

AE 6353: Orbital Mechanics

Homework #1

Prof. G. Lightsey

18 August 2025

Due: In person section: 27 August 2025, 11:59 pm ET on Canvas.

Due: Distance learning section: 3 September 2025, 11:59 pm ET on Canvas.

Reading: P&C Chapter 1, BMWS Chapter 1.

Submit your work as a single PDF file on Canvas. Show all work and box final answers for clarity. Material for grading should go at the front with codes and printouts appended and labeled by problem at the end.

If you want to use chatGPT or another software tool to assist you with the homework, it is allowed as long as you give attribution. It is best used to check your work after you have solved the problem first by hand. If you get the wrong answer with chatGPT, you will not receive partial credit. If you use chatGPT to assist with the solution to a problem, include your session printout with prompts and responses with your name and date on the printout indicating that this is your work.

When you write Matlab code to solve a problem, include a printout of your Matlab scripts with your solution. Write your name and date on all pages, indicating that this is your original work.

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Problem 1.

For this problem, use:

- $m_{\text{Earth}} = \text{mass of Earth} = 5.9722 \times 10^{24} \text{ kg}$
- $m_{\text{Moon}} = \text{mass of Moon} = 7.3477 \times 10^{22} \text{ kg}$
- $d_{\text{EM}} = \text{distance between Earth and Moon} = 3.8440 \times 10^5 \text{ km}$

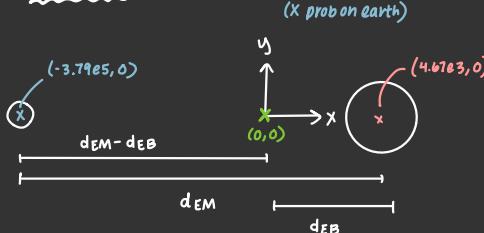
Assume the Moon and Earth co-orbit their barycenter in circular orbits using the two-body approximation.

Let the Earth be initially located at $t = 0$ on the positive x-axis of an inertial reference frame with its origin located at the barycenter of the Earth-Moon system, and the Moon is on the negative x-axis of this reference frame at $t = 0$. The xy-plane of this reference frame is the orbit plane of the Earth-Moon system.

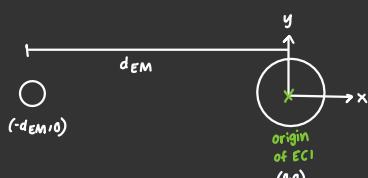
- Solve for the initial position coordinates of the Earth and Moon at $t = 0$, $\underline{x}_E(t=0)$ and $\underline{x}_M(t=0)$ in this Earth-Moon-Inertial (EMI) frame.
- The observed lunar period of motion around Earth relative to the stars is $T_{\text{EM}} = 27.322$ days = 2.3606×10^6 sec. Solve for the initial velocity coordinates of the Earth and Moon at $t = 0$, $\underline{v}_E(t=0)$ and $\underline{v}_M(t=0)$ in this Earth-Moon-Inertial (EMI) frame.
- Now transform these results to an Earth Centered Inertial (ECI) reference frame whose origin is located at the center of mass of the Earth and axis directions are aligned with the EMI frame at $t = 0$ (Thought question: why did we put the word 'Inertial' in quotes?). What are the initial values of \underline{x}_E , \underline{v}_E , \underline{x}_M , and \underline{v}_M in this ECI frame?
- What is the shape and period of the orbit trajectory of the Moon in this ECI reference frame?
- Using the two body equation of relative motion, calculate the semimajor axis a_{EM} in km and period T_{EM} in sec. Use $G = 6.6743 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$.
- Redo the calculation for a_{EM} and T_{EM} from part (e), assuming $m_{\text{Earth}} \gg m_{\text{Moon}}$ and neglecting the mass of the Moon.
- Using the observed average distance of $d_{\text{EM}} = 3.8440 \times 10^5$ km as the correct value, how much difference in percent is there between this value and the answers you obtained in part (e) and (f)?



