

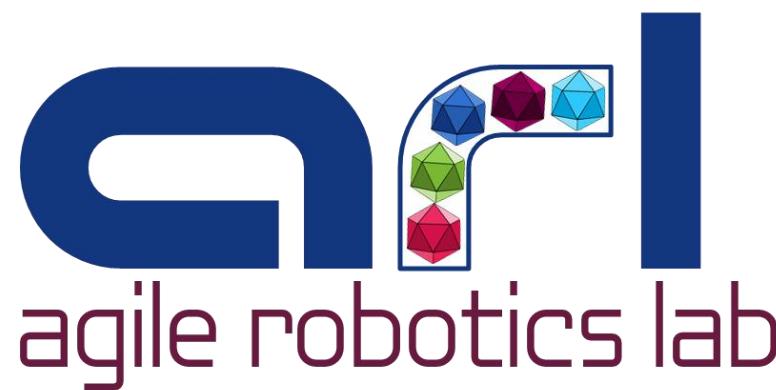
# Geometric Static Modeling Framework for Piecewise-Continuous Curved-Link Multi Point-of-Contact Tensegrity Robots

Lauren Ervin and Vishesh Vikas

Agile Robotics Laboratory

arl.ua.edu

The University of Alabama, USA



# Motivation

Discontinuous, curved morphology well suited for space applications

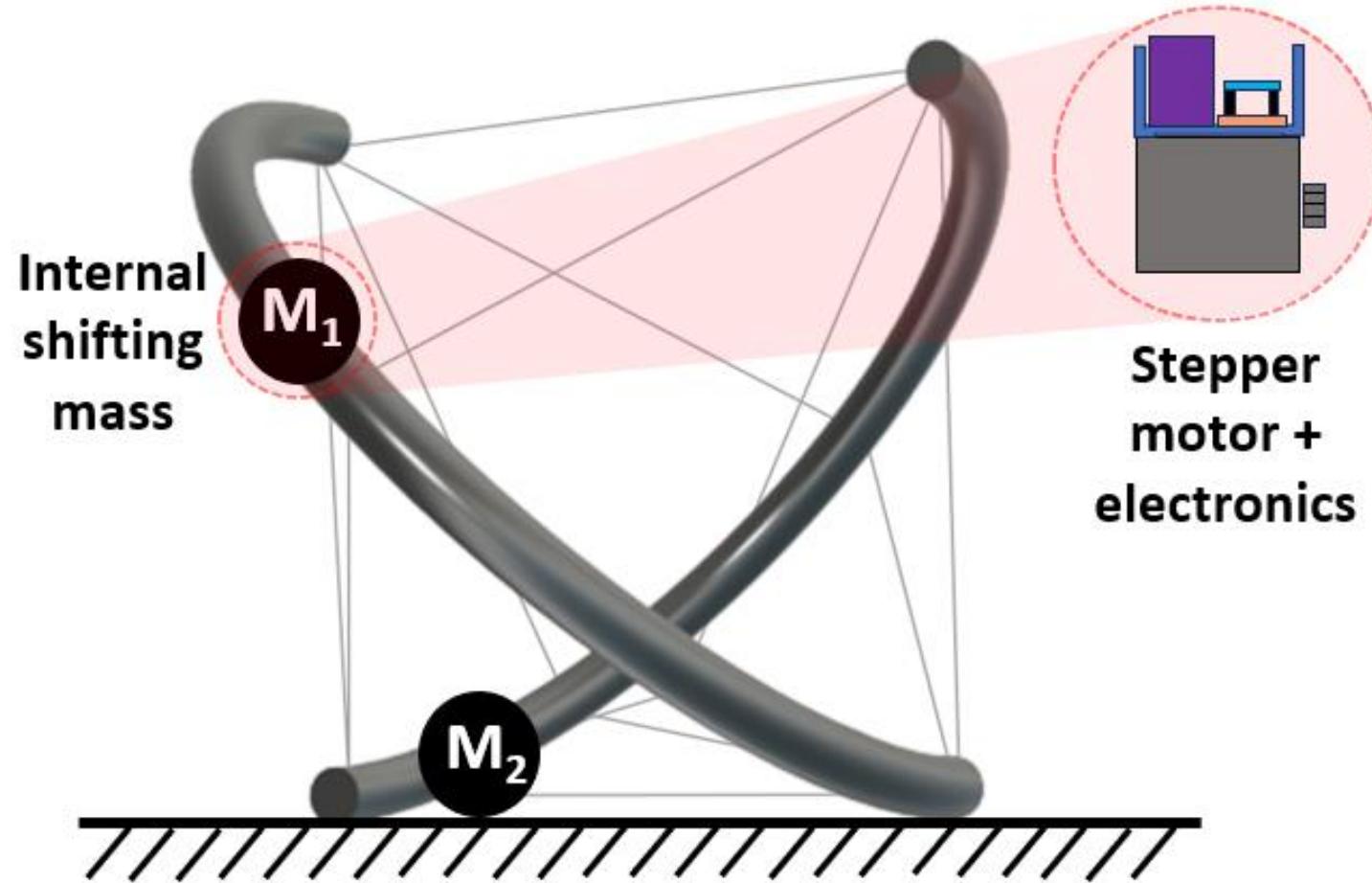


# Critical Terminology

- **Tensegrity mechanism** – Synergistically combine *tension* elements (pre-stressed cables) with *compression* elements (rigid rods) to achieve *structural integrity*
- **Screw Theory**– Framework used to geometrically model motion of rigid bodies as twists along a screw axis
- **Hybrid system** – The robot moves in four different states: it always rolls along the entirety of one arc while pivoting about an endpoint of the other until swapping arcs
- **Piecewise continuous movement** – Rolling produces continuous movement in different directions that instantaneously flip when updating pivot points; when combined, the movement generates a straight path
- **Stability** – A robot orientation where tip-over does not occur regardless of internal mass positions

