Assignment-1(STAT 638)

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```
tinytex::install_tinytex()
```

```
## tlmgr install grffile
```

Problem 2.1

Marginal and conditional probability: The social mobility data from Section 2.5 gives a joint probability distribution on (Y1, Y2)= (father's occupation, son's occupation). Using this joint distribution, calculate the following distributions:

a) the marginal probability distribution of a father's occupation;

In order to get marginal probability of father's occupation, you need to get the summation of son's occupation for each margin of father's occupation.

```
farm = 0.018 + 0.035 + 0.031 + 0.008 + 0.018
operatives = 0.002 + 0.112 + 0.064 + 0.032 + 0.069
craftsmen = 0.001 + 0.066 + 0.094 + 0.032 + 0.084
sales = 0.001 + 0.018 + 0.019 + 0.010 + 0.051
professional = 0.001 + 0.029 + 0.032 + 0.043 + 0.130

cat('Marginal Probability of the Father:\n\n')
```

```
## Marginal Probability of the Father:
```

```
cat('Farm:', farm ,'\nOperatives:',operatives,'\nCraftsmen:',craftsmen,'\nSales:',sales,'\nProfessional
```

```
## Farm: 0.11
## Operatives: 0.279
## Craftsmen: 0.277
## Sales: 0.099
## Professional: 0.235
```

b) the marginal probability distribution of a son's occupation;

In order to get marginal probability of son's occupation, you need to get the summation of father's occupation for each margin of son's occupation.

```r
farm = 0.018 + 0.002 + 0.001 + 0.001 + 0.001
operatives = 0.035 + 0.112 + 0.066 + 0.018 + 0.029
craftsmen = 0.031 + 0.064 + 0.094 + 0.019 + 0.032
sales = 0.008 + 0.032 + 0.032 + 0.010 + 0.043
professional = 0.018 + 0.069 + 0.084 + 0.051 + 0.130


cat('Marginal Probability of the Father:\n\n')
```

```
## Marginal Probability of the Father:
```

```r
cat('Farm:', farm ,'\nOperatives:',operatives,'\nCraftsmen:',craftsmen,'\nSales:',sales,'\nProfessional
```

```
## Farm: 0.023
## Operatives: 0.26
## Craftsmen: 0.24
## Sales: 0.125
## Professional: 0.352
```

c) the conditional distribution of a son's occupation, given that the father is a farmer;

In order to find the conditional distribution of son given that the father is a farmer, we need to first add up the marginal probability of which the father's occupation is farm:

```r
f_farm = 0.018 + 0.035 + 0.031 + 0.008 + 0.018
```

Then divide each son's marginal occupation where father's occupation is farm by the total marginal probability of the father's occupation (farm)

```r
son_farm_father_farm = 0.018
son_operatives_father_farm = 0.035
son_craftsmen_father_farm = 0.031
son_sales_father_farm = 0.008
son_professional_father_farm = 0.018
cond_farm = son_farm_father_farm/f_farm
cond_operatives = son_operatives_father_farm/f_farm
cond_craftsmen = son_craftsmen_father_farm/f_farm
cond_sales = son_sales_father_farm/f_farm
cond_professional = son_professional_father_farm/f_farm

cat('Conditional Probability of the Son (given that father is farmer):\n\n')
```

```
## Conditional Probability of the Son (given that father is farmer):
```

```r
cat('Farm:', cond_farm ,'\nOperatives:',cond_operatives,'\nCraftsmen:',cond_craftsmen,'\nSales:',cond_sa
```

```
## Farm: 0.1636364
## Operatives: 0.3181818
## Craftsmen: 0.2818182
## Sales: 0.07272727
## Professional: 0.1636364
```

d) the conditional distribution of a father's occupation, given that the son is a farmer.

In order to find the conditional distribution of father given that the son is a farmer, we need to first add up the marginal probability of which the son's occupation is farm:

```
s_farm = 0.018+0.002+0.001+0.001+0.001
```

Then divide each father's marginal occupation where father's occupation is farm by the total marginal probability of the son's occupation (farm)

```
son_farm_father_farm = 0.018
son_farm_father_operatives = 0.002
son_farm_father_craftsmen = 0.001
son_farm_father_sales = 0.001
son_farm_father_professional = 0.001
cond_farm = son_farm_father_farm/s_farm
cond_operatives = son_farm_father_operatives/s_farm
cond_craftsmen = son_farm_father_craftsmen/s_farm
cond_sales = son_farm_father_sales/s_farm
cond_professional = son_farm_father_professional/s_farm

cat('Conditional Probability of the Father (given that son is farmer):\n\n')
```

```
## Conditional Probability of the Father (given that son is farmer):
```

```
cat('Farm:', cond_farm ,'\nOperatives:',cond_operatives,'\nCraftsmen:',cond_craftsmen,'\nSales:',cond_sa
```

```
## Farm: 0.7826087
## Operatives: 0.08695652
## Craftsmen: 0.04347826
## Sales: 0.04347826
## Professional: 0.04347826
```

Problem 2.2

Expectations and variances: Let Y1 and Y2 be two independent random variables, such that $E[Y_i] = \mu_i$ and $Var[Y_i] = \sigma_i^2$.

Using the definition of expectation and variance, compute the following quantities, where a1 and a2 are given constants:

a) $E[a_1Y_1 + a_2Y_2]$, $Var[a_1Y_1 + a_2Y_2]$;

First let's consider $E[a_1Y_1]$ alone. By the formula of expectation $E[a_1Y_1] = \sum a_1y_1p(y_1) = a_1(\sum y_1p(y_1)) = a_1E[Y_1]$ when discrete and $E[a_1Y_1] = \int a_1y_1p(y_1) = a_1(\int y_1p(y_1)) = a_1E[Y_1]$ when continuous.

In any case we can tell that $E[a_1Y_1] = a_1E[Y_1]$ pretty simply. <- #1

Now consider $E[Y_1 + Y_2]$. By the formula $E[Y_1 + Y_2] = \sum y_1p(y_1) + y_2p(y_2) = \sum y_1p(y_1) + \sum y_2p(y_2) = E[Y_1] + E[Y_2]$ when discrete and $E[Y_1 + Y_2] = \int y_1p(y_1) + y_2p(y_2) = \int y_1p(y_1) + \int y_2p(y_2) = E[Y_1] + E[Y_2]$ when continuous.

In any case we can tell that $E[Y_1 + Y_2] = E[Y_1] + E[Y_2]$. <- #2

Using the two properties above (#1,#2) we can say that $E[a_1Y_1 + a_2Y_2] = a_1E[Y_1] + a_2E[Y_2] = a_1\mu_1 + a_2\mu_2$