Assignment #2

Yutae Lee (626005947)

2022-09-20

Problem 3.1

a)

$$P(Y_1, \ldots\ldots, Y_100|\theta) = \theta^{\sum_{i=1}^{100} y_i}(1-\theta)^{100-\sum_{i=1}^{100} y_i}$$

$$Pr(\sum Y_i = y|\theta) = \binom{100}{y}\theta^y(1-\theta)^{100-y}$$

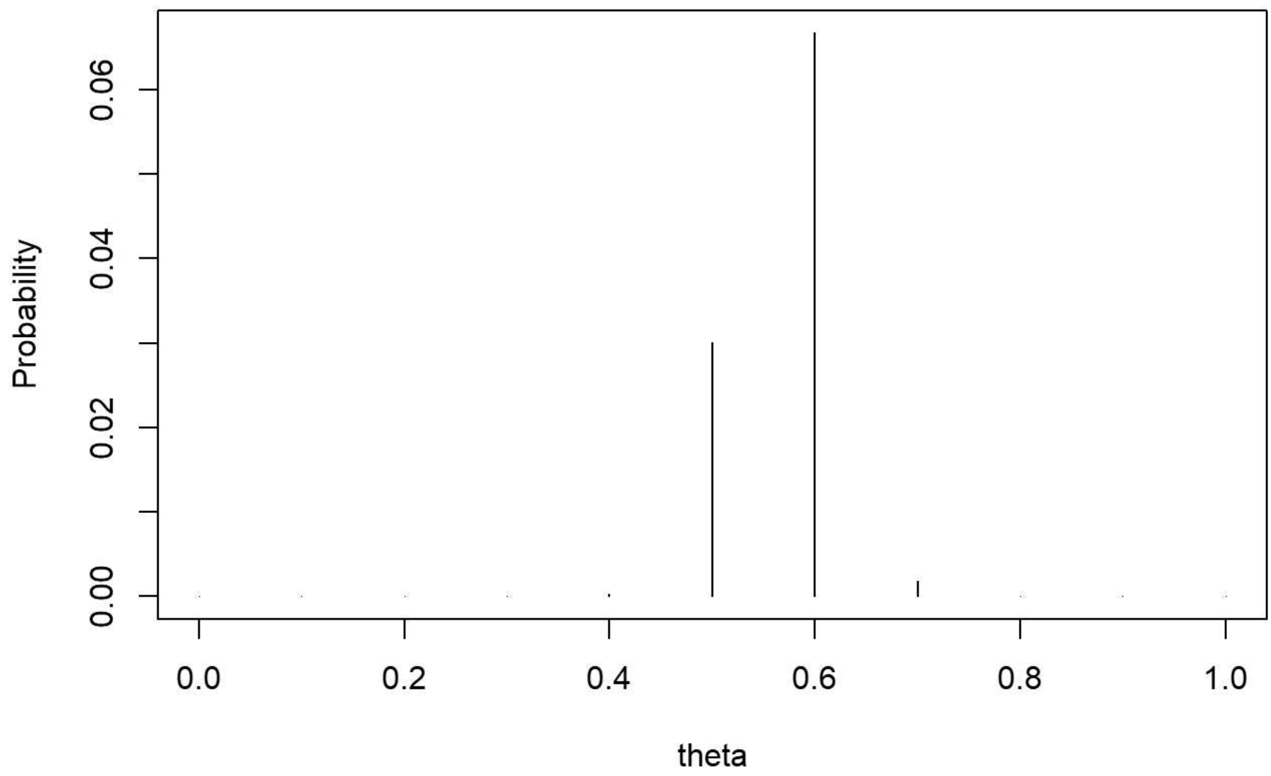
b)

$$Pr(\sum Y_i = 57|\theta) = \binom{100}{57}\theta^{57}(1-\theta)^{43}$$

```
theta=seq(0,1,by = 0.1)
probability = (choose(100,57)) * (theta ^ 57) * ((1-theta) ^ 43)
probability
```

```
##  [1] 0.000000e+00 4.107157e-31 3.738459e-16 1.306895e-08 2.285792e-04
##  [6] 3.006864e-02 6.672895e-02 1.853172e-03 1.003535e-07 9.395858e-18
## [11] 0.000000e+00
```

```
plot(theta,probability,type = 'h', ylab = 'Probability')
```