# Tensegrity eXploratory Robot (TeXploR) Design

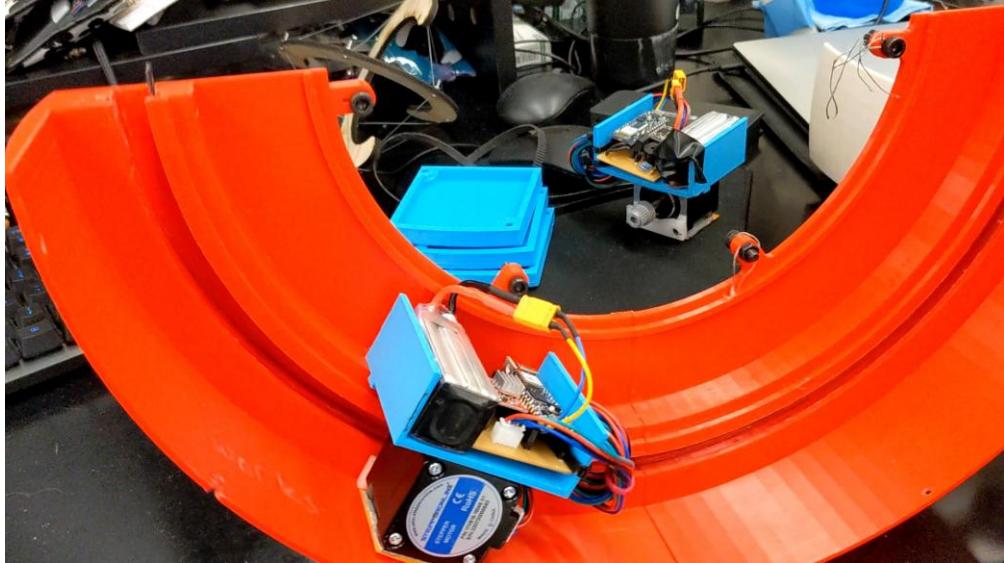
Curved-links with two points of contact



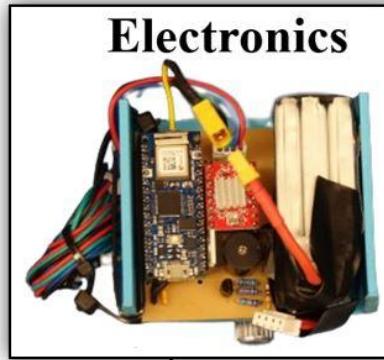
# TeXploR Prototype Mechatronics

All electronics mounted internally and shift along arc

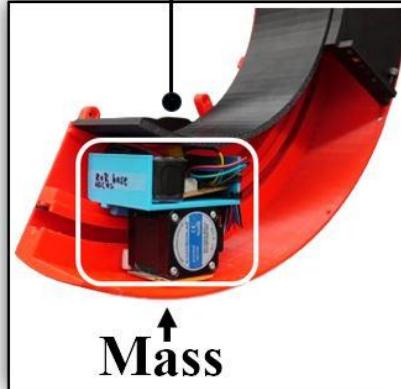
**Internal Mass Shifting**



**Electronics**



**Mass**



**Arc**

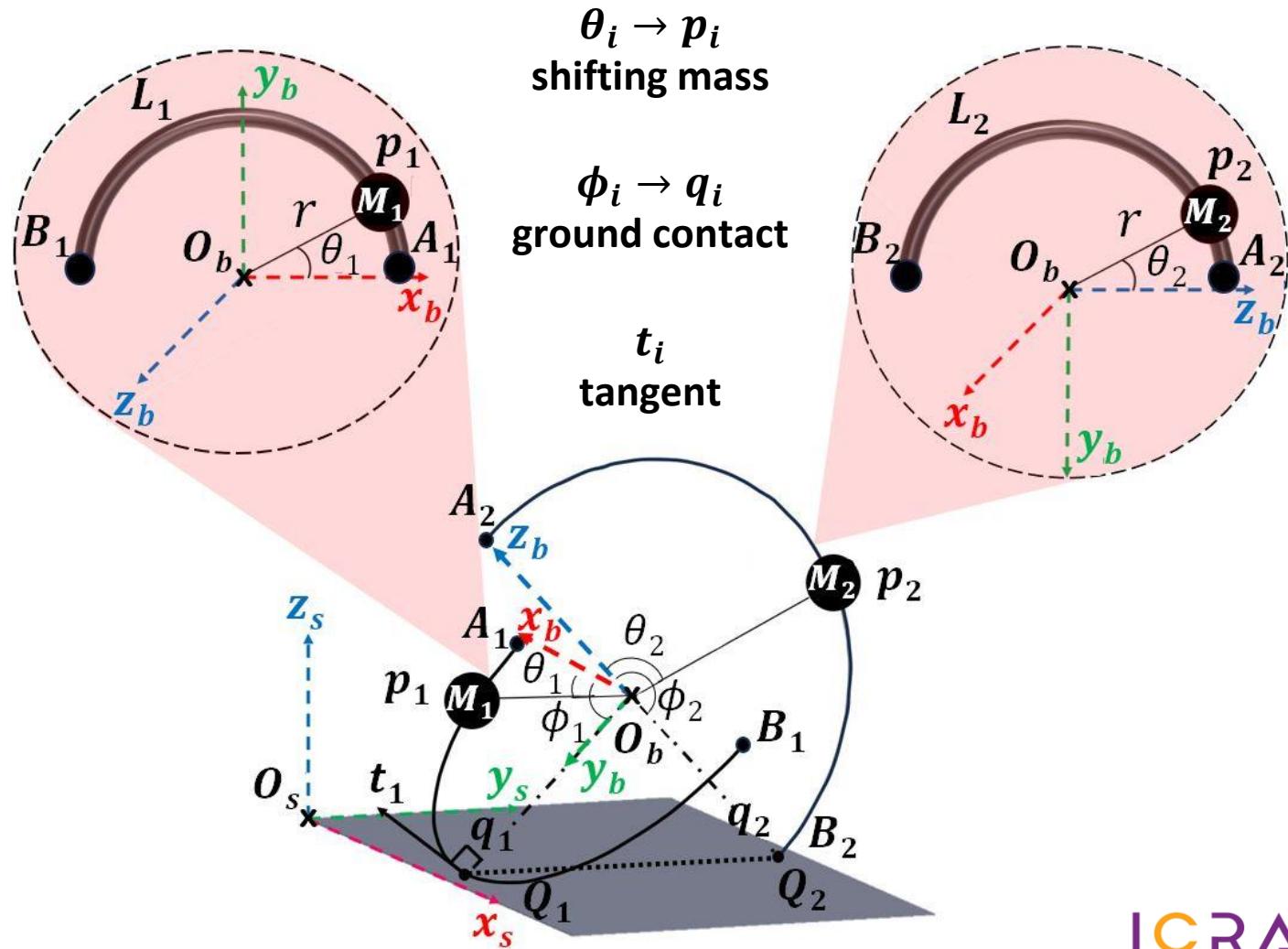


# Kinematics

Geometric representation using Screw Theory

