Assignment 2

ELEC 446 Mobile Robotics

Problem 1 of 1

In this assignment, you will design a navigation system for a differential-drive mobile robot by using a Luenberger-like observer. Although we know that real sensors are not perfect, we initially assume so in this assignment (i.e., you do not need incorporate or quantify uncertainty).

The kinematic model for the differential-drive mobile robot is given by

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\cos\theta & \frac{1}{2}\cos\theta \\ \frac{1}{2}\sin\theta & \frac{1}{2}\sin\theta \\ -\frac{1}{\ell} & \frac{1}{\ell} \end{bmatrix} \begin{bmatrix} v_L \\ v_R \end{bmatrix}, \tag{1}$$

using the coordinate shown in Figure 1. The state (configuration) of the system is given by $\boldsymbol{q} = (x, y, \theta) \in \mathbb{R}^2 \times \mathbb{S}^1$. The robot is equipped with incremental wheel encoders that measure the left and right wheel speeds v_L and v_R, respectively (which are also our model's inputs). Let $\ell = 0.25$ m.

(a) **[2 marks]** Write a discrete-time approximation for the mobile robot model (1) with a sample time $T = 0.1$ s, which you will use for your navigation/estimation system design.

(b) **[10 marks]** Suppose that the mobile robot is equipped with a sensor system that can measure the range from (x, y) on the robot to two beacons located at $(x_{b,1}, y_{b,1}) = (4, 3)$ m and $(x_{b,2}, y_{b,2}) = (3, -7)$ m in the robot's environment. Design a Luenberger-like observer for the mobile robot model (1) to estimate the full state $\boldsymbol{q}_k$ at each time step $k = 1, 2, \ldots$ You may also assume that the range measurements are perfect (i.e., no noise). For full marks, show your work and clearly write the steps of your design/algorithm.

(c) **[6 marks]** Simulate the navigation system with your observer design from part (b) for 20 seconds with an initial observer condition $\hat{\boldsymbol{q}}_0 = (0, 0, 0)$ and robot state $\boldsymbol{q} = (-3, 2, \frac{\pi}{6})$. Do

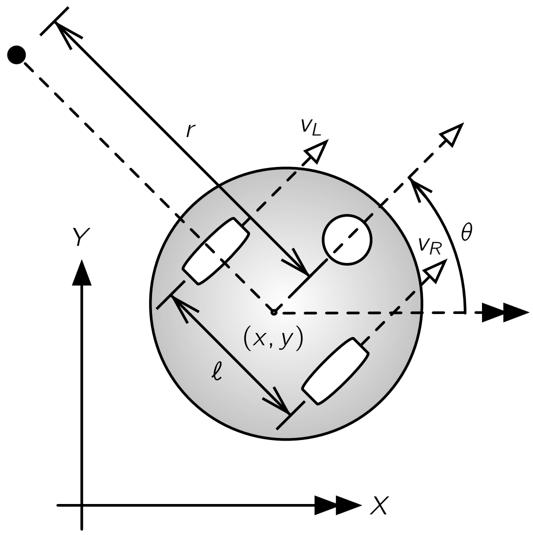

Figure 1: Differential drive vehicle with left and right wheels speeds v_L and v_R, respectively. The instantaneous centre of rotation (solid circle) lies at a distance r from $(x, y) \in \mathbb{R}^2$.

so by driving the vehicle around a circle with radius 5.0 m at 0.25 m/s. Plot the true and estimated states x, y, and θ over time on the same axes. For your simulation, place the poles of your observer at locations $\lambda = 0.7, 0.8, 0.9$ (all inside the unit circle).

(d) **[2 marks]** Add a bit of noise to the range measurements in your simulation from part (c) and plot the true and estimated states x, y, and θ over time on the same axes. To simulate the presence of some normally distributed noise to a range measurement r, you can use $r_{\text{noisy}} = r + \sigma\epsilon$, where $\epsilon \sim \mathcal{N}(0, 1)$ and σ is the standard deviation of the noise. Let's pick $\sigma^2 = 0.01$ m for this problem (in practice we would get this from a specifications sheet of the sensor or from experiments). Plot the true and estimated states x, y, and θ over time on the same axes. Any difference in the results from part (c)?

(e) **[1 marks BONUS]** Add more beacons to the environment and repeat your simulation from part (d). Plot the true and estimated states x, y, and θ over time on the same axes, as before. Any difference in the results from part (d)?