted as:

$$E(x_t) = \mu$$
$$Var(x_t) = E(x_t - \mu)^2 = \gamma_0$$
$$Cov(x_t, x_{t-\ell}) = E\{(x_t - \mu)(x_{t-\ell} - \mu)\} = \gamma_\ell$$

where the constants μ and γ_0 represents the mean and variance of a weak stationary time series x_t. The covariance, γ_ℓ, between x_t and $x_{t-\ell}$ is only dependent of ℓ and measures how values of x_t are related to its past values $x_{t-\ell}$. These properties provide a simple framework for future observations and can be exploited in mean reverting strategies. As a stationary series would fluctuate around the mean, with constant variance over time, the investor can enter positions when the value of x_t differs from the mean value.

[1] Tsay (2010) splits stationarity into strict or weak, where strictly stationary conditions are so rigid that it is hard to verify empirically. Therefore, in time series analysis we have apply a more practical version in the weakly stationarity.

Unfortunately, prices and time series are likely to be nonstationary. This is understandable when observing companies share prices as it never has a set price level. Such nonstationary series are more commonly named unit-root nonstationary, where the most renowned model is the random walk model (Tsay, 2013). Inherently, nonstationary series has infinite variance as time goes to infinity.

The expected time between crossings of mean value is also infinite, meaning a nonstationary series would wander away from the mean and rarely, if ever, cross the mean value. Under such a model it is not possible to forecast future stock price movements. Thus, we are unable to take advantageous trading positions. It is further possible to transfer a time series that is unit-root nonstationary to stationarity by differencing. Differencing involves the use of the changed series of a time series. In finance one often uses the logarithmic return series;

$$x_t = \ln(P_t) - \ln(P_{t-1})$$

When a nonstationary series can be differenced d times before it becomes stationary, it is said to be integrated of order d or $x_t \sim I(d)$ (Engle and Granger, 1987). Thus, a stationary time series d will be equal to zero. For d equal to one, the change will be stationary after one differencing, and if the series was integrated by order 2, one would have to take the difference of the first difference to make a stationary series.

The combination of a stationary time series with constant variance and a nonstationary series will always be integrated of order 1. Generally, the sum of two $I(d)$ series is also $I(d)$. This combination can be noted as

$$z_t = x_t - ay_t$$

Where x_t and y_t are time series and where a is a constant. There is a possible case where the linear combination equals a series integrated by order $I(d-b)$, where $b > 0$. The simplest case being two nonstationary series, $x_t \sim I(d)$ and $y_t \sim I(d)$, where both d and b are equal to one and their combination equals, $z_t \sim I(d-b) = I(0)$, a stationary series.

In this case, the constant a provides the scaling for which long parts of the series x_t and y_t are cancelled out. That is, the infinite variance condition of nonstationary series is nullified. Admittedly, there is not a consistent a which makes z_t stationary. When it occurs, the term co-integration is used to describe this. An important notion of co-integration series is that it can be represented and modelled as error correction models.

This is proposed and proved in the Granger Representation Theorem (Engle and Granger, 1987). In an error correction model, there exists a long-run equilibrium and whenever there is a deviation from this equilibrium in one period, it is corrected in the following period. Combined with a co-integrated time series, the two series will be stationary and a long-run equilibrium would exist. Also, when a deviation occurs, the pairs' time series revert and restore the long-run equilibrium (Vidyamurthy, 2004). To create a way to test the relationship for co-integration Engle and Granger (1987) constructed the Engle-Granger two-step approach.

One of the proposed ways for co-integrated pairs to emerge is in the commodity- and interest-rate markets. If prices of gold on different exchanges drifted too far apart from one another, there would be opportunities for an arbitrageur (Granger, 1981). Vidyamurthy (2004) contributed to pairs trading literature by creating an approach to find pairs of stocks that are co-integrated and where anomalies occurred. The profit would hence be made with a correction of their relative prices.

The co-integration model is represented as an error correction model and applied to the logarithm of prices to two co-integrated stocks. This is denoted as

$$i)\ \log(p_t^A) - \log(p_{t-1}^A) = \alpha_A \log(p_{t-1}^A) - \gamma \log(p_{t-1}^B) + \varepsilon_A$$
$$ii)\ \log(p_t^B) - \log(p_{t-1}^B) = \alpha_B \log(p_{t-1}^A) - \gamma \log(p_{t-1}^B) + \varepsilon_B$$

The left-hand side of the equation is the return of the respective shares, A and B. With the spread, $\log(p_{t-1}^A) - \gamma \log(p_{t-1}^B)$, in both equations expressing the long-run equilibrium between stock A and B. As both price series of stock A and B are nonstationary, the spread will be stationary if the shares are cointegrated. The error correction parameters, α_A and α_B, as well as the cointegration coefficient γ, are what really determines the model and when estimating it their values are crucial.

Potential pairs are identified by comparing fundamental or statistical historic data. Vidyamurthy (2004) applies a modified version of the Engle-Granger two-step approach to test if pairs of stocks are cointegrated. First, one of the time series is regressed against the other and results in estimating the linear relationship between the two series. In the approach suggested by Vidyamurthy (2004), the logarithmic price of stock A is regressed against the logarithmic price of stock B. This gives the linear relationship

$$\log(p_t^A) - \gamma \log(p_t^B) = \mu + \varepsilon_t$$

The co-integration coefficient γ represents the beta value of the regression, where γ shares of stock B are equal to one share of stock A. With this, the equilibrium value μ is the intercept, in which Vidyamurthy (2004) states μ is also the premium for holding stock A over an equal value of stock B. Following this, the residual series from the regression are tested for stationarity by measuring the degree of mean reversion to determine if the pairs are tradeable, where a high number of equilibrium crossing suggests a stationary series. Trading signals can then be adopted from Gatev et al.'s (1999; 2006) original thresholds. When spread of the pair diverges by a set amount, a long and short position is taken and the portfolio closes at mean reversion.

Do et al. (2006) argues that a high zero crossing rate in the spread time series is not necessarily indicative of a co-integrating pair. The spread is interpreted as the return on a portfolio, while co-integration implies a long-run mean in the spread series, where a deviation indicates a disequilibrium from the mean value. As a solution, it is suggested to add more stringent cointegration tests.

3.3 Other approaches to pairs trading

In this section we briefly discuss the non-classical approaches to pairs trading. As pairs trading has gained in popularity, practitioners and researchers have proposed various approaches to pairs trading that differs from more orthodox research.

The time series approach introduced by Elliot, Van Der Hoek, and Malcom (2005). The time series model explains the spread as a mean-reverting Gaussian Markov chain, observed in Gaussian noise as described by Krauss (2017). Avellaneda and Lee (2010) also apply this method with mean reverting portfolios creating trading signals using Principal Component Analysis. This time series approach shows some promising results but fails to address the problem of matching optimal pairs, according to Krauss (2017).

Do et al. (2006) suggests modeling the spread in a continuous time setting to quantify the mean reversion of the spread using theoretical pricing methods. These approaches' main contributions are the forecast and quantitative ability that, in case of mean reverting properties and the presence of theoretical assets pricing, suggest more accurate trading thresholds to open and unwind positions. Similar to the time series, the stochastic approach model fails to address the problem of matching pairs. These methods are rarely used in empirical studies, and therefore not given further consideration in this paper.

4. Data

For the empirical part of the study we use high frequency minute-by-minute data from the constituents of the S&P 500 index in the period of December 13, 2017 to March 12, 2018. The dataset is adjusted for dividends and splits. In the event of no trades in a given minute, the dataset lack price information. We solve this by creating an index for all the trading minutes of the day from 09:30 to 16:00. For the time intervals lacking price information we use the closing price from the previous time point. To ensure that the starting price at each day is correct, we use the first closing price in each respective day. With this we achieve a comparable index containing 391 data points for each stock per day for 60 trading days. This generates a dataset containing 11 730 000 stock prices.

The S&P 500 index consists of the 500 largest companies by market capitalization listed on the NYSE and NASDAQ. The index captures roughly 80% of the US stock market capitalization[2]. These stocks are highly liquid which in turn reduce the arbitrage risks and is favorable for pairs trading strategy. Five companies on the S&P 500 are listed with two different share classes, making the total number of stocks in the index 505. In line with Do and Faff (2010) we observe that these stocks are often repeated as pairs when included in the pair formation algorithm. When we test a sample for the effect of excluding these securities, we note that a higher number of pairs are being traded and there is a greater volume of roundtrips in total. The percentage of positions closed on time are far less, resulting in a lower average holding time per roundtrip. Therefore, we have chosen to keep the class A stocks of companies with two different share classes, even though this may eliminate some potential payoff from intra-company arbitrage opportunities.

4.1 Pairs formation

Before identifying and constructing preferable pairs, we need to define the length of the historic co-movement and the length of the trading period that we want to investigate. The most common framework is to apply formation periods twice the length of trading periods. Another common feature is to overlap the periods by shifting them by one day as demonstrated in Figure 1. We follow Gundersen (2014) and apply two separate frameworks for formation and trading periods. The first framework being two days of formation and one trading day, and second with

[2] https://eu.spindices.com/indices/equity/sp-500 (08.05.2017)

four days of formation followed by two days of trading. Given that formation periods do not use any future data and the separation of different trading periods, we exclude looking-ahead bias. As a result of this, the dataset is divided into 58 periods (2:1 framework) and 55 periods (4:2 framework).