$$\boldsymbol{p}_i^i = r \begin{bmatrix} c\theta_i \\ s\theta_i \\ 0 \end{bmatrix}, \quad \boldsymbol{q}_i^i = r \begin{bmatrix} c\phi_i \\ s\phi_i \\ 0 \end{bmatrix}$$

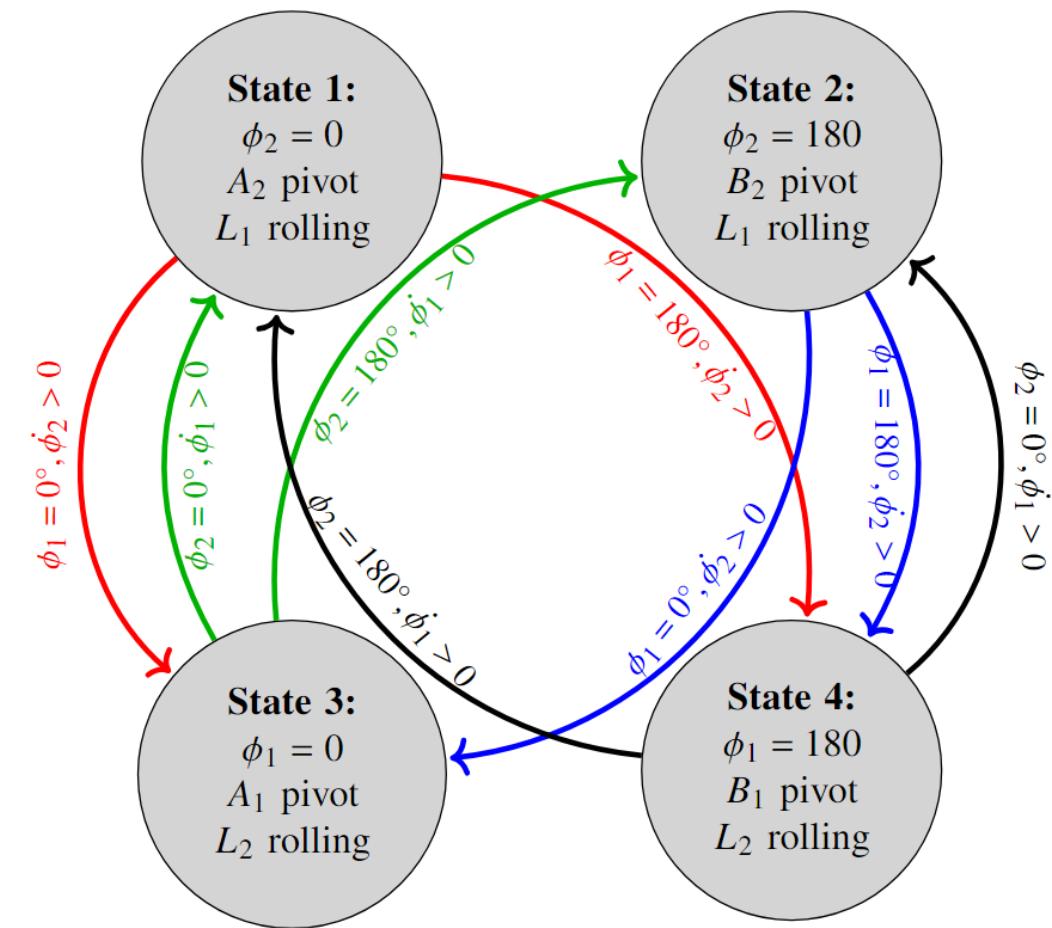
$$T_{12} = \begin{bmatrix} R_{12} & \boldsymbol{o}_{12}^1 \\ \mathbf{0}_{1 \times 3} & 1 \end{bmatrix}, \quad R_{12} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad \boldsymbol{o}_{12}^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$



