

$$\begin{aligned}
\frac{d}{dx}(f(x) + g(x)) &= \lim_{h \rightarrow 0} \frac{f(x+h) + g(x+h) - (f(x) + g(x))}{h} \\
&= \lim_{h \rightarrow 0} \frac{f(x+h) + g(x+h) - f(x) - g(x)}{h} \\
&= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x) + g(x+h) - g(x)}{h} \\
&= \lim_{h \rightarrow 0} \left( \frac{f(x+h) - f(x)}{h} + \frac{g(x+h) - g(x)}{h} \right) \\
&= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} + \lim_{h \rightarrow 0} \frac{g(x+h) - g(x)}{h} \\
&= f'(x) + g'(x).
\end{aligned}$$