Game Theory in Pragmatics:
Evolution, Rationality & Reasoning

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1 Summary & Keywords

Game theory provides formal means of representing and explaining action choices in social decision situations where the choices of one participant depend on the choices of another. Game theoretic pragmatics approaches language production and interpretation as a game in this sense. Patterns in language use are explained as optimal, rational, or at least nearly optimal or rational, solutions to a communication problem. Three intimately related perspectives on game theoretic pragmatics are sketched here: (i) the evolutionary perspective explains language use as the outcome of some optimization process, (ii) the rationalistic perspective pictures language use as a form of rational decision making, and (iii) the probabilistic reasoning perspective considers specifically speakers' and listeners' beliefs about each other. There are clear commonalities behind these three perspectives, and they may in practice blend into each other.

At the heart of game theoretic pragmatics lies the idea that speaker and listener behavior, when it comes to using a language with a given semantic meaning, are attuned to each other. By focussing on the evolutionary or rationalistic perspective we can then give a functional account of general patterns in our pragmatic language use. The probabilistic reasoning perspective invites modeling actual speaker and listener behavior, e.g., as it shows in quantitative aspects of experimental data.

Keywords: pragmatic inferences, language use, context, evolution of language, conversational implicature

2 Introduction

Game theoretic pragmatics, in the broad sense that I conceive of it here, is a set of approaches loosely unified by the desire to explain certain interesting regularities in language use and interpretation as the result of goal-oriented, purposeful communicative behavior of speakers and listeners that is, in a sense to be specified presently, optimal, rational, or at least nearly or approximately so. By this loose associative definition, different approaches within this paradigm can differ substantially in a number of ways. One important dimension of difference is the assumed goal or purpose of conversation for which a particular pragmatic pattern is hypothesized to be an (approximately) optimal or rational adaptation or choice. Another distinction lies in how different approaches assume that the observed and allegedly optimal behavior is to be separated from other conceivable ways of "solving the language game." According to *the evolutionary picture*, existing pragmatic practices are the outcome of gradual adaptation, habitualization or preferred conventionalization through processes of cultural evolution, such as imitation, occasional innovation, reinforcement and the like. According to *the rationalistic picture*, pragmatic language use is demarcated as, crudely speaking, that behavior which rational agents should choose in a normative sense. Finally, *the reasoning picture* would construe pragmatic language use as approximately rational reasoning, but would feel free to allow for occasional limitations of reasoning power or the effects of general cognitive biases, e.g., from perception, limited memory or other.

In what follows, I will try to provide a rough sketch of the landscape of possible approaches within game theoretic pragmatics. The goal is to show their diversity alongside their commonalities. Effectively, I argue that the evolutionary, rationalistic and reasoning perspectives are at best fuzzy distinctions for the purpose of exposition, but not necessarily of substance. This is what allows subsumption under a common header. The main commonality in all of these approaches is that they explicitly consider production and comprehension side by side, and see in particular production as attuned to comprehension and comprehension as attuned to production. This latter seemingly circular mutual dependency is what motivates the use of game theory.

All approaches that I here classify as game theoretic pragmatics are formal. Formalization comes with the known benefits and will inspire the known worries. Any formalization must make simplifications and abstractions, or else it would be as unwieldy as reality itself. Given a set of simplifying assumptions, formal modeling allows to see clearly the consequences and implications of these. A model's simplifying assumptions may even be false (they most likely are) and known to be false (they often are). This is acceptable, if we do not see the model as the ulterior goal, but as a means to an end. This way, even models that are known to be too simplistic or even false *can* be useful. They can show unexpected consequences of our initial assumptions (e.g., that they are inconsistent), or show where exactly the model's picture of reality goes astray.

To convey an idea of game theoretic pragmatics, it is necessary to introduce at least a small number of mathematical notions, some of which may be unfamiliar to linguists. The most basic ones are that of a *game* in its technical sense and that of an *equilibrium*, which is the most prominent of game theoretic *solution concepts*. These are introduced in Section 3. Section 4 introduces a number of games as models of contexts in which patterns of theoretically interesting pragmatic language use should arise. Sections 5, 6 and 7 will then give examples of solutions to the games from Section 4 from the evolutionary, rationalistic and reasoning perspective.[1] Section 8 and concludes with some pointers to recent interesting developments.

3 Games and their Solutions

A game in the technical sense is an abstract description of features relevant to a situation of interactive decision making. Situations of interactive decision making are those in which at least two agents must choose how to act and in which choices may mutually influence the outcome for both agents. One example is a speaker's utterance and the listener's subsequent uptake and interpretation of that utterance. The simplest formal model for this is a *signaling game* (Lewis, 1969; Sobel, 2008). The speaker knows what the actual world state is. The listener does not know this. The speaker then makes an utterance and the listener tries to guess what the world state is. If the guess is correct, communication succeeded. Whether it did, however, depends on both the speaker's and the listener's behavior.

[1] An implementation in programming language R of the game models and solution concepts used here is available online at `https://github.com/michael-franke/game_theoretic_pragmatics_ORE`.

Here is a definition of signaling games.[2] Let T be a set of mutually exclusive world states that captures what distinctions are currently relevant for the interlocutors. The prior probability of these states is $\Pr \in \Delta(T)$. The sender knows the true state $t \in T$ but the receiver does not. The sender selects a message $m \in M$ to, intuitively speaking, communicate t. The receiver observes message m and chooses some response act $a \in A$. An outcome of one round of playing this game is a triple $\langle t, m, a \rangle$. A numerical measure of how good such an outcome is is the utility function $\mathrm{U}: T \times M \times A \rightarrow \mathbb{R}$.[3]

Here is a first example of a signaling game. There are two world states. It is windy in t_{wind} and calm in t_{calm}. It's equally likely a priori that either state obtains. The sender has checked the weather, the receiver has not. The sender can make one out of two utterances m_{yai} and m_{nei} and the receiver must then choose whether two bring a surfboard a_{surf} or a skateboard a_{skate}. The former is appropriate for a windy, the latter for a calm day. We can model this by utilities that do not depend on the message that was used, e.g., for all $m \in \{m_{\text{yai}}, m_{\text{nei}}\}$:

$$\begin{aligned} \mathrm{U}(t_{\text{wind}}, m, a_{\text{surf}}) &= 1 & \mathrm{U}(t_{\text{wind}}, m, a_{\text{skate}}) &= 0 \\ \mathrm{U}(t_{\text{calm}}, m, a_{\text{surf}}) &= 0 & \mathrm{U}(t_{\text{calm}}, m, a_{\text{skate}}) &= 1 \end{aligned}$$

So far, the signaling game describes a particular context for communicative behavior. It is a *context model* that includes, on a crude level of abstraction, what interlocutors care about and what the spaces of possible actions and beliefs are. The context model does not by itself select any of the acts and beliefs within these spaces. That is the task of a *solution concept*.

Solution concepts appeal to agents' behavior and, sometimes, their beliefs. These are captured via the notion of a strategy. There are different kinds of strategies. We will consider non-probabilistic *pure strategies* and probabilistic *behavioral strategies* here. Generally speaking, a pure strategy for agent X maps each of X's choice points onto a single action choice (suitable for that choice point). A behavioral strategy for X maps each choice point on a probability distribution over all possible action choices (suitable for that choice point). A sender's pure strategy $s \in M^T$ maps each state to a message; a receiver's pure strategy $r \in A^M$ maps each message to an act. A sender's behavioral strategy $\sigma \in (\Delta(M))^T$ maps each state to a probability distribution over messages; a recevier's behavioral strategy $r \in (\Delta(A))^M$ maps each message to a probability distribution over acts.