# Hybrid State System

Four states of locomotion for rolling sequence from holonomic constraints

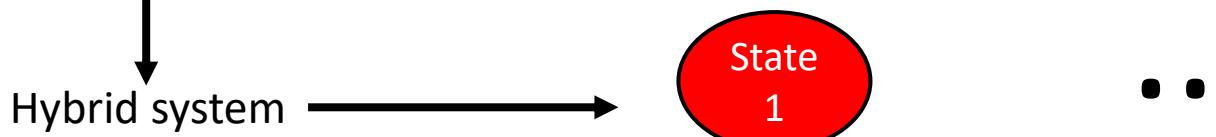
- Case 1:  $t_2$  undefined  $\rightarrow$  State 1:  $\phi_2 = 0$   
or  
State 2:  $\phi_2 = 180$
- Case 2:  $t_1$  undefined  $\rightarrow$  State 3:  $\phi_1 = 0$   
or  
State 4:  $\phi_1 = 180$
- Case 3:  $t_1, t_2$  defined  $\rightarrow \phi_2 = -\phi_1$   
or  
 $\phi_2 = \phi_1 + 180$



# Geometric Static Modeling Framework

Internal mass inputs → static equilibrium position output

Holonomic constraints



$$\mathcal{F} = \begin{bmatrix} \mathbf{f} \\ \mathbf{m} \end{bmatrix} = \sum_{i=1}^2 \left[ (M_i + m_i)g \mathbf{z}_s + \mathbf{F}_{ri} \right] = 0 \quad \text{s.t.}$$

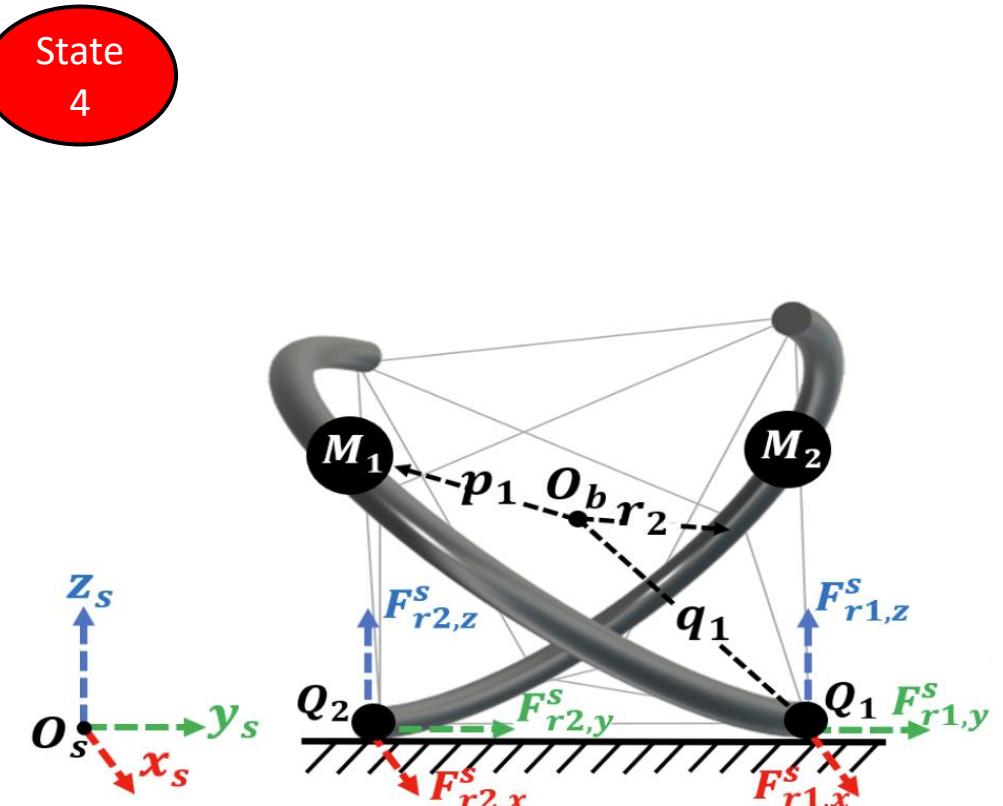
$$\mathbf{f}^s = \begin{bmatrix} F_{r1,x}^s - F_{r2,x}^s \\ F_{r1,y}^s - F_{r2,y}^s \\ (M_1 + M_2 + m_1 + m_2)g - (F_{r1,z}^s + F_{r2,z}^s) \end{bmatrix}$$

$$\mathbf{m}^b = \sum_{i=1}^2 \left( M_i g \hat{\mathbf{p}}_i^b + m_i g \hat{\mathbf{r}}_i^b - F_{ri,z}^s \hat{\mathbf{q}}_i^b \right) \mathbf{z}_s^b + \left( F_{r1,x}^s \hat{\mathbf{q}}_1^b - F_{r2,x}^s \hat{\mathbf{q}}_2^b \right) \mathbf{x}_s^b + \left( F_{r1,y}^s \hat{\mathbf{q}}_1^b - F_{r2,y}^s \hat{\mathbf{q}}_2^b \right) \mathbf{y}_s^b$$