Following Gatev et al. (1999; 2006) and Stübinger and Bredthauer (2017) we identify pairs by applying the original distance approach. This is the approach that Stübinger and Bredthauer (2017) reports as the best performing approach. Hence, we first construct a cumulative total return index for each security and normalize it to the first day for each period. Secondly, we calculate the Euclidean squared distance between every stock. With no restrictions on pair formation, it is possible to create 124 750 unique pairs. However, we only choose the top 5 and top 20 pairs with the lowest distances for each period. These top pairs are then traded in their respective trading periods.

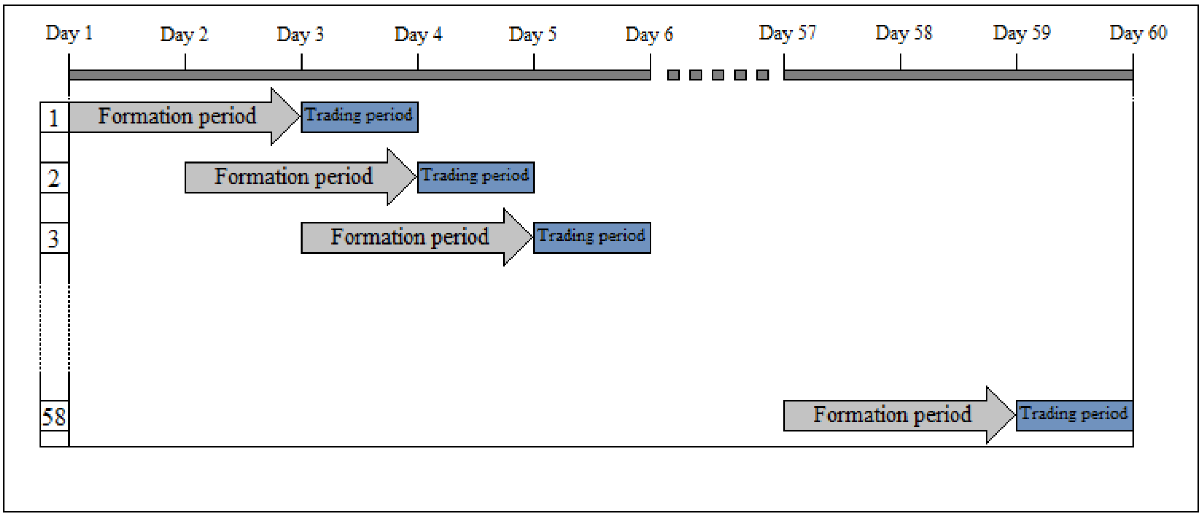

Figure 1: Demonstration of formation and trading periods with two days formation and one day of trading.

4.2 Trading rules

Pairs are traded with predetermined rules during the trading period. As the formation and trading subsets are independent from each other, it is unproblematic to create combinations of multiple pairs trading strategies. Static thresholds are the original trading thresholds used by Gatev et al. (1999; 2006), as described in section 3. Recent papers suggest that the most favorable risk and return is achieved by implementing the Euclidean distance method with varying trading thresholds (Kishore, 2012 and Stübinger and Bredthauer, 2017).

When implementing static trading thresholds to open and unwind positions, we need to first calculate the normalized return spread between the selected pairs from the formation period. With this, the equilibrium value is calculated as the mean of the normalized spread and the upper and lower bands illustrates the distance from the equilibrium calculated as k times the standard deviation of the spread.

For varying thresholds, the equilibrium, upper and lower bands are calculated from the rolling mean and standard deviation of the past n minutes. This type of dynamic thresholds is often referred to as Bollinger bands (Bollinger, 1992; 2001). Consequently, for every pair the thresholds will be updated at every time point in their respective trading periods. Whereas, Stübinger and Bredthauer (2017) use an n of 391 for their 5-days of trading, we carry out a somewhat larger ratio by calculating the bands based on the past 120 and 240 minutes for respectively one and two days of trading. Thus, the bands at the start of each trading period is determined from the last 120 (240) minutes in each respective formation period. Note that our dataset does not contain after-hours trading quotes, which in turn may affect the calculations.

Following the completion of established trading thresholds, we monitor the spread for each minute in the trading period. If the spread crosses the upper or lower band in time t, a long and short position is opened at time $t + 1$. The effect of having a one-minute lag before acting on the signal is termed as wait-one-period and is further discussed in the transaction costs subsection. The positions are closed upon convergence of the spread to its equilibrium value. In the spirit of Nath (2003) and Gundersen (2014) we test the algorithm both with and without a stop-loss trigger. When a position is stopped out the pair cannot trade in the same period. As with opening signals, the closing of positions is actualized with a one-minute lag. This means that losses from stop-loss signals can be much greater than the stop-loss level, depending on the volatility of the spread. Granted that the position is still open at the minute to the last in the trading period, the position is automatically closed at 16:00.

As displayed in Figure 2, we depict an example of static and varying thresholds where the selected pair, trading period and parameters are identical with the only difference being the trading thresholds.

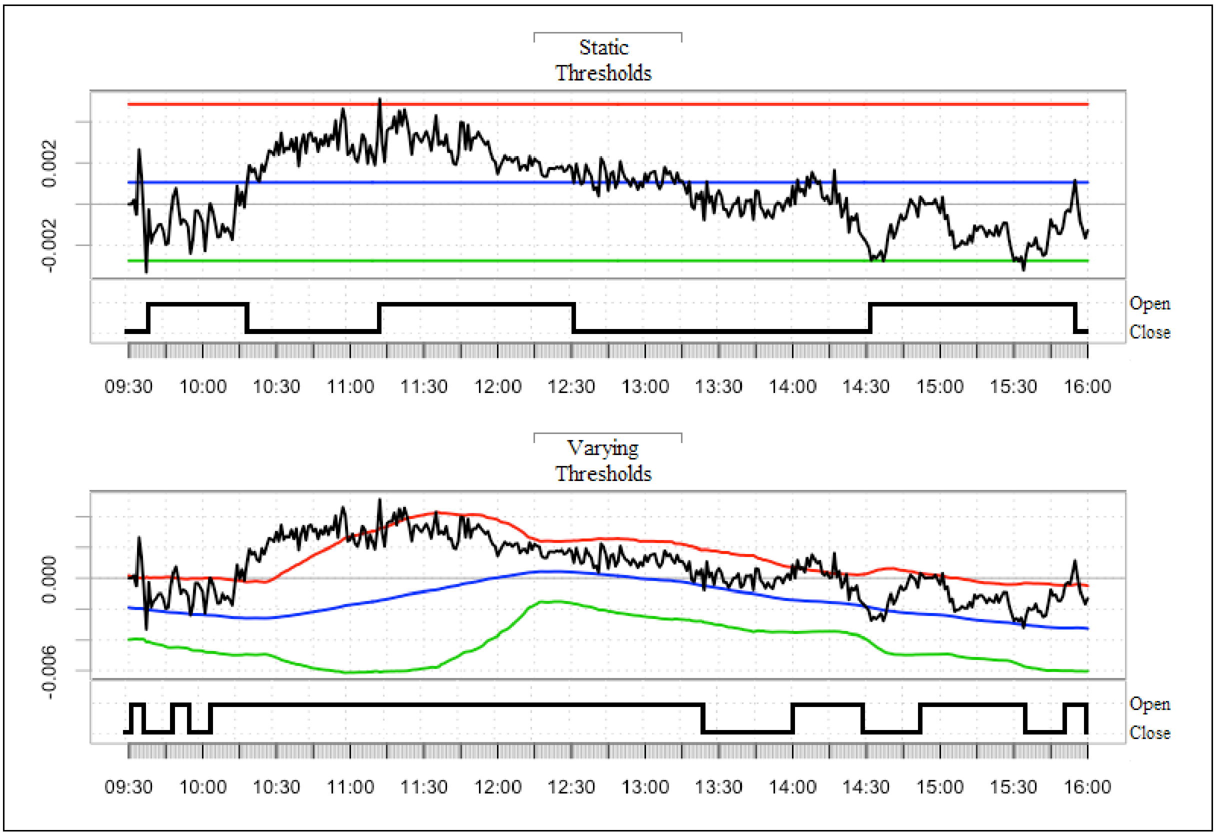

Figure 2: Demonstration of static and varying upper and lower thresholds and calculated mean, trading period 37, one day of trading, and the normalized spread between company Consolidated Edison Inc. and Western Digital Corp.

4.3 Return computation

As pairs trading is a strategy with zero net initial investment, return calculations may differ between market operators. Firstly, the possibility of financing the long position with the short proceeds creates a possible leverage effect that may influence the return calculation. As follows, it is uncertain how much cash is needed to implement the strategy. Our return calculation follows Gatev et al. (1999; 2006) where a $1-long, $1-short position requires a $1 deposit. This is a conservative approach as opposed to Avellaneda and Lee (2010) and Liu, Chang, Geman (2017) which respectively require $ 0.5 and $ 0.4 deposit for every $2 gross market exposure.

Secondly, we need to consider if the daily profit and loss is divided by how much capital you commit to the strategy, or how much capital the strategy requires per trade. These two methods are the most cited return calculations. Return on committed capital refers to when cashflows are divided by the number of possible traded pairs. Return on employed capital refers to when cashflows are divided by each active traded pair (Gatev et al. 1999; 2006). They also claim that return on committed capital is obviously much more conservative when it considers the opportunity cost of having to commit capital to a trading strategy even if the capital is not employed. On the other hand, they suggest as hedge funds are flexible in their capital allocation, return on employed capital may give a more realistic measure of profits. In this paper we report the return on committed capital if not stated otherwise, as this is the most conservative approach.