We can interpret behavioral strategies in several ways. One is to think of them as the specification of what a single agent actually does. If confronted with t the probability that a speaker with strategy σ will respond with m would be $\sigma(m \mid t)$. Another way of interpreting behavioral strategies is as population-level averages. The probability $\sigma(m \mid t)$ would then be the probability of sampling an arbitrary sender from the population and observing that sender respond with m if she is confronted with t. This view is compatible with the assumption that no agent in the population actually has a probabilistic strategy. By this frequentist conception, behavioral strategies capture the modeller's beliefs about occurrence probabilities of action choices in the whole population. This is to be kept separate from yet another construal of behavioral strategies, namely as beliefs of the agents themselves. Under this third interpretation, a sender's behavioral strategy σ specifies the receiver's possibly uncertain beliefs

[2] A few remarks on notation and terminology. The literature on signaling games speaks of senders and receivers when linguists would speak of speakers and listeners. Biologists and economists like to think of what I call states here as types, hence the variable t. The notation $\Delta(X)$ is for the set of all probability distributions over set X. For simplicity, I will consider only games with finite sets T, M and A.

[3] I assume throughout this paper that speaker and hearer have perfectly aligned interests and so only consider one utility function. Game theory is not confined to this assumption. It can deal with situations of partially or fully diverging preferences just as well. Game theoretic modeling of non-cooperative language use is a highly successful enterprise that nicely complements the usual prevalent assumption of full cooperativity (e.g. Merin, 1999; Rubinstein, 2000; van Rooij, 2004; Franke, de Jager, and van Rooij, 2012; de Jaegher and van Rooij, 2014; Wagner, 2015).

about the sender's behavior. In this case, $\sigma(m \mid t)$ is the probability that the receiver assigns to the event that m is chosen by the sender if the actual state is t. There are important conceptual differences between these views. For our purposes, however, we can brush over most of these. It suffices to note here that the evolutionary, rationalistic and reasoning perspectives may wish to make use of different interpretations of behavioral strategies at different times.

The most basic solution concept of game theory is that of an equilibrium. Just as there are many kinds of games, there are many kinds of equilibria. Common to all is that we consider the players' behavior to be in equilibrium if no player has a positive incentive to behave differently, given the behavior of all other players. To define such a notion for our signaling games, let's first fix some auxiliary concepts. The *expected utility* of the sender's choice of m in state t, given that the receiver behaves as specified by ρ, is the weighted sum, given how likely each act a is given by ρ, of the utilities that result from the outcome $\langle t, m, a \rangle$:

$$\mathrm{EU}_S(m, t, \rho) = \sum_{a \in A} \rho(a \mid m) \times \mathrm{U}(t, m, a)\,.$$

Likewise, if the sender's behavior is σ, the receiver's expected utility of choosing act a in response to message m is:

$$\mathrm{EU}_R(a, m, \sigma) = \sum_{t \in T} \mu(t \mid m) \times \mathrm{U}(t, m, a)\,,$$

where $\mu \in (\Delta(T)^M$ is the posterior distribution over states, given a message, under the prior Pr and the speaker's behavioral strategy σ. This is constrained by Bayes' rule, whenever Bayes' rules is applicable:[4]

$$\mu(t \mid m) = \begin{cases} \frac{\Pr(t)\cdot\sigma(m|t)}{\sum_{t' \in T} \Pr(t')\cdot\sigma(m|t')} & \text{if denominator is defined} \\ \frac{1}{|T|} & \text{otherwise} \end{cases}$$

The higher the expected utility of a choice, the better it appears to be able to yield a high payoff, given uncertainty about the co-player's behavior. Rational agents, for instance, would only choose options that maximize expected utility (see also Section 6). This is reflected in the definition of equilibrium. A pair of strategies $\langle \sigma, \rho \rangle$ is in equilibrium if for all t, m, a:

(i) $\sigma(m \mid t) > 0$ implies $\mathrm{EU}_S(m, t, \rho) \geq \mathrm{EU}_S(m', t, \rho)$ for all m', and

(ii) $\rho(a \mid m) > 0$ implies $\mathrm{EU}_R(a, m, \sigma) \geq \mathrm{EU}_R(a', m, \sigma)$ for all a'.

In words, a strategy pair is in equilibrium if all choices at all choice points that occur with some probability must be rational in the sense that they maximize expected utility given the co-player's behavior.

Figure 1 gives four examples of strategy pairs for the surf-or-skate game. The sender's strategies are visualized by the arrows from states to messages; the receiver's from messages to acts. The only non-trivial probabilistic strategy is that of the sender in Figure 1a, where the numbers on the dashed arrows indicate the probability with which the sender sends each message in state t_{wind}. This

[4]When Bayes' rule does not apply, the formulation here assumes a flat distribution over states. This is a choice of convenience, which is crude but enough for our current general purposes. Different notions of equilibrium derive from more complex definitions of receiver responses to unexpected messages (e.g. Kreps and Wilson, 1982; Cho and Kreps, 1987; Banks and Sobel, 1987; Battigalli and Siniscalchi, 2002; Battigalli, 2006). Different ways of forming beliefs about unexpected speaker choices may entail different patterns of pragmatic language use (e.g. Franke, 2009).

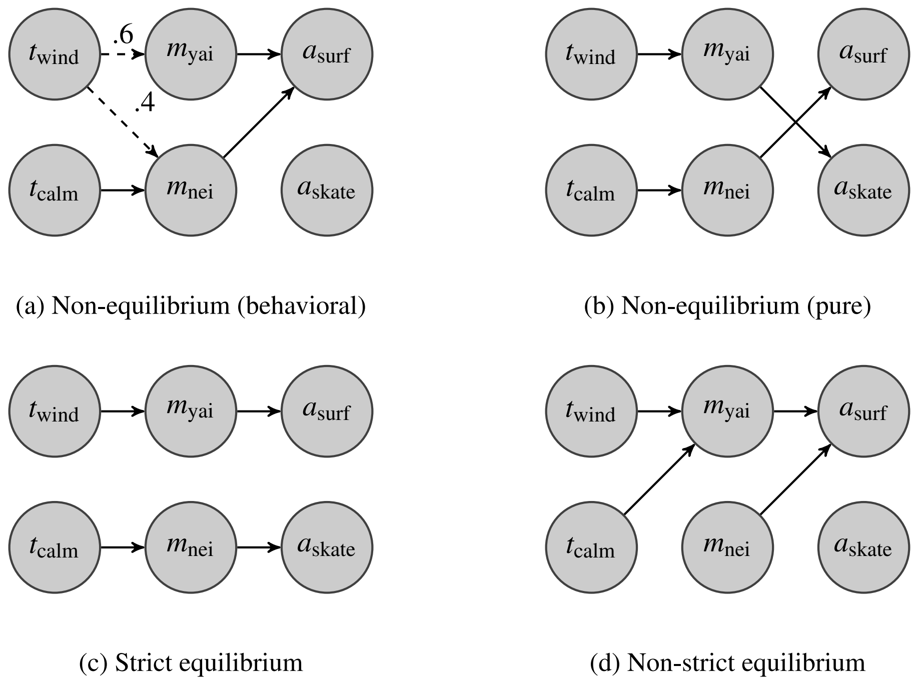

Figure 1: Strategy pairs for the surf-or-skate game

strategy pair is not an equilibrium, because the receiver's response to m_{nei} does not satisfy the second condition of our definition. The reason is that, since the speaker uses m_{nei} with a higher frequency in state t_{calm} than in state t_{wind} (and since states are equally likely a priori), the receiver's expected utility for a_{skate} is higher than that for a_{surf}. Similarly, the pure strategy profile in Figure 1b is not an equilibrium either. Intuitively, given the receiver's behavior, the sender would gain by swapping his use of messages; likewise, the receiver could gain by swapping act choices. The pair in Figure 1c is an equilibrium where players always get the maximum possible payoff. This is not so for the strategy pair in Figure 1d, although this is an equilibrium as well. It is an equilibrium because our definition only requires that choices that occur with positive probability should not be worse than others, but not that they must be strictly better than all others. The equilibrium in Figure 1d is therefore a *non-strict equilibrium*. This particular case is called a *pooling equilibrium* because the speaker pools several states under one message.