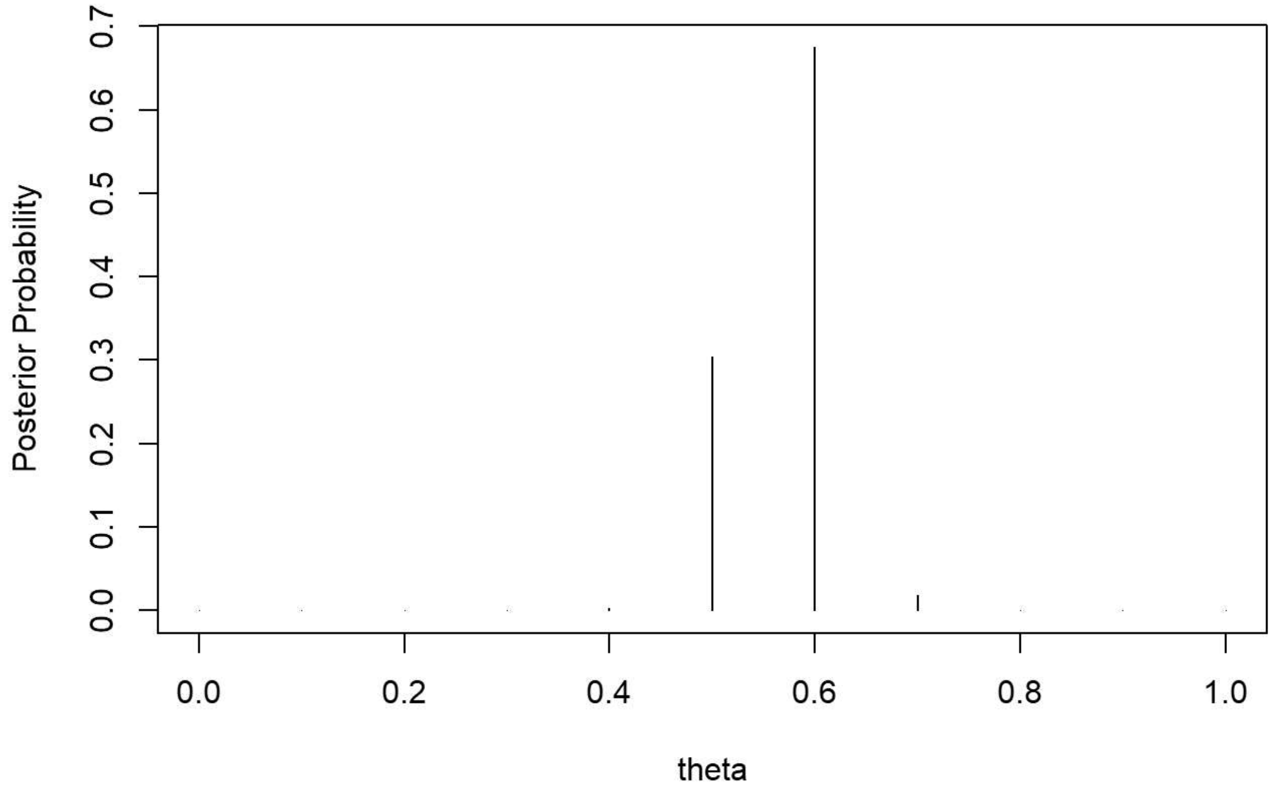
c)

$$P(\theta|y) = \frac{\binom{100}{57}\theta^{57}(1-\theta)^{43}(1/11)}{p(y)} \propto \theta^{57}(1-\theta)^{43}(1/11) = c * \theta^{57}(1-\theta)^{43}(1/11)$$

```
prior_probability = rep(1/11,11)
post_probability = theta ^ 57 * (1-theta) ^ 43 * (1/11)
post_probability = post_probability/sum(post_probability)
post_probability
```

```
##  [1] 0.000000e+00 4.153701e-30 3.780824e-15 1.321705e-07 2.311695e-03
##  [6] 3.040939e-01 6.748515e-01 1.874172e-02 1.014907e-06 9.502335e-17
## [11] 0.000000e+00
```

```
plot(theta,post_probability, type = 'h', ylab = "Posterior Probability")
```

