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Online digital filters for biological signals: some fast designs for a small computer

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Abstract—*After reviewing the design of a class of lowpass recursive digital filters having integer multiplier and linear phase characteristics, the possibilities for extending the class to include high pass, bandpass, and bandstop ('notch') filters are described. Experience with a PDP 11 computer has shown that these filters may be programmed simply using machine code, and that online operation at sampling rates up to about 8 kHz is possible. The practical application of such filters is illustrated by using a notch design to remove mains-frequency interference from an e.c.g. waveform*

Keywords—*Digital filters, On-line computing, Signal processing, Notch filter*

1 Introduction

THE INCREASING use of digital computers in biomedical research has been accompanied by a growing awareness of the advantages of digital filters over their more conventional analogue counterparts for a variety of signal processing applications. These advantages include freedom from drift in filter performance due to ageing or temperature change, and the ease with which filter characteristics may be modified by adjusting a set of programmed numerical coefficients. In an early report on an important biomedical application, WEAVER *et al.* (1968) described a suite of computer programs for filtering e.c.g. data on an IBM 7090 computer. More recently, TAYLOR and MACFARLANE (1974) have investigated some of the practical problems of programming digital filters on a small computer such as the PDP8, and have discussed the relative speed of operation of a variety of lowpass filters applied, once again, to the processing of e.c.g. waveforms.

The speed of operation of a particular digital filter is likely to be a decisive factor when online working is required, and many otherwise attractive designs prove too slow for applications such as the real-time processing of e.c.g. or e.m.g. signals. The user of a small digital computer—even if it has high-level language facilities such as Basic or Fortran—will often find it necessary to return to machine-code programming to get a digital filter operating at anything like its maximum speed. Once in machine code, he will probably be armed with little more than the basic manipulations of binary arithmetic. At first sight, therefore, the possibilities for online filtering of biological signals on a small computer look distinctly limited.

However, there is one class of recursive digital filters which uses only small integer multipliers, and which is therefore both simple to program and fast in execution (LYNN, 1975). The processing of biological signals with lowpass filters of this type has been discussed both by LYNN (1971) and TAYLOR and MACFARLANE (1974), although without specific reference to online operation. The aim of this paper is to review the basis of design of such filters, to extend the class to include highpass, bandpass and bandstop characteristics, and finally to report some practical results obtained with an online PDP11 computer.

2 Class of lowpass recursive digital filters

The basis of design of the class of filters covered by this paper may be introduced by considering the filter recurrence formula

$$y_n = x_n - x_{n-m}$$

where y_n represents the current (filtered) output sample value from the filter, x_n represents the current (raw-data) input sample, and x_{n-m} represents the input sample delivered to the filter m sampling periods previously. This time-domain description of the filter may be converted into a transfer function using z-transform notation (JURY, 1964: BOGNER and CONSTANTINIDES, 1975). Thus

$$Y(z) = 1 . X(z) - z^{-m} . X(z),$$

from which the transfer function $G(z)$ is derived as

$$G(z) = \frac{Y(z)}{X(z)} = (1 - z^{-m})$$

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where $z = e^{sT}$ and T is the sampling interval. $G(z)$ may be represented by a set of zeros in the complex z-plane, corresponding to the roots of the equation

$$(1-z^{-m}) = 0$$

In this case, there are m zeros equally spaced around the unit circle, each of which gives rise to a transmission zero in the corresponding filter frequency response. The zero pattern and frequency-response magnitude function for the case when $m = 16$ are illustrated in Figs. 1*a* and *b*. Although this frequency characteristic is of little value as it stands, cancellation of one of the zeros by a coincident pole may be used to give a passband. For example, if the zero at $z = (1, 0)$ is cancelled as shown in Fig. 1c, the lowpass characteristic of Fig. 1d is obtained. The addition of the pole causes $G(z)$ to be modified to:

$$G(z) = \frac{(1-z^{-m})}{(1-z^{-1})} = \frac{Y(z)}{X(z)}$$

So that: $X(z).(1-z^{-m}) = Y(z).(1-z^{-1})$ and the time-domain recurrence formula becomes:

$$x_n - x_{n-m} = y_n - y_{n-1}$$

or

$$y_n = y_{n-1} + x_n - x_{n-m}$$

The filter is now recursive, since each output depends upon a previous output as well as inputs. The integer