The complementing notion of a strict equilibrium is this. A pair of strategies $\langle \sigma, \rho \rangle$ is a strict equilibrium if for all t, m, a:

(i) $\sigma(m \mid t) > 0$ implies $\text{EU}_S(m,t,\rho) > \text{EU}_S(m',t,\rho)$ for all m', and

(ii) $\rho(a \mid m) > 0$ implies $\text{EU}_R(a,m,\sigma) > \text{EU}_R(a',m,\sigma)$ for all a'.

This can only be true of a pair of degenerate behavioral strategies that assign all probability to just one choice, i.e., that are essentially pure strategies. Interestingly, this notion of strict equilibrium coincides with the notion of an *evolutionary stable state* (Maynard Smith and Price, 1973; Maynard Smith, 1982).[5] Intuitively speaking, an evolutionary stable state is a state of a population which cannot

[5] It is not true in general that strict equilibria coincide with evolutionary stability, but it is true for the approach taken here in which both players' behavior at each choice point is assumed to be adjustable independently from behavior at other choice points (e.g. Selten, 1980).

be invaded by a small number of mutants. The underlying idea is that expected utility is a measure of expected, or average, fitness. The higher the fitness the higher the chances of survival and the amount of offspring. To see how this connects, suppose that a population is in a state that is a strict equilibrium in the above sense and that mutations happen independently at only one choice point (be it for the sender or the receiver). Such local mutations would be driven out since the incumbent population does strictly better in terms of fitness.

There are two important things to notice here, before concluding this brief survey of basic game theoretic notions. Firstly, it is possible to justify game theoretic analyses of the very same game by appealing either to basic intuitions about rational choice or by appealing to evolutionary selection. Secondly, it should be pointed out that the example discussed in this section also demonstrates how game theoretic analysis can lead to a naturalistic account of the evolution of conventional meaning, which was the prime motivation of David Lewis in his seminal book *Convention* which introduced signaling games for this purpose (Lewis, 1969). There are two strict equilibria in of the surf-or-skate game: one is in Figure 1c, the other would reverse message use for the sender and act choice for the receiver. Both of these associate each message, which we assumed to be entirely meaningless so far, with a single state and with a single act. Intuitively, the joint behavior of sender and receiver seems to give meaning to messages. While Lewis himself thought to give a naturalization of conventional meaning by appealing to a rationalization of behavior in strict equilibria, much recent work has also looked at evolutionary accounts for the emergence of meaningful signaling in non-human animals where ascriptions of rationality may appear dubious (for overview see, e.g., Skyrms, 2010; Franke and Wagner, 2014). In the remainder, we will explore the orthogonal question of how game theory can help explain the linguistic behavior of agents who have a language with commonly known semantic meaning at hand.

To sum up so far, a signaling game can be seen as an abstract representation of a context in which a speaker makes an utterance and the listener chooses some response to that utterance. The outcome for both interlocutors depends on the true world state, which only the speaker knows, and the response of the listener. Game theory offers ways of talking about behavior, beliefs and (modellers') expectations of behavior of interlocutors in these situations, as well as a measure for what counts as good behavior for the interlocutors whose preferences are part of the context model. Game theoretic solution concepts can then single out particular ways of behaving by appeal to rationality or evolution.

4 Games as Pragmatic Context Models

Here are some more signaling game models that will help explain general patterns of pragmatic inference. Following Grice (1989), Neo-Gricean pragmatics distinguishes three main types of pragmatic inference, namely *Q*-, *I*- and *M*-implicatures (e.g. Atlas and Levinson, 1981; Horn, 1984; Levinson, 2000). Let us use these categories to build a stock of interesting examples, just for the purpose of demonstrating what game theoretic approaches to pragmatics could look like. This section introduces a signaling game model for each of these three inference patterns, plus some others that add interesting dimensions that might otherwise be missed.

Scalar implicatures. Quantity implicatures are inferences derived from Grice's Maxim of Quantity which requires speakers to adequately pitch the level of informativity of their utterances. The most basic case is that of a *scalar implicature* associated with the lexical contrast between *some* and *all*, which form an implicational scale (whence the naming). Usually, an utterance of a sentence like (1a) would invite reasoning why the speaker did not say (1b) to the conclusion that actually (1c) is true (given that this is relevant information and that the speaker is sufficiently likely informed about whether (1b) or (1c) is the case).

(1) a. I own some of Johnny Cash's albums.

b. I own all of Johnny Cash's albums.

c. I own some but not all of Johnny Cash's albums.

A game model that captures some of the important pieces of a context in which to derive this implicature has two states: in $t_{\exists\neg\forall}$ the speaker owns some but not all and in $t_{\forall}$ she owns all of Johnny Cash's albums. Let's assume for simplicity that the prior probability of both states are $1/2$. There are two messages m_{some} and m_{all}, corresponding to (1a) and (1b) respectively. The conventional meaning of a message is captured in the set of states in which it is true, namely $[\![m_{\text{some}}]\!] = \{t_{\exists\neg\forall}, t_{\forall}\}$ and $[\![m_{\text{all}}]\!] = \{t_{\forall}\}$. We assume that the game is one of interpretation. The listener chooses acts that correspond exactly with the relevant state distinctions: $A = T$. Communication is succesful if the chosen interpretation matches the true state: $\mathrm{U}(t, m, t') = 1$ if $t = t'$ and 0 otherwise.

This is a simple context model. The expected kind of behavior would be a pure strategy pair in which the sender uses m_{some} in $t_{\exists\neg\forall}$ and m_{all} in $t_{\forall}$, while the receiver interprets m_{some} as $t_{\exists\neg\forall}$ and m_{all} as $t_{\forall}$. While this is a strict equilibrium, thereby an evolutionary stable state, it is not the only one. Flipping the use of messages and interpretations is also one, albeit one that does not correspond to intuition. A suitable game theoretic explanation would tell us why we see the one but not the other.

Additional alternatives. Deriving scalar implicatures from considerations of rationality or optimality of communicative behavior is particularly challenging when we assume that the speaker could have used an expression to communicate a putative implicature explicitly, e.g., by saying:

(2) I own some but not all of Johnny Cash's albums.

We should therefore also consider a variant of the scalar implicature game which includes a third message m_{sbna} with $[\![m_{\text{sbna}}]\!] = \{t_{\exists\neg\forall}\}$. As (2) is more complex than (1b) and (1a), we should assume that its use incurs a small additional cost. Keeping utilities as before, we factor in economy of message choice in the form of a factor $0 < c < 1$ for the use of the more complex message m_{sbna}: $\mathrm{U}(t, m, t') = 1 \times c_m$ if $t = t'$ and 0 otherwise, where $c_m = 0.8$ if $m = m_{\text{sbna}}$ and 1 otherwise.[6]

I-Implicatures. I-implicatures are inferences to stereotypes. The word *milk* in an utterance of (3a) will most likely be interpreted as *cow's milk* in most dialects of English, not as *goat's milk*. This may be, so the game theoretic explanation will run, because cow's milk is the more prevalent or stereotypical instance of milk in these speech communities. Where this is not so, a similar utterance might be interpreted as referring to goat's milk.