EMI frame



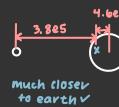
ECI frame



a) origin is barycenter; moon+earth pos wrt barycenter

$\Theta t=0$

$$\begin{aligned}\underline{x}_{E-\text{EMI}} &= [4.6719e3, 0] \text{ km} \\ \underline{x}_{M-\text{EMI}} &= [-3.7973e5, 0] \text{ km}\end{aligned}$$



$$\text{barycenter at: } \frac{m_2}{m_1 + m_2} \mathbf{r}$$

b) velocity in EMI given period

$$\omega = 2\pi/T$$

$$T_{\text{EMI}} = 2.3606e6 \text{ sec}$$

$\Theta t=0$

$$\begin{aligned}\underline{v}_{E-\text{EMI}} &= [0, 0] \text{ km/s} \\ \underline{v}_{M-\text{EMI}} &= [-1.0107, 0] \text{ km/s}\end{aligned}$$

same T, moon has larger r: $r_m \propto v_m \propto \omega$
checks ✓



c) pos and vel in ECI

$\Theta t=0$

$$\begin{aligned}\underline{x}_{E-\text{ECI}} &= [0, 0] \text{ km} \\ \underline{v}_{E-\text{ECI}} &= [0, 0] \text{ km/s} \\ \underline{x}_{M-\text{ECI}} &= [-3.8440, 0] \text{ km} \\ \underline{v}_{M-\text{ECI}} &= [-1.0232, 0] \text{ km/s}\end{aligned}$$

CM/origin
no motion wrt CM ($r=0$)

$$T = \frac{2\pi}{\sqrt{\mu}} a^{3/2} = 2.3718e6 \text{ sec}$$

circular frame
change not impacting
 $r = d_{\text{EM}}$
 $a = d_{\text{EM}}$
 $\mu = \mu_E = 3.986e5 \text{ km}^3/\text{s}^2$

shape: circular
period: $T_{\text{EMI}} = 2.3718e6 \text{ s}$
 $\approx 27.45 \text{ days}$

observed = 27.32 days

e) semimajor axis + period calc

$$\ddot{\underline{r}} = -\frac{\mu}{r^2} \underline{r}$$

$a=r$ bc circular

$$\mu = G(m_1+m_2)$$

$$a_{\text{EMI}} = \|\underline{v}\|$$

$$a_{\text{EMI}} = d_{\text{EM}}$$

$$T = \frac{2\pi}{\sqrt{\mu}} a^{3/2}$$

$$\begin{aligned}T_{\text{EMI}} &= 2.358e6 \text{ sec} \\ a_{\text{EMI}} &= 3.844e5 \text{ km}\end{aligned}$$

f) redo part e without the moon's mass

$$m_1 \gg m_2$$

$$M = Gm_{\text{Earth}}$$

$$\begin{aligned}T_{\text{EMI}} &= 2.373e6 \text{ sec} \\ a_{\text{EMI}} &= 3.844e5 \text{ km}\end{aligned}$$

NOTE:

didn't recalculate a_{EMI} , dr. lightsey said to just look at T for part g since it's circular in the ECI frame so even if mass changes the distance between planets won't. $a_{\text{EMI}} = d_{\text{EM}}$ mass assumptions

g) % difference

$$\frac{|2.358e6 - 2.373e6|}{\text{avg}} \times 100 = 0.61 \%$$

$$\text{diff} = 0.61\%$$

```
% AE 6353 - Homework 1
% Lauren Forcey
% 8/29/25

% =====
% Problem 1

% assumptions: moon and earth co-orbit their barycenter
% in circular orbits using the two body approx

% used this vid to visualize:
% https://www.youtube.com/watch?v=7hMfCCqSdFc&t=21s

% planet masses
mearth = 5.9722e24; % kg
mmoon = 7.3477e22; % kg

% distance between earth and moon
dEM = 3.844e5; % km

% distance from earth to barycenter
bc_loc = (mmoon/(mmoon + mearth))* dEM;

disp('1a: initial position coordinates at t=0')
```