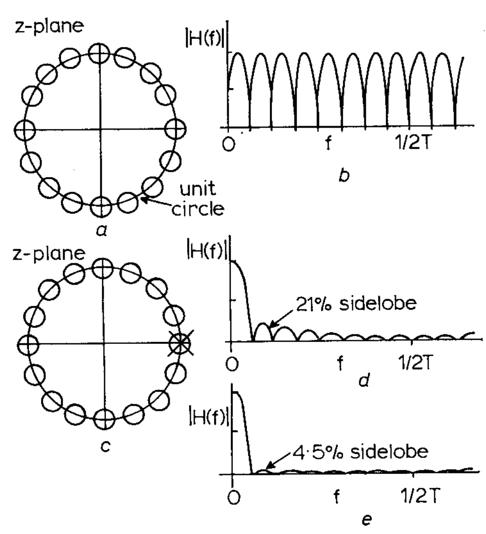

Fig. 1a z-plane zero configuration

Fig. 1b Frequency-response-magnitude characteristic corresponding to Fig. 1a

Fig. 1c Cancelling pole is added at z = (1, 0)

Fig. 1d Low-pass filter characteristic

Fig. 1e Reduced sidelobe levels obtained with second-order poles and zeros

m can be adjusted to give the desired cutoff frequency, and it may be shown that the frequency-response characteristic tends to a sin (x) form as m becomes large.

It is perhaps worth emphasising that an equivalent nonrecursive filter may be specified by rearranging $G(z)$ as a numerator power series in z^{-1}. Thus

$$G(z) = \frac{(1-z^{-m})}{(1-z^{-1})}$$

$$= 1 + z^{-1} + z^{-2} + z^{-3} + \ldots z^{-m+1} = \frac{Y(z)}{X(z)}$$

which gives the nonrecursive recurrence formula

$$y_n = x_n + x_{n-1} + x_{n-2} + x_{n-3} + \ldots x_{n-m+1}$$

This result shows that each output sample value is the sum of m consecutive inputs, and that (apart from a scale factor) the filter is of the simple 'moving-average' type. The peak gain, which occurs at zero frequency, is equal to m. It is also clear that the nonrecursive form is less economic in computation than the recursive one—especially if m is large—since it requires m additions in place of just one addition and one subtraction.

Filters of this type have the advantage of pure linear phase characteristics, because their poles and zeros are confined to the unit circle in the z-plane (LYNN, 1972). This means that all frequency components in the signal experience the same transmission delay through the filter, which helps preserve the relative timing of peaks or features in the output waveform. Unfortunately, the frequency-response amplitude characteristic is rather poor, suffering from a poorly defined cutoff and substantial sidelobes, the first of which has a peak level about 21% (−13·5 dB) of that of the main lobe (see Fig. 1d). Although little can be done about the cutoff slope, the sidelobes may be greatly reduced by using second-(or higher) order zeros, and cancelling pole, instead of the first-order ones shown in Fig. 1c. For example, if 16 second-order zeros, plus a second-order cancelling pole at $z = (1, 0)$, are specified, the transfer function becomes

$$G(z) = \frac{(1-z^{-16})^2}{(1-z^{-1})^2} = \frac{1-2z^{-16}+z^{-32}}{1-2z^{-1}+z^{-2}} = \frac{Y(z)}{X(z)}$$

which gives the filter recurrence formula

$$x_n - 2x_{n-16} + x_{n-32} = y_n - 2y_{n-1} + y_{n-2}$$

or

$$y_n = 2y_{n-1} - y_{n-2} + x_n - 2x_{n-16} + x_{n-32}$$

The first sidelobe level is now about 4·5% (−27 dB) of the main lobe, as shown in Fig. 1e, but this improved performance is paid for by increased computation since the recurrence formula involves five terms, two of which must be multiplied by two.

3 Extending the design

In spite of the rather slow cutoff rates of these lowpass filters, their integer multipliers make them so fast in operation that they are attractive for applications—such as the detection of cardiac arrhythmias mentioned by TAYLOR and MACFARLANE (1974)—which do not require a tightly specified amplitude characteristic. To extend the range of such applications, it seems worthwhile to describe the possibilities for deriving equivalent filters with highpass, bandpass and bandstop characteristics.