4.4 Transaction costs

Exploiting relative mispricing between assets often yield a low and modest return. Therefore, a certain number of trades is critical to achieve feasible annualized returns. Thus, transaction costs are not negligible. Do and Faff (2012) prove that almost all return provided from the simple model of Gatev et al. (1999; 2006) disappear after controlling for various transaction costs.

Transaction costs mainly consist of commissions, bid-ask spread, and short selling costs and the magnitude of these cannot be determined definitively (Do and Faff, 2012). These vary in time, with liquidity and between different market participants (Liu et al., 2017). There are many ways to control for transaction costs both directly and indirectly. Bid-ask spread, also referred to as bid-ask bounce or slippage, for example, is often controlled for as either using a fixed basis-point (bps) cost or use a time-lag as a proxy. Gatev et al. (1999; 2006) use a wait-one-period as a proxy for transaction costs.

In a contrarian strategy, price divergence may be a potential spread in the bid-ask price. The winners price is more likely to be an ask quote and the losers price is more likely to be a bid quote (Gatev et al., 1999; 2006). To cope with this problem, we implement a lag when testing our strategy. We restrict our algorithms to open or unwind a position in a pair the minute after a signal is given. This wait-one-period proxy is estimated by Liu et al. (2017) as 10 bps per roundtrip when testing the strategy return for both fixed bps as transaction costs, and a signal-lag as a proxy. In addition, following Avellaneda and Lee (2010), Krauss (2017) and Stübinger and Bredthauer (2017), we choose to test our strategy for 5 basis-points per trade per stock, as transaction costs. Additionally, we test for 2.5 bps and no fixed transaction costs. This combined with a wait-one-period lag in our algorithm will represent our transaction costs.

5. Results

In this section we present our results from the different strategies. First, Table 1 through 4 depicts all 288 portfolios. Second, we discuss the top and worst performing portfolios. Third, we present risk and return characteristics from 8 conservative portfolios. Fourth, we present some holding statistics for specific portfolios. In addition, we test a portfolio to traditional risk exposure. Thereafter, we test our algorithm to find the break-even point for different top performing portfolios. Finally, we add a volatility timing measure restricting our algorithm trading rules to optimize point of entry.

To find our optimal portfolios we test various parameters of k, stop-loss and transaction costs. Stübinger and Bredthauer's (2017) optimal k parameter proved to be 2.5 and they conclude a k of 2 as used by Gatev et al. (1999; 2006), is too aggressive when handling high frequency data. Nevertheless, we implement $k's$ equal to $2, 2.5$ and 5. With a k equal to 5, the spread would have to diverge five times the standard deviation from the equilibrium before taking a position. We acknowledge with this parameter the pair may already diverge beyond convergence, as there may no longer be an equilibrium relationship. However, as transaction costs are predominately important in pairs trading, a high k could lead to fewer transactions with higher profits per trade.

Stop-loss triggers are differentiated by applying stop-loss at 2%-level and without stop-loss signals. For transaction costs, we perform the back-test using transaction costs of 2.5 basis-points per transaction, 5 basis-points and without any transaction costs.

5.1 Unrestricted case

In our unrestricted case we allow all our stocks to be paired with each other in all formation periods. We create a 500 x 500 matrix which translate to 124 750 possible pairs. We then choose the top 5 and 20 pairs of stocks based on the minimum-distance criterion and construct a portfolio to test in the subsequent trading period.

In the spirit of Gundersen (2014) we report our results in tables illustrating the relationship between different configuration and returns. As shown in the table below transaction costs totally deteriorate the returns. The table is read from left to right arranged by number of formation and trading days, transaction costs per trade, number of standard deviation from its calculated mean before a position in a pair is entered, and a stop-loss configuration. The returns, standard deviations, and Sharpe ratios are reported on an annualized basis. Lastly, number of roundtrips for the whole period is reported where one roundtrip equals four trades, i.e. buying and selling a stock for both the long and the short position. When discussing results from the following tables we refer to the different portfolio configurations in numbers.

Table 1: Annualized risk and returns measures for top 5 pairs in the no restriction case using static and varying entry thresholds

	Parameters				Static Thresholds				Varying Thresholds			
Configuration	Format	TC	k	SL	Return	SD	SR	Roundtrips	Return	SD	SR	Roundtrips
1	2:1	5 bps	2	2 %	-48,30 %	4,59 %	-10,91	562	-62,29 %	4,43 %	-14,44	751
2	2:1	5 bps	2	NA	-48,75 %	4,67 %	-10,81	562	-62,79 %	4,45 %	-14,50	751
3	2:1	5 bps	2,5	2 %	-38,96 %	4,91 %	-8,28	455	-54,16 %	4,61 %	-12,13	615
4	2:1	5 bps	2,5	NA	-39,48 %	4,98 %	-8,26	455	-54,78 %	4,65 %	-12,14	615
5	2:1	5 bps	5	2 %	-20,35 %	3,96 %	-5,57	188	-27,06 %	3,67 %	-7,84	246
6	2:1	5 bps	5	NA	-21,36 %	4,10 %	-5,62	188	-28,04 %	3,76 %	-7,91	246
7	2:1	2,5 bps	2	2 %	-15,66 %	4,89 %	-3,55	562	-27,51 %	4,62 %	-6,33	751
8	2:1	2,5 bps	2	NA	-16,38 %	4,97 %	-3,64	562	-28,48 %	4,64 %	-6,50	751
9	2:1	2,5 bps	2,5	2 %	-9,29 %	5,14 %	-2,14	455	-21,72 %	4,72 %	-4,97	615
10	2:1	2,5 bps	2,5	NA	-10,07 %	5,20 %	-2,26	455	-22,77 %	4,76 %	-5,14	615
11	2:1	2,5 bps	5	2 %	-6,19 %	3,78 %	-2,09	188	-9,66 %	3,61 %	-3,15	246
12	2:1	2,5 bps	5	NA	-7,38 %	3,93 %	-2,31	188	-10,87 %	3,68 %	-3,41	246
13	2:1	0 bps	2	2 %	37,58 %	5,31 %	6,75	562	39,31 %	5,18 %	7,26	751
14	2:1	0 bps	2	NA	36,39 %	5,39 %	6,43	562	37,45 %	5,21 %	6,87	751
15	2:1	0 bps	2,5	2 %	34,78 %	5,44 %	6,08	455	33,64 %	4,98 %	6,40	615
16	2:1	0 bps	2,5	NA	33,62 %	5,50 %	5,80	455	31,85 %	5,02 %	6,00	615
17	2:1	0 bps	5	2 %	10,47 %	3,64 %	2,40	188	11,89 %	3,60 %	2,82	246
18	2:1	0 bps	5	NA	9,07 %	3,80 %	1,94	188	10,38 %	3,67 %	2,37	246
19	4:2	5 bps	2	2 %	-25,72 %	4,85 %	-5,66	564	-31,93 %	3,22 %	-10,43	546
20	4:2	5 bps	2	NA	-20,31 %	4,53 %	-4,86	566	-31,64 %	3,21 %	-10,40	548
21	4:2	5 bps	2,5	2 %	-21,64 %	4,50 %	-5,20	434	-25,46 %	3,13 %	-8,69	456
22	4:2	5 bps	2,5	NA	-15,75 %	4,03 %	-4,34	436	-25,29 %	3,09 %	-8,75	456
23	4:2	5 bps	5	2 %	2,25 %	4,36 %	0,12	193	-10,11 %	2,19 %	-5,40	174
24	4:2	5 bps	5	NA	7,39 %	4,51 %	1,26	193	-10,09 %	2,19 %	-5,40	174
25	4:2	2,5 bps	2	2 %	-3,75 %	5,11 %	-1,07	564	-12,55 %	3,34 %	-4,27	546
26	4:2	2,5 bps	2	NA	3,34 %	4,87 %	0,34	566	-12,10 %	3,32 %	-4,16	548
27	4:2	2,5 bps	2,5	2 %	-4,36 %	4,59 %	-1,32	434	-8,12 %	3,22 %	-3,05	456
28	4:2	2,5 bps	2,5	NA	2,91 %	4,20 %	0,29	436	-7,90 %	3,18 %	-3,02	456
29	4:2	2,5 bps	5	2 %	11,71 %	4,52 %	2,21	193	-2,64 %	2,19 %	-1,99	174
30	4:2	2,5 bps	5	NA	17,33 %	4,68 %	3,33	193	-2,62 %	2,19 %	-1,98	174
31	4:2	0 bps	2	2 %	24,71 %	5,43 %	4,23	564	12,34 %	3,53 %	3,01	546
32	4:2	0 bps	2	NA	34,01 %	5,28 %	6,12	566	13,02 %	3,50 %	3,24	548
33	4:2	0 bps	2,5	2 %	16,72 %	4,72 %	3,18	434	13,25 %	3,37 %	3,42	456
34	4:2	0 bps	2,5	NA	25,70 %	4,42 %	5,43	436	13,51 %	3,33 %	3,55	456
35	4:2	0 bps	5	2 %	22,04 %	4,71 %	4,32	193	5,44 %	2,22 %	1,68	174
36	4:2	0 bps	5	NA	28,18 %	4,89 %	5,41	193	5,46 %	2,21 %	1,69	174

Configuration: Portfolio configuration number
Format: Formation and trading period in days
TC: Transaction cost per trade in basis points
k: Upper and lower band for opening a position in standard deviations
SL: Stop-loss (NA equals no stop-loss)
Return: Annualized return
SD: Annualized standard deviation for returns
SR: Annualized Sharpe ratio
Roundtrips: Number of total roundtrips

5.2 Top 5 pairs unrestricted case

The highest return generated from our top 5 pairs in the unrestricted case is portfolio configuration 13. Using varying thresholds and generating annualized return of 39.31% with a standard deviation of 5.18%, we obtain a Sharpe ratio of 7.26. This is achieved through 751 roundtrips. The worst performing portfolio is, not surprisingly, the portfolio with the same configuration when adding transaction costs of 5 bps and with no stop-loss, generating an annualized return of -62.29% and a standard deviation of 4.43%.