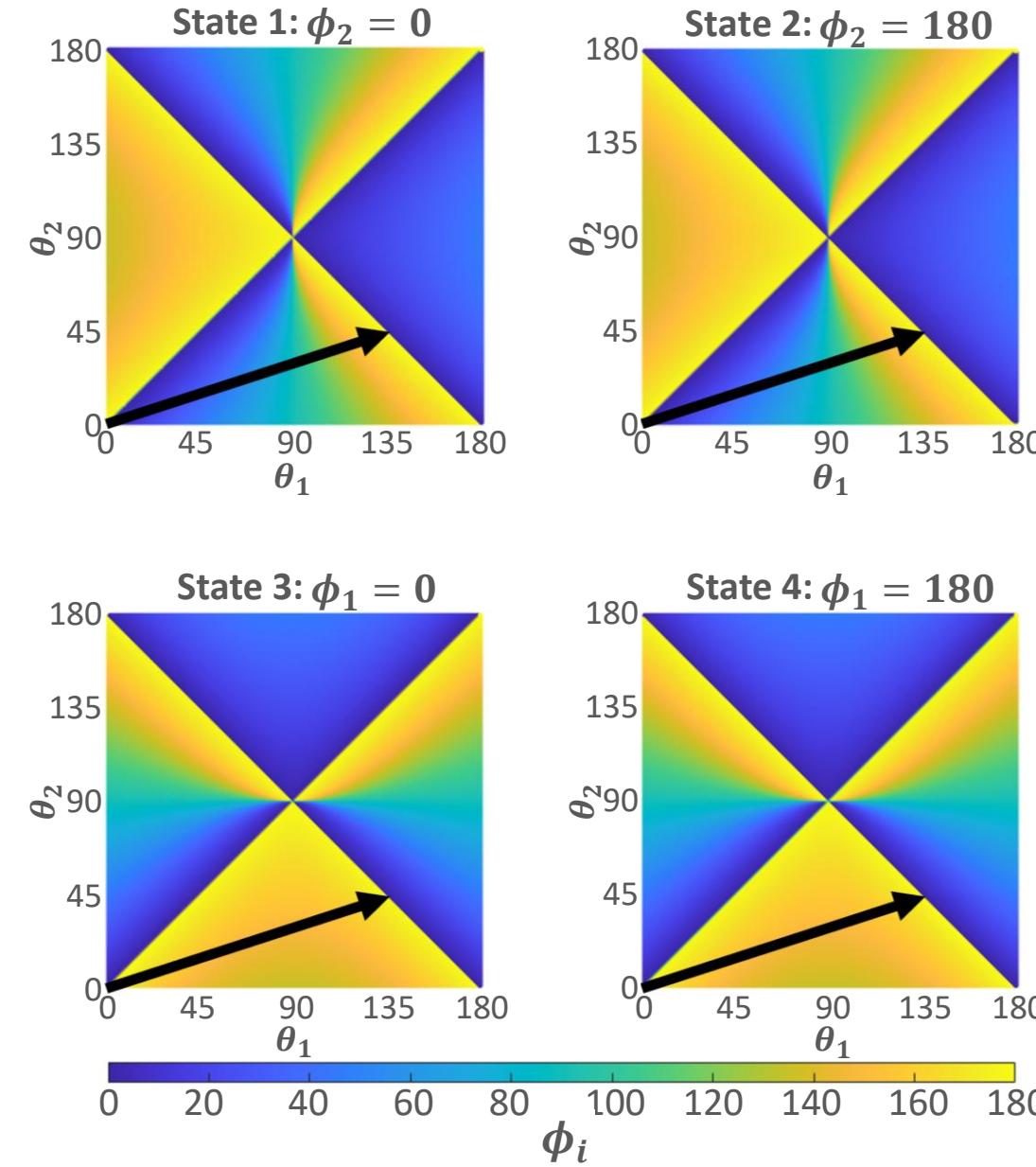
$$\mathbf{z}_s^b = -\frac{1}{\sqrt{2}} [c_{\phi_1}, s_{\phi_1}, 1]^T, \quad \tan \phi_1 = \frac{s_{\theta_1} - s_{\theta_2}}{c_{\theta_1}}$$

$$F_{r1,z} = Mg \left[ -1 + \frac{c_{\phi_1+\theta_2}}{2} - c_{\theta_2} + c_{\phi_1-\theta_1} - \frac{c_{\phi_1-\theta_2}}{2} \right] - mg$$

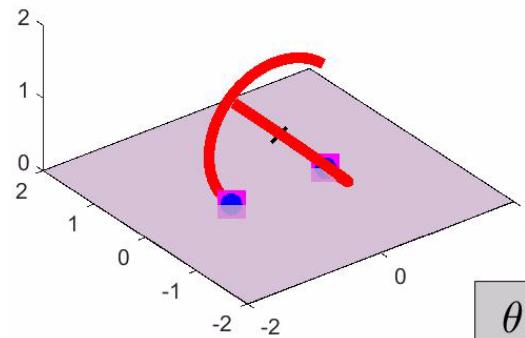
$$F_{r2,z} = Mg \left[ -1 - \frac{c_{\phi_1+\theta_2}}{2} + c_{\theta_2} - c_{\phi_1-\theta_1} + \frac{c_{\phi_1-\theta_2}}{2} \right] - mg$$