3.1 Highpass filters

Highpass counterparts of the lowpass filters already described may be easily obtained by placing the cancelling pole(s) at the point $z = (-1, 0)$ rather than at $z = (1, 0)$. This gives a passband centred at $f = \frac{1}{2T}$ Hz, which is the highest frequency present in any analogue signal prior to adequate sampling. Consider, for example, a filter having 10 double zeros equally spaced around the unit circle, with a double cancelling pole at $z = (-1, 0)$. The transfer function is given by

$$G(z) = \frac{(1-z^{-10})^2}{(1+z^{-1})^2} = \frac{1-2z^{-10}+z^{-20}}{1+2z^{-1}+z^{-2}} = \frac{Y(z)}{X(z)}$$

which yields the filter recurrence formula

$$y_n = -2y_{n-1} - y_{n-2} + x_n - 2x_{n-10} + x_{n-20}$$

The pole-zero configuration and frequency-response-magnitude characteristic of this filter are shown in Fig. 2; it has a similar side-lobe performance to the low pass filter of Fig. 1e, but a rather broader main lobe because of the use of a smaller number of double zeros.

3.2 Bandpass filters

Bandpass characteristics may be obtained by placing the cancelling pole(s) elsewhere on the unit circle in the z-plane. However, there are considerable restrictions if the advantages of integer arithmetic are to be retained. Consider a complex-conjugate pole pair on the unit circle at the points $z = e^{j\theta}$ and $z = e^{-j\theta}$, giving rise to a denominator term

$$(e^{j\theta}-z^{-1})(e^{-j\theta}-z^{-1})$$
$$=1-z^{-1}(e^{j\theta}+e^{-j\theta})+z^{-2}=1-2\cos\theta . z^{-1}+z^{-2}$$

in the corresponding transfer function. If the coefficient $2\cos\theta$, which becomes a multiplier for one of the y (output) terms in the corresponding recurrence formula, is to be an integer, θ must be restricted to 0°, 60°, 90°, 120°, or 180°. 0° and 180° represent poles on the real axis in the z-plane, and therefore refer to filters of the lowpass and highpass types already discussed. The other possible angles for cancelling poles, 60°, 90°, and 120°, give rise to bandpass filters with centre frequencies of $1/6T$, $1/4T$, and $1/3T$ Hz, respectively. As an example, suppose a bandpass function having a centre frequency $f = 1/6T$ Hz is required, with a nominal bandwidth (between the transmission zeros to either side of the centre frequency) of $1/12T$ Hz. 24 zeros must therefore be placed around the unit circle, with cancelling poles placed at angles $\theta = \pm 60°$ to the real axis. The transfer function is given by

$$G(z) = \frac{(1-z^{-24})}{(1-2\cos\theta . z^{-1}+z^{-2})}$$
$$= \frac{(1-z^{-24})}{(1-z^{-1}+z^{-2})} = \frac{Y(z)}{X(z)}$$

from which the recurrence formula of the filter is derived as

$$y_n = y_{n-1} - y_{n-2} + x_n - x_{n-24}$$

The pole-zero configuration and frequency-response-magnitude characteristic are shown in Fig. 3. Like the lowpass and highpass designs, such filters have pure linear phase characteristics, and their sidelobe levels may be reduced by specifying second- (or higher-) order poles and zeros, at the expense of increased computation.

The limited choice of centre frequency for such fast bandpass filters must restrict their practical application rather severely. On the other hand, it is worth bearing in mind that the centre frequency

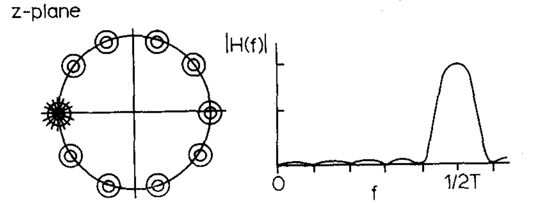

Fig. 2 Pole-zero configuration and response-amplitude characteristic of a highpass recursive digital filter

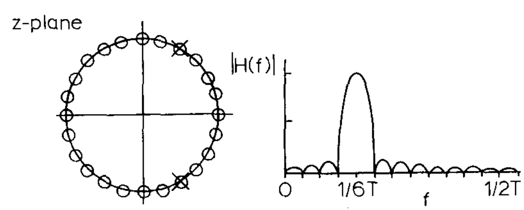

Fig. 3 A bandpass filter with a centre frequency of 1/6T Hz

of any digital bandpass filter depends upon the sampling interval as well as on the choice of pole-zero configuration: in many cases it is therefore possible to trim the centre frequency by adjusting the sampling rate.