[6] There is another common way of formalizing message preferences, namely in terms of *additive* costs rather than multiplicative ones. There is a big difference. If m_1 and m_2 differ only with respect to their cost, then additive costs will lead to the same difference in expected utilities between these messages if they have the same chance of communicative success, no matter whether expectations of success are high or low. Multiplicative costs weigh higher the higher the expectation of success. The motivation for the latter is to capture a trade-off of exploration and exploitation. In case of low chances of communicative success, agents do not mind "experimenting" with uncommon and possibly more cumbersome expressions. In case of high chances of communicative success, agents may which to "hone their blade" to perfection. The use of multiplicative costs helps select the "right" equilibria in some examples considered here, but for all cases where it does, alternative ways are available. For instance, the iterated quantal response model and the probabilistic reasoning model defined here will not select the desired solution for additive costs, but do so for multiplicative costs (see Bergen, Levy, and Goodman, 2012; Franke and Jäger, 2014; Bergen, Levy, and Goodman, to appear, for discussion and alternative solutions). I do not want to make any commitment here as to whether multiplicative costs are conceptually better than additive ones.

(3) a. Kate drank a glass of milk.

b. Kate drank a glass of cow's milk.

c. Kate drank a glass of goat's milk.

Our simple game model for this case assumes that there are two states: in t_{cow} Kate drank cow's milk, while in t_{goat} it was goat's milk. Let's assume for concreteness that the prior probability of a person drinking cow's milk, if she drinks milk, is $\Pr(t_{\text{cow}}) = 0.8$ and so $\Pr(t_{\text{goat}}) = 0.2$. The speaker could have made any of the three utterances in (3), represented here at m_{milk}, m_{cow} and m_{goat} respectively. The semantic meaning of these is just the obvious: $[\![m_{\text{milk}}]\!] = \{t_{\text{cow}}, t_{\text{goat}}\}$, $[\![m_{\text{cow}}]\!] = \{t_{\text{cow}}\}$ and $[\![m_{\text{goat}}]\!] = \{t_{\text{goat}}\}$. We assume that the game is one of interpretation: $A = T$. Utilities for communicative success are as before: 1 for a matching interpretation, 0 otherwise. Additionally, we factor in message preferences in the form of a factor $0 < c < 1$ for the use of the more complex messages (3b) and (3c): $\mathrm{U}(t, m, t') = 1 \times c_m$ if $t = t'$ and 0 otherwise, where $c_m = 1$ if $m = m_{\text{milk}}$ and 0.8 otherwise.

The behavioral pattern that we would like to single out as an account of I-implicature is the strategy pair in which the sender uses m_{milk} in state t_{cow} and m_{goat} in state t_{goat}, and where the receiver interprets accordingly and also interprets m_{cow} as t_{cow} even if it does not get used (frequently) by the sender.

M-Implicatures. M-implicatures are complementary to I-implicatures. While the latter associate stereotypical expressions with stereotypical interpretations, the former associate marked expressions with marked interpretations. This double inference pattern is also sometimes referred to as *Horn's division of pragmatic labor* (Horn, 1984). A standard example is the pragmatic meaning difference between (4a) and (4b). These expressions are arguably true in exactly the same situations by semantic meaning alone. But pragmatically, while (4a) suggests an intentional killing with standard means, (4b) suggests perhaps unintentional killing or killing with non-standard means. Intuitively, given the cumbersome description in (4b), listeners expect that something out of the ordinary will have caused the speaker not to use a common description.

(4) a. Black Bart killed the sherif.

b. Black Bart caused the sherif to die.

One way to formalize a context model for this case is to assume two world states: in t a normal killing event took place, while in t^* some non-stereotypical killing event happened. To capture this typicality difference, we assume that the prior probability of t is higher than that of t^*. For concreteness, let us assume that $\Pr(t) = 0.75$ and hence $\Pr(t^*) = 0.25$. There are two messages, one normal m, and one marked m^*, corresponding to utterances (4a) and (4b) respectively. As for semantic meaning, the assumption is that both messages are true in either state. The game is a game of interpretation with $A = T$. Utilities are as before —1 for a matching interpretation, 0 otherwise— but also factor in message preferences in the form of a factor $c = 0.8$ for the use of the marked message.

We would like a solution concept to pick out the behavioral pattern that associates t with m and t^* with m^*. Sender and receiver behavior that follows this pattern is a strict equilibrium, but so is the reverse pattern that associates t with m^* and t^* with m.

Free-Choice implicatures. A pattern of inference that has inspired a lot of work in theoretical pragmatics are free-choice inferences which arise from the use of disjunctions embedded under existential deontic modals (e.g. Kamp, 1973, 1978; Zimmermann, 2000; Kratzer and Shimoyama, 2002; Asher and Bonevac, 2005; Geurts, 2005; Schulz, 2005; Fox, 2007; Eckardt, 2007; Klinedinst, 2007; Fusco, 2014). An utterance of a sentence like (5a) would usually quite clearly communicate (5b). Unfortunately, this inference does not follow from a standard possible worlds semantics for the existential modal paired with a standard logical semantics for disjunction.[7] This is why a number of authors have argued that the free-choice inference should be derived as a pragmatic inference.

(5) a. You may take an apple or a banana.
b. You may take an apple and you may take a banana.
c. You may take an apple.
d. You may take a banana.

Our game theoretic context model assumes that there are three states: in t_{A} the listener may take an apple but no banana, in t_{B} taking a banana is fine, but not an apple, while in t_{AB} taking an apple and taking a banana is permitted.[8] Let's assume that all three states are equally likely *a priori*. The set of possible messages is made up of (5a), (5c) and (5d), to be written in short as: m_{AorB}, m_{A} and m_{B}. The assumed semantic meaning for these, in terms of our state distinctions, is: $[\![m_{\mathrm{AorB}}]\!] = \{t_{\mathrm{A}}, t_{\mathrm{B}}, t_{\mathrm{AB}}\}$, $[\![m_{\mathrm{A}}]\!] = \{t_{\mathrm{A}}, t_{\mathrm{AB}}\}$ and $[\![m_{\mathrm{B}}]\!] = \{t_{\mathrm{B}}, t_{\mathrm{AB}}\}$. Again, let's assume that the game is one of interpretation with $A = T$ and the usual zero-or-one utilities for successful communication.

We would like to select the intuitive pragmatic sender-receiver behavior in which m_{AorB} is associated with t_{AB}, while m_{A} goes with t_{A} and m_{B} goes with t_{A}. Again, this is a strict equilibrium, but it is not the only one. Any way of matching states and messages one-to-one in pure sender and receiver strategies is a strict equilibrium.

Numerosity. All of the games introduced so far had simple utility functions according to which communication was either clearly a success or clearly a failure. In order to stress that game theoretic modeling is much more flexible than that, and to show that this flexibility can be potentially useful, we should consider at least one more complex case.

Suppose that there are ten balls in an urn. Balls are either black or white. Someone gives you a vague description:

(6) Some of the balls are white.

How many balls do you think are white? To answer this question, you might put yourself into the shoes of a speaker. Would (6) be a natural thing to say if there was only one white ball? Would it be natural for nine white balls?

A number of empirical studies have measured how natural or typical a description like (6) is for varying numbers of white balls (e.g. Degen and Tanenhaus, 2015; van Tiel, to appear). Subjects indicated on a suitable rating scale how well sentences like (6) fit a given picture (e.g., of black and white balls). Results indicate that (6) is not natural at all for zero or one white ball, but increasingly natural for two or three, peaking at around four or five and then slowly decreasing for more white balls. Game theoretic tools help explain the empirically measured typicality values (e.g. Franke, 2014b, 2016). Suffice it here to sketch in rather rough outline a context model in which such an explanation could be advanced.

[7] To see this, suppose that the set of worlds which captures what is permitted in the actual world contains only one world. In this world the addressee takes an apple, but not a banana. Then there is a world in which the disjunction "addressee takes an apple or a banana" is true. So (5a) is true, but, by our assumption that this is the only world in the relevant set, (5b) is false. Hence, (5a) does not entail (5b) by virtue of its standard semantics.

[8] To keep matters simple here, this context model does not distinguish as an additional relevant state whether taking both a banana *and* and apple at the same time is permitted. See Franke (2011) for an approach that does.