When *k* decreases, more positions are opened. As these trades are profitable by themselves they do not generate sustainable returns to justify the transaction costs added. When we stretch the trading bands outwards, using higher *k*, the number of total roundtrips naturally decreases. Moreover, we achieve positive annualized returns of 7.39% with a standard deviation of 4.51% for portfolio configuration 24 (5 bps), with no stop-loss using static entry-thresholds, resulting in an annualized Sharpe ratio of 1.26. The same portfolio with added stop-loss, returns 2.25% and a standard deviation of 4.36%. In this case, the stop-loss exits profitable trades prematurely. Both these portfolios achieved 193 roundtrips in the 55-day period.

Something we find peculiar with our results are the impact of the stop-loss limit. There are several occasions like the scenario above, where only one stop-loss is triggered in the same period. Making the total roundtrips equal in both cases. When these pairs are stopped out, a common outcome is that the pair would have converged, making the total profit greater, or potential loss smaller. This makes the stop-loss trigger arbitrary. Furthermore, the results suggest a negative effect of restricting pairs that are stopped out, not being able to open new positions. As a result of this, we assume a stop-loss threshold of 2% to be too strict, as it does not provide enough room for pairs to converge. This assumption is mainly for frameworks with two trading days, as these stand out with the negative effect of our stop-loss trigger. A possible solution could be higher stop-loss thresholds to ensure the pair actually have diverged sufficient enough where we could assume there are no longer an equilibrium relationship. With higher stop-loss thresholds we are unsure if restriction of new positions in closed pairs should be applied. However, if the pair converges to equilibrium, there is likely that profits can be made on new roundtrips.

As shown in Table 1 the highest number of roundtrips in the top 5 unrestricted case is 751 as opposed to the lowest number of roundtrips of 174, with varying thresholds of k equal to 2 and k equal to 5. When the number of roundtrips increases, the impact of transaction costs on returns rise. This is discussed more thoroughly in the following sub-section discussing break-even points for the return and transaction cost relationship of specific portfolios.

5.3 Top 20 pairs unrestricted case

The following Table 2 shows the portfolios of top 20 pairs in the unrestricted case. The table is read in the same fashion as Table 1. Note that increasing the number of pairs in our portfolios decreases the standard deviations, giving a diversification effect in our portfolios and yielding better Sharpe ratios. The portfolio yielding the highest return now shifts to static thresholds yielding an annualized return of 35.01% with a standard deviation of 3.57%, with portfolio configuration 13. However, the same portfolio using varying thresholds return 33.50% with a standard deviation of 2.91%, achieving a Sharpe ratio of 10.94 making this the preferred portfolio of all portfolios.

As in the top 5 no restriction case, returns diminish as we add a fixed transaction cost. In fact, only four portfolios yield annualized returns above zero, as opposed to six in the top 5 pairs with added transaction costs. These four portfolios all have the same configuration, 29 and 30, using both static and varying entry-thresholds. The highest annualized return is 5.77% with a Sharpe ratio of 1.23 using static thresholds and no stop-loss.

The worst performing portfolio is with varying thresholds, k equal to 2, one trading day, and 5 bps added transaction costs are applied. These portfolio configurations generate large losses, making them unfit for trading. Naturally, as the number of pairs in our portfolio increases, the number of roundtrips simultaneously increases. The portfolio with the highest number of total roundtrips is 3 733 roundtrips, which translates to 14 932 individual trades in the 55-day period.

Table 2: Annualized risk and returns measures for top 20 pairs in the no restriction case using static and varying entry thresholds

	Parameters				Static Thresholds				Varying Thresholds			
Configuration	Format	TC	k	SL	Return	SD	SR	Roundtrips	Return	SD	SR	Roundtrips
1	2:1	5 bps	2	2 %	-49,23 %	3,21 %	-15,85	2245	-64,26 %	2,61 %	-25,32	3028
2	2:1	5 bps	2	NA	-49,55 %	3,29 %	-15,60	2245	-64,79 %	2,71 %	-24,53	3048
3	2:1	5 bps	2,5	2 %	-40,44 %	3,35 %	-12,57	1806	-54,55 %	2,47 %	-22,79	2415
4	2:1	5 bps	2,5	NA	-40,96 %	3,38 %	-12,63	1806	-55,12 %	2,59 %	-21,97	2430
5	2:1	5 bps	5	2 %	-21,91 %	2,64 %	-8,94	746	-27,20 %	1,69 %	-17,09	954
6	2:1	5 bps	5	NA	-22,23 %	2,72 %	-8,80	746	-27,99 %	1,73 %	-17,15	954
7	2:1	2,5 bps	2	2 %	-17,20 %	3,36 %	-5,64	2245	-30,92 %	2,69 %	-12,13	3028
8	2:1	2,5 bps	2	NA	-17,73 %	3,42 %	-5,69	2245	-31,65 %	2,76 %	-12,08	3048
9	2:1	2,5 bps	2,5	2 %	-11,74 %	3,46 %	-3,89	1806	-23,13 %	2,58 %	-9,64	2415
10	2:1	2,5 bps	2,5	NA	-12,51 %	3,48 %	-4,09	1806	-23,85 %	2,67 %	-9,57	2430
11	2:1	2,5 bps	5	2 %	-8,16 %	2,57 %	-3,83	746	-10,42 %	1,74 %	-6,99	954
12	2:1	2,5 bps	5	NA	-8,52 %	2,65 %	-3,87	746	-11,39 %	1,79 %	-7,32	954
13	2:1	0 bps	2	2 %	35,01 %	3,57 %	9,33	2245	33,50 %	2,91 %	10,94	3028
14	2:1	0 bps	2	NA	34,16 %	3,62 %	8,96	2245	32,66 %	2,94 %	10,52	3048
15	2:1	0 bps	2,5	2 %	30,77 %	3,60 %	8,07	1806	29,97 %	2,74 %	10,31	2415
16	2:1	0 bps	2,5	NA	29,62 %	3,62 %	7,70	1806	29,18 %	2,81 %	9,78	2430
17	2:1	0 bps	5	2 %	8,02 %	2,55 %	2,48	746	10,21 %	1,81 %	4,70	954
18	2:1	0 bps	5	NA	7,59 %	2,61 %	2,25	746	9,03 %	1,87 %	3,90	954
19	4:2	5 bps	2	2 %	-28,42 %	3,81 %	-7,91	2173	-44,08 %	3,37 %	-13,57	3655
20	4:2	5 bps	2	NA	-24,02 %	3,44 %	-7,47	2196	-43,15 %	3,37 %	-13,30	3733
21	4:2	5 bps	2,5	2 %	-25,33 %	3,48 %	-7,78	1701	-34,45 %	3,32 %	-10,88	2914
22	4:2	5 bps	2,5	NA	-20,77 %	3,31 %	-6,78	1718	-33,39 %	3,29 %	-10,67	2962
23	4:2	5 bps	5	2 %	-6,89 %	3,19 %	-2,70	759	-10,79 %	2,81 %	-4,44	1086
24	4:2	5 bps	5	NA	-3,08 %	3,26 %	-1,47	761	-7,84 %	2,73 %	-3,49	1088
25	4:2	2,5 bps	2	2 %	-8,08 %	3,86 %	-2,54	2173	-14,92 %	3,46 %	-4,81	3655
26	4:2	2,5 bps	2	NA	-2,17 %	3,50 %	-1,11	2196	-12,73 %	3,40 %	-4,25	3733
27	4:2	2,5 bps	2,5	2 %	-9,19 %	3,45 %	-3,16	1701	-8,40 %	3,41 %	-2,97	2914
28	4:2	2,5 bps	2,5	NA	-3,46 %	3,34 %	-1,55	1718	-6,42 %	3,36 %	-2,42	2962
29	4:2	2,5 bps	5	2 %	1,59 %	3,19 %	-0,04	759	1,05 %	2,87 %	-0,23	1086
30	4:2	2,5 bps	5	NA	5,77 %	3,30 %	1,23	761	4,41 %	2,81 %	0,96	1088
31	4:2	0 bps	2	2 %	18,03 %	3,96 %	4,13	2173	29,46 %	3,63 %	7,65	2655
32	4:2	0 bps	2	NA	25,94 %	3,62 %	6,69	2196	33,95 %	3,52 %	9,15	3733
33	4:2	0 bps	2,5	2 %	10,43 %	3,46 %	2,52	1701	27,99 %	3,53 %	7,43	2914
34	4:2	0 bps	2,5	NA	17,62 %	3,41 %	4,67	1718	31,45 %	3,48 %	8,55	2962
35	4:2	0 bps	5	2 %	10,83 %	3,22 %	2,83	759	14,45 %	2,95 %	4,32	1086
36	4:2	0 bps	5	NA	15,42 %	3,36 %	4,08	761	18,27 %	2,90 %	5,72	1088