1a: initial position coordinates at t=0

```
% distance from earth's center to barycenter
re_EMI = [bc_loc,0]
```

```
re_EMI = 1x2
103 x
    4.6719      0
```

```
% distance from moon's center to barycenter
rm_EMI = [-(dEM-bc_loc),0]
```

```
rm_EMI = 1x2
105 x
    -3.7973      0
```

```
%   m ----- bc e
```

Tem = 2.3606e6 % lunar period around earth

Tem = 2360600

```
% b: find initial velocity coordinates in EMI frame
```

```
% ang vel = 2pi/ T
% earth and moon orbital period equal about barycenter
wm = 2*pi / Tem;
we = 2*pi / Tem;
```

```
disp('1b: initial velocity coordinates')
```

```
1b: initial velocity coordinates
```

```
vm_EMI = wm*rm_EMI
```

```
vm_EMI = 1x2  
-1.0107 0
```

```
ve_EMI = we*re_EMI
```

```
ve_EMI = 1x2  
0.0124 0
```

```
% transform into the earth centered inertial frame  
% origin: CM of earth
```

```
disp('1c: r and v in ECI')
```

```
1c: r and v in ECI
```

```
re_ECI = [0,0]
```

```
re_ECI = 1x2  
0 0
```

```
rm_ECI = [-dEM,0]
```

```
rm_ECI = 1x2  
-384400 0
```

```
ve_ECI = 2*pi*re_ECI/Tem % zero since origin
```

```
ve_ECI = 1x2  
0 0
```

```
vm_ECI = 2*pi*rm_ECI/Tem % nonzero since r >0
```

```
vm_ECI = 1x2  
-1.0232 0
```

```
% what is the shape and period of the orbit trajectory
```

```
% semi major axis = rp = rm_ECI  
disp('1d: calculated period')
```

```
1d: calculated period
```

```
T_calc = (2*pi/sqrt(3.986e5))*(norm(rm_ECI))^(3/2)
```

```
T_calc = 2.3718e+06
```

```
T_days = T/60/60/24;
```

```
% a_EM and period  
disp('1e:a_EM and T_EM')
```

1e:a_EM and T_EM

```
G = 6.67e-20; % km^3/kg s^2  
a_EM = dEM % circular
```

a_EM = 384400

```
mu_EM = G*(mearth+mmoon);  
T_EM = (2*pi/sqrt(mu_EM))*(a_EM)^(3/2)
```

T_EM = 2.3581e+06

```
% a_EM and period  
disp('1f:a_EM and T_EM without moon')
```

1f:a_EM and T_EM without moon

```
G = 6.67e-20; % km^3/kg s^2  
a_EM = dEM % circular
```

a_EM = 384400

```
mu_EM2 = G*(mearth);  
T_EM2 = (2*pi/sqrt(mu_EM2))*(a_EM)^(3/2)
```

T_EM2 = 2.3726e+06

```
% g find the % difference in answers e and f  
disp('1f: percent difference')
```

1f: percent difference

```
perdiff = (abs(T_EM-T_EM2)/((T_EM + T_EM2)/2))*100
```

perdiff = 0.6114

% =====

Problem 2.