3.3 Bandstop ('notch') filter

Probably more valuable than the bandpass filters themselves is the possibility they offer for designing narrow bandstop ('notch') filters. Such filters have been widely used for rejecting mains-frequency interference from recorded biological signals such as the e.c.g. However, the design method invariably used in the past is based upon z-plane poles placed just inside the unit circle, leading to a recurrence formula with floating-point multipliers which need specification to perhaps five or six decimal figure accuracy (WEAVER *et al.*, 1968). An alternative, novel approach which yields recurrence formulas with integer coefficients will now be outlined. It rests upon the fact that the frequency response of a composite filter formed by adding the outputs of two (or more) linear-phase networks with the same transmission delay is equal to the simple algebraic sum of the individual responses. Specifically, if the output of a linear-phase bandpass filter is subtracted from that of a pure delay network with constant gain, an equivalent bandstop filter may be produced. This is illustrated by Fig. 4.

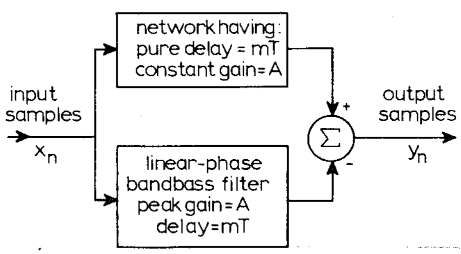

Fig. 4 Derivation of a bandstop characteristic

The design of a 50 Hz notch filter for rejection of supply-frequency interference in the e.c.g. therefore involves specifying a bandpass filter with a 50 Hz centre frequency. If, as before, the cancelling pole pair is placed at $\theta = \pm 60°$, the sampling rate must be 300 Hz, which is rather low for accurate preservation of the diagnostically important QRS complex. 600 Hz is more appropriate, but requires a cancelling pole pair at $\theta = \pm 30°$. On its own, such a pole pair gives rise to floating-point multipliers in the recurrence formula. However, three pole pairs—at $\theta = \pm 30°$, $\pm 90°$, and $\pm 150°$—may be placed on the unit circle by incorporating the simple term $(1+z^{-6})$ in the denominator of $G(z)$. This gives rise, of course, to unwanted passbands at 150 and 250 Hz; but the former can be eliminated by including an extra zero pair at $\theta = \pm 90°$, and the latter is unlikely to be of practical significance in the final design. The other decision to be made concerns the number of zeros to be placed around the unit circle. A bandwidth of ± 1 Hz may well be suitable, implying 600 zeros for a 600 Hz sampling rate. However, the use of 600 zeros produces an impulse response of antisymmetric form, which implies a 90° phase shift at all frequencies in addition to the constant transmission delay. The difficulties which this would cause when subtracting the output of the bandpass filter from that of the all-pass network are avoided by increasing the number of zeros to 606. The transfer function of the bandpass filter is therefore

$$G(z) = \frac{(1+z^{-606})(1+z^{-2})}{(1+z^{-6})}$$

$$= \frac{(1+z^{-2}+z^{-606}+z^{-608})}{(1+z^{-6})} = \frac{Y(z)}{X(z)}$$

giving a recurrence formula:

$$y_n = -y_{n-6}+x_n+x_{n-2}+x_{n-606}+x_{n-608}$$

The peak gain of the filter, which is $101\sqrt{(3)} \simeq 175$, may be found by evaluating the magnitude of $G(z)$ for $z = e^{j\pi/6}$ (which represents the sinusoidal frequency 50 Hz). Its transmission delay may be inferred by noting that multiplication of $G(z)$ by z^{301} makes it an even function of z. Since any even function of z represents an (unrealisable) zero-delay system, the delay of the actual bandpass filter must be 301 sampling periods—just over 0·5 s.