# Quasi-Static Simulations

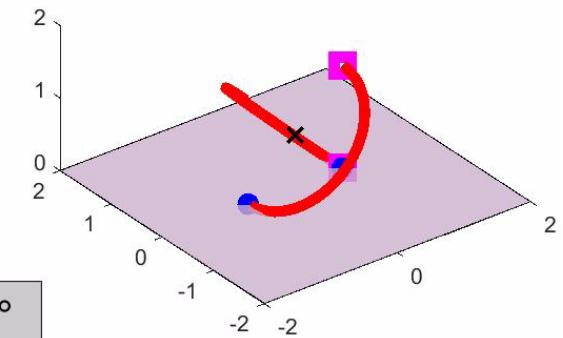


**State 1,  $\phi_1=0^\circ$ ,  $\phi_2=0^\circ$**

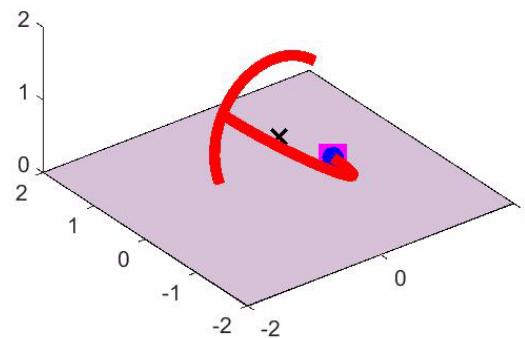


$\theta_1=0^\circ$ ,  $\theta_2=0^\circ$

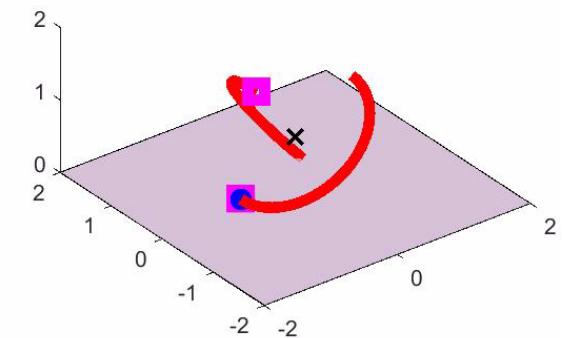
**State 2,  $\phi_1=0^\circ$ ,  $\phi_2=180^\circ$**



**State 3,  $\phi_1=0^\circ$ ,  $\phi_2=0^\circ$**



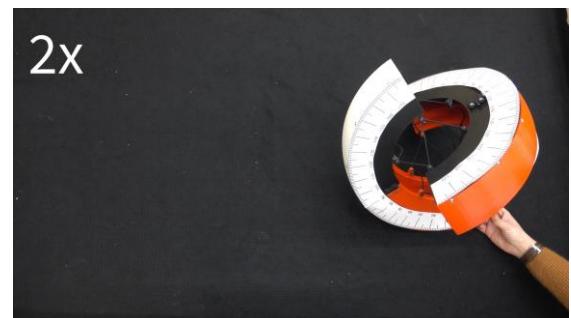
**State 4,  $\phi_1=180^\circ$ ,  $\phi_2=0^\circ$**



# Experimental Validation

**State 3,  $\theta_1 = 135^\circ, \theta_2 = 135^\circ$**

2x



**State 4,  $\theta_1 = 45^\circ, \theta_2 = 135^\circ$**

2x



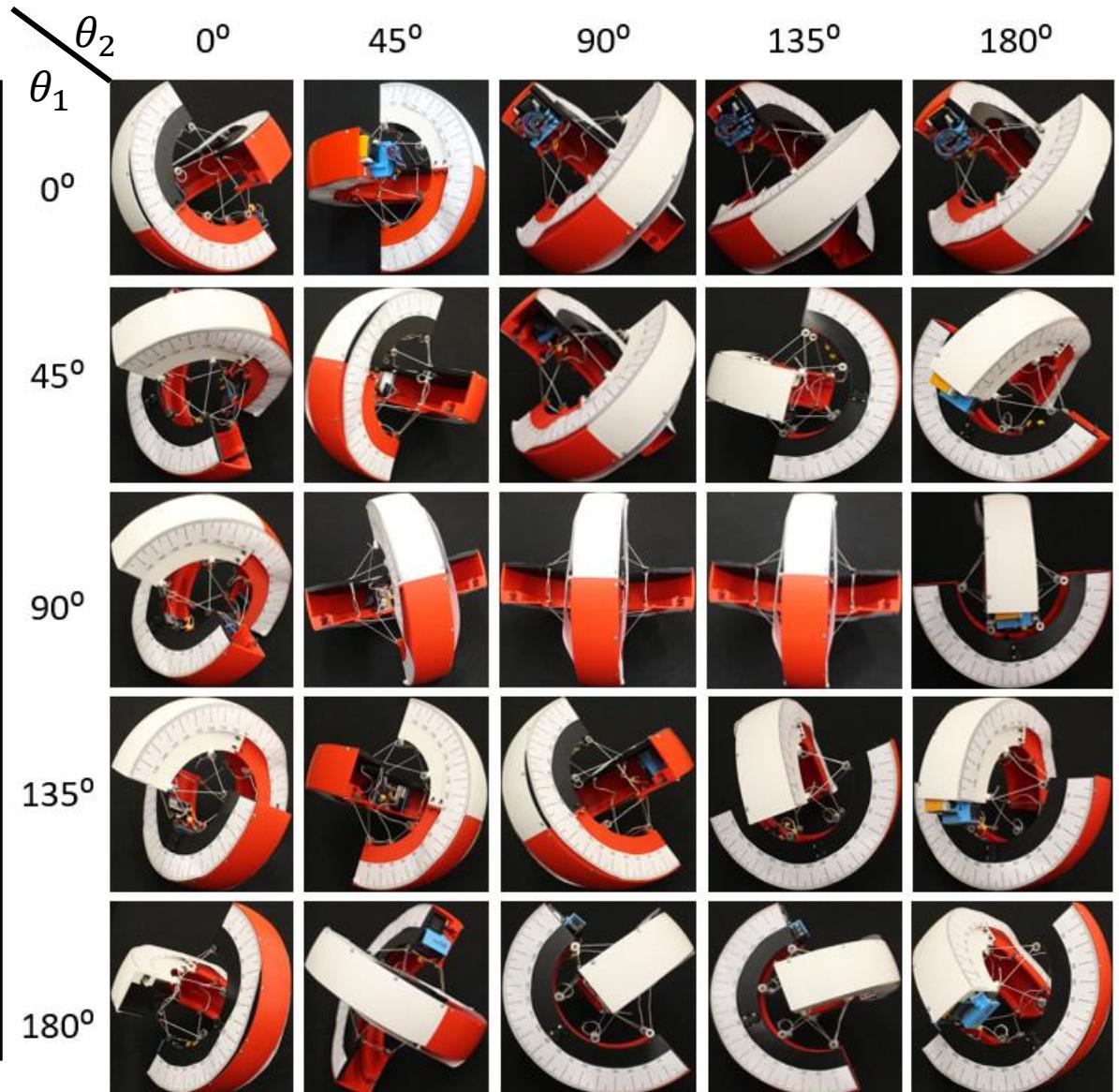
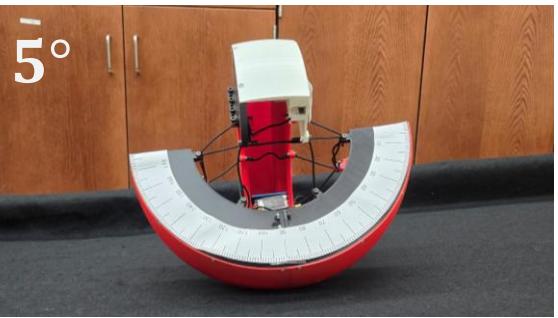
**$\phi_1 = 90^\circ, \phi_2 = 0^\circ, 0^\circ$  incline**