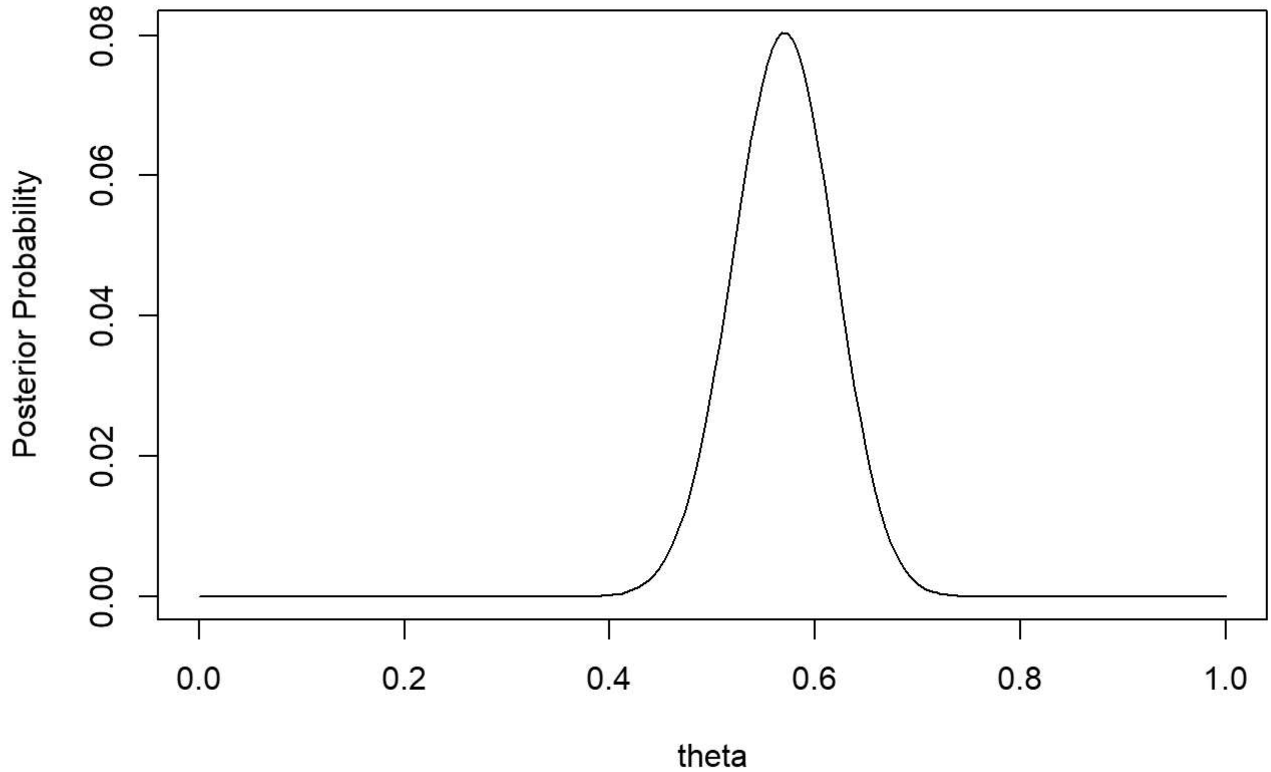


d)

```
theta = seq(0,1,length = 200)
prior_probability = 1
posterior_probability = choose(100,57) * theta ^ 57 * (1-theta) ^ 43 * 1
posterior_probability
```

```
##   [1]  0.000000e+00 2.834059e-103  3.285211e-86  2.875599e-76  3.052907e-69
##   [6]  8.176555e-64  2.134968e-59  1.117756e-55  1.804554e-52  1.185861e-49
##  [11]  3.834099e-47  6.981943e-45  7.912721e-43  6.020512e-41  3.262031e-39
##  [16]  1.318785e-37  4.130871e-36  1.033865e-34  2.120886e-33  3.643164e-32
##  [21]  5.335754e-31  6.766524e-30  7.529203e-29  7.435961e-28  6.583891e-27
##  [26]  5.272346e-26  3.848301e-25  2.577885e-24  1.594580e-23  9.157886e-23
##  [31]  4.907333e-22  2.464458e-21  1.164562e-20  5.196945e-20  2.197423e-19
##  [36]  8.830184e-19  3.381546e-18  1.237228e-17  4.334943e-17  1.457631e-16
##  [41]  4.713045e-16  1.468048e-15  4.412659e-15  1.281933e-14  3.604718e-14
##  [46]  9.824490e-14  2.598566e-13  6.678177e-13  1.669418e-12  4.063558e-12
##  [51]  9.640633e-12  2.231308e-11  5.042452e-11  1.113533e-10  2.404773e-10
##  [56]  5.082369e-10  1.051896e-09  2.133395e-09  4.242513e-09  8.277057e-09
##  [61]  1.585125e-08  2.981309e-08  5.509554e-08  1.000899e-07  1.788201e-07
##  [66]  3.143210e-07  5.437893e-07  9.262933e-07  1.554102e-06  2.569023e-06
##  [71]  4.185541e-06  6.722957e-06  1.064923e-05  1.663957e-05  2.565336e-05
##  [76]  3.903285e-05  5.862735e-05  8.694604e-05  1.273412e-04  1.842229e-04
##  [81]  2.633021e-04  3.718589e-04  5.190239e-04  7.160619e-04  9.766348e-04
##  [86]  1.317022e-03  1.756267e-03  2.316211e-03  3.021386e-03  3.898716e-03
##  [91]  4.977006e-03  6.286177e-03  7.856233e-03  9.715967e-03  1.189141e-02
##  [96]  1.440406e-02  