For this problem, use:

- $\mu_{\text{Moon}} = 4902.8 \text{ km}^3/\text{sec}^2$

An object is observed orbiting the Moon at $t = 0$ with the following position and velocity in a Moon Centered 'Inertial' reference frame:

$$\underline{r}_{MCI}(t=0) = \begin{bmatrix} -1085.9 \\ 2659.8 \\ 1085.9 \end{bmatrix} \text{ km}, \quad \underline{v}_{MCI}(t=0) = \begin{bmatrix} -0.94760 \\ -0.77371 \\ 0.94760 \end{bmatrix} \text{ km/sec}$$

The object passes behind the Moon and can no longer be tracked. Four hours later ($t = 14,400 \text{ sec}$), a new object is found with these parameters:

$$\underline{r}_{MCI}(t=+4 \text{ hours}) = \begin{bmatrix} -4420.3 \\ -6449.9 \\ -3475.6 \end{bmatrix} \text{ km}, \quad \underline{v}_{MCI}(t=+4 \text{ hours}) = \begin{bmatrix} 0.33809 \\ -0.27958 \\ -0.41117 \end{bmatrix} \text{ km/sec}$$

Could this object reasonably be the same one that was tracked earlier, or not? Justify your answer with quantitative analysis.

within the precision of the problem, these two measurements **DO NOT** belong to the same object.

$$\mathcal{E} = T + V = \frac{1}{2}V^2 - \frac{\mu}{r}$$

supporting evidence:

✓ $\mathcal{E} \% \text{ diff} = 1.735\%$ ← % diff of total energy] checking total energy conserved

✗ $\theta = 41.9^\circ$ ← angle between the angular momentum vectors

✓ $\mathbf{h} \% \text{ diff} = 1.87\%$ ← % diff of angular momentum magnitudes (norm($r \times v$))] checking angular momentum conserved

→ orbit plane is based on angular momentum vector and that's inertially fixed ∵ shouldn't change

→ energy, angular momentum

* plug trajectory from Q4 to double check
↳ see if it passes through the 2nd point

if same orbit:

- total energy conserved
- angular momentum conserved } must check both!
- ↳ magnitude & direction

↳ NOT conserved in this case

$$\cos \theta = \frac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{a}\| \|\mathbf{b}\|}$$

$$\theta = \cos^{-1} \left(\frac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{a}\| \|\mathbf{b}\|} \right)$$

```
% =====
% Problem 2

disp('2: checking if same object')

2: checking if same object

disp('answer: no, direction of ang mom not conserved')

answer: no, direction of ang mom not conserved

mu_moon = 4902.8; % km^3/s^2

% initial
```

```
rMCI0 = [-1085.9, 2659.8, 1085.9];
vMCI0 = [-0.94760,-0.77371,0.94760];

hMCI0 = cross(rMCI0,vMCI0);
h0mag = norm(hMCI0);

% after 4 hours
rMCI4 = [-4420.3,-6449.9,-3475.6];
vMCI4 = [0.33809,-0.27958,-0.41117];

hMCI4 = cross(rMCI4,vMCI4);
h4mag = norm(hMCI4);

hmagdiff = (h4mag-h0mag)/((h0mag+h4mag)/2) * 100
```

hmagdiff = 1.8766

```
% angle between the angular momentum vectors
theta = acosd(dot(hMCI0,hMCI4)/(h0mag*h4mag))
```

theta = 41.9083

```
r0mag = norm(rMCI0);
r4mag = norm(rMCI4);
v0mag = norm(vMCI0);
v4mag = norm(vMCI4);

% energy
e0 = 0.5*v0mag^2 - mu_moon/r0mag;
e4 = 0.5*v4mag^2 - mu_moon/r4mag;

% percent diff of total energy
ediff = (abs(e0-e4))/(abs(e0+e4)/2)*100
```

ediff = 1.7354

```
% =====
```

Problem 3.

