Geometric Static Modeling Framework for Piecewise-Continuous Curved-Link Multi Point-of-Contact Tensegrity Robots

Lauren Ervin and Vishesh Vikas

Agile Robotics Laboratory
arl.ua.edu
The University of Alabama, USA

Motivation

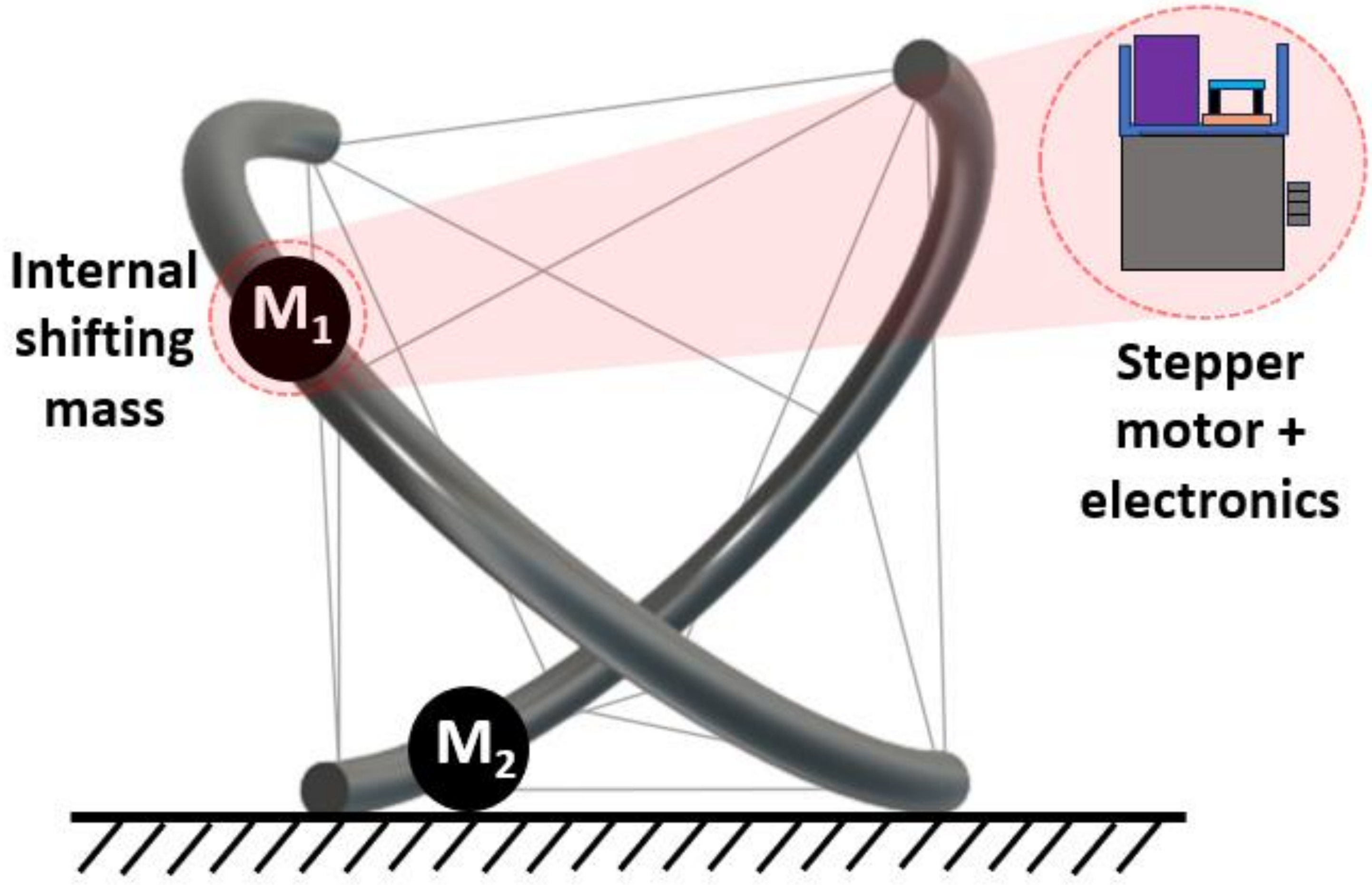
Discontinuous, curved morphology well suited for space applications

Critical Terminology

- **Tensegrity mechanism** – Synergistically combine *tension* elements (pre-stressed cables) with *compression* elements (rigid rods) to achieve *structural integrity*
- **Screw Theory**– Framework used to geometrically model motion of rigid bodies as twists along a screw axis
- **Hybrid system** – The robot moves in four different states: it always rolls along the entirety of one arc while pivoting about an endpoint of the other until swapping arcs
- **Piecewise continuous movement** – Rolling produces continuous movement in different directions that instantaneously flip when updating pivot points; when combined, the movement generates a straight path
- **Stability** – A robot orientation where tip-over does not occur regardless of internal mass positions

