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ELEC446 Assignment 2 + KB - 19kobl - 20220414

Luenberger-like observer: diff. drive robot kinematics are—

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\end{bmatrix} = \begin{bmatrix}\frac{1}{2}\cos\theta & \frac{1}{2}\cos\theta\\ \frac{1}{2}\sin\theta & \frac{1}{2}\sin\theta\\ -\frac{1}{l} & \frac{1}{l}\end{bmatrix}\begin{bmatrix}V_L\\ V_R\end{bmatrix} \quad \text{where } q=(x,y,\theta)\in\mathbb{R}^2\times S^1, \; l=0.25\text{m}$$

a) discrete time approximation with $T=0.1s$ $(q \to q_k)$

$$\frac{\Delta q_k}{T} \doteq \dot{q} \quad \text{where } \Delta q_k = q_{k+1} - q_k$$

$$\therefore \hat{q}_{k+1} = q_k + T\,G(q_k)\,v_k$$

ie.

$$\begin{bmatrix} x_{k+1} = x_k + T\dot{x} = x_k + T\left(\frac{1}{2}\cos\theta_k[V_L+V_R]\right)\\ y_{k+1} = y_k + T\dot{y} = y_k + T\left(\frac{1}{2}\sin\theta_k[V_L+V_R]\right)\\ \theta_{k+1} = \theta_k + T\dot{\theta} = \theta_k + T\left(\frac{1}{l}[V_R-V_L]\right)\end{bmatrix}$$

with $T=0.1$, $l=0.25$:

$$\hat{q}_{k+1} = \begin{bmatrix}x_{k+1}\\ y_{k+1}\\ \theta_{k+1}\end{bmatrix} = \begin{bmatrix}x_k\\ y_k\\ \theta_k\end{bmatrix} + \begin{bmatrix}0.05\cos\theta_k & 0.05\cos\theta_k\\ 0.05\sin\theta_k & 0.05\sin\theta_k\\ -0.4 & 0.4\end{bmatrix}\begin{bmatrix}V_L\\ V_R\end{bmatrix}$$

$$q_{k+1} = q_k + T\,G(q_k) \cdot v = f(q_k, v_k)$$

b) beacons: $(x_{b1}, y_{b1}) = (4,3)$, $(x_{b2}, y_{b2}) = (3,-7)$

estimate full state q_k with Luenberger-like observer.

Assume measurements are perfect and certain (no noise or uncertainty)

at each step $k = 1, 2, \ldots$, do the following:

1 – receive sensor data for $V_L, V_R, \hat{r}_n$ (range to beacon $n=1,2$)

2 – express predicted range $\hat{r}_n$ as vector $h(q_k)$ → sensor model

3 – find gain matrix L to correct error in r_k (measured range)

4 – add weighted difference $L(r_{nk} - h(q_k))$ to q_{k+1} function

b) cont. - final observer should be in form

$$\hat{q}_{k+1} = q_k + TG(q_k)v_k + L(r_k - h(\hat{q}_k))$$

range prediction: $r_{nk} = \sqrt{(x_k - x_{bn})^2 + (y_k - y_{bn})^2}$

beacons at $b_1 = (4,3)$ $b_2 = (3,-7)$

$$\therefore \left.\begin{aligned} \hat{r}_{1k} &= \sqrt{(\hat{x}_k - 4)^2 + (\hat{y}_k - 3)^2} \\ \hat{r}_{2k} &= \sqrt{(\hat{x}_k - 3)^2 + (\hat{y}_k + 7)^2} \end{aligned}\right\} = h(\hat{q}_k)$$

to find L for $L(r_k - h(\hat{q}_k))$ → linearize about $\hat{q}_k$ at each k

$$F_k = \frac{\partial f}{\partial q_k}, \quad H_k = \frac{\partial h}{\partial q_k}$$

F_k: $\frac{\partial x_{k+1}}{\partial x_k} = 1$, $\frac{\partial x_{k+1}}{\partial y_k} = 0$, $\frac{\partial x_{k+1}}{\partial \theta} = -0.05\theta_k \sin\theta_k (v_L + v_R)$