Table 1 on the next page contains four orbits around Solar System objects with incomplete information. Reproduce Table 1 in your assignment and complete the missing entries. If a parameter cannot be determined from the information given, say 'Unknown'. Note: '∞' is not the same thing as 'Unknown'. Classify each Trajectory as either 'circle', 'ellipse', 'hyperbola', or 'parabola'. Include units in all your answers. To receive full credit, show your work.

a) given:

$$\mu = 3.986e5 \text{ km}^3/\text{s}^2$$

$$r_p = 10000 \text{ km}$$

$$a = -20000 \text{ km}$$

$$\epsilon = \frac{-\mu}{2a}$$

$$r_p = a(1-\epsilon)$$

$$\epsilon = \frac{-3.986e5}{2(-20000)}$$

$$10000 = -20000(1-\epsilon)$$

$$10000 = -20000 + 20000\epsilon$$

$$30000 = 20000\epsilon$$

$$\epsilon = 0.9985 \text{ km}^3/\text{s}^2$$

$$\boxed{\epsilon = 1.5}$$

central mass:

$$\mu = GM$$

$$M = \frac{\mu}{G} = \frac{3.986e14 \text{ m}^3/\text{s}^2}{6.67e-11 \text{ Nm}^2/\text{kg}^2}$$

$$T = ?$$

not a closed orbit; unknown

$$M_{earth} = 5.97e24 \text{ kg} \quad \therefore \text{earth}$$

$$M_{earth} = 5.97e24 \text{ kg} \quad (BMH)$$

c) given: $a = \infty$
 $\epsilon = 1$
 $\mu = 0$
 parabolic

$$\frac{-\mu}{2a} \rightarrow 0 = \epsilon$$

$$M_{Titan} \approx 1.35e23 \text{ kg}$$

not closed :: period is unknown

$$\mu = MG \quad \text{km}^3/\text{s}^2$$

$$\mu = (1.35e23 \text{ kg})(6.67e-11 \text{ Nm}^2/\text{kg}^2)$$

$$\mu = \frac{MG \cdot \frac{4\pi^2 a^2 \cos^2(\theta)}{T^2}}{a^3}$$

$$a = 9.005e12 \text{ m}^3/\text{s}^2$$

$$\mu = 9.005e3 \text{ m}^3/\text{s}^2$$

$$\boxed{\mu = 9.005e3 \text{ m}^3/\text{s}^2}$$

$$r_p = a(1-\epsilon)$$

by this $r_p = 0$
 (not true)
 need velocity

$$v_p = \frac{h^2}{2\mu}$$

b) given:

$$\epsilon = 0$$

$$r_p = 2.346e4 \text{ km}$$

$$T = 1.0908e5 \text{ s}$$

$$T = \frac{2\pi}{\sqrt{\mu}} a^{3/2}$$

$$\frac{T}{a^{3/2}} = \frac{2\pi}{\sqrt{\mu}} \Rightarrow \mu = \left(\frac{2\pi}{T} \right)^2 a^{3/2}$$

$$r_p = a(1-\epsilon)$$

$$r_p = a = 2.346e4 \text{ km}$$

$$\boxed{a = 2.346e4 \text{ km}}$$

$$\mu = \left[\frac{2\pi}{T} \right]^2 a^{3/2} = \left[\frac{2\pi}{1.0908e5} (2.346e4)^{3/2} \right]^2$$

central mass:

$$\mu = GM$$

$$M = \frac{\mu}{G} = \frac{4.284e13 \text{ m}^3/\text{s}^2}{6.67e-11 \text{ Nm}^2/\text{kg}^2}$$

$$\boxed{M = 6.423e23 \text{ kg} \leftarrow \text{Mars}}$$

$$M_{Mars} = 6.419e23 \text{ kg}$$

d) given:

$$\epsilon = 0.33$$

$$\epsilon = -246.4 \text{ km}^3/\text{s}^2$$

$$T = 7.6214e7 \text{ s}$$

$$\mu, a, r_p \quad T = \frac{2\pi}{\sqrt{\mu}} a^{3/2} \quad \begin{cases} 2 \text{ yrs} \\ 3 \text{ yrs} \end{cases}$$