Tensegrity eXploratory Robot (TeXploR) Design

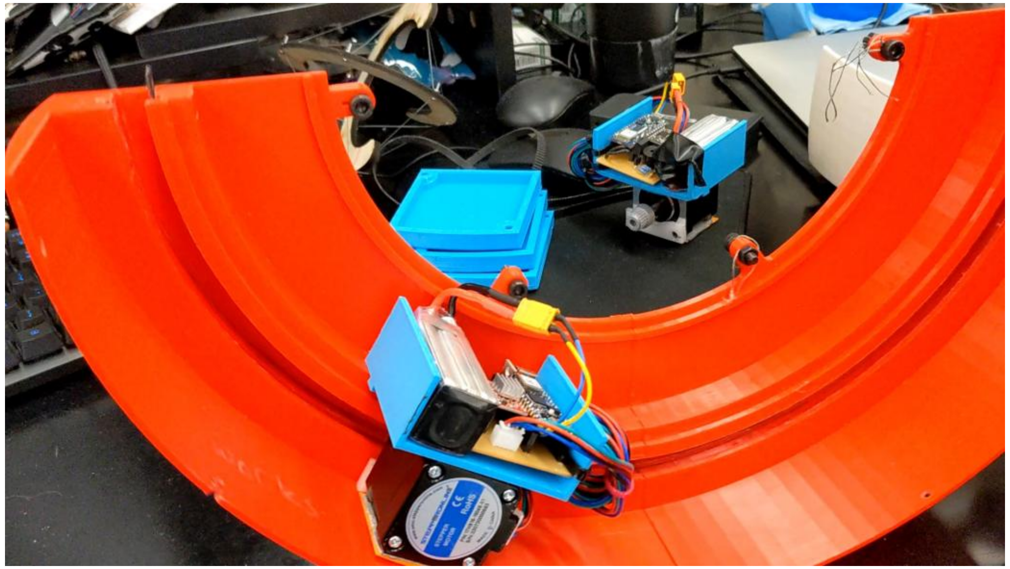
Curved-links with two points of contact

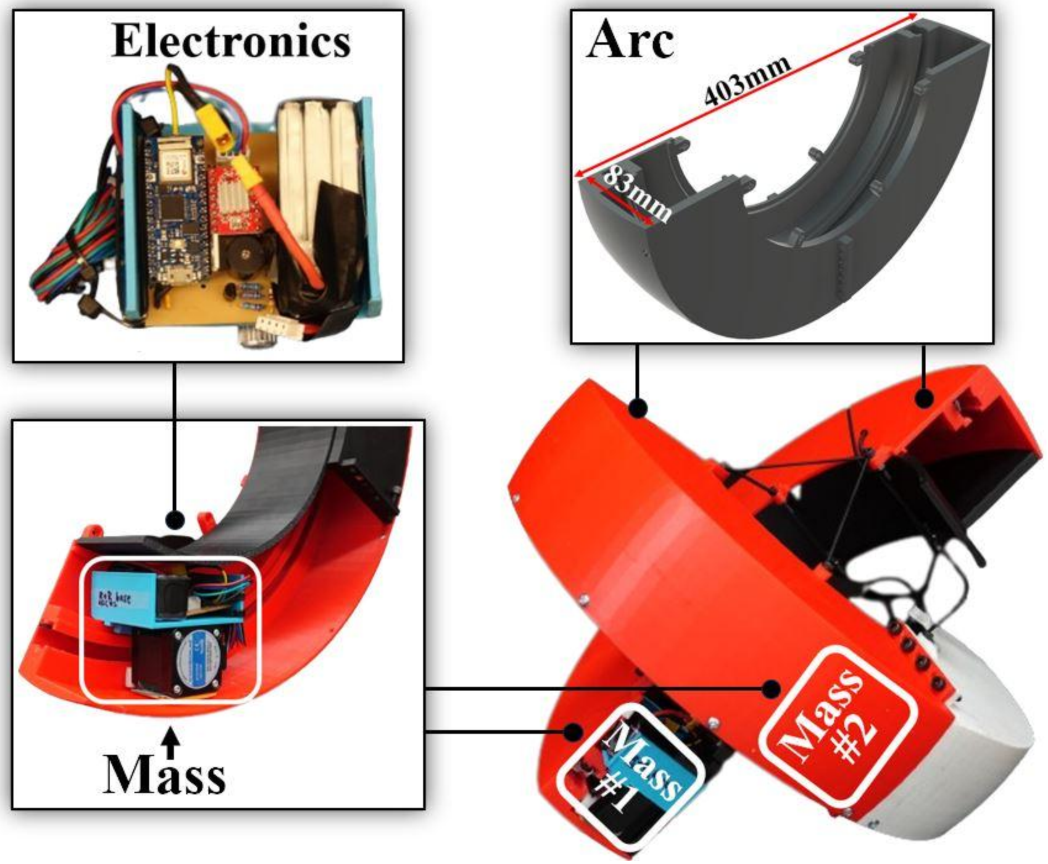


TeXploR Prototype Mechatronics

All electronics mounted internally and shift along arc

Internal Mass Shifting



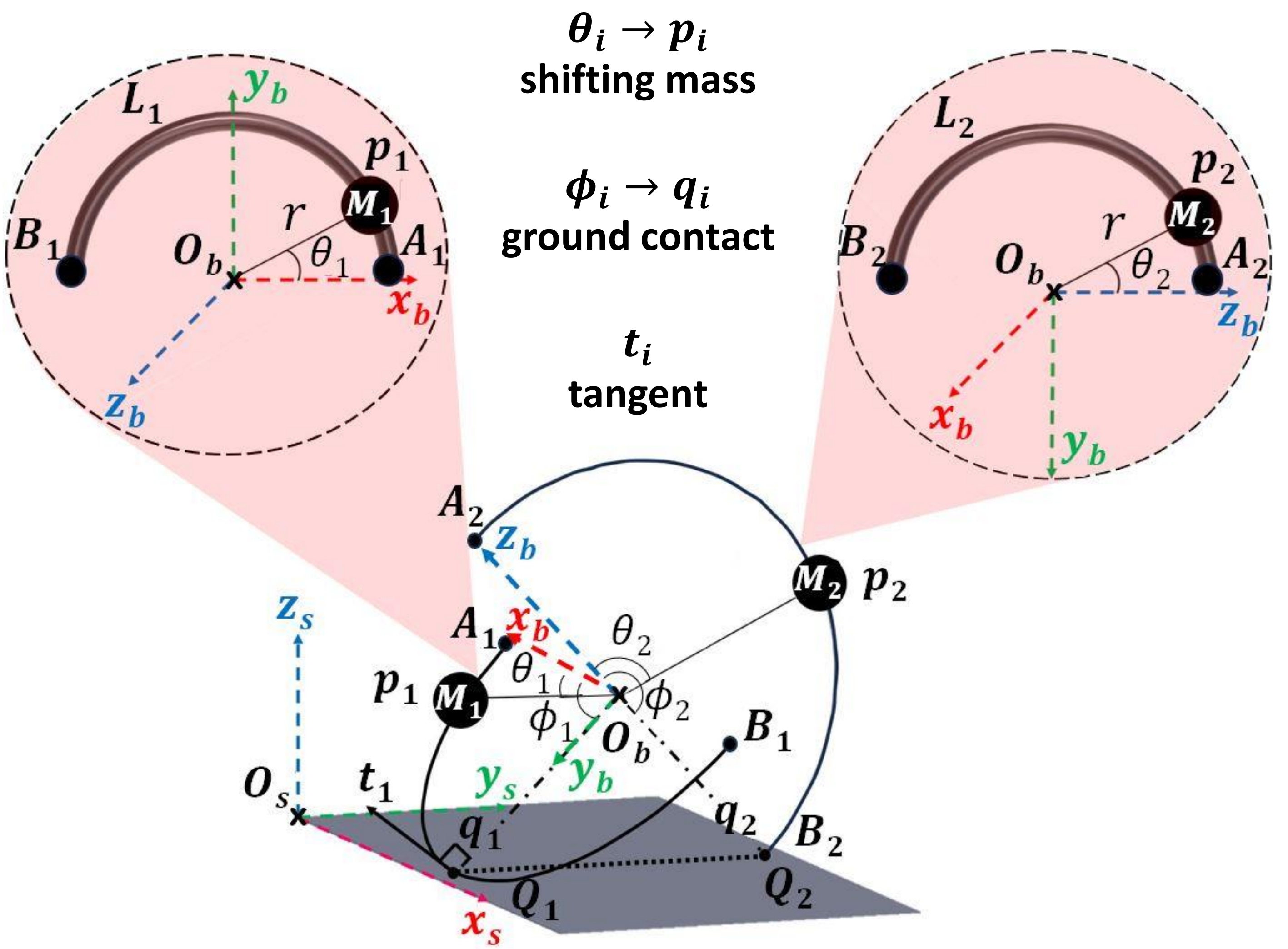


Kinematics

Geometric representation using Screw Theory

$$\boldsymbol{p}_i^i = r\begin{bmatrix} c_{\theta_i} \\ s_{\theta_i} \\ 0 \end{bmatrix}, \quad \boldsymbol{q}_i^i = r\begin{bmatrix} c_{\phi_i} \\ s_{\phi_i} \\ 0 \end{bmatrix}$$