States represent the number of white balls $T = \{t_0, \ldots, t_{10}\}$. We assume that each state is equally likely *a priori*. While there may be uncountable others, let us focus attention on a small set of messages. Suppose that there are just these seven:

(7) (none | one | two | three | some | most | all) of the balls are white.

Represent these as m_x with x being *none*, *one*, ..., *all*. Some justification for assuming these and not others is that *none*, *some*, *most* and *all* are frequently assumed to be salient lexical alternatives. The exclusion of number terms beyond *three* is a terribly crude implementation of the idea that the perception of higher numbers (usually) requires counting, so that these alternative utterances should not be as salient as the ones that we assume here (see Franke, 2014b, 2016, for an approach that tries to estimate latent salience of alternatives from empirical data). The semantic meaning assumed for these is just the obvious one, where *some* means at least one and *most* means *more than half*:

$$[\![m_{\text{none}}]\!] = \{t_0\} \qquad [\![m_{\text{one}}]\!] = \{t_1\} \qquad [\![m_{\text{two}}]\!] = \{t_2\} \qquad [\![m_{\text{three}}]\!] = \{t_3\}$$
$$[\![m_{\text{some}}]\!] = T \setminus \{t_0\} \qquad [\![m_{\text{most}}]\!] = \{t_i \mid i \geq 6\} \qquad [\![m_{\text{all}}]\!] = \{t_{10}\}$$

Let us assume that the game is one of interpretation, so that $A = T$. Unlike before, utilities are defined in terms of *similarity between states* (e.g. Jäger and van Rooij, 2007; Jäger, 2007; Jäger, Metzger, and Riedel, 2011; Franke, Jäger, and van Rooij, 2011; O'Connor, 2014). If t_a is the actual state and t_i is the listener's guess, then communication need not be a total failure if $t_a \neq t_i$. If t_a and t_i are similar enough, communication might still be somewhat successful. For concreteness, let us consider the utilities in Table 1. These are rather arbitrarily chosen, albeit it with the intention of capturing the intuition that similarity is not uniform in difference between any two numbers, but more diffuse for median values than at the margins.[9] Let us also assume, again without any implied theoretical commitments but merely for the sake of an interesting example, that number terms (m_{one}, m_{two}, m_{three}) incur increasing multiplicative processing costs with $c_{m_{\text{one}}} = 0.8$, $c_{m_{\text{two}}} = 0.7$ and $c_{m_{\text{three}}} = 0.6$.

5 Evolutionary Dynamics

Evolutionary approaches to solving games model the effects of gradual changes or minor adaptations of agents' behavior within a population over time. These may lead to successful ways of playing the game, where what counts as successful is defined by the game's utility function. The *replicator dynamic* is a prominent, versatile and appealing representative of this approach. It was introduced as a general model of asexual reproduction where individuals with a higher fitness have higher chances of survival (Taylor and Jonker, 1978). But the replicator dynamic can also be derived as an abstract description of learning or imitation processes, i.e., as a *cultural evolutionary dynamic* (e.g. Schlag, 1998; Helbing, 1996; Börgers and Sarin, 1997; Sandholm, 2010). The latter perspective is more plausible for the question that we are after here: is it plausible to consider patterns of pragmatic inferences, like those described in Section 4, as the outcome of a gradual process of adaptation towards efficient communication, given a set of conventional semantic meanings?

[9]The way in which these utilities were constructed is loosely inspired by computational models of the *approximate number system* (e.g. Dehaene, 1997). The idea is that number representations are probability distributions on a *mental number line*. If n is the actual number of white balls in a given display, activation on the (continuous) mental number line is given by a normal distribution with mean n and a standard deviation sd_n. The higher the number n, the higher the standard deviation of its representation function $sd_n = w \times n$, where the *Weber fraction* w is a free parameter. The utilities in Table 1 are derived from the probabilities of confusing the number of white/black balls in one state for the number of white/black balls in another. However, no claim is intended here that this is a good model of perceptual similarity between displays of white and black dots.

	t_0	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}
t_0	1.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
t_1	0.005	1.000	0.183	0.006	0.000	0.000	0.000	0.000	0.000	0.000	0.000
t_2	0.000	0.456	1.000	0.431	0.085	0.007	0.000	0.000	0.000	0.000	0.000
t_3	0.000	0.026	0.756	1.000	0.598	0.207	0.035	0.001	0.000	0.000	0.000
t_4	0.000	0.000	0.205	0.823	1.000	0.713	0.301	0.048	0.000	0.000	0.000
t_5	0.000	0.000	0.020	0.318	0.792	1.000	0.792	0.318	0.020	0.000	0.000
t_6	0.000	0.000	0.000	0.048	0.301	0.713	1.000	0.823	0.205	0.000	0.000
t_7	0.000	0.000	0.000	0.001	0.035	0.207	0.598	1.000	0.756	0.026	0.000
t_8	0.000	0.000	0.000	0.000	0.000	0.007	0.085	0.431	1.000	0.456	0.000
t_9	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.006	0.183	1.000	0.005
t_{10}	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	1.000

Table 1: Utilities for the numerosity game. The table shows the utility obtained when the row state is actual, and the receiver guesses the column state. Utilities are less diffuse towards the end points.

The simplest formulation of the replicator dynamic considers discrete update steps. If σ_n and ρ_n describe the average sender and receiver behavior in the relevant population at present, then the average behavior σ_{n+1} and ρ_{n+1} at the next time step is:[10]

$$\sigma_{n+1}(m \mid t) = \frac{\sigma_n(m \mid t) \times \text{EU}_S(m,t,\rho_n)}{\sum_{m'} \sigma_n(m' \mid t) \times \text{EU}_S(m',t,\rho_n)}\,, \quad \rho_{n+1}(a \mid m) = \frac{\rho_n(a \mid m) \times \text{EU}_R(a,m,\sigma_n)}{\sum_{a'} \rho_n(a' \mid m) \times \text{EU}_R(a',m,\sigma_n)}\,. \quad (1)$$

The intuition behind this formulation is this. We interpret expected utility as expected fitness. For a sender's choice of message at state t, we then look at how well each m fares in terms of expected fitness. The probability that an arbitrary sender will play m in state t after the update is just proportional to the probability of seeing it before the update, times the expected fitness of m. The normalizing constants in the denominators make sure that we compare expected fitnesses of alternative choices. Intuitively put, the probabilities of choices increase or decrease along a gradient of relative expected fitness. It is this property which makes the replicator dynamic a forerunner for the plainest and most straightforward implementation of fitness-based selection.

Consider the scalar implicature game from Section 4 as a first example. There are two states, m_{some} and $t_\forall$, and two message m_{some} and m_{all}. We are interested in whether signaling behavior in line with scalar implicature production and comprehension is a prominent outcome of evolutionary selection under the replicator dynamic, given the standardly assumed conventional meanings of m_{some} and m_{all}. To do so, let us imagine an initial population in which speakers and hearers produce and interpret with nothing but semantic meaning to guide them. The average sender and receiver behavior in such a population is described by the following *literal strategies*:[11]

$$\sigma_0 = \sigma_{\text{literal}} : \begin{array}{cc} & \begin{array}{cc} m_{\text{some}} & m_{\text{all}} \end{array} \\ \begin{array}{c} t_{\exists\neg\forall} \\ t_\forall \end{array} & \begin{bmatrix} 1 & 0 \\ .5 & .5 \end{bmatrix} \end{array} \qquad \rho_0 = \rho_{\text{literal}} : \begin{array}{cc} & \begin{array}{cc} t_{\exists\neg\forall} & t_\forall \end{array} \\ \begin{array}{c} m_{\text{some}} \\ m_{\text{all}} \end{array} & \begin{bmatrix} .5 & .5 \\ 0 & 1 \end{bmatrix} \end{array}. \quad (2)$$

[10]This formulation presupposes that expected fitness is always positive. Since this is not always the case for the games from Section 4, where expected utility can be 0, all calculations reported here add a small constant of 0.01 as a baseline minimum fitness that every trait has. Nothing of substance hinges on this.