$$\epsilon = \frac{-\mu}{2a}$$

$$r_p = a(1-\epsilon)$$

$$\mu = -2\epsilon a$$

$$T = \frac{2\pi}{\sqrt{-2\epsilon a}} a^{3/2}$$

$$r_p = a(1-\epsilon)$$

$$T = \frac{2\pi a^{3/2}}{\sqrt{-2\epsilon a}} \quad \boxed{r_p = 2.493e8 \text{ km}}$$

$$T = \frac{2\pi a}{\sqrt{-2\epsilon a}} \Rightarrow \frac{2\pi a}{\sqrt{-2\epsilon a}}$$

$$\frac{\sqrt{-2\epsilon a}}{2\pi} = a \quad \boxed{a = 1.989e30 \text{ m}}$$

$$a = \frac{\sqrt{-2(-246.4)} \cdot 7.6214e7}{2\pi}$$

$$\boxed{a = 2.693e8 \text{ km}}$$

$$\mu = GM$$

$$M = \frac{\mu}{G} = \frac{1.327e20 \text{ m}^3/\text{s}^2}{6.67e-11 \text{ Nm}^2/\text{kg}^2}$$

$$\boxed{M_{Sun} = 1.989e30 \text{ kg} \leftarrow \text{Sun}}$$

Table 1. Fill in the missing orbital parameters.
 (Reproduce this table in your assignment, showing additional work where needed.)

Part	Central Mass	μ	e	r_p	a	ϵ	T	Trajectory
a	Earth	$3.986e5 \text{ km}^3/\text{s}^2$	1.5	10000 km	-20000 km	$9.945 \text{ km}^3/\text{s}^2$	Unknown	Hyperbola
b	Mars	$4.284e4 \text{ km}^3/\text{s}^2$	0	2.346e4 km	$2.346e4 \text{ km}$	$-0.913 \text{ km}^3/\text{s}^2$	$1.0908e5 \text{ s}$	Circular
c	Titan	$8.005e3 \text{ km}^3/\text{s}^2$	1	unknown	∞	0	0	Parabola
d	Sun	$1.327e11 \text{ km}^3/\text{s}^2$	0.33	1.804e8 km	$2.693e8 \text{ km}$	-246.4 km^3/s^2	$7.6214e7 \text{ s}$	Ellipse

Problem 4.

In class, we derived the equation of motion for the two-body gravity problem:

$$\ddot{r} = -\frac{\mu}{r^3} \hat{r}$$

Write your own Matlab program that uses the `ode45` integrator to numerically propagate an orbit forward in time by $t_f = 43,082$ seconds (Thought question: why this number?). Use the following initial conditions in an ECI reference frame:

$$r(t=0) = [2120.3, 7642.6; -3964.1] \text{ km}$$

$$v(t=0) = [-5.4694; -5.0287; -4.4382] \text{ km/s}$$

Note that the equation of motion is a three-dimensional vector second order ODE, whereas `ode45` in Matlab solves a system of first order ODEs. Therefore, you need to convert the vector equation of motion into a system of first order ODEs. Here are some links that will help you learn how to use `ode45` to solve a second order ODE:

- http://www.math.umd.edu/~petersd/460/html/ode45_demo2.html
- <http://www.math.psu.edu/~walther/teach/MA366labs/ode45.pdf>

Use relative and absolute tolerances of $1e-12$ for your integration-these can be set using the Matlab `odeset` command. An example function call might be:

```
options = odeset('RelTol',1e-12,'AbsTol',1e-12);
[t,x]=ode45(@F,[t0, tf],x0,options);
```

- ✓ a. Plot the orbit in 3D using the Matlab `plot3` command. Use the following code to plot the Earth as a sphere. Overlay your orbit on this plot. Label your x, y, and z axes and specify units. Include the problem number, your name and date on the title indicating this is your own work. Turn in a copy of your finished plot.

- ✓ b. Report the final position and velocity vectors, r_f and v_f at time t_f .

- ✓ For fun. (Not for credit) What is the name of this type of orbit and how is it useful?