$$T_{12} = \begin{bmatrix} R_{12} & \boldsymbol{o}_{12}^1 \\ \boldsymbol{0}_{1\times 3} & 1 \end{bmatrix}, R_{12} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \boldsymbol{o}_{12}^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$



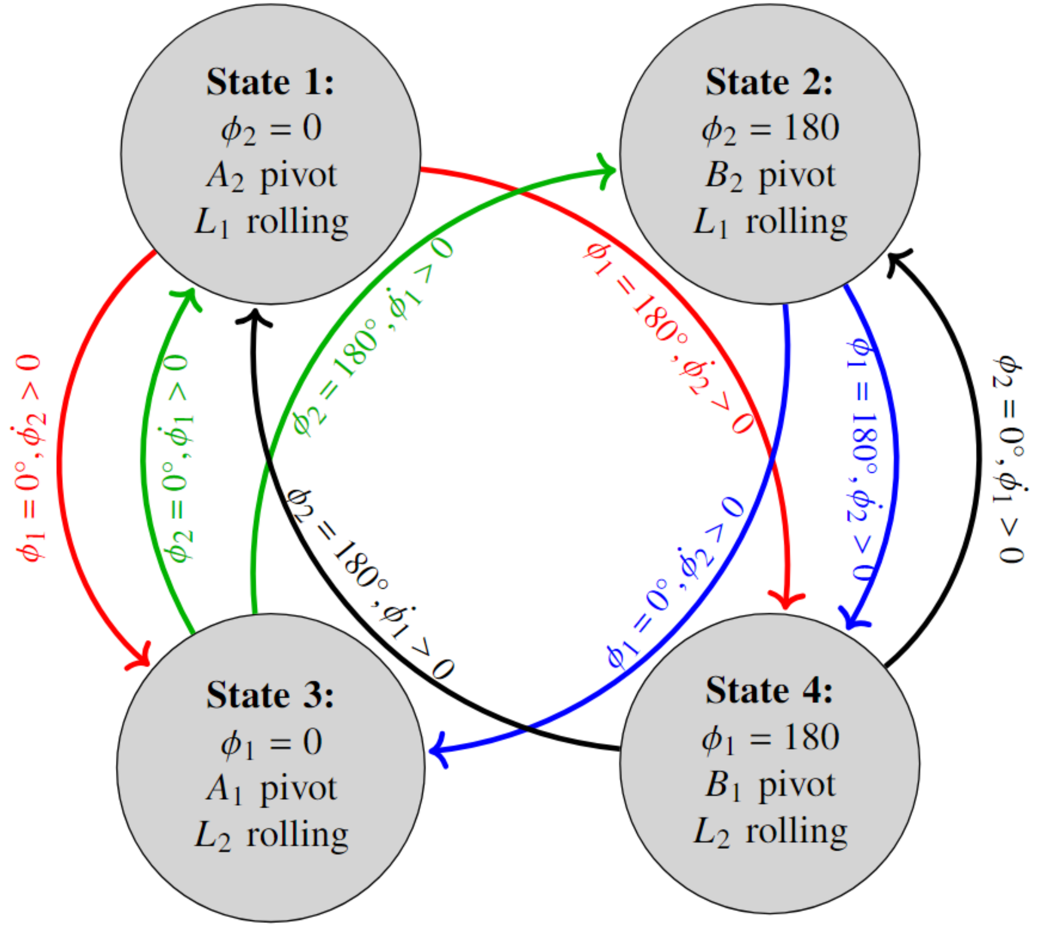
Hybrid State System

Four states of locomotion for rolling sequence from holonomic constraints

Case 1: t_2 undefined $\rightarrow$ State 1: $\phi_2 = 0$ or State 2: $\phi_2 = 180$