$\frac{\partial y_{k+1}}{\partial x_k} = 0$, $\frac{\partial y_{k+1}}{\partial y_k} = 1$, $\frac{\partial y_{k+1}}{\partial \theta} = 0.05\theta_k \cos\theta_k (v_L + v_R)$

$\frac{\partial \theta_{k+1}}{\partial x_k} = 0$, $\frac{\partial \theta_{k+1}}{\partial y_k} = 0$, $\frac{\partial \theta_{k+1}}{\partial \theta} = 1$

$$\rightarrow F_k = \begin{bmatrix} 1 & 0 & -0.05\sin\theta_k(v_L + v_R) \\ 0 & 1 & 0.05\cos\theta_k(v_L + v_R) \\ 0 & 0 & 1 \end{bmatrix}$$

H_k: beacon 1: $\frac{\partial r_{1k}}{\partial x_k} = \frac{x_k - 4}{\sqrt{(x_k-4)^2 + (y_k-3)^2}}$, $\frac{\partial r_{1k}}{\partial y_k} = \frac{y_k - 3}{\sqrt{(x_k-4)^2 + (y_k-3)^2}}$, $\frac{\partial r_{1k}}{\partial \theta} = 0$

beacon 2: $\frac{\partial r_{2k}}{\partial x_k} = \frac{x_k - 3}{\sqrt{(x_k-3)^2 + (y_k+7)^2}}$, $\frac{\partial r_{2k}}{\partial y_k} = \frac{y_k + 7}{\sqrt{(x_k-3)^2 + (y_k+7)^2}}$, $\frac{\partial r_{2k}}{\partial \theta} = 0$

$$H_k = \begin{bmatrix} \frac{x_k - 4}{r_{1k}} & \frac{y_k - 3}{r_{1k}} & 0 \\ \frac{x_k - 3}{r_{2k}} & \frac{y_k + 7}{r_{2k}} & 0 \end{bmatrix}$$

Finding L_k such that $F_k - L_k H_k$ has eigenvalues $|\lambda_i| < 1$:

$\underset{\text{observer matrix}}{A} = F_k - L_k H_k \Rightarrow LH$ must be 3x3, ie L is 2x3 matrix

Finding Gain matrix L_k: (cont.)

$$L_kH_k = \begin{bmatrix} L_{11}\cdot\frac{x_k-4}{r_{1k}} + L_{12}\cdot\frac{x_k-3}{r_{2k}} & L_{11}\cdot\frac{y_k-3}{r_{1k}} + L_{12}\frac{y_k+7}{r_{2k}} & 0 \\ L_{21}\cdot\frac{x_k-4}{r_{1k}} + L_{22}\frac{x_k-3}{r_{2k}} & L_{21}\frac{y_k-3}{r_{1k}} + L_{22}\frac{y_k+7}{r_{2k}} & 0 \\ L_{31}\cdot\frac{x_k-4}{r_{1k}} + L_{32}\frac{x_k-3}{r_{2k}} & L_{31}\frac{y_k-3}{r_{1k}} + L_{32}\frac{y_k+7}{r_{2k}} & 0 \end{bmatrix}$$

$$A = F - LH = \begin{bmatrix} 1 - L_{11}\frac{x-4}{r_1} - L_{12}\frac{x-3}{r_2} & -L_{11}\frac{y-3}{r_1} - L_{12}\frac{y+7}{r_2} & -0.05\sin\theta(v_L+v_R) \\ -L_{21}\frac{x-4}{r_1} - L_{22}\frac{x-3}{r_2} & 1 - L_{21}\frac{y-3}{r_1} - L_{22}\frac{y+7}{r_2} & 0.05\cos\theta(v_L+v_R) \\ -L_{31}\frac{x-4}{r_1} - L_{32}\frac{x-3}{r_2} & -L_{31}\frac{y-3}{r_1} - L_{32}\frac{y+7}{r_2} & 1 \end{bmatrix}$$

(↑ all x, y, θ are x_k, y_k, θ_k: omitted k for writing clarity)