1.726898e-02  2.049283e-02  2.407194e-02  2.799058e-02
## [101]  3.221952e-02  3.671515e-02  4.141905e-02  4.625840e-02  5.114711e-02
## [106]  5.598781e-02  6.067459e-02  6.509650e-02  6.914164e-02  7.270161e-02
## [111]  7.567616e-02  7.797770e-02  7.953548e-02  8.029905e-02  8.024095e-02
## [116]  7.935823e-02  7.767283e-02  7.523077e-02  7.210012e-02  6.836786e-02
## [121]  6.413597e-02  5.951683e-02  5.462825e-02  4.958857e-02  4.451190e-02
## [126]  3.950394e-02  3.465850e-02  3.005489e-02  2.575627e-02  2.180896e-02
## [131]  1.824264e-02  1.507137e-02  1.229522e-02  9.902355e-03  7.871429e-03
## [136]  6.174040e-03  4.777131e-03  3.645195e-03  2.742184e-03  2.033074e-03
## [141]  1.485047e-03  1.068314e-03  7.565893e-04  5.272845e-04  3.614627e-04
## [146]  2.436203e-04  1.613544e-04  1.049633e-04  6.702566e-05  4.198878e-05
## [151]  2.578926e-05  1.551904e-05  9.143199e-06  5.269921e-06  2.969101e-06
## [156]  1.633720e-06  8.771047e-07  4.589923e-07  2.338652e-07  1.158835e-07
## [161]  5.577285e-08  2.603606e-08  1.177161e-08  5.146438e-09  2.171856e-09
## [166]  8.830474e-10  3.451963e-10  1.294475e-10  4.645021e-11  1.590603e-11
## [171]  5.182128e-12  1.600956e-12  4.672713e-13  1.283180e-13  3.300147e-14
## [176]  7.907841e-15  1.755193e-15  3.584736e-16  6.686141e-17  1.129063e-17
## [181]  1.709005e-18  2.291935e-19  2.686448e-20  2.708177e-21  2.303275e-22
## [186]  1.614730e-23  9.070009e-25  3.940678e-26  1.266743e-27  2.845303e-29
## [191]  4.141798e-31  3.528394e-33  1.524725e-35  2.710667e-38  1.432896e-41
## [196]  1.307249e-45  7.423621e-51  2.658707e-58  4.033867e-71  0.000000e+00
```

```
plot(theta,posterior_probability,type = 'l' , ylab = 'Posterior Probability')
```



