GEOEEM WS 15/16

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Contents

1 Introduction

Common to each method is the fact that the current flow is used in the subsurface. The aim is the determination of the conductivity distribution of the subsurface from the Earth's surface down to several 100 km depth.

Application areas:

- Near surface exploration (0 - 300 m depth):
  - Application for the environment: Waste site exploration, search for suitable landfill sites, ...
  - Groundwater exploration
  - Archaeology
  - Exploration for deposits, engineering applications (e.g. cativity detection,...)
- Exploration of deep structures (> 300 m)
  - Geothermal fields, oil and gas exploration
  - tectonic questions, shear zones
  - deep crust and upper mantle

1.1 Classification of methods

Classifications possible as:

- According to the source (artificial or natural)
- Inclusion of magnetic field or not?
- Direct current or alternating current?

DC-resistivity methods: Direct current resistivity (DC), Induced polarization (IP), Self potential (SP)

Electromagnetic methods:

- *Frequency domain*: Magnetotellurics (MT), Audiomagnetotellurics (AMT), Controlled source AMT (CSAMT), Radiomagnetotellurics (RMT)
- *Time domain:* Transient electromagnetics (TEM), Long offset transient electromagnetics (LOTEM)

Electromagnetic methods using high frequencies ($f > 10$ MHz): Ground penetrating radar (GPR)

2 Conductivity

The conductivity σ of the minerals in the nature covers a range of 25 decades! For example:

$$10^{-18} S/m \rightarrow \text{Diamand}$$
$$10^{7} S/m \rightarrow \text{Copper}$$

Instead of the conductivity, the resistivity $\rho = \frac{1}{\sigma} \Omega m$ is often used.

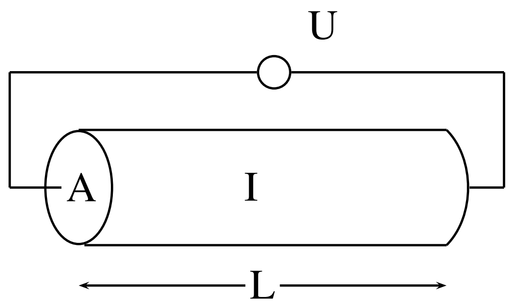

Figure 2.1: Schematic derivation of Ohm's law

Definition: Ohm's law

Let us consider a rock sample of length L, resistivity ρ and cross section A.
A current $I[A]$ flows by applying a voltag $U[V]$ to the rock sample:

$$I = \frac{AU}{\rho L}$$
$$\Leftrightarrow \rho \underbrace{\frac{I}{A}}_{\text{current density } j} = \underbrace{\frac{U}{L}}_{\text{electric Field } E}$$
$$\vec{j}\rho = \vec{E} \tag{2.1}$$

We measure I and U, A and L are known, so we can calculate ρ.

2.1 Mechanisms of electrical conductivity

Metallic conductivity: Current flows by free electrons $\rho \equiv T$

Electrolytic conductivity: Charge carriers are cations and anions: ρ decreases with temperature T.

Semi-conductors: Charge carriers must be activated by heat, light or EM-radiation. Strongly dependent on temperature T. Important for mantle (deep earth structures)

Boundary layer conductivity: Occurs due to the interaction of the pore liquid with the rock matrix. This is the source of SP-anomalies!

3 DC-resistivity method

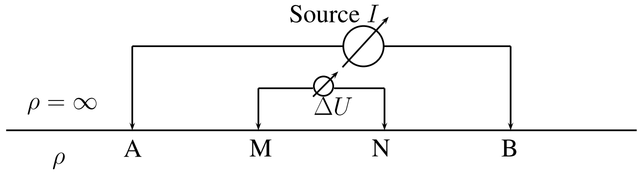

Figure 3.1: Four point measurement

Resistivity ρ of the subsurface derived from I (which is known), ΔU (which is measured) and the geometrical factor K (which is also known).