[11]Choice points are given in the row, with choices in the columns. Each row therefore denotes a probability distribution over choices for each choice point.

Since m_{all} is false in m_{some}, literal senders have no choice in m_{some} but to produce m_{some}. On the other hand, both messages are true in state $t_{\forall}$ and so the average expectation for literal language use is that either message is used with probability .5. Similarly for literal interpretation.

Applying the discrete time replicator dynamic in Equation (1) to an initial population of literal language users, we obtain a new population state, namely:

$$\sigma_1 : \begin{array}{c c} & \begin{array}{cc} m_{\text{some}} & m_{\text{all}} \end{array} \\ \begin{array}{c} t_{\exists\neg\forall} \\ t_{\forall} \end{array} & \begin{bmatrix} 1 & 0 \\ .34 & .66 \end{bmatrix} \end{array} \qquad \rho_1 : \begin{array}{c c} & \begin{array}{cc} t_{\exists\neg\forall} & t_{\forall} \end{array} \\ \begin{array}{c} m_{\text{some}} \\ m_{\text{all}} \end{array} & \begin{bmatrix} .66 & .34 \\ 0 & 1 \end{bmatrix} \end{array}$$

Why is this? — Given literal interpretation ρ_{literal}, m_{all} is interpreted as $t_{\forall}$ with certainty while m_{some} only has a 50% chance of being interpreted as $t_{\forall}$. Therefore, the use of m_{all} in $t_{\forall}$ has a higher expected utility than that of m_{some}. So the proportions of message use in that state are adjusted by evolutionary selection to reflect the proportional fitness advantage, yielding the sender strategy σ_1. Similarly for the receiver side.

The next application of Equation (1) has slightly different measures of expected fitness, because what counts as, say, good sender behavior now is no longer measured with respect to ρ_{literal} but with respect to ρ_1. After another update step the population state is:

$$\sigma_2 : \begin{array}{c c} & \begin{array}{cc} m_{\text{some}} & m_{\text{all}} \end{array} \\ \begin{array}{c} t_{\exists\neg\forall} \\ t_{\forall} \end{array} & \begin{bmatrix} 1 & 0 \\ .15 & .85 \end{bmatrix} \end{array} \qquad \rho_2 : \begin{array}{c c} & \begin{array}{cc} t_{\exists\neg\forall} & t_{\forall} \end{array} \\ \begin{array}{c} m_{\text{some}} \\ m_{\text{all}} \end{array} & \begin{bmatrix} .85 & .15 \\ 0 & 1 \end{bmatrix} \end{array}$$

Probabilities of message use and interpretation have shifted even more towards "scalar implicature"-like behavior. Further update steps push us ever closer: starting from literal language use, the replicator dynamic leads to signaling behavior that we would expect if agents intended and computed scalar implicatures.

This is a noteworthy result. Even if agents themselves do not reason about optimal language use or behave rationally themselves, a general process of gradual adaptation leads us from literal language use to pragmatic language use of exactly the kind that we see in reality. Seen in this light, scalar implicatures surface as an optimal adaptation under selective pressure to communicate efficiently. In provocative slogan form: the practice is rational, even if the practitioners might not be.

Similar results hold for the other games considered in Section 4. If we start from literal sender and receiver behavior, the replicator dynamic leads to exactly the sender and receiver behavior that we flagged as the to-be-explained pragmatic language use for the I-implicature, M-implicature and free-choice game, as well as for the scalar implicature game with the additional alternative. These results are also summarized in Table 2.

The strategy pair for the numerosity game that results from 100 iterations of the replicator dynamic when starting from literal strategies is:

$$\sigma_{100}: \begin{array}{c} \\ t_0 \\ t_1 \\ t_2 \\ t_3 \\ t_4 \\ t_5 \\ t_6 \\ t_7 \\ t_8 \\ t_9 \\ t_{10} \end{array} \begin{array}{c} \begin{array}{ccccccc} m_{\text{none}} & m_{\text{one}} & m_{\text{two}} & m_{\text{three}} & m_{\text{some}} & m_{\text{most}} & m_{\text{all}} \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

$$\rho_{100}: \begin{array}{c} \\ m_{\text{none}} \\ m_{\text{one}} \\ m_{\text{two}} \\ m_{\text{three}} \\ m_{\text{some}} \\ m_{\text{most}} \\ m_{\text{all}} \end{array} \begin{array}{c} \begin{array}{ccccccccccc} t_0 & t_1 & t_2 & t_3 & t_4 & t_5 & t_6 & t_7 & t_8 & t_9 & t_{10} \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

This, too, is a very plausible result. The use of *some* is restricted "from above," so to speak, by *most* and "from below" by number expressions. Interestingly, we obtain something like a stereotypical interpretation of quantifiers, in that the receiver interprets *some* as t_5 and *most* as t_7 (see van Tiel, 2014; Tiel, Bååth, and Sauerland, 2016, for further considerations of stereotypical interpretations of quantifiers).

6 Rational & Approximately Rational Reasoning

Many explanations of pragmatic inferences appeal to rationality of interlocutors themselves, not just to the optimality of the system of pragmatic inferences on an abstract level. Such a rationalistic explanation could run as follows: the speaker said "you may do *A* or *B*"; if she wanted the listener to infer that *A* but not *B* is permitted, the utterance "you may do *A*" would have been sufficient; a rational speaker who mentions *B* must have in mind something that goes beyond what a rational listener infers from "you may do *A*;" presumably, *A* and *B* are both allowed. This is just a very vague example of pragmatic reasoning that appeals to the actual rational reasoning process that could be called upon in support of pragmatic inferences. It is perhaps what first comes to mind in connection with the requirement of Grice that conversational implicatures be *calculable* (Grice, 1989), i.e., supportable by some argument why they should hold.

There are many ways of cashing out rational reasoning about language use and interpretation. Here is a sketch of the general idea behind a family of approaches (e.g. Rabin, 1990; Benz, 2006; Stalnaker, 2006; Benz and van Rooij, 2007; Franke, 2011; Jäger, 2012, 2014). Just like with the evolutionary approach from Section 5, let us assume that pragmatic reasoning departs from literal semantic production and comprehension $\sigma_0 = \sigma_{\text{literal}}$ and $\rho_0 = \rho_{\text{literal}}$, as defined in Equation (2). Unlike before, we may now interpret this as an initial belief about what a hypothetical literal speaker or listener would do, even if there never was any such purely literal language user. Rational agents maximize expected utility with respect to their beliefs about what the co-player will do. A *best response* is an action that maximizes expected utility in this sense:

$$\mathrm{BR}_S(t,\rho) = \arg\max_{m'} \mathrm{EU}_S(m',t,\rho) \qquad\qquad \mathrm{BR}_R(m,\sigma) = \arg\max_{a'} \mathrm{EU}_R(a',m,\sigma)$$

If more than one option is contained in a best response set, a rational agent will be indifferent between all best responses. A sequence of *iterated best responses* is therefore inductively defined as follows:

$$\sigma_{n+1}(m \mid t) = \begin{cases} \frac{1}{|\mathrm{BR}_S(t,\rho_n)|} & \text{if } m \in \mathrm{BR}_S(t,\rho_n) \\ 0 & \text{otherwise} \end{cases} \qquad \rho_{n+1}(a \mid m) = \begin{cases} \frac{1}{|\mathrm{BR}_R(m,\sigma_n)|} & \text{if } a \in \mathrm{BR}_R(m,\sigma_n) \\ 0 & \text{otherwise} \end{cases}$$

We may interpret each σ_{n+1} as an expectation about what a rational sender would do, if she believes that the receiver's behavior is ρ_n. This way we implement a chain of iterated reasoning that is motivated by an ever deeper nesting of beliefs about mutual rational choice.