Case 2: t_1 undefined $\rightarrow$ State 3: $\phi_1 = 0$ or State 4: $\phi_1 = 180$

~~Case 3: t_1, t_2 defined $\rightarrow$ $\phi_2 = -\phi_1$ or $\phi_2 = \phi_1 + 180$~~



Geometric Static Modeling Framework

Internal mass inputs → static equilibrium position output

Holonomic constraints

↓

Hybrid system → State 1 ... State 4

$$\mathcal{F}=\begin{bmatrix}\boldsymbol{f}\\ \boldsymbol{m}\end{bmatrix}=\sum_{i=1}^{2}\begin{bmatrix}(M_i+m_i)g\boldsymbol{z}_s+\boldsymbol{F}_{ri}\\ M_i g\hat{\boldsymbol{p}}_i\boldsymbol{z}_s+m_i g\hat{\boldsymbol{r}}_i\boldsymbol{z}_s+\hat{\boldsymbol{q}}_i\boldsymbol{F}_{ri}\end{bmatrix}=0 \quad \text{s.t.}$$

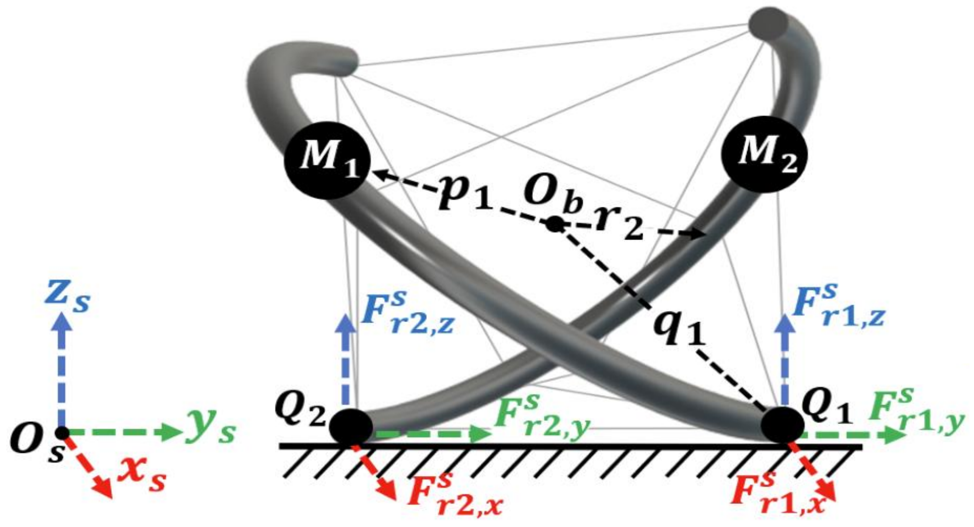
$$\boldsymbol{f}^s=\begin{bmatrix}\cancel{F^s_{r1,x}-F^s_{r2,x}}\\ \cancel{F^s_{r1,y}-F^s_{r2,y}}\\ (M_1+M_2+m_1+m_2)g-(F^s_{r1,z}+F^s_{r2,z})\end{bmatrix}$$

$$\boldsymbol{m}^b=\begin{matrix}\sum_{i=1}^{2}\left(M_i g\hat{\boldsymbol{p}}_i^b+m_i g\hat{\boldsymbol{r}}_i^b-F^s_{ri,z}\hat{\boldsymbol{q}}_i^b\right)\boldsymbol{z}_s^b\\ +\cancel{\left(F^s_{r1,x}\hat{\boldsymbol{q}}_1^b-F^s_{r2,x}\hat{\boldsymbol{q}}_2^b\right)\boldsymbol{x}_s^b}+\cancel{\left(F^s_{r1,y}\hat{\boldsymbol{q}}_1^b-F^s_{r2,y}\hat{\boldsymbol{q}}_2^b\right)\boldsymbol{y}_s^b}\end{matrix}$$