Each output sample from the notch filter (k_n) is now found as the difference between the output of an all-pass network having gain 175 and delay 301 T, and that of the bandpass filter. Hence the recurrence formulas for the notch filter are

$$y_n = -y_{n-6}+x_n+x_{n-2}+x_{n-606}+x_{n-608}$$

and

$$k_n = 175x_{n-301}-y_n$$

Multiplication by the integer 175 is not, of course, as fast as a simple addition or subtraction; however, since

$$175x_{n-301} = (128+32+16-1)x_{n-301}$$

it may be achieved by seven binary shifts left, three additions and one subtraction. In the normal type of notch filter inaccurate specification of one of the floating-point multipliers can lead to instability; there is no such danger in approximating the true gain of $101\sqrt{3}$ by the integer value 175. The only effect is marginally to degrade the rejection performance of the filter at its centre frequency, which is now attenuated 2800 times (69 dB) compared with signals in the passband. Some practical results obtained with this filter are described later.

Table 1 summarises the properties of the various types of filter so far discussed, and includes three notch filter designs to provide 50 Hz rejection at sampling frequencies of 300, 600, or 1200 Hz (filter types -12, -11, and -13, respectively). As already mentioned, type-12 produces an extra notch at 250 Hz; and type-13 will produce extra notches at 250, 350, and 550 Hz. However, these unwanted notches are, like that at 50 Hz, only about ± 1 Hz wide and seem very unlikely to cause any practical difficulties. The table is not exhaustive and could easily be extended to include other filters based upon the foregoing design techniques.

4 Online operation on a small computer

Several of the filters already described have been programmed on a PDP11 digital computer to assess the possibilities for fast online operation. The particular machine used has a 16 k core store with a basic cycle time of 900 ns, a disc-operating system, and a laboratory peripheral system with a real-time clock and a.d.c. and d.a.c. channels for interfacing with the laboratory environment.

To make the results obtained as relevant as possible for users of other small computers, the complete digital-filtering operation was confined within the range of the machine's basic 16 bit word, overflow being avoided both by restricting the dynamic range of the input signal and, where necessary, by suitable scaling of signals at intermediate points in the computation. Instead of programming in real-time Basic or Fortran, which would have been relatively slow, the PDP11's Macro assembler language was used. In this language, the machine-code operations of binary number manipulation are specified by a convenient set of mnemonics (ADD for add, SUB for subtract, MOV for move, ASR for arithmetic shift right ($\div 2$), BR for branch etc.). Sampling of the input signal was controlled by the real-time clock, with the sampling rate set on the console bitswitches and capable of alteration during online operation. A portion of core was reserved for storage of the present input sample (x_n) and the required number of previous inputs (x_{n-1}, x_{n-2} etc.), and for previous output(s). Each time the input was sampled, the corresponding filtered output value was computed and output via a digital-to-analogue convertor for display on an oscilloscope or chart recorder.

The maximum sampling rate achieved with the various filters listed in Table 1 varied between about 6·5 kHz for the simplest recurrence formulas (e.g. filters of types-1 and -3) and about 3·5 kHz for the more complicated ones (e.g. the notch filter, type-11). However, it is important to stress that the programs incorporated a number of features, such as scaling of the input signal, a display routine and online alteration of the sampling rate using the console bitswitches, which were not strictly part of the digital filtering process itself. Elimination of these features would have caused the sampling rates to rise to about 8 and 5 kHz, respectively, and a