Consider the scalar implicature game for illustration again. The literal strategies are repeated here:

$$\sigma_0 = \sigma_{\text{literal}} : \begin{array}{c c} & \begin{array}{cc} m_{\text{some}} & m_{\text{all}} \end{array} \\ \begin{array}{c} t_{\exists\neg\forall} \\ t_{\forall} \end{array} & \begin{bmatrix} 1 & 0 \\ .5 & .5 \end{bmatrix} \end{array} \qquad\qquad \rho_0 = \rho_{\text{literal}} : \begin{array}{c c} & \begin{array}{cc} t_{\exists\neg\forall} & t_{\forall} \end{array} \\ \begin{array}{c} m_{\text{some}} \\ m_{\text{all}} \end{array} & \begin{bmatrix} .5 & .5 \\ 0 & 1 \end{bmatrix} \end{array}.$$

Applying one step of best response reasoning to these literal strategies already yields the desired result, which is also a fixed point of further applications of best response reasoning:

$$\sigma_1 : \begin{array}{c c} & \begin{array}{cc} m_{\text{some}} & m_{\text{all}} \end{array} \\ \begin{array}{c} t_{\exists\neg\forall} \\ t_{\forall} \end{array} & \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{array} \qquad\qquad \rho_1 : \begin{array}{c c} & \begin{array}{cc} t_{\exists\neg\forall} & t_{\forall} \end{array} \\ \begin{array}{c} m_{\text{some}} \\ m_{\text{all}} \end{array} & \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{array}$$

Iterated best response reasoning also selects "intuitive behavior" for the M-implicature, the free-choice and the numerosity game. It's predictions for the I-implicature game are fine if we start the reasoning sequence with a literal speaker, but not if we start with a literal listener. Under the simple formulation given here, iterated best response reasoning fails to select the intuitively correct pragmatic behavior for the scalar implicature game with an additional alternative *some but not all* (see Franke, 2009; Rothschild, 2013; Franke, 2014a; Jäger, 2014, for discussion and alternative solutions to this problem in related models).

Iterated best response reasoning assumes that interlocutors behave rationally and believe in each other's rationality. There is no room for slips, mistakes, errors or occasional idioticity in this model. This is as it should be, some might argue, for an abstract rationalization of general patterns of pragmatic language use. On the other hand, it is at least telling to see what would happen if we allowed for a weakening of the assumption of flawless rationality. *Iterated quantal response* models allow for (mutual beliefs in) approximately rational choice (e.g. Franke, Jäger, and van Rooij, 2011; Degen, Franke, and Jäger, 2013; Franke and Jäger, 2014). As before, the starting point of iterated quantal response reasoning is literal production and comprehension. Best responses are replaced by quantal responses. Quantal responses, also known as logit or soft-max responses, are standard probabilistic approximations to binary selection functions (e.g. Luce, 1959; Sutton and Barto, 1998; McFadden,

	replicator dynamic	iterated best resp.	iterated quantal resp.	probabilistic reasoning
scalar implicature	✓	✓	✓	✓
scalar imp. added altern.	✓	–	✓	✓
I-implicature	✓	✓ / –	✓	✓
M-implicature	✓	✓	✓	✓
free choice	✓	✓	✓	✓
numerosity	✓	✓	✓	?

Table 2: Which models give intuitive predictions for which case? A checkmark ✓ indicates an intuitive prediction. For iterated best and quantal response models, predictions may depend on whether we start with a literal production or a literal comprehension strategy. This matters only for the I-implicature game under iterated best responses (see main text).

1976; Goeree, Holt, and Palfrey, 2005; Rogers, Palfrey, and Camerer, 2009; Train, 2009). One motivation for these is that expected utilities are computed only approximately. If the distribution of errors in the computation of expected utilities has a particular shape, agents who try to choose optimally effectively realize probabilistic behavior that is characterized by:

$$\sigma_{n+1}(m \mid t) = \frac{\exp(\lambda \times \mathrm{EU}_S(m,t,\rho_n))}{\sum_{m'} \exp(\lambda \times \mathrm{EU}_S(m',t,\rho_n))} \qquad \rho_{n+1}(a \mid m) = \frac{\exp(\lambda \times \mathrm{EU}_R(a,m,\sigma_n))}{\sum_{a'} \exp(\lambda \times \mathrm{EU}_R(a',m,\sigma_n))}$$

The parameter $\lambda \geq 0$ in this formulation governs how noisy choices are. The smaller λ the more deviance from rational best responses. If $\lambda \to \infty$, we retrieve the previously defined best response approach. Thus conceived, iterated best response reasoning is just a special case of iterated quantal response reasoning.

Consider the example of the scalar implicature game after one round of quantal response reasoning for $\lambda = 5$ (an arbitrarily chosen value):

$$\sigma_1: \begin{array}{cc} & \begin{array}{cc} m_{\text{some}} & m_{\text{all}} \end{array} \\ \begin{array}{c} t_{\exists\neg\forall} \\ t_{\forall} \end{array} & \begin{bmatrix} .92 & .08 \\ .08 & .92 \end{bmatrix} \end{array} \qquad \rho_1: \begin{array}{cc} & \begin{array}{cc} t_{\exists\neg\forall} & t_{\forall} \end{array} \\ \begin{array}{c} m_{\text{some}} \\ m_{\text{all}} \end{array} & \begin{bmatrix} .84 & .15 \\ .01 & .99 \end{bmatrix} \end{array}$$

What is interesting is that as long as expected utilities and λ are finite, quantal responses will assign a positive probability to every choice, including choices that deviate from semantic convention. That applies as well for the results of further iteration, of course. Still, iterated quantal response reasoning can select strategy pairs that are just noisy versions of the "correct" pragmatic strategies in all of our example games for appropriate values of λ. If λ is too low, the system can collapse: every choice is equally likely. If λ is too high, we end up with some of the wrong predictions of iterated best response reasoning. But for, e.g., $\lambda = 5$, iterated quantal response reasoning "solves" all of the games from Section 4 (see Table 2).

7 Probabilistic Reasoning

Models of probabilistic reasoning are popular in cognitive science in general (e.g. Tenenbaum, Griffiths, and Kemp, 2006; Tenenbaum et al., 2011) and in pragmatics in particular (e.g. Frank and Goodman, 2012; Goodman and Stuhlmüller, 2013; Kao et al., 2014; Lassiter and Goodman, 2015; Bergen and Goodman, 2015; Franke and Degen, to appear; Franke and Jäger, to appear). Most of the latter work takes the *rational speech act model* of Frank and Goodman (2012) as a starting point. Models in this tradition deviate from the models introduced previously in a number of aspects: (i) RSA models usually only consider a sequence starting with literal interpretation and disregard literal production as a starting point of pragmatic reasoning; (ii) the speaker's utilities are defined in terms of the beliefs of the listener, not his action choices; (iii) the listener chooses acts based on his posterior beliefs without explicit appeal to a utility function. Here is a definition that parallels previous model definitions:

$$\begin{aligned}
\rho_0 &= \rho_{\text{literal}} \\
\text{EU}^*_S(m,t,\rho_n) &= \log(\rho_n(t \mid m)) + \rho_n(t \mid m) \times c_m \\
\sigma_{n+1}(m \mid t) &= \frac{\exp(\lambda * \text{EU}^*_S(m,t,\rho_n))}{\sum_{m'} \exp(\lambda * \text{EU}^*_S(m,t,\rho_n))} \\
\rho_{n+1}(t \mid m) &= \frac{\Pr(t) \times \sigma_{n+1}(m \mid t')}{\sum_{t'} \Pr(t') \times \sigma_{n+1}(m \mid t')}
\end{aligned}$$

The speaker's expected utilities are defined in terms of $\log(\rho_n(t \mid m))$. This can be derived from a standard measure of divergence between the speaker's belief (who knows the actual state t) and the listener's belief $\rho_n(t \mid m)$.[12] The additive component $\rho_n(t \mid m) \times c_m$ is to implement the desired exploration-exploitation behavior for log-transformed utilities, just as multiplicative costs did before (see Footnote 6). The speaker's choice probabilities are given by a soft-max function, just as in iterated quantal response models. The listener's choice probabilities are given by the listener's posterior beliefs about states, as derived by Bayes' rule. One way to think about this is that listeners implement *probability matching* (e.g. Vulkan, 2000). Essentially, this means that subtle manipulations of utilities, such as in the numerostity game, are lost: the game's utility function plays no role in above definition. Qing and Franke (2015) use statistical model comparison based on empirical data to contrast some of these modeling choices with those seen in the previous section.