→ find eigenvalues $\in (0,1)$ to satisfy $\det(A - \lambda I) = 0$

$$A - \lambda I = \begin{bmatrix} 1 - L_{11}\frac{x-4}{r_1} - L_{12}\frac{x-3}{r_2} - \lambda & -L_{11}\frac{y-3}{r_1} - L_{12}\frac{y+7}{r_2} & -0.05\sin\theta(v_L+v_R) \\ -L_{21}\frac{x-4}{r_1} - L_{22}\frac{x-3}{r_2} & 1 - L_{21}\frac{y-3}{r_1} - L_{22}\frac{y+7}{r_2} - \lambda & 0.05\cos\theta(v_L+v_R) \\ -L_{31}\frac{x-4}{r_1} - L_{32}\frac{x-3}{r_2} & -L_{31}\frac{y-3}{r_1} - L_{32}\frac{y+7}{r_2} & 1-\lambda \end{bmatrix}$$

$$\begin{aligned} \det(A - \lambda I) = {} & \left(1 - L_{11}\frac{x-4}{r_1} - L_{12}\frac{x-3}{r_2} - \lambda\right) \det \begin{bmatrix} 1 - L_{21}\frac{y-3}{r_1} - L_{22}\frac{y+7}{r_2} - \lambda & 0.05\cos\theta(v_L+v_R) \\ -L_{31}\frac{y-3}{r_1} - L_{32}\frac{y+7}{r_2} & 1-\lambda \end{bmatrix} \\ & - \left(-L_{11}\frac{y-3}{r_1} - L_{12}\frac{y+7}{r_2}\right) \det \begin{bmatrix} -L_{21}\frac{x-4}{r_1} - L_{22}\frac{x-3}{r_2} & 0.05\cos\theta(v_L+v_R) \\ -L_{31}\frac{x-4}{r_1} - L_{32}\frac{x-3}{r_2} & 1-\lambda \end{bmatrix} \\ & + (-0.05\sin\theta(v_L+v_R)) \det \begin{bmatrix} -L_{21}\frac{x-4}{r_1} - L_{22}\frac{x-3}{r_2} & 1 - L_{21}\frac{y-1}{r_1} - L_{22}\frac{y+7}{r_2} - \lambda \\ -L_{31}\frac{x-4}{r_1} - L_{32}\frac{x-3}{r_2} & -L_{31}\frac{y-3}{r_1} - L_{32}\frac{y+7}{r_2} \end{bmatrix} \end{aligned}$$

$$\begin{aligned} 0 = {} & \left(1 - L_{11}\frac{x-4}{r_1} - L_{12}\frac{x-3}{r_2} - \lambda\right)\left(1 - L_{21}\frac{y-3}{r_1} - L_{22}\frac{y+7}{r_2} - \lambda\right)(1-\lambda) \\ & - \left(1 - L_{11}\frac{x-4}{r_1} - L_{12}\frac{x-3}{r_2} - \lambda\right)(0.05\cos\theta(v_L+v_R))\left(-L_{31}\frac{y-3}{r_1} - L_{32}\frac{y+7}{r_2}\right) \\ & + \left(L_{11}\frac{y-3}{r_1} + L_{12}\frac{y+7}{r_2}\right)\left(-L_{21}\frac{x-4}{r_1} - L_{22}\frac{x-3}{r_2}\right)(1-\lambda) \\ & - \left(L_{11}\frac{y-3}{r_1} + L_{22}\frac{y+7}{r_2}\right)(0.05\cos\theta(v_L+v_R))\left(-L_{31}\frac{x-4}{r_1} - L_{32}\frac{x-3}{r_2}\right) \\ & + (-0.05\sin\theta(v_L+v_R))\left(-L_{21}\frac{x-4}{r_1} - L_{22}\frac{x-3}{r_2}\right)\left(-L_{31}\frac{y-3}{r_1} - L_{32}\frac{y+7}{r_2}\right) \\ & - (-0.05\sin\theta(v_L+v_R))\left(1 - L_{21}\frac{y-1}{r_1} - L_{22}\frac{y+7}{r_2} - \lambda\right)\left(-L_{31}\frac{x-4}{r_1} - L_{32}\frac{y-3}{r_2}\right) \end{aligned}$$

from here: ① choose $\lambda \in [0,1)$, ② find L matrix values to satisfy ↑ above.

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c) see code + observations (lines 5-9)

d) see code + observations (lines 6-12)

e) incomplete —

see code + comments