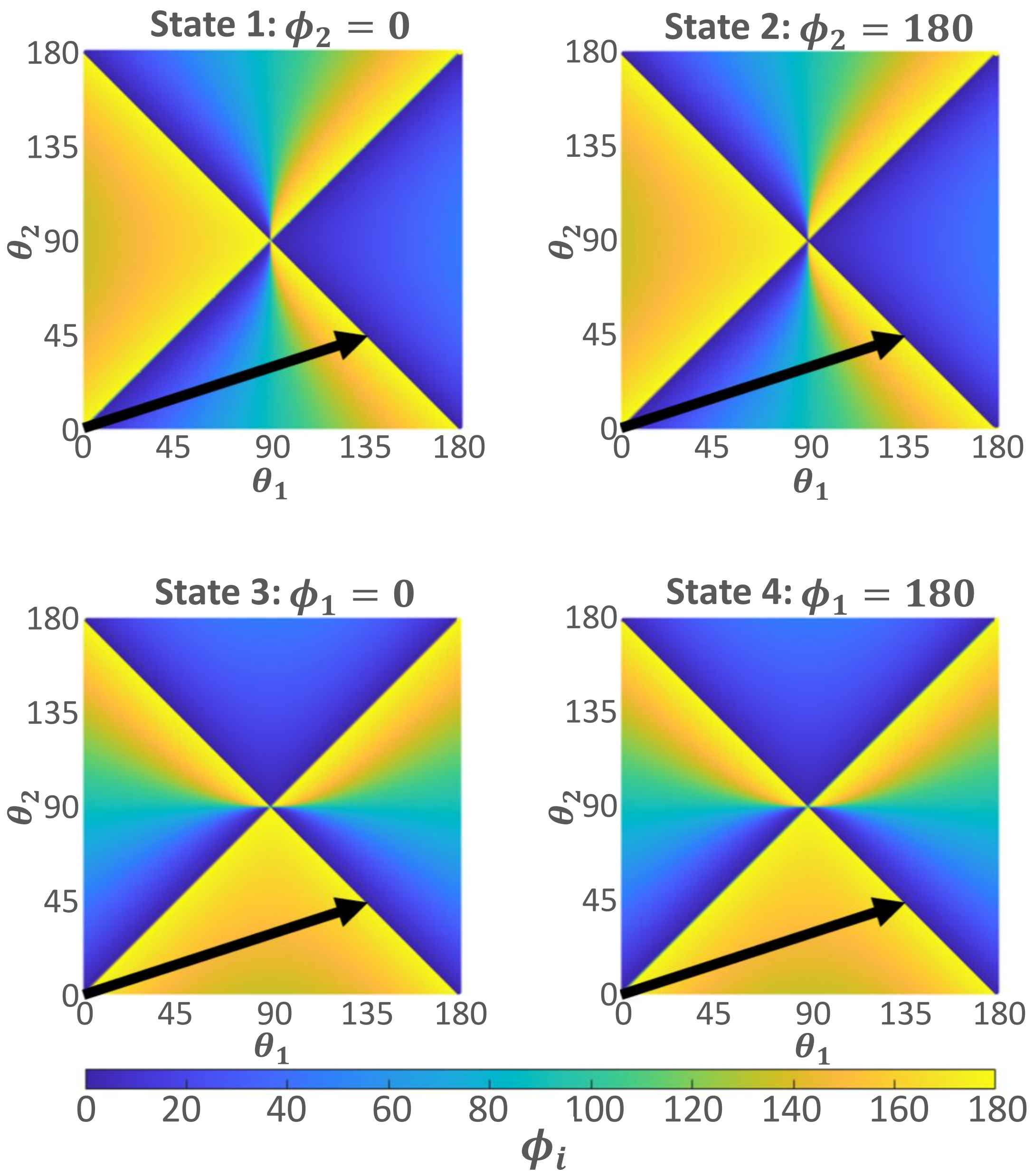
$$\boldsymbol{z}_s^b=-\frac{1}{\sqrt{2}}[c_{\phi_1},s_{\phi_1},1]^T,\quad \boxed{\tan\phi_1=\frac{s_{\theta_1}-s_{\theta_2}}{c_{\theta_1}}}$$

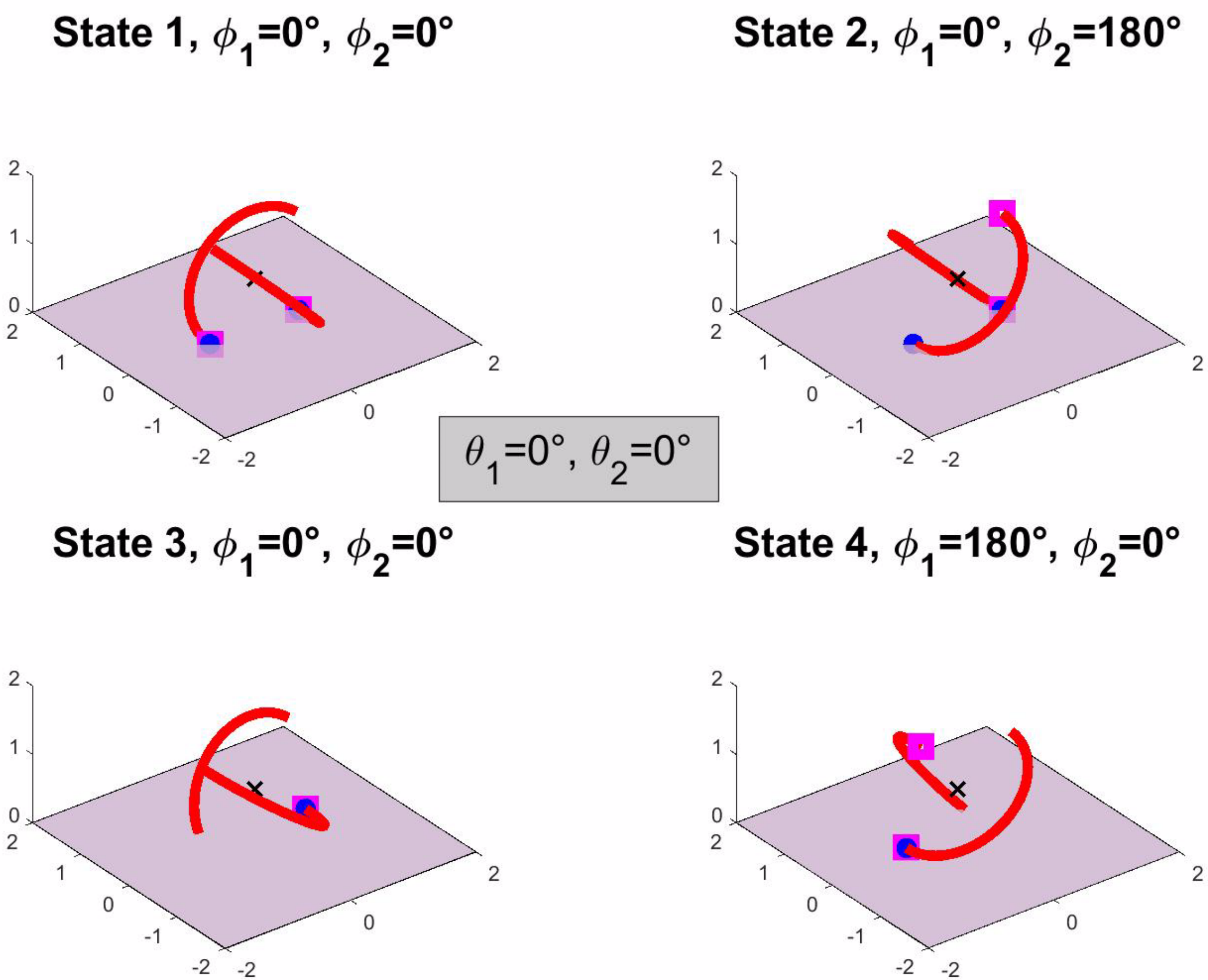
$$F_{r1,z}=Mg\left[-1+\frac{c_{\phi_1+\theta_2}}{2}-c_{\theta_2}+c_{\phi_1-\theta_1}-\frac{c_{\phi_1-\theta_2}}{2}\right]-mg$$