Table 1

Filter type	Centre frequency Hz	Nominal bandwidth Hz	First sidelobe level (approx.)	Maximum gain	Recurrence formula $y_n = \dots$
1. lowpass	0	$\pm 1/mT$	0.21	m	$y_{n-1}+x_n-x_{n-m}$
2. lowpass	0	$\pm 1/mT$	0.045	m^2	$2y_{n-1}-y_{n-2}+x_n-2x_{n-m}+x_{n-2m}$
3. highpass	$1/2T$	$\pm 1/mT$	0.21	m	$-y_{n-1}+x_n+x_{n-m}$ (m odd) $-y_{n-1}+x_n-x_{n-m}$ (m even)
4. highpass	$1/2T$	$\pm 1/mT$	0.045	m^2	$-2y_{n-1}-y_{n-2}+x_n+2x_{n-m}+x_{n-2m}$ (m odd) $-2y_{n-1}-y_{n-2}+x_n-2x_{n-m}+x_{n-2m}$ (m even)
5. bandpass	$1/6T$	$\pm 1/6mT$	0.21	$2m\sqrt{3}$	$y_{n-1}-y_{n-2}+x_n-x_{n-6m}$
6. bandpass	$1/6T$	$\pm 1/6mT$	0.045	$12m^2$	$2y_{n-1}-3y_{n-2}+2y_{n-3}-y_{n-4}+x_n$ $-2x_{n-6m}+x_{n-12m}$
7. bandpass	$1/4T$	$\pm 1/4mT$	0.21	$2m$	$-y_{n-2}+x_n-x_{n-4m}$
8. bandpass	$1/4T$	$\pm 1/4mT$	0.045	$4m^2$	$-2y_{n-2}-y_{n-4}+x_n-2x_{n-4m}+x_{n-8m}$
9. bandpass	$1/3T$	$\pm 1/6mT$	0.21	$2m\sqrt{3}$	$-y_{n-1}-y_{n-2}+x_n-x_{n-6m}$
10. bandpass	$1/3T$	$\pm 1/6mT$	0.045	$12m^2$	$-2y_{n-1}-3y_{n-2}-2y_{n-3}-y_{n-4}+x_n$ $-2x_{n-6m}+x_{n-12m}$
11. bandstop (notch)	$1/12T$	$\pm 1/606T$		$101\sqrt{3}$ ≈ 175	$-y_{n-6}+x_n+x_{n-2}+x_{n-606}+x_{n-608}$ and final output: $k_n = 175x_{n-301}-y_n$
12. bandstop (notch)	$1/6T$	$\pm 1/303T$		101	$-y_{n-3}+x_n+x_{n-303}$ and final output: $k_n = 101x_{n-150}-y_n$
13. bandstop (notch)	$1/24T$	$\pm 1/1212T$		$101\sqrt{3}$ ≈ 175	$y_{n-24}-x_n-x_{n-4}+x_{n-12}+x_{n-16}-x_{n-1212}$ $-x_{n-1216}+x_{n-1224}+x_{n-1228}$ and final output: $k_n = 175x_{n-602}+y_n$

further modest increase in speed could have been obtained by use of a faster analogue-to-digital convertor.

One problem which arises when trying to contain the complete computation within a 16 bit word is that of filter gain. Assuming that 8 bit accuracy (256 distinguishable levels) is adequate for specifying the amplitude of most biological signals at the filter input, then a gain of up to 256 is possible without running the risk of overflow on the output side. This obviously restricts the use of filters such as those of types-2 and -4 in Table 1, which have high gain when their bandwidths are small. It also explains why Table 1 lists only notch designs based upon bandpass filters with first-order transfer-function poles and zeros; for although second-order poles and zeros would give a better notch characteristic, the gain would be too great to accommodate.

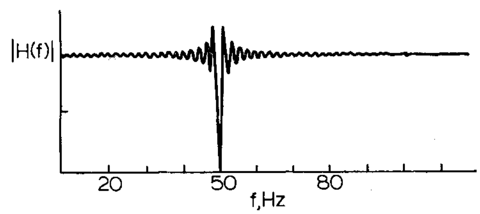

Fig. 5 Part of the response-magnitude characteristic of a digital notch filter, measured online with a PDP 11 computer. Sampling rate : 600/s

Fig. 5 shows part of the frequency-response-magnitude characteristic of the notch filter (type-11). This characteristic was measured online at a sampling rate of 600 Hz, by feeding a swept-frequency sinusoidal signal from an analogue signal generator into the computer via an analogue-to-digital convertor, the filtered output passing via a digital-to-analogue convertor and peak-detector circuit to an analogue chart recorder. This result confirms the theoretical prediction—that the frequency response of the notch filter is equal to that of an all-pass network minus that of a narrow-bandpass filter with an approximately $\sin(x)$ magnitude characteristic. It should be emphasised that only a very small portion of the total frequency range is covered by Fig. 5: the response is essentially flat from 0 to 300 Hz (the Nyquist frequency), apart from the required notch centred on 50 Hz, and an additional one centred on 250 Hz. The practical effect of using this filter on a typical e.c.g. waveform contaminated with 50 Hz mains interference is shown in Fig. 6.

5 Discussion and conclusions

The two major advantages offered by the class of digital filters described in this paper—integer multipliers which allow very fast operation, and a linear-phase response—are not surprisingly offset to some extent by corresponding disadvantages, the most obvious of which is a rather poor amplitude response. In addition, the filters involve greater transmission delays than are generally produced by recursive designs based upon z-plane poles and zeros placed inside the unit circle: to take a rather extreme example, the filter illustrated by Fig. 6 has a delay of about 0·5 s, whereas with the usual type of notch design (WEAVER *et al.*, 1968; LYNN, 1971) the delay is negligible. Whether such delay is important in practice obviously depends upon the particular application. A further potential disadvantage is the relatively large amount of computer storage required. Whereas a typical recursive filter based upon z-plane poles and zeros inside the unit circle may require storage of just a few input and output sample values, the notch filters listed in Table 1—which once again represent an extreme case—require the storage of hundreds of input values. This could be a disadvantage if computer storage is limited.