Molniya Orbit (I think)

highly elliptical, good for communications satellites, constant perigee + apogee

`eqnofmotion.m`

```
Editor - eqnofmotion.m
open_part1_student.mw  cals_and_plots_lab4.mw  hw1.mw  eqnofmotion.m  Project-Fall25
1 %function stateder = eqnofmotion(t, state)
2 % Lauren, 8/25
3 % the original equation is a 3D vector second order ODE
4 % need to translate into a system of 6 first order ODEs
5
6 % position
7 posVec = state(1:3); % (x; y; z)
8
9 % r = distance from center of earth to the satellite
10 % r = rm - rM (BEM 1-20)
11 r = norm(posVec);
12
13 % velocity
14 velVec = state(4:6); % (vx; vy; vz)
15 mu = 3.986e5; % km^3/s^2 (the earth is the central mass
16
17 % state = [r; dxdydz; dxdydz];
18 % aka state = [velocity; accel] where accel = (-mu/(mag(r)^3))*rvec
19 stateder = [velVec; -mu/r^3 * posVec];
20 end
```

(function to calculate stateder)

★ asked AI to generate an image of what it would expect wr these inputs as a check



PLANNING

$$\ddot{r} = -\frac{\mu}{r^3} \hat{r}$$

by $t_f = 43,082$ seconds

displacement: (x_0, y_0, z_0) km
velocity: (v_{x0}, v_{y0}, v_{z0}) km/s

need eqn of motion function
to feed into ODE45

$$\vec{r} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \dot{\vec{r}} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix}$$

$$\vec{r} = -\frac{\mu}{r^3} \hat{r} \quad r(t=0) = r_{x0}, r_{y0}, r_{z0}$$

$$\vec{v}(t=0) = v_{x0}, v_{y0}, v_{z0}$$

$$\text{state} = \begin{bmatrix} x \\ y \\ z \\ \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} \quad \left. \begin{array}{l} \text{velocity } \vec{v} = \dot{\vec{r}} \\ \text{position } \vec{r} \end{array} \right\}$$

state derivative

$$\text{stateder} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ -\mu/r^3(x) \\ -\mu/r^3(y) \\ -\mu/r^3(z) \end{bmatrix} \quad \left. \begin{array}{l} \text{velx} \\ \text{vely} \\ \text{velz} \\ -\mu/r^3(x) \\ -\mu/r^3(y) \\ -\mu/r^3(z) \end{array} \right\}$$

use to get updated state

Initial state \rightarrow ODE45 \rightarrow index final pos+vel \rightarrow plot all pos
 ↳ state vec
 ↳ calculate derivative
 ↳ get next acceleration
 ↳ output array w/ state
 at each timestamp

RESULT

b: final position and velocityvecs:

$$rf = 1 \times 3$$

$$10^3 \times$$

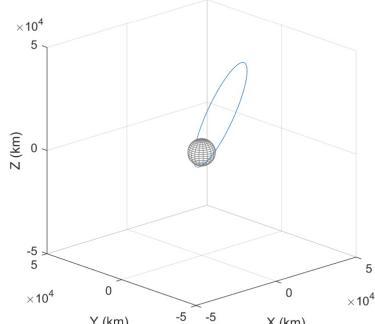
$$2.0568 \quad 7.5840 \quad -4.0154$$

$$vf = 1 \times 3$$

$$-5.4833 \quad -5.0795 \quad -4.4116$$

a: plot in 3D

Problem 4: Lauren Forcey - 8/25/25



```

% AE 6353 - Homework 1
% Lauren Forcey
% 8/29/25

% =====
% Problem 4

% references:
% https://www.math.umd.edu/~petersd/460/html/ode45_demo2.html
% https://www.math.psu.edu/~walther/teach/MA366labs/ode45.pdf

% given: initial conditions and propagation time
ri = [2120.3; 7642.6;-3964.1]; % km
vi = [-5.4694;-5.0287;-4.4382]; % km/s
tprop = 43082; % propagation time

% making col vector w all ICs
x0 = [ri;vi];

% propagaton time
t0 = 0;
tf = tprop;

% solve with ode45
options = odeset('RelTol',1e-12,"AbsTol",1e-12);
[t, state] = ode45('eqnofmotion',[t0,tf], x0, options);

% PART B: grab the final position and velocity vectors
% last row = state at tf
disp('b: final position and velocity vecs:')