$$F_{r2,z}=Mg\left[-1-\frac{c_{\phi_1+\theta_2}}{2}+c_{\theta_2}-c_{\phi_1-\theta_1}+\frac{c_{\phi_1-\theta_2}}{2}\right]-mg$$



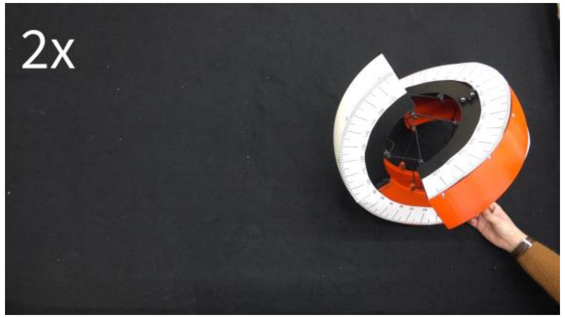
Quasi-Static Simulations





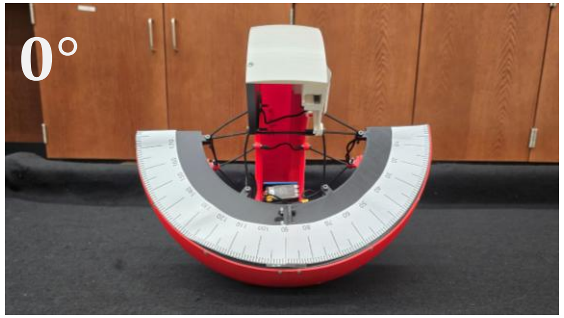
Experimental Validation

State 3, $\theta_1 = 135°, \theta_2 = 135°$

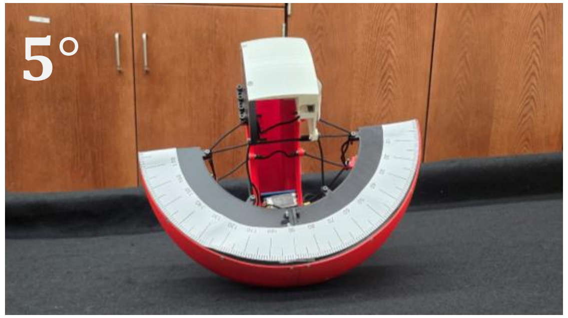


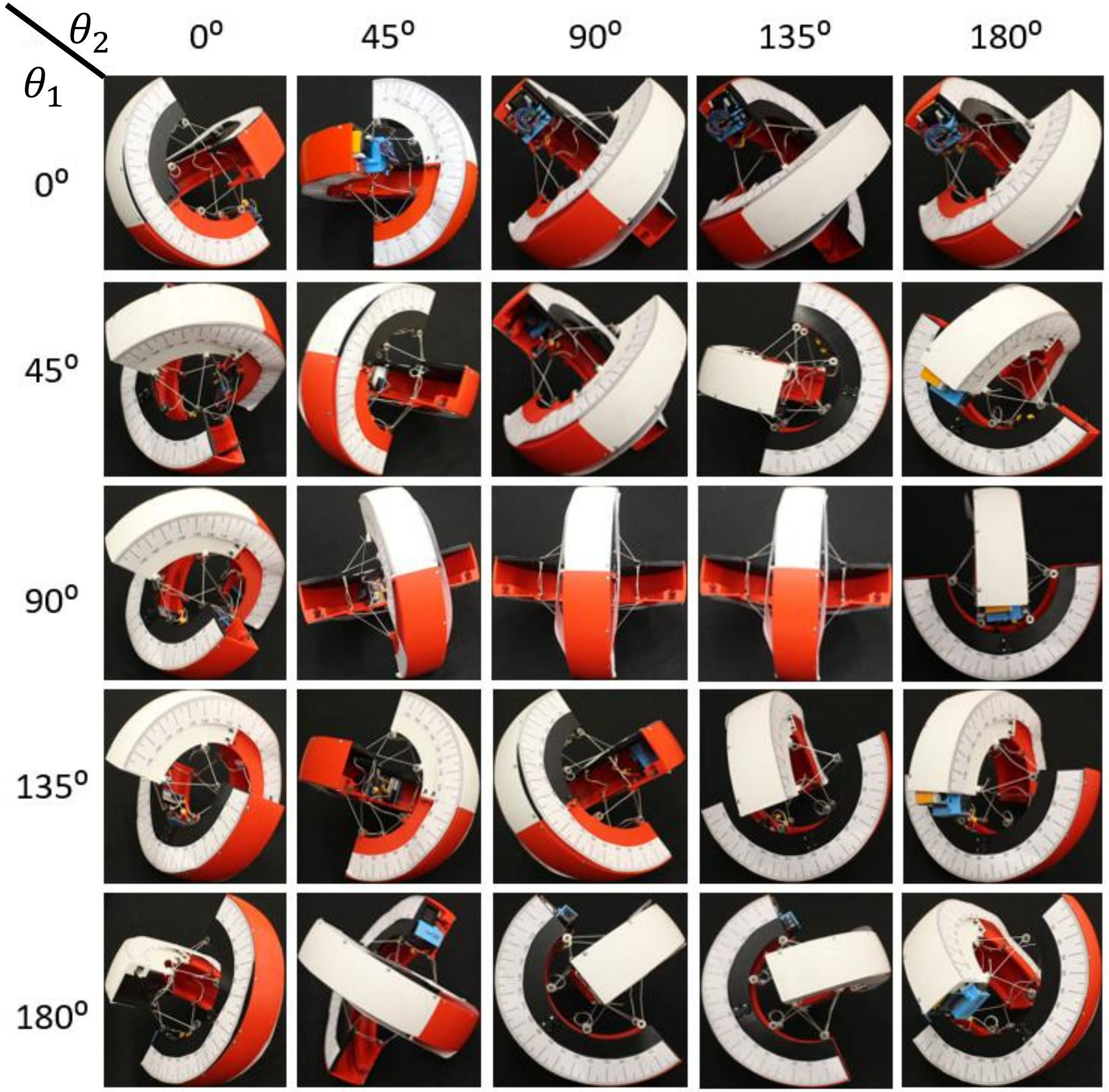
State 4, $\theta_1 = 45°, \theta_2 = 135°$