e)

```
theta = seq(0,1,length = 200)
posterior_probability = dbeta(theta,58,44)
posterior_probability
```

```
##   [1]  0.000000e+00 2.862400e-101  3.318063e-84  2.904355e-74  3.083436e-67
##   [6]  8.258321e-62  2.156317e-57  1.128933e-53  1.822599e-50  1.197719e-47
##  [11]  3.872440e-45  7.051763e-43  7.991848e-41  6.080717e-39  3.294651e-37
##  [16]  1.331973e-35  4.172180e-34  1.044204e-32  2.142095e-31  3.679596e-30
##  [21]  5.389112e-29  6.834190e-28  7.604495e-27  7.510321e-26  6.649730e-25
##  [26]  5.325070e-24  3.886784e-23  2.603664e-22  1.610526e-21  9.249465e-21
##  [31]  4.956407e-20  2.489103e-19  1.176208e-18  5.248914e-18  2.219397e-17
##  [36]  8.918486e-17  3.415361e-16  1.249600e-15  4.378293e-15  1.472207e-14
##  [41]  4.760176e-14  1.482729e-13  4.456785e-13  1.294753e-12  3.640765e-12
##  [46]  9.922735e-12  2.624551e-11  6.744959e-11  1.686112e-10  4.104193e-10
##  [51]  9.737039e-10  2.253621e-09  5.092876e-09  1.124668e-08  2.428821e-08
##  [56]  5.133193e-08  1.062415e-07  2.154729e-07  4.284939e-07  8.359828e-07
##  [61]  1.600976e-06  3.011123e-06  5.564649e-06  1.010908e-05  1.806083e-05
##  [66]  3.174642e-05  5.492272e-05  9.355563e-05  1.569643e-04  2.594714e-04
##  [71]  4.227396e-04  6.790186e-04  1.075572e-03  1.680597e-03  2.590990e-03
##  [76]  3.942318e-03  5.921363e-03  8.781550e-03  1.286146e-02  1.860651e-02
##  [81]  2.659351e-02  3.755775e-02  5.242142e-02  7.232225e-02  9.864011e-02
##  [86]  1.330193e-01  1.773830e-01  2.339373e-01  3.051599e-01  3.937703e-01
##  [91]  5.026776e-01  6.349039e-01  7.934795e-01  9.813126e-01  1.201032e+00
##  [96]  1.454810e+00  1.744167e+00  2.069776e+00  2.431266e+00  2.827048e+00
## [101]  3.254172e+00  3.708230e+00  4.183324e+00  4.672098e+00  5.165858e+00
## [106]  5.654769e+00  6.128134e+00  6.574747e+00  6.983306e+00  7.342863e+00
## [111]  7.643292e+00  7.875748e+00  8.033083e+00  8.110204e+00  8.104336e+00
## [116]  8.015181e+00  7.844955e+00  7.598308e+00  7.282112e+00  6.905154e+00
## [121]  6.477733e+00  6.011200e+00  5.517453e+00  5.008445e+00  4.495702e+00
## [126]  3.989898e+00  3.500509e+00  3.035544e+00  2.601384e+00  2.202705e+00
## [131]  1.842507e+00  1.522209e+00  1.241817e+00  1.000138e+00  7.950143e-01
## [136]  6.235780e-01  4.824902e-01  3.681647e-01  2.769606e-01  2.053404e-01
## [141]  1.499898e-01  1.078997e-01  7.641552e-02  5.325574e-02  3.650773e-02
## [146]  2.460565e-02  1.629680e-02  1.060130e-02  6.769592e-03  4.240867e-03
## [151]  2.604716e-03  1.567423e-03  9.234631e-04  5.322620e-04  2.998792e-04
## [156]  1.650057e-04  8.858758e-05  4.635822e-05  2.362038e-05  1.170423e-05
## [161]  5.633058e-06  2.629643e-06  1.188932e-06  5.197902e-07  2.193574e-07
## [166]  8.918778e-08  3.486483e-08  1.307420e-08  4.691471e-09  1.606509e-09
## [171]  5.233950e-10  1.616966e-10  4.719440e-11  1.296012e-11  3.333148e-12
## [176]  7.986919e-13  1.772744e-13  3.620584e-14  6.753003e-15  1.140354e-15
## [181]  1.726095e-16  2.314854e-17  2.713313e-18  2.735259e-19  2.326308e-20
## [186]  1.630877e-21  9.160709e-23  3.980084e-24  1.279411e-25  2.873756e-27
## [191]  4.183216e-29  3.563678e-31  1.539972e-33  2.737773e-36  1.447225e-39
## [196]  1.320321e-43  7.497857e-49  2.685294e-56  4.074206e-69  0.000000e+00
```

```
plot(theta,posterior_probability,type = 'l', ylabs = "Posterior Probability")
```

```
## Warning in plot.window(...): "ylabs"는 그래픽 매개변수가 아닙니다
```