```

b: final position and velocity vecs:

```
rf = state(end,1:3) % first 3 vals in the last row
```

```
rf = 1x3
103 ×
2.0568    7.5840   -4.0154
```

```
vf = state(end,4:6) % last 3 vals in the last row
```

```
vf = 1x3
-5.4833   -5.0795   -4.4116
```

```
% PART A: plotting it in 3D
% Extract components of the array
% first 3 cols are pos, second 3 are vel
disp('a: plot in 3D')
```

a: plot in 3D

```
% state vec : col 1 = x, col 2 = y, col 3 = z
plot3(state(:,1), state(:,2), state(:,3))
```

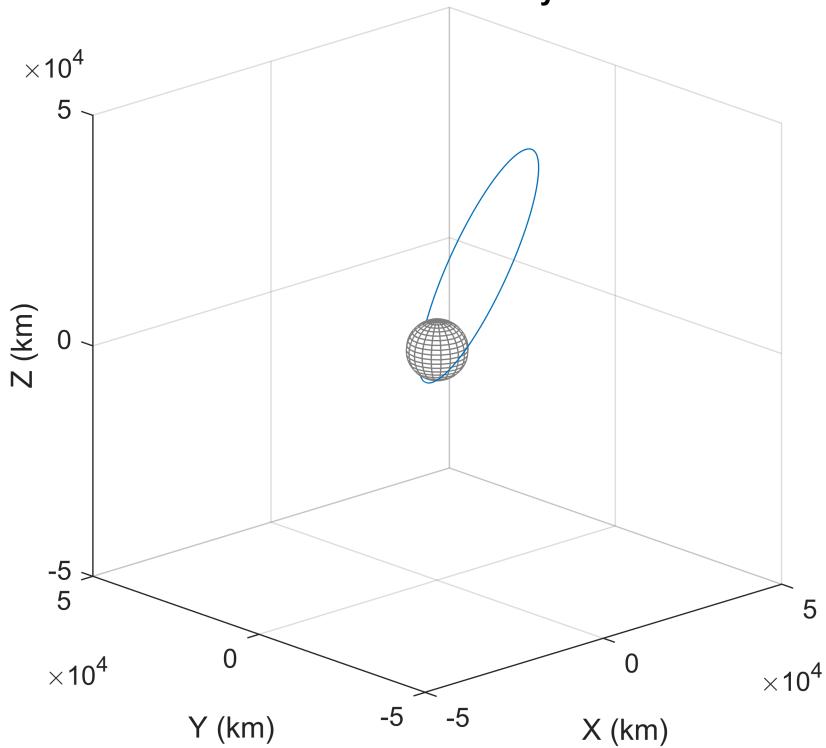
```

title({'Problem 4: Lauren Forcey - 8/25/25'})
xlabel('X (km)')
ylabel('Y (km)')
zlabel('Z (km)')

% code block from hw to plot the earth as a sphere
% (copy pasted)
figure(1)
npanels=20;
erad = 6378.1; % equatorial radius (km)
prad = 6356.8; % polar radius (km)
axis(5e4*[-1 1 -1 1 -1 1]);
view(-43,19);
grid;
hold on;
axis vis3d;
[xx,yy,zz ] = ellipsoid(0, 0, 0, erad, erad, prad,npanels);
globe = surf(xx, yy, -zz, 'FaceColor', 'w', 'EdgeColor',0.5*[1 1 1]);
hold off;

```

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```
% =====
```



Lauren Forcey
8/25/25