$\phi_1 = 90°, \phi_2 = 0°$, 0° incline



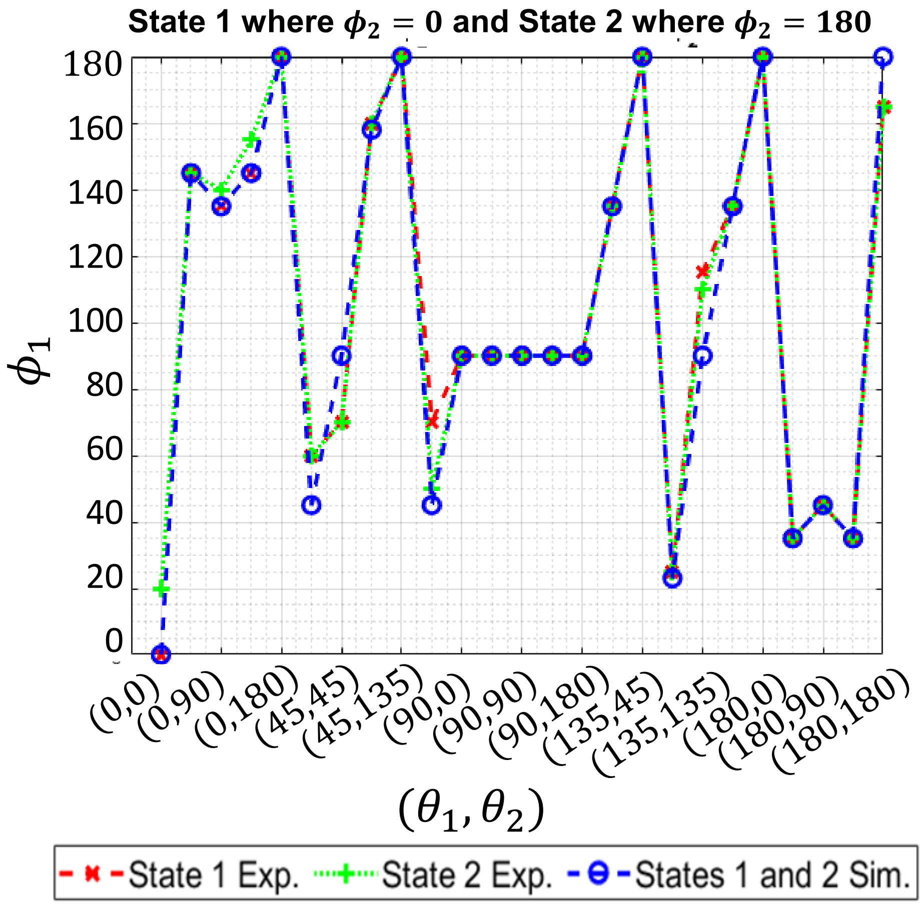
$\phi_1 = 90°, \phi_2 = 0°$, 5° incline

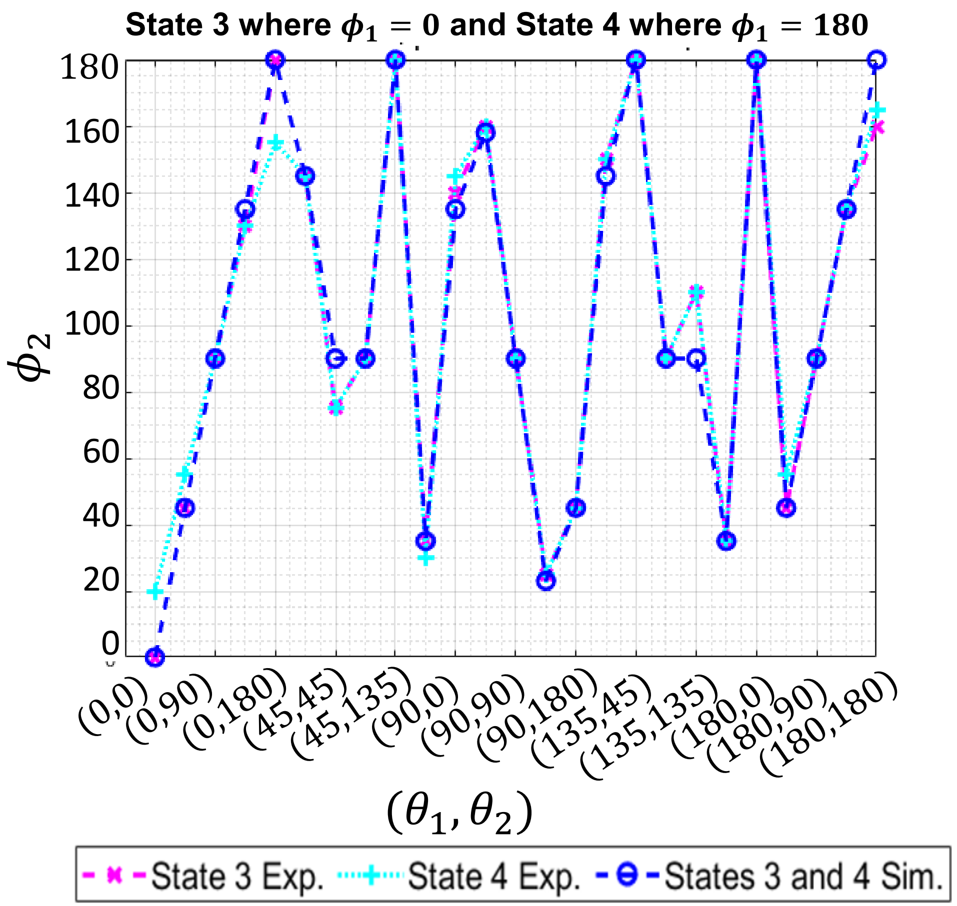




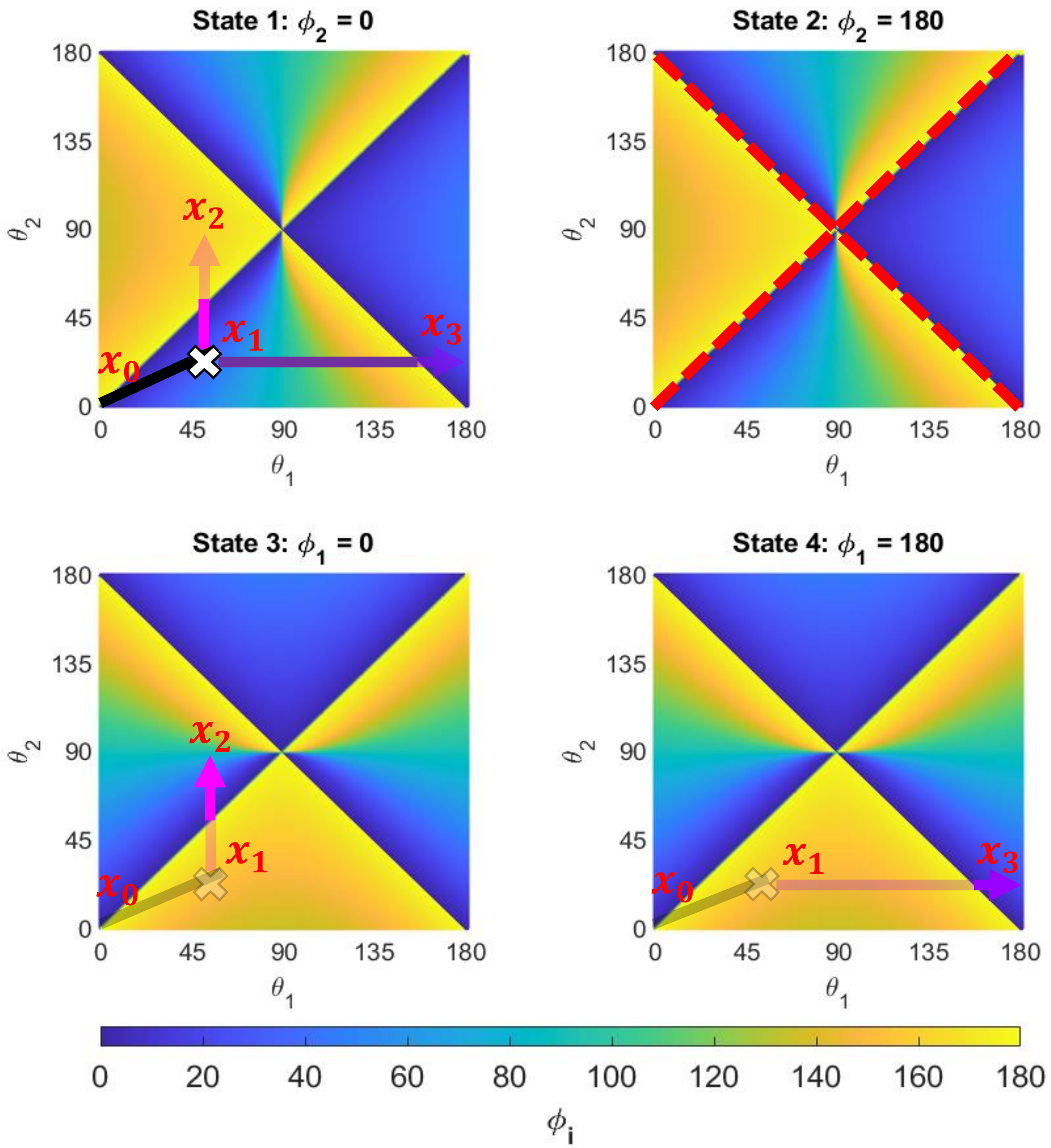
Simulated vs. Experimental Results

MAE 4.36°





Quasi-Static Simulations



Generalizable to Variable Morphologies

Shape morphing

Number of curved links

Curved link length

Takeaways and Research Impact

1. The paper statically models a mobile, curved-link tensegrity robot using a **geometric representation** and provides subsequent simulation and validation. The **generalizable modeling framework** is adaptable to robots with multiple points of contact and different morphologies;

2. The model analytically **proves the hybrid nature** of the system where the robot exists in **four states**. Each state corresponds to the motion where the robot instantaneously pivots about the end of one curved-link and rolls along the other;

3. **Static simulations** provide input-output relationship between the internal mass position and ground contact points, equivalently, the robot orientation;

4. A tetherless robot prototype **experimentally validates** the static model with **high accuracy**.

Questions?

L. Ervin and V. Vikas, "Geometric Static Modeling Framework for Piecewise-Continuous Curved-Link Multi Point-of-Contact Tensegrity Robots," in *IEEE Robotics and Automation Letters*, vol. 9, no. 12, pp. 11066-11073, Dec. 2024, doi: 10.1109/LRA.2024.3486199.