Configuration: Portfolio configuration number
Format: Formation and trading period in days
TC: Transaction cost per trade in basis points
k: Upper and lower band for opening a position in standard deviations
SL: Stop-loss (NA equals no stop-loss)
Return: Annualized return
SD: Annualized standard deviation for returns
SR: Annualized Sharpe ratio
Roundtrips: Number of total roundtrips

5.4 Restricted case

The matching of pairs in the unrestricted case is purely based on the minimum-distance criterion. Thus, we may find stocks from different sectors being matched and paired together. While this is not necessarily problematic, we may see that stocks that are exposed to common factors express similar return patterns and makes them more likely to fit in a pairs trading strategy (Gatev et al. 1999; 2006). To find suitable pairs within the same sector we use the Global Industry Classification Standard (GICS), which classify companies both quantitatively and qualitatively into four tiers. The standard sorts securities in 11 sectors, 24 industry groups, 68 industries and 157 sub-industries. In this empirical study we implement GICS sector restrictions in our pairs formation.

5.5 Top 5 restricted case

Several papers report that sector restrictions improve return from different pairs trading strategies (Do and Faff, 2010 and Gundersen, 2014). Table 3 shows top 5 pairs where formation and trading between stocks are restricted by the 11 sectors Consumer Discretionary, Consumer staples, Energy, Financials, Health care, Industrials, Information technology, Materials, Real estate, Telecommunication services, and Utilities. The table is set up in the same manner as Table 1 and 2.

In Table 3 we see that configuration 32 yields the highest return and most favorable Sharpe ratio with portfolio using varying thresholds. The annualized return is 40.63% with a Sharpe ratio of 8.37. Comparing this result with the top performer from the top 5 unrestricted case, we see that the return is only slightly higher from the restricted case while the standard deviation has dropped 0.53%-points. Overall, the difference between the results are not considerable, although the number of portfolios yielding a positive return rise from six to eight. Many of the companies match in the unrestricted case as well as in the restricted case. These companies are exposed to low cross-differences and variances are likely to be formed in both cases (Gatev et al., 1999; 2006)

Interestingly, it is worth noting that portfolio configuration 24 still yield positive returns using static thresholds, but with a lower annualized return of 2.59% and a standard deviation of 3.64%. This is also the case for the other portfolios with added transaction costs. However, two of the portfolios applying varying threshold also yields return above zero.

Table 3: Annualized risk and returns measures for top 5 pairs in the restricted case using static and varying entry thresholds

	Parameters				Static Thresholds				Varying Thresholds			
Configuration	Format	TC	k	SL	Return	SD	SR	Roundtrips	Return	SD	SR	Roundtrips
1	2:1	5 bps	2	2 %	-46,43 %	4,51 %	-10,68	526	-64,63 %	4,35 %	-15,25	747
2	2:1	5 bps	2	NA	-46,74 %	4,58 %	-10,59	526	-65,24 %	4,33 %	-15,46	748
3	2:1	5 bps	2,5	2 %	-36,47 %	4,58 %	-8,34	429	-56,48 %	4,23 %	-13,76	625
4	2:1	5 bps	2,5	NA	-37,33 %	4,64 %	-8,41	429	-57,16 %	4,20 %	-14,01	626
5	2:1	5 bps	5	2 %	-14,21 %	4,04 %	-3,94	178	-29,73 %	3,54 %	-8,87	244
6	2:1	5 bps	5	NA	-15,29 %	4,11 %	-4,14	178	-30,68 %	3,62 %	-8,95	244
7	2:1	2,5 bps	2	2 %	-15,32 %	4,80 %	-3,55	526	-32,24 %	4,39 %	-7,72	747
8	2:1	2,5 bps	2	NA	-15,81 %	4,87 %	-3,60	526	-33,35 %	4,37 %	-8,02	748
9	2:1	2,5 bps	2,5	2 %	-7,71 %	4,69 %	-2,01	429	-25,03 %	4,37 %	-6,12	625
10	2:1	2,5 bps	2,5	NA	-8,96 %	4,74 %	-2,25	429	-26,13 %	4,34 %	-6,42	626
11	2:1	2,5 bps	5	2 %	0,16 %	3,94 %	-0,39	178	-13,12 %	3,48 %	-4,26	244
12	2:1	2,5 bps	5	NA	-1,11 %	4,00 %	-0,70	178	-14,29 %	3,54 %	-4,51	244
13	2:1	0 bps	2	2 %	33,83 %	5,19 %	6,19	526	29,80 %	4,72 %	5,95	747
14	2:1	0 bps	2	NA	33,05 %	5,26 %	5,96	526	27,78 %	4,69 %	5,55	748
15	2:1	0 bps	2,5	2 %	34,03 %	4,88 %	6,63	429	29,11 %	4,65 %	5,89	625
16	2:1	0 bps	2,5	NA	32,23 %	4,92 %	6,21	429	27,33 %	4,61 %	5,56	626
17	2:1	0 bps	5	2 %	16,91 %	3,88 %	3,92	178	7,41 %	3,46 %	1,65	244
18	2:1	0 bps	5	NA	15,44 %	3,94 %	3,48	178	5,97 %	3,52 %	1,21	244
19	4:2	5 bps	2	2 %	-24,66 %	4,56 %	-5,78	556	-42,44 %	4,52 %	-9,76	918
20	4:2	5 bps	2	NA	-20,33 %	4,40 %	-5,01	557	-39,85 %	4,11 %	-10,10	926
21	4:2	5 bps	2,5	2 %	-20,61 %	4,25 %	-5,26	416	-32,62 %	4,28 %	-8,03	732
22	4:2	5 bps	2,5	NA	-15,67 %	3,92 %	-4,43	417	-29,86 %	3,79 %	-8,32	737
23	4:2	5 bps	5	2 %	-1,41 %	3,46 %	-0,90	172	-9,53 %	2,91 %	-3,86	272
24	4:2	5 bps	5	NA	2,59 %	3,64 %	0,24	172	-7,60 %	2,63 %	-3,54	272
25	4:2	2,5 bps	2	2 %	-2,73 %	4,91 %	-0,90	556	-12,31 %	4,75 %	-2,95	918
26	4:2	2,5 bps	2	NA	2,89 %	4,84 %	0,24	557	-8,02 %	4,28 %	-2,28	926
27	4:2	2,5 bps	2,5	2 %	-3,90 %	4,43 %	-1,27	416	-5,73 %	4,47 %	-1,67	732
28	4:2	2,5 bps	2,5	NA	2,12 %	4,19 %	0,10	417	-1,65 %	3,96 %	-0,85	737
29	4:2	2,5 bps	5	2 %	6,69 %	3,53 %	1,41	172	2,49 %	2,98 %	0,26	272
30	4:2	2,5 bps	5	NA	11,00 %	3,75 %	2,48	172	4,67 %	2,72 %	1,09	272
31	4:2	0 bps	2	2 %	25,55 %	5,34 %	4,47	556	33,60 %	5,17 %	6,17	918
32	4:2	0 bps	2	NA	32,87 %	5,34 %	5,83	557	40,63 %	4,65 %	8,37	926
33	4:2	0 bps	2,5	2 %	16,30 %	4,66 %	3,13	416	31,87 %	4,76 %	6,34	732
34	4:2	0 bps	2,5	NA	23,64 %	4,50 %	4,87	417	37,88 %	4,23 %	8,56	737
35	4:2	0 bps	5	2 %	15,44 %	3,63 %	3,79	172	16,09 %	3,10 %	4,64	272
36	4:2	0 bps	5	NA	20,11 %	3,90 %	4,72	172	18,56 %	2,88 %	5,86	272

Configuration: Portfolio configuration number
Format: Formation and trading period in days
TC: Transaction cost per trade in basis points
k: Upper and lower band for opening a position in standard deviations
SL: Stop-loss (NA equals no stop-loss)
Return: Annualized return
SD: Annualized standard deviation for returns
SR: Annualized Sharpe ratio
Roundtrips: Number of total roundtrips

5.6 Top 20 Restricted case

Table 4 is interpreted in the same way as the previous tables. The top performing portfolio now use static thresholds with portfolio configuration 32. This portfolio yields an annualized return of 33.78%, with a standard deviation of 3.26%, and obtains a Sharpe ratio of 9.84. The total number of portfolios yielding a positive return with added transaction costs is still eight, although the configuration has shifted. The portfolio using static thresholds with portfolio configuration 27, yields a return 0.06% per annum, standard deviation of 2.84%, through 1 606 roundtrips. The same portfolio without stop-loss returns 5.09% with almost the same standard deviation of 2.89% and 13 more roundtrips.