0°



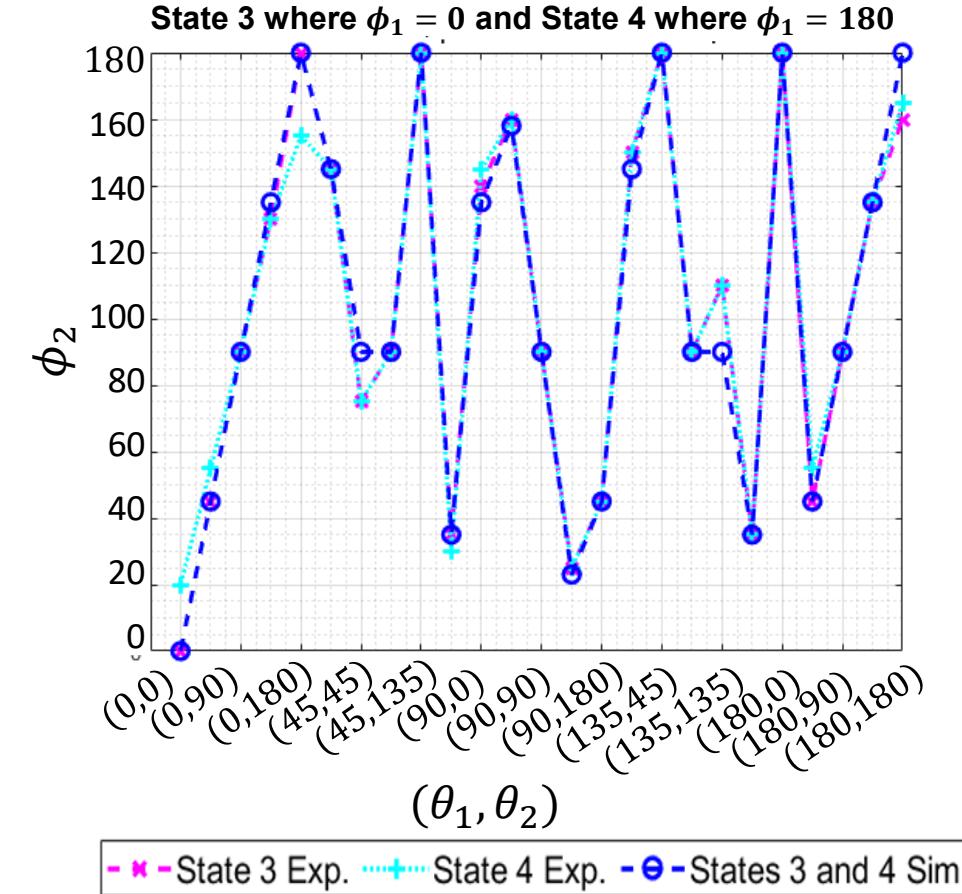
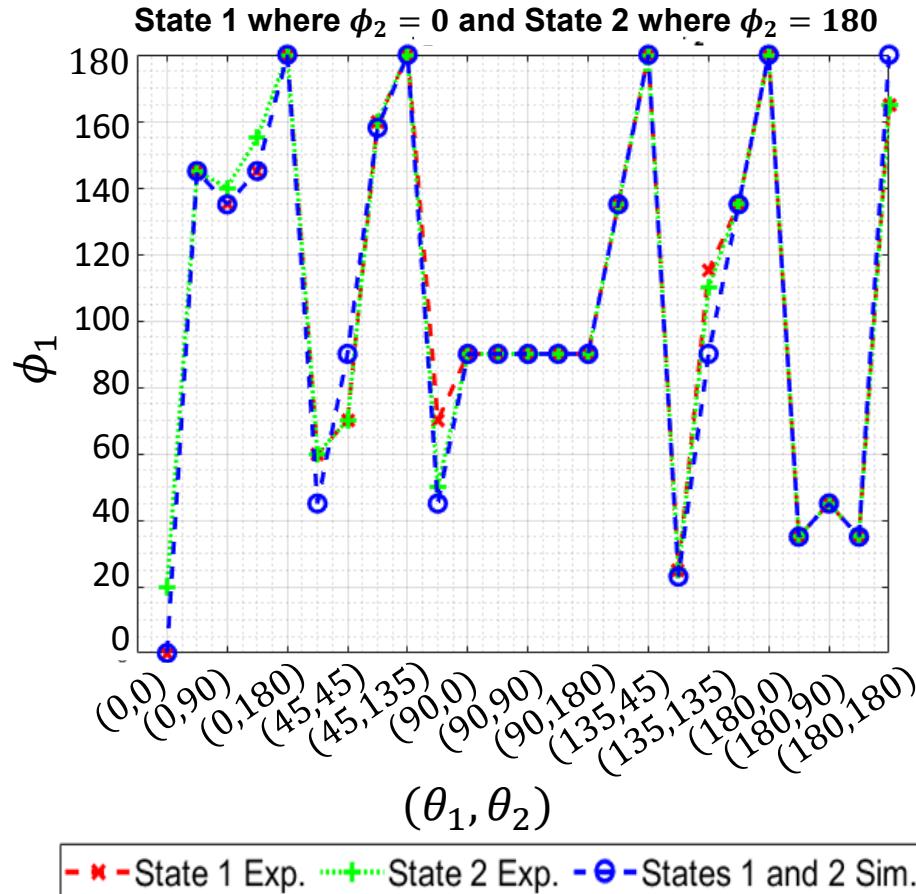
**$\phi_1 = 90^\circ, \phi_2 = 0^\circ, 5^\circ$  incline**