Since the speed of operation of these filters has been discussed at some length, it seems appropriate to mention the alternative approach to digital filtering based upon the fast Fourier transform (f.f.t.). This method, which has been discussed in recent texts such as those of BOGNER and CONSTANTINIDES (1975) and OPPENHEIM and SCHAFER (1975), and with more specific reference to biological signals by YOGANATHAN *et al.* (1976), allows great flexibility in the choice of filter characteristics and is sometimes held to give a faster filtering action than the equivalent time-domain recurrence formula. However, the core-storage requirements of typical f.f.t. algorithms are greater than those of the filters listed in Table 1, and, as TAYLOR and MACFARLANE (1974) have pointed out, the speed of the f.f.t. cannot rival that of a time-domain recurrence formula which involves just a few integer multipliers. The filters described in this paper therefore seem a natural choice for those

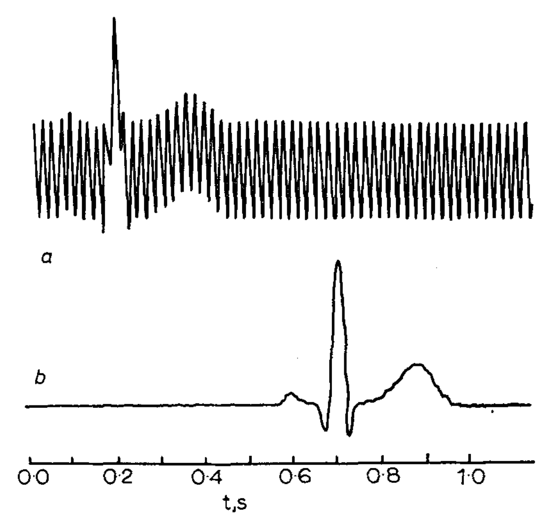

Fig. 6 A typical e.c.g. waveform, (a) heavily contaminated with 50 Hz interference, and (b) after processing online with the notch filter of Fig. 5

applications in which operating speed is more important than a tightly specified response-amplitude characteristic.

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Filtres digitaux en ligne pour signaux biologiques: quelques dispositifs rapides pour petit ordinateur

Sommaire—Après avoir passé en revue la conception d'un type de filtres digitaux récurrents passe-bas à multiplicateurs incorporés et à caractéristiques de phase linéaires, cet article décrit les possibilités d'extension de ce type aux filtres passe-haut, passe-bande et à élimination de bande. Une expérience menée avec un ordinateur PDP 11 a indiqué que ces filtres peuvent être programmés de manière simple avec un code machine, et qu'il est possible d'effectuer des opérations en ligne avec des taux d'échantillonnage jusqu'à environ 8 kHz. L'application pratique de tels filtres est illustrée par un exemple dans lequel un filtre à élimination de bande est utilisé pour éliminer les interférences dues à la fréquence du courant d'alimentation dans un tracé d'e.c.g.

On-Line Digitalfilter für biologische Signale: Einige Schnellausführungen für einen Kleincomputer

Zusammenfassung—Nach einer Untersuchung der Konstruktion einer Gruppe von Rekursivdigitalfiltern mit niedrigem Durchlässigkeitsbereich und mit ganzzahligen Multipliziereinrichtungen und Linearphaseneigenschaften werden die Möglichkeiten beschrieben, die Gruppe so zu erweitern, daß sie Hochfilter, Bandpaßfilter und Bandstopfilter ("Kerbfilter") einschließt. Erfahrungen mit einem PDP 11-Computer haben gezeigt, daß diese Filter auf einfache Weise unter Verwendung von Maschinenkode programmiert werden können und daß On-Line-Betrieb bei Entnahmegeschwindigkeiten von bis zu 8 kHz möglich ist. Die praktische Anwendung solcher Filter wird durch Verwendung einer Kerbkonstruktion zur Ausscheidung von Netzfrequenzstörungen von einer ECG-Wellenform illustriert.