Frequently used electrode arrays

Industrial standard of measuring is via an *Multielectrode array.*

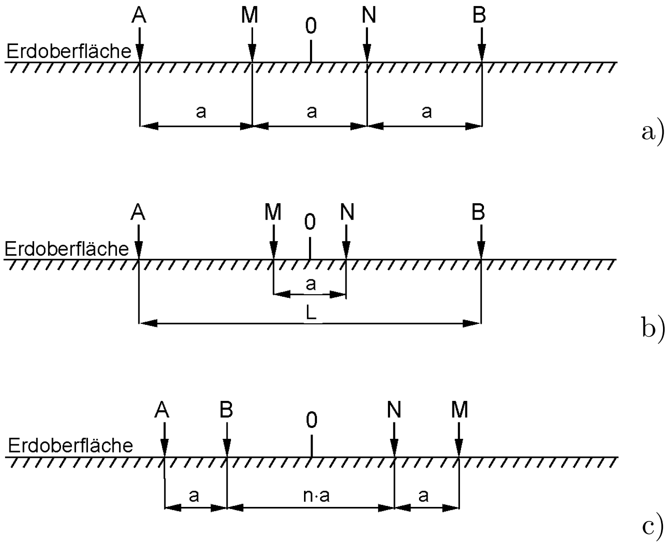

Figure 3.2: a) Wenner, Half-Wenner; b) Schlumberger, Half-Schlumberger; c) Dipole-dipole, source???

3.1 Basic equations of DC-resistivity

The first assumption of DC-resistivity methods and the major difference to EM-methods is the assumption of stationary currents:

$$\frac{\partial}{\partial t} = 0$$

The fields do not depend on time.

Looking at the *Maxwell's equations*:

$$\nabla \times \vec{E} = \frac{\partial \vec{B}}{\partial t} = 0 \tag{3.1}$$

This means irrotational electric field and from that follows, that the electric field vector can be derived by a scalar potential:

$$\vec{E} = -\nabla V \tag{3.2}$$

Insert equation (3.2) into eq. (2.1):

$$\vec{j} = -\sigma \nabla V \tag{3.3}$$

Continuity equation:

$$\nabla \cdot \vec{j} + \frac{\partial q}{\partial t} = 0 \tag{3.4}$$

Now new charges are generated in the course of time

$$\nabla \cdot \vec{j} = 0 \tag{3.5}$$

which is valid outside of the source.

If we insert eq. (3.3) into (3.5):

$$-\nabla \cdot (\sigma \nabla V) = 0$$
$$\nabla \sigma \nabla V + \sigma \nabla^2 V = 0$$

$\nabla \sigma = 0$ for areas with constant conductivity, so:

$$\nabla^2 V = 0 \tag{3.6}$$

which is called the *Laplace-equation*, the basic equation of DC-resistivity.

Derivation of solutions of this elliptic partial differential equation using different boundary conditions:

Assume a current source with strength I at point $\vec{r}_0$, then the spatial current distribution can be given as:$\nabla \cdot \vec{j} = I\delta(\vec{r} - \vec{r}_0$ and so:

$$\nabla \cdot (\sigma \nabla V) = -I\delta(\vec{r} - \vec{r}_0 \tag{3.7}$$

This equation can be solved numerically for arbitrary distribution of conductivity ratio.

3.1.1 Potential of a current electrode

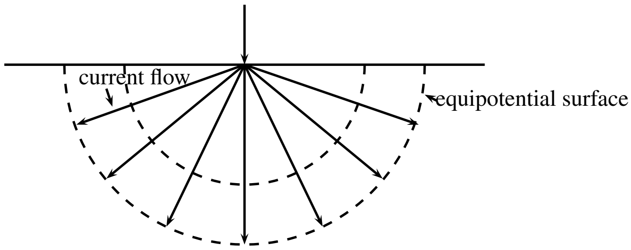

Figure 3.3: Single current source

Using *Ohm's law*: $\vec{E} = \rho\vec{j} = \rho\frac{I}{2\pi r^2}$, where $2\pi r^2$ is the surface of the half sphere. Using $E = -\frac{dV}{dr}$ follows the potential of a homogeneous half space:

$$V = \frac{\rho I}{2\pi r} \tag{3.8}$$

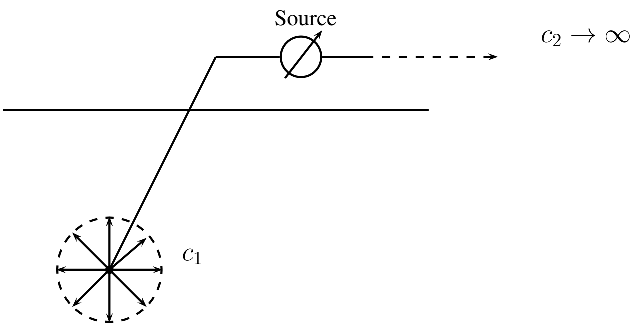

Figure 3.4: Mise a la masse method

In the case of the *Mise a la masse method* the potential of the homogeneous full space is:

$$V = \frac{\rho I}{4\pi r} \tag{3.9}$$

The same result can be derived by using the Laplace-equation (3.6) and the use of spherical coordinates:

$$\nabla^2 V = \frac{d^2 V}{dr^2} + \frac{2}{r}\frac{dV}{dr}$$

From the symmetry of the system the potential is a function of the distance to the source r only. Multiplying by r^2 and integrating, we get:

$$\frac{dV}{dr} = \frac{c_1}{r^2}$$

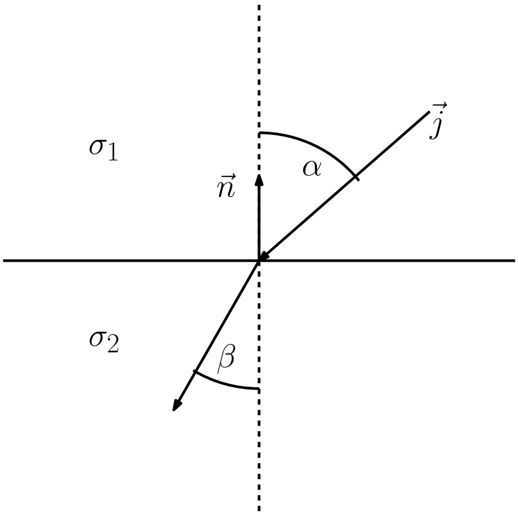

Figure 3.5: Boundary with dip angles GEOING s5

Integrating over r again leads to the solution:

$$V = -\frac{c_1}{r} + c_2 \qquad\qquad c_1, c_2 = const.$$

To determine the constants we have to use boundary conditions: From $\lim_{r\to\infty} V(r) = 0$ follows that $c_2 = 0$. Using the current density: $j = \frac{I}{A} \Leftrightarrow I = jA$:

$$I = 4\pi r^2 j = -4\pi r^2 \sigma \frac{dV}{dr} = -4\pi\sigma c_1$$

From this equation we can derive c_1:

$$V = \frac{I\rho}{4\pi r} \tag{3.10}$$

Boundary equations

Boundary with different conductivities.

Two boundary conditions which must hold at any contact between two regions of different conductivity.

- Potential is continuous across the boundary
- j_n is also continuous.

$$V^1 = V^2, \quad \left(\frac{\partial V}{\partial x}\right)^1 = \left(\frac{\partial V}{\partial x}\right)^2, \quad j_n^1 = j_n^2$$

$$E_t^1 = E_t^2, \quad \sigma_1 E_n^1 = \sigma_2 E_n^2$$

$$\sigma_1 \frac{E_n^1}{E_t^1} = \sigma_2 \frac{E_n^2}{E_t^2}$$
$$\sigma_1 \cot\alpha = \sigma_2 \cot\beta$$
$$\frac{\tan\alpha}{\tan\beta} = \frac{\sigma_1}{\sigma_