5°

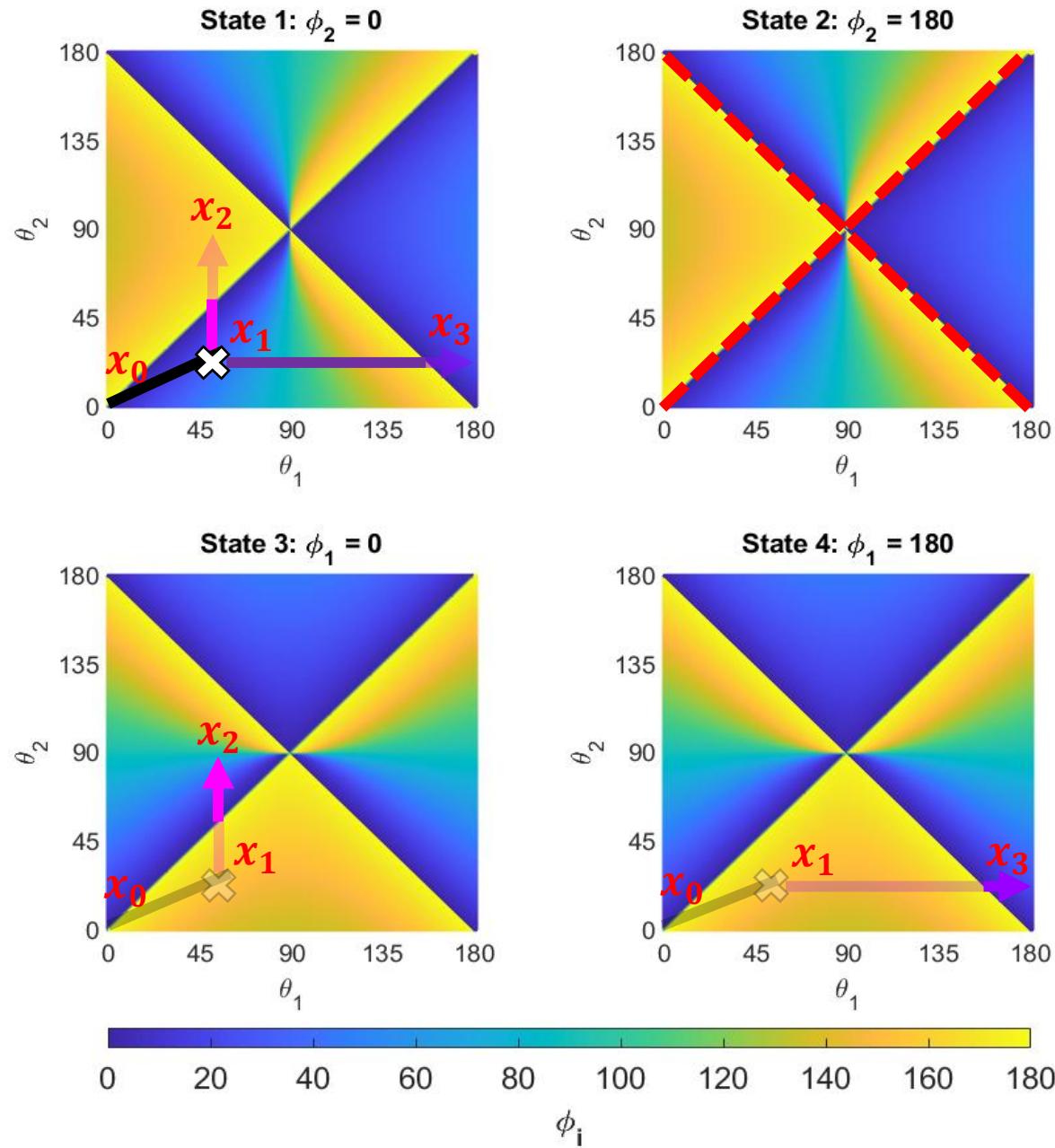


# Simulated vs. Experimental Results

MAE 4.36°



# Quasi-Static Simulations



# Generalizable to Variable Morphologies



Shape morphing



Number of curved links



Curved link length

# Takeaways and Research Impact

1. The paper statically models a mobile, curved-link tensegrity robot using a **geometric representation** and provides subsequent simulation and validation. The **generalizable modeling framework** is adaptable to robots with multiple points of contact and different morphologies;
2. The model analytically **proves the hybrid nature** of the system where the robot exists in **four states**. Each state corresponds to the motion where the robot instantaneously pivots about the end of one curved-link and rolls along the other;
3. **Static simulations** provide input-output relationship between the internal mass position and ground contact points, equivalently, the robot orientation;
4. A tetherless robot prototype **experimentally validates** the static model with **high accuracy**.

# Questions?



L. Ervin and V. Vikas, "Geometric Static Modeling Framework for Piecewise-Continuous Curved-Link Multi Point-of-Contact Tensegrity Robots," in *IEEE Robotics and Automation Letters*, vol. 9, no. 12, pp. 11066-11073, Dec. 2024, doi: 10.1109/LRA.2024.3486199.