The RSA model, as defined here, predicts well for all of the example games defined in Section 4. It selects the desired pragmatic behavior, modulo some margin of error induced by the soft-max function, for all games relevant here, except, perhaps, for the interpretation of messages in the numerosity game. Since the definitions above do not take utilities, and thereby similarity between states, into account, the predicted listener interpretation of *some* and *most* does not single out a prototype. Rather it assings equal probability to states t_4 and t_5 for m_{some} and t_6, t_7, t_8 and t_9 for m_{most}.

While the probabilistic reasoning approach may loose some flexibility due to the omission of specific utility information, its focus on interlocutors' beliefs gives us a lot more flexibility when it comes to predicting empirical data from psycholinguistic experiments. For example, Frank and Goodman (2012) applied an RSA model to experimental data from a task that presented subjects with a small set of referents (geometric shapes). The speakers' task was to describe a designated referent, the listeners' task was to guess which referent was meant. The salience of objects in a given referential context was empirically measured and integrated as different prior beliefs $\Pr(t)$ of the listeners. Similarly, Kao et al. (2014) fed an empirical measure of subjects' prior beliefs about everyday events (e.g., the prize of a watch) into an RSA-style model that aims to predict non-literal interpretations (e.g., for a sentence like *That watch cost me a million.*).

Many applications of RSA-style models, especially those that aspire to a fit to experimental data,

[12] The speaker, who knows that the actual state t_k, has a degenerate belief $P_S \in \Delta(T)$ with $P_S(t_k) = 1$. If the listener has belief $P_L \in \Delta(T)$, utility in terms of negative Kullback-Leibler divergence reduces to: $U_S(P_S, P_L) = -\text{KL}(P_S \mid P_L) = -\sum_i P_S(t_i) \log \frac{P_S(t_i)}{P_L(t_i)} = -\log \frac{1}{P_L(t_k)} = \log P_L(t_k)$.

only look at a level-1 sender σ_1 and the corresponding level-1 receiver ρ_1.[13] A level-1 sender closely corresponds to a Gricean speaker. Level-1 senders prefer true messages over false ones and prefer more informative true messages, i.e., semantically stronger ones, over less informative true messages. This is baked-in for RSA-style models, but also holds for games with payoff functions like in the games from Section 4 (leaving the numerosity game aside) for iterated best/quantal response models. This, then, implements, more or less directly, the Gricean requirements of truthfulness (Maxim of Quality) and informativeness (Maxim of Quantity). Additionally, a level-1 sender's choice preferences can be influenced by considerations of markedness or brevity of expression (Maxim of Manner). A receiver whose interpretation behavior is attuned to such a Gricean(-like) sender captures Gricean interpretation in the sense that potential meaning enrichments are derived from the assumption that speakers adhere to certain patterns of behavior, as captured by the Gricean maxims. It is, of course, an empirical issue as to whether exactly these sender and receiver types best capture attested pragmatic behavior (Franke and Degen, to appear).

8 Summary

Common to all approaches subsumed here under the label "game theoretic pragmatics" is that they take the speaker's and the listener's stance explicitly into account. Production and comprehension are considered mutually dependent, which motivates the use of game theory as a formal approach to social decision making. Moreover, speaker and listener behavior are explained as optimal or rational with respect to each other, at least to some extent or to some approximation. Three main flavors of game theoretic approaches were introduced, so as to highlight their commonalities and differences. Having seen examples for these, let us consider them together, compare, go further and reflect.

The evolutionary perspective, exemplified in Section 5 by the replicator dynamic, assumes gradual adaptations to agents' behavior over time. Agents are not necessarily rational decision makers. Gradual optimization could proceed by low-level processes such as imitation of others' behavior in such a way as to make imitation of succesful behavior more likely than imitation of less successful behavior. Evolutionary dynamics that assume that agents are more sophisticated exist as well, of course. This way, the evolutionary perspective can blend into the other approaches introduced here, e.g., by asking how rational-like behavior described by the rationalistic perspective, characterized in Section 6, can be acquired or sustained (e.g. Vogel et al., 2014).

Section 6 introduced two related ideas of modeling idealized pragmatic reasoning from a rationalistic perspective. Game theoretic pragmatics has seen many more approaches to rationalizing pragmatic inferences and language use. The seminal work of Prashant Parikh must be emphasized here. Parikh's achievements include accounts of pragmatic meaning enrichments and a game theoretic reconstruction of Gricean speaker meaning (Parikh, 1991, 1992, 2001). Asher, Sher, and Williams (2001) seek to derive Gricean maxims from game theoretic considerations. Insightful approaches to rationalizing implicatures are plenty (e.g. Benz, 2012; van Rooij and de Jager, 2012; Rothschild, 2013; Pavan, 2013).

The reasoning perspective sketched in Section 7 inspires a lot of recent work that seeks a tight integration with experimental data. In other words, the focus is less on abstract rationalization of idealized behavior and more on predicting, by means of probabilistic models of speaker and listener

[13] Notice that the indexing is different in RSA and iterated best/quantal response models. In RSA, a level-1 receiver believes that his co-player is a level-1 sender. This would be a level-2 receiver in iterated best/quantal response models. The latter's indexing tracks depth of belief in rationality, while in RSA it tracks applications of joint speaker-listener inference steps.

behavior, relevant quantitative patterns in experimental data. A key element in many such approaches is to allow the listener to make *joint inferences* about the actual world state t and also about other uncertain pragmatic parameters of interest. For example, Kao et al. (2014) describe listeners' reasoning about the topic question that a speaker may have wanted to convey by an utterance. Likewise, Bergen, Levy, and Goodman (to appear) and Potts et al. (to appear) consider models that capture intricate and complex implicatures by describing listeners' reasoning about potential lexical enrichments that the speaker may have had in mind.

Applications of game theoretic pragmatics are bountiful. Clark (2012) uses game theory to shed light on such diverse phenomena as politeness or pronoun resolution (c.f. Clark and Parikh, 2007; Mühlenbernd and Quinley, 2013). Grosz (2014) gives a rationale for the use and interpretation of optative markers using signaling games. McCready (2012) uses game theoretic modeling to explain disambiguation of expressive meaning components. Bergen and Goodman (2015) and Stevens (2016) examine the potential effects of noise, intonation and omissions on pragmatic reasoning (c.f. Benz, 2012). Computational approaches turn towards game theoretic pragmatics to model optimal question-answer dialogues (e.g. Stevens et al., 2014, 2015). Evolutionary game theory helps explore pragmatic pressure on language change and grammaticalization (e.g. Ahern and Clark, 2014; Deo, 2015; Enke, Mühlenbernd, and Yanovich, 2016).

Further Resources

1. An implementation in programming language R of all game models and solution concepts used in this paper is available here:
`https://github.com/michael-franke/game_theoretic_pragmatics_ORE`
2. An implementation in Python of several versions of probabilistic reasoning models are here:
`https://github.com/cgpotts/pypragmods`
3. Implementations in Church of many other probabilistic pragmatic models are here:
`http://forestdb.org`

Further Reading

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