Table 4: Annualized risk and returns measures for top 20 pairs in the restricted case using static and varying entry thresholds

	Parameters				Static Thresholds				Varying Thresholds			
Configuration	Format	TC	k	SL	Return	SD	SR	Roundtrips	Return	SD	SR	Roundtrips
1	2:1	5 bps	2	2 %	-45,89 %	2,77 %	-17,21	2062	-65,89 %	2,79 %	-24,22	3055
2	2:1	5 bps	2	NA	-46,42 %	2,88 %	-16,74	2062	-66,36 %	2,81 %	-24,23	3069
3	2:1	5 bps	2,5	2 %	-35,94 %	2,91 %	-12,96	1542	-55,99 %	2,68 %	-21,57	2468
4	2:1	5 bps	2,5	NA	-36,78 %	2,99 %	-12,87	1652	-56,42 %	2,70 %	-21,50	2478
5	2:1	5 bps	5	2 %	-15,64 %	2,32 %	-7,49	644	-29,22 %	1,66 %	-18,66	953
6	2:1	5 bps	5	NA	-17,09 %	2,42 %	-7,76	644	-29,77 %	1,70 %	-18,52	953
7	2:1	2,5 bps	2	2 %	-15,21 %	2,96 %	-5,72	2062	-33,69 %	2,86 %	-12,40	3055
8	2:1	2,5 bps	2	NA	-16,05 %	3,06 %	-5,79	2062	-34,41 %	2,86 %	-12,63	3069
9	2:1	2,5 bps	2,5	2 %	-8,22 %	3,08 %	-3,22	1652	-24,71 %	2,78 %	-9,51	2468
10	2:1	2,5 bps	2,5	NA	-9,42 %	3,17 %	-3,52	1652	-25,29 %	2,79 %	-9,66	2478
11	2:1	2,5 bps	5	2 %	-2,95 %	2,30 %	-2,03	644	-12,92 %	1,65 %	-8,86	953
12	2:1	2,5 bps	5	NA	-4,63 %	2,41 %	-2,63	644	-13,60 %	1,69 %	-9,04	953
13	2:1	0 bps	2	2 %	32,82 %	3,22 %	9,66	2062	28,89 %	3,05 %	8,90	3055
14	2:1	0 bps	2	NA	31,51 %	3,32 %	8,98	2062	27,88 %	3,04 %	8,60	3069
15	2:1	0 bps	2,5	2 %	31,48 %	3,31 %	8,99	1652	28,78 %	2,93 %	9,23	2468
16	2:1	0 bps	2,5	NA	29,75 %	3,39 %	8,27	1652	28,06 %	2,94 %	8,96	2478
17	2:1	0 bps	5	2 %	11,63 %	2,34 %	4,24	644	7,12 %	1,67 %	3,23	953
18	2:1	0 bps	5	NA	9,70 %	2,44 %	3,27	644	6,29 %	1,71 %	2,67	953
19	4:2	5 bps	2	2 %	-22,82 %	2,65 %	-9,26	2084	-44,93 %	2,70 %	-17,28	3561
20	4:2	5 bps	2	NA	-17,49 %	2,89 %	-6,65	2103	-43,75 %	2,48 %	-18,33	3609
21	4:2	5 bps	2,5	2 %	-16,80 %	2,81 %	-6,58	1606	-35,88 %	2,60 %	-14,48	2860
22	4:2	5 bps	2,5	NA	-12,75 %	2,78 %	-5,21	1619	-34,60 %	2,41 %	-15,06	2892
23	4:2	5 bps	5	2 %	0,57 %	2,43 %	-0,47	649	-13,33 %	2,13 %	-7,07	1110
24	4:2	5 bps	5	NA	1,96 %	2,56 %	0,10	649	-11,05 %	1,93 %	-6,62	1110
25	4:2	2,5 bps	2	2 %	-1,93 %	2,73 %	-1,33	2084	-17,11 %	2,85 %	-6,61	3561
26	4:2	2,5 bps	2	NA	5,07 %	3,04 %	1,10	2103	-14,88 %	2,59 %	-6,42	3609
27	4:2	2,5 bps	2,5	2 %	0,06 %	2,84 %	-0,58	1606	-10,96 %	2,74 %	-4,62	2860
28	4:2	2,5 bps	2,5	NA	5,09 %	2,89 %	1,17	1619	-8,85 %	2,52 %	-4,19	2892
29	4:2	2,5 bps	5	2 %	8,34 %	2,51 %	2,64	649	-1,56 %	2,20 %	-1,49	1110
30	4:2	2,5 bps	5	NA	9,83 %	2,65 %	3,06	649	1,02 %	1,99 %	-0,34	1110
31	4:2	0 bps	2	2 %	24,61 %	2,89 %	7,93	2084	24,76 %	3,09 %	7,47	3561
32	4:2	0 bps	2	NA	33,78 %	3,26 %	9,84	2103	28,81 %	2,79 %	9,70	3609
33	4:2	0 bps	2,5	2 %	20,32 %	2,92 %	6,37	1606	23,64 %	2,92 %	7,51	2860
34	4:2	0 bps	2,5	NA	26,55 %	3,04 %	8,17	1619	27,03 %	2,67 %	9,48	2892
35	4:2	0 bps	5	2 %	16,70 %	2,61 %	5,75	649	11,80 %	2,29 %	4,41	1110
36	4:2	0 bps	5	NA	18,32 %	2,77 %	6,00	649	14,73 %	2,08 %	6,25	1110

Configuration: Portfolio configuration number
Format: Formation and trading period in days
TC: Transaction cost per trade in basis points
k: Upper and lower band for opening a position in standard deviations
SL: Stop-loss (NA equals no stop-loss)
Return: Annualized return
SD: Annualized standard deviation for returns
SR: Annualized Sharpe ratio
Roundtrips: Number of total roundtrips

First, we note that all portfolio configurations with transaction cost and 2:1 formation-trading period consistently yields negative returns, except for portfolio configuration 11 using static thresholds in the unrestricted top 5 case, which yield an annualized return of 0.16% and a standard deviation of close to four. This shows that two days is not enough time to measure and quantify the distance, i.e. the co-movement of stocks, or that one day is not enough time to allow the prices to adjust in the market. Thus, profitable positions will be unwound at the close of the trading day prematurely of convergence. However, Bowen et al. (2010) shows that most of the return from high frequency pairs trading arise from the first hour of trading.

Moreover, if the market is strongly efficient the price history is not an estimator for the future. In addition, there is substantial trading after the traditional close at 16:00. This is not captured by our algorithm, making a gap in our data set when calculating the distance between pairs. This also affects the calculation of the rolling mean and entry standard deviation calculation, which in turn may have an impact on our formation periods, trading periods and results.

Our sector restriction does not provide more profitable results than the unrestricted case. This indicates formation of similar pairs as suggested by Gatev et al. (1999; 2006) or that GICS sector tier is not strict enough when optimizing pair selection approach. Suggesting more fundamental factors should be applied when restricting formation.

In the next subsection we pick out some portfolios where we report some more thorough risk and return measures and exposures. We also estimate the break-even point for transaction costs, as transaction cost vary from retail to institutional investors, liquidity, timing and size of the order, and market conditions (Liu et al., 2017). Therefore, it is interesting to find out how high transaction costs one can have before the strategies are unsustainable. As we have shown from our results there are obviously profits to be made from pairs trading when subtracting additional transaction costs. Thus, making the optimal portfolio depends on what kind of transaction costs and entry-thresholds the investor can achieve and apply.

5.7 Risk and return characteristics of specific portfolio configuration

In Table 5 we report daily return and risk measurements in the spirit of Stübinger and Bredthauer (2017) in all trading periods. We consider a conservative portfolio configuration opening a position when the spread deviates from its calculated mean with standard deviation of 2.5 for both static (S) and varying (V) thresholds, adding a 10 basis-points transaction cost per roundtrip.

The formation period consists of four days followed by a two-day trading period. The table consists of top 5 (T5) and top 20 (T20) portfolios, where pairs both can be paired unrestricted (NR) and restricted (R). We compare this to a buy and hold strategy in the S&P 500-index in the same period.

Table 5: Daily return characteristic and risk metrics for top 5 and top 20 pairs with static and varying entry thresholds of k(2.5), formation and trading period of 4:2, transaction cost 2.5 bps and no stop-loss compared S&P-long returns in the period December 12. 2017 to March 13. 2018

Measure	T5 NR S	T5 NR V	T5 R S	T5 R V	T20 NR S	T20 NR V	T20 R S	T20 R V	S&P
Mean return	0,0001	-0,0003	0,0001	-0,0001	-0,0001	-0,0003	0,0002	-0,0004	0,0007
SE mean return	0,0003	0,0002	0,0003	0,0002	0,0002	0,0002	0,0002	0,0002	0,0015
t-statistic	0,4587	-1,7046	0,3373	-0,2710	-0,6908	-1,2983	1,1417	-2,4249	0,4941
Median	0,0000	-0,0003	0,0000	0,0002	-0,0002	-0,0004	0,0002	-0,0004	0,0015
Minimum	-0,0047	-0,0032	-0,0047	-0,0045	-0,0037	-0,0030	-0,0026	-0,0027	-0,0408
Maximum	0,0048	0,0047	0,0047	0,0028	0,0042	0,0040	0,0047	0,0019	0,0174
Quartile 1	-0,0009	-0,0013	-0,0010	-0,0011	-0,0009	-0,0012	-0,0004	-0,0011	-0,0014
Quartile 3	0,0013	0,0003	0,0014	0,0012	0,0006	0,0004	0,0007	0,0002	0,0071
Standard deviation	0,0019	0,0014	0,0019	0,0018	0,0015	0,0015	0,0013	0,0011	0,0112
Skewness	0,0964	0,7843	0,0125	-0,5755	0,4333	0,7596	0,8197	0,1629	-1,5722
Kurtosis	0,3212	1,6971	0,1508	-0,0716	1,1501	0,6927	2,1262	-0,4926	3,8135
Historical VaR 1%	-0,0042	-0,0030	-0,0043	-0,0047	-0,0034	-0,0028	-0,0023	-0,0027	-0,0379
Historical VaR 5%	-0,0029	-0,0022	-0,0029	-0,0032	-0,0023	-0,0023	-0,0015	-0,0022	-0,0211
Historical Cvar 1%	-0,0051	-0,0030	-0,0051	-0,0059	-0,0040	-0,0031	-0,0023	-0,0031	-0,0545
Historical Cvar 5%	-0,0037	-0,0026	-0,0038	-0,0040	-0,0029	-0,0028	-0,0018	-0,0025	-0,0336
Maximum drawdown	0,0241	0,0353	0,0201	0,0277	0,0214	0,0332	0,0110	0,0427	0,1013

Note: VaR and CVaR describes the daily historical value at risk and the conditional value at risk respectively, i.e. tail risk and expected shortfall, at specific probability levels. Maximum drawdown describe max decline from peak to a through.