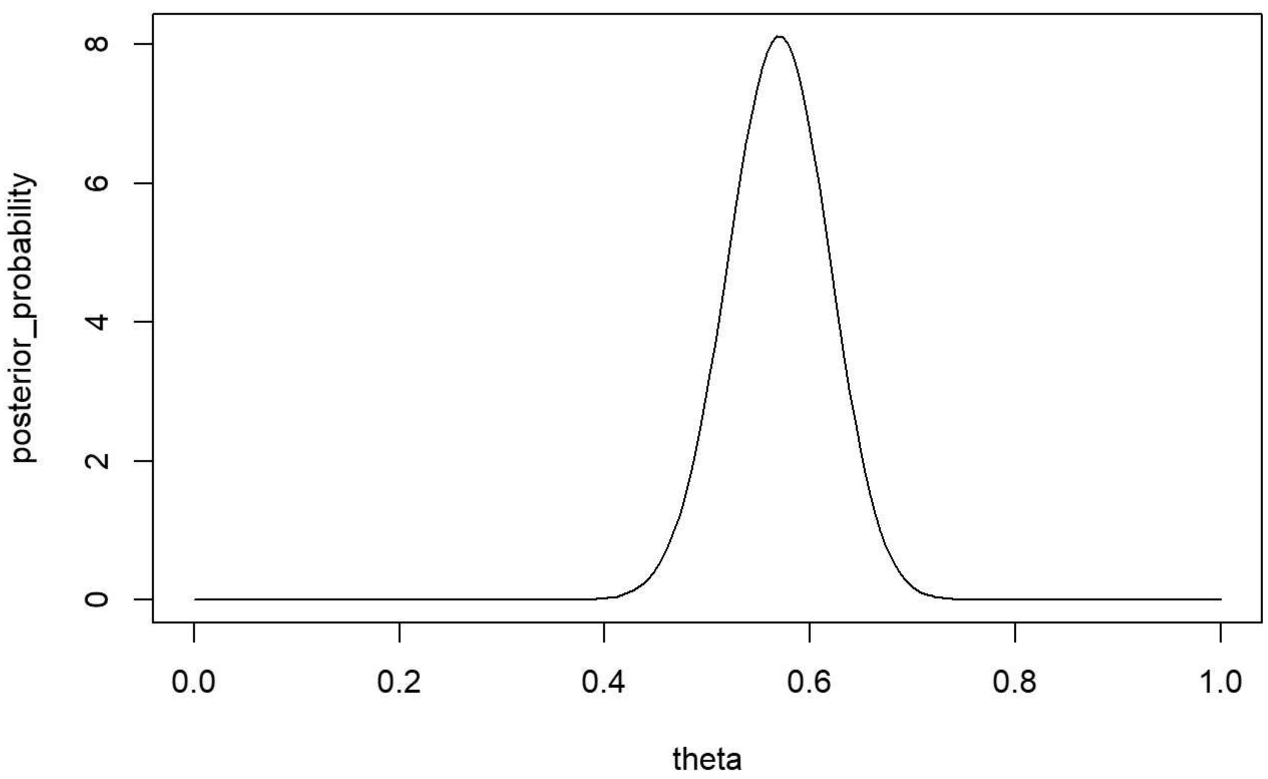
```
## Warning in plot.xy(xy, type, ...): "ylabs"는 그래픽 매개변수가 아닙니다
```

```
## Warning in axis(side = side, at = at, labels = labels, ...): "ylabs"는 그래픽 매
## 개변수가 아닙니다

## Warning in axis(side = side, at = at, labels = labels, ...): "ylabs"는 그래픽 매
## 개변수가 아닙니다
```

```
## Warning in box(...): "ylabs"는 그래픽 매개변수가 아닙니다
```

```
## Warning in title(...): "ylabs"는 그래픽 매개변수가 아닙니다
```




```
theta[posterior_probability == max(posterior_probability)]
```

```
## [1] 0.5678392
```

Using the beta distribution we are able to get more accuracy in estimating θ. We can also see that all of the graph shows similar values within themselves. Unlike other priors which gave us $\theta = 0.6$, in beta we can get $\theta = 0.5678392$.

Problem 3.3

a)

$x_A \sim poi(\theta_A), x_B \sim poi(\theta_B)$

$\theta_A \sim \gamma(120, 10), \theta_B \sim \gamma(12, 1)$

$f(\theta_A) = \frac{10^{120}}{\sqrt{120}} e^{-10\theta_A} * \theta_A^{120-1}$ $\beta(x_A) = e^{-\theta_A} \frac{\theta_A^{x_A}}{x_A!}$

$Y_A = \sum_{i=1}^{10} x_{Ai} \sim poi(10\theta_A)$ $\beta(y_A) = e^{-\theta_A} \frac{(10\theta_A)^{y_A}}{y_A!}$

$f(y_A, \theta_A) = \frac{10^{120}}{\sqrt{120}} e^{-10\theta_A} \theta_A^{119} * \frac{e^{-10\theta_A * (10\theta_A)^{y_A}}}{y_A!}$

$f(y_A) = \int_0^{\text{inf}} f(y_A, \theta_A) d\theta_A$

$= \frac{10^{120}}{\sqrt{120} y_A!} * 10^{y_A} \int_0^{inf} e^{-20} \theta_A \theta_A^{120+y_A-1} d\theta_A$

$= \frac{10^{120} 10 y_A}{\sqrt{120} y_A!} * \frac{\sqrt{120+y_A}}{20^{120+y_A}}$

$f(\theta_A | y_A) = \frac{f(y_A * \theta_A)}{f(y_A)} = \frac{e^{-20} \theta_A \theta_A^{120 y_A - 1}}{\sqrt{120+y_A}/(20^{120+y_A})}$

$\gamma(120 + y_A, 20)$

= $\gamma(120 + 117, 20) = \gamma(237, 20)$

mean = 11.85

var = 0.5929

95% C.I. = (10.38925,13.40545)

Doing the same process for θ_B I get $\gamma(125, 19)$

I get mean = 8.9286

Var = 0.0.6378

95% C.I. = (7.432064,10.56031)

Sorry for the unfinished work but this is all I have upto.