Portfolios using varying thresholds all yield an average daily return below zero. The top performing portfolios in respect of return is the top 5 pairs both the restricted and unrestricted case, along with the top 20 pairs from the restricted case. All using static thresholds to enter trades in these specific portfolios. S&P 500 yields an average daily return of 0.07% (t-stat = 0.494) which is clearly higher than our best performing portfolio T 20 R S with an average daily return of 0.02% (t-stat = 1.142).

Only one of the portfolios is statistically significant different from zero, T20 R V, with a t-stat of -2,4249, indicating a return significantly different from zero. Moreover, like Gatev et al. (1999; 2006) we note that all our returns are positively skewed in respect of the normal distribution. Goetzmann, Ingersoll, Spiegel and Welch (2004) show that negative skewness may bias the Sharpe ratio upwards. In these specific portfolios this imply that our Sharpe ratios is biased downwards. The excess kurtosis in our distributions is fairly small compared to the S&P in the same period. All tails in our sample show a leptokurtic distribution indicating long, skinny tails.

The value at risk at different probability levels depict that our strategies have far less daily value at risk compared to the S&P 500. The maximum drawdown which measures the decline from a global peak in the portfolio to a through. The S&P 500 in this period have a 10.13% decline compared to T20 R V where the maximum drawdown is 4.27%.

Table 6 illustrate trading and holding statistics from the portfolios mentioned above. We see that return on employed capital increase as the number of pairs never traded increase, as described in return computation subsection. Number of roundtrips range from 436 to 737 in the top 5 pairs, and from 1718 to 2962 in the top 20 pairs. In the portfolio T20 R V we see that from one single roundtrip the maximum profit is 8.57% and the maximum loss is 7.22%. Interestingly, we see a clear pattern of average minutes of active pairs and pairs closed by force at the end of a trading period in respect of static and varying thresholds. This indicates that static thresholds signal convergence less often, as opposed to the dynamic threshold, leading to higher horizon risk.

Table 6: Trading and holding characteristic for top 5 and top 20 pairs with static and varying entry thresholds of k(2.5), formation and trading period of 4:2, transaction cost 2.5 bps and no stop-loss in the period December 12. 2017 to March 13. 2018

Feature	T5 NR S	T5 NR V	T5 R S	T5 R V	T20 NR S	T20 NR V	T20 R S	T20 R V
Period cummulative return (committed)	0,0126	-0,0353	0,0092	-0,0072	-0,0152	-0,0286	0,0219	-0,0396
Period standard deviation (commited)	0,0037	0,0028	0,0037	0,0035	0,0030	0,0030	0,0026	0,0022
Period cummulative return (employed)	0,0147	-0,0363	0,0095	-0,0072	-0,0150	-0,0286	0,0250	-0,0397
Period standard deviation (employed)	0,0041	0,0029	0,0041	0,0035	0,0031	0,0030	0,0027	0,0022
Number of roundtrips	436	456	417	737	1718	2962	1619	2892
Number of pairs never traded	14	2	19	0	52	1	78	2
Maximum roundtrip profit	0,0144	0,0301	0,0144	0,0301	0,1331	0,1331	0,0794	0,0857
Maximum roundtrip loss	-0,0384	-0,0205	-0,0384	-0,0314	-0,0853	-0,0722	-0,0538	-0,0722
Pairs closed by time (percentage)	0,4771	0,4145	0,4916	0,2687	0,4936	0,2606	0,5022	0,2732
Average minutes of open pairs	327	153	321	191	329	187	329	196

To test the excess return of the best portfolio, T20 R S, against traditional systematic risk factors summarized in Table 7. As Stübinger and Bredthauer (2017), we use three different type of regressions based on different factors. First, the standard three-factor model (FF3) Fama and French (1996), where the sensitivity to the market, the difference between small-cap and large-cap stocks (SMB) and the difference between stocks with high book-to-market ratio and low book-to-market ratio (HML) are captured. Second, as Gatev et al. (1999; 2006), we regress the same three factors above in addition to a momentum factor and a short-term reversal factor (FF3+2). Finally, as described in Fama and French (2015), we extend the FF3 model (FF5) with a factor of robust minus weak profitability stocks (RMW), and a factor of a portfolio with companies with conservative minus aggressive investment behavior (CMA)[3].

[3] All data is provided and downloaded from Kenneth R. French’s website. (05.24.2018) http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

Table 7: Daily returns systematic risk exposure for T20, R, S, F(4:2), TC(2.5 bps), k(2.5) December 13. 2017 to March 12. 2018. T-statiststics is expressed in paranthesis.

Feature	FF3	FF3+2	FF5
Intercept	0,0186 (1,03)	0,0184 (1,04)	0,0172 (0,93)
Market	-0,0103 (-0,61)	-0,0136 (-0,78)	0,0010 (0,05)
SMB	0,0044 (-0,12)	0,0208 (0,58)	
HML	0,0577 (0,96)	0,1082 (1,70)	
Momentum		0,0752 (1,71)	
Reversal		0,0830 (1,93)	
SMB5			0,0176 (0,46)
HML5			0,1139 (1,67)
RMW5			0,0738 (1,09)
CMA5			-0,0967 (-1,19)
R^2	0,0241	0,1334	0,0799
Number of observations	55	55	55

SMB: Small Minus Big
HML: High minus low book-to-market ratio
RMW5: Robust minus weak
CMA: Conservative minus agressive

****P<0.001, **P<0.01, *P<0.05*

From Table 7 the returns show no significant loadings on common traditional risk factors, nor do they yield any significant alpha. However, the excess return show some, though insignificant, loading from the reversal, momentum and HML factor in FF3+2. Leading us to conclude that this strategy is not significantly exposed to traditional risk factors and does not express any significant excess return. The r-squared also depicts that the regression explain little of the excess return variance.

5.8 Zero return transaction cost estimation

Pairs trading profitability have been in a declining trend simultaneously with the decrease of transaction costs according to Gatev et al. (1999; 2006). It is essential that one considers transaction costs when optimizing a pairs trading strategy. Do and Faff (2012) report institutional commissions in recent years to be lower than 10 bps per half round trip. Comparing this to our configurations, this is equal 5 bps per transaction. They also point out that short fees are not included to their commission estimation. Likewise, Gatev et al. (1999; 2006) assumes shorting costs do not change the potential profits of pairs trading strategies for hedge funds and institutional investors as these fees are negligible. They also state that short fees are more relevant for retail investors. According to Bogomolov's (2013) calculations in the US equity market common commission costs per transaction for securities on the S&P500 are less than 10 basis points.

High frequency trading can constitute several thousand trades a year thus it is imperative how much transaction costs the different strategies can tolerate before being unprofitable. For the three best performing portfolios in the no-restriction and sector restriction case, with zero, 2.5 and 5 basis points transaction cost subtracted per trade we iterate returns to zero on a three-decimal basis.

Table 8 demonstrate estimations for break-even point and transaction costs added additional to our built-in proxy. We find that these specific portfolios yield close to zero annualized returns when adding 1.096 bps up to 7.015 bps per trade. Moreover, these results again illustrate the dependence of the number of roundtrips. This indicates, in line with Bowen et al. (2010), that the investor should adjust the strategy to which transaction cost level the one can achieve to find the optimal balance between transaction costs and the entry-threshold. As these levels are varying in time and in different market conditions, this may prove challenging.

Table 8: Break-even estimation points for top performing portfolios in relation to transaction costs

TC	Format	Threshold	k	SL	Return	SR	Roundtrips	Break-even TC	Return
				No Restriction Case					
0 bps	Top 20, 2:1	Varying	2	2 %	33,50 %	10,94	3028	1,096 bps	0,01 %
2,5 bps	Top 5, 4:2	Static	5	NA	17,33 %	3,33	193	7,015 bps	0,00 %
5 bps	Top 5, 4:2	Static	5	NA	7,39 %	1,26	193	7,015 bps	0,00 %
				Sector Restriction Case					
0 bps	Top 20, 4:2	Static	2	NA	33,78 %	9,84	2103	3,011 bps	0,00 %
2,5 bps	Top 20, 4:2	Static	5	NA	9,83 %	3,06	649	5,651 bps	0,00 %
5 bps	Top 5, 4:2	Static	5	NA	2,59 %	0,24	172	5,809 bps	0,00 %

TC: Transaction cost per trade in basis points
Format: Number of pair in portfolio, and formation and trading period in days
k: Upper and lower band for opening a position in standard deviations
SL: Stop-loss (NA equals no stop-loss)
Return: Annualized return
SR: Annualized Sharpe ratio
Roundtrips: Number of total roundtrips
Break-even TC : The transaction costs added to the strategy per trade in basis-points closest to zero

In Figure 3 below, we show the compounded return from the three top performing portfolios with respective transaction cost level of zero, 2.5 bps, and 5 bps. Compared to the return from S&P 500 in the same period. The portfolios show no excess return when adding transaction costs. In line with Stübinger and Bredthauer (2017) more recent results.

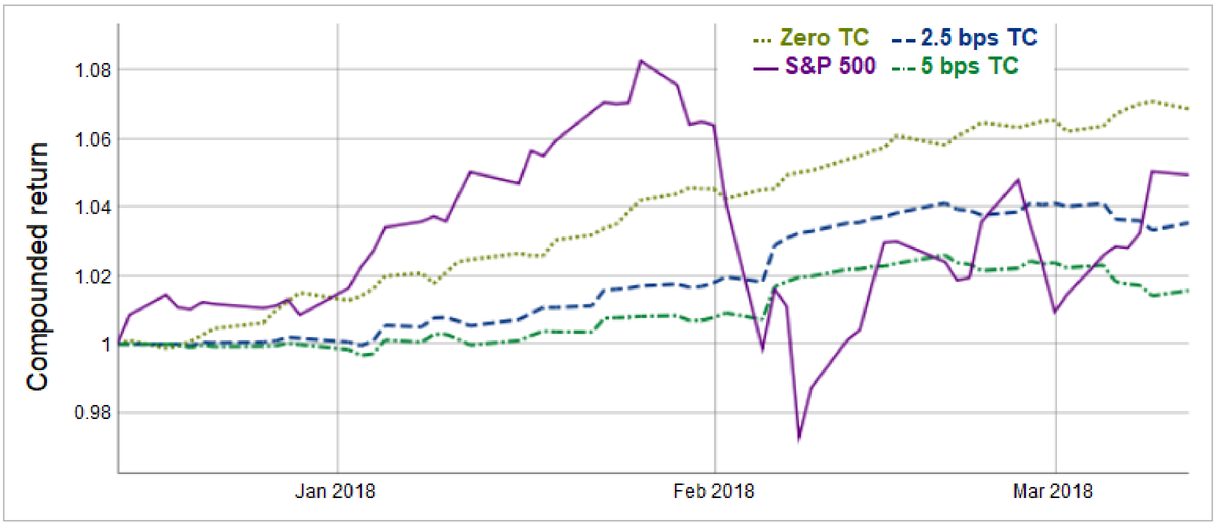

Figure 3: Top performing portfolios in respect of transaction cost level versus the S&P 500 -index from mid-December 2017 to mid-March 2018

5.9 Volatility timing

To optimize the pairs trading strategy, we implement a volatility timing for opening positions in co-moving pairs. Recent studies suggest pairs trading performing relatively well under market downfalls with a higher volatility level than adjacent periods (Do and Faff, 2010 and Stübinger and Bredthauer, 2017). Do and Faff's (2010) results suggest volatility levels do not significantly increase the profitability of pairs trading. Huck (2015) points out that Do and Faff's (2010) methodology can be improved by implementing volatility as trading thresholds for opening positions in pairs.

The Chicago Board Options Exchange (CBOE) Market Volatility Index (VIX) measures the estimated volatility of the S&P500 for the next 30-days. The index indicates the level of insurance hedged investors take when they expect a downfall in the market (Whaley, 2009). Following Huck (2015) we implement volatility timing to our pairs trading. The approach measures the percentage change of the VIX relative to a rolling mean of the index and use this as a threshold for opening positions. Four thresholds are determined:

1) If the percentage change of the VIX is below -10% (decreasing)
2) between -10% and 10% (stable)
3) above 10% (increasing)
4) above 20% or if the VIX index closed above 30

Calculating a rolling mean of the VIX index for the past 14 days. If the daily VIX impact closed at time $t-1$ within our pre-set threshold, a potential opening of positions would be possible in the following day. We apply the volatility timing on our most profitable portfolio with an annualized Sharp ratio of 10.94. The specific characteristics of the portfolio can be viewed in Table 2, configuration 13. The results are presented in Table 9.

Table 9: Trading, holding and risk metrics for our top performing portfolio with and without implementing volatility timing thresholds

	VIX Thresholds				
Measure	None	Decreasing	Stable	Increasing	High
Annualized return	33,50 %	2,00 %	23,45 %	6,02 %	5,93 %
Annualized Sharpe ratio	10,94	0,09	8,65	1,37	1,88
Roundtrips	3028	585	1615	828	491
Pairs closed by time (percentage)	0,2962	0,3231	0,2848	0,2995	0,2811
Average minutes of open pairs	109	118	105	110	104
Mean return	0,0011	0,0004	0,0016	0,0008	0,0015
SE mean return	0,0002	0,0006	0,0003	0,0005	0,0005
t-statistic	4,7812	0,6583	5,5978	1,7063	3,1275
Median	0,0014	0,0008	0,0017	0,0006	0,0010
Minimum	-0,0031	-0,0026	-0,0031	-0,0027	-0,0003
Maximum	0,0045	0,0034	0,0037	0,0045	0,0038
1. Quartile	0,0005	-0,0015	0,0012	-0,0001	0,0005
3. Quartile	0,0024	0,0018	0,0026	0,0020	0,0026
Stdev	0,0018	0,0020	0,0016	0,0020	0,0014
Skewness	-0,6603	-0,2362	-1,3945	0,0844	0,3465
Kurtosis	-0,0905	-1,4675	2,3873	-0,7905	-1,5506
Historical VaR 1%	-0,0037	-0,0039	-0,0034	-0,0033	-0,0008
Historical VaR 5%	-0,0022	-0,0030	-0,0014	-0,0023	-0,0006
Historical Cvar 1%	-0,0051	-0,0047	-0,0073	-0,0040	-0,0056
Historical Cvar 5%	-0,0029	-0,0033	-0,0027	-0,0029	-0,0008
Maximum drawdown	0,0031	0,0026	0,0031	0,0031	0,0003

Not surprisingly the compounded return on each volatility thresholds are lower opposed to no volatility timing. As they set limits on days of trading the total number of roundtrips decrease, leading to lower cumulative returns when no transaction costs are involved. As a consequence of this, it is more preferable to investigate the mean returns. Both Stable and High outperform our best portfolio without volatility timing-thresholds with lower standard deviations. In line with Huck (2015) there are more frequent trading signals for the Stable-threshold.

The roundtrips using High-threshold are less than a third when applying the Stable-threshold. As observed in subsection 5.8, a high volatility threshold would be more robust to transaction costs. These results coincide with Huck (2015) who report a slight increase in performance when implementing volatility timing as an entry-threshold.

Huck (2015) argues that a volatility timing could delay the optimal opening signal, if the thresholds are not in the correct state when the spread diverge crossing the upper or lower band. Our results when using volatility timing does not outperforming our best portfolio when annualizing the returns. Therefore, it would be disadvantageous to apply this as a sole trading threshold for opening positions. suggesting that an implementation of volatility thresholds may be used as a signal for leveraging up forthcoming positions. Although, Stein (2009) explains how arbitrageurs with high leverage levels may be forced to exit positions due to shock in their portfolios. Which in turn can create upwards pressure in arbitrage possibilities create further divergence.

6. Conclusion

We have tested the statistical arbitrage strategy pairs trading using high frequency data from the S&P 500 in the period December 13, 2017 to March 12, 2018. In our algorithm we implement a lag of one period to control for the bid-ask-bounce. Before controlling for additional transaction costs, our best portfolio yields a positive annualized return of 33.50%, with an annualized standard deviation of 2.91% obtaining a Sharpe ratio of 10.94. However, when adding a transaction costs of 2.5 bps per transaction, the best performing portfolio generate an annualized return of 17.33%, with a standard deviation of 4.68% providing a Sharpe ratio of 3.33. Furthermore, when subtracting 5 bps per transaction, the best portfolio achieved an annualized return of 7.39%, with a standard deviation of 4.51% providing a Sharpe ratio of 1.26.

We confirm Stübinger and Bredthauer's (2017) evidence of varying thresholds being superior to static thresholds. Implementation of pair formation restrictions have no effect on the profitability when applied to our pairs trading strategy. We find that that pairs trading reduces tail risk (VaR1% -0.27%) compared to the S&P 500 (VaR1% -3.79%) in the sample period. When testing a conservative portfolio excess returns to systematic traditional risk loading, using three different models, and find no significant exposure to these risk factors.

In line with Bowen et al. (2010) our findings suggest practitioners should adjust their pairs trading strategy given the amount of transaction costs they are eligible for. With high (low) transaction costs, higher (lower) entry-thresholds are preferable.

Further research could include a more fundamental approach finding optimal pairs. Additionally, investigate the profitability of implementing different volatility timing-thresholds as a signal for leveraging forthcoming positions in their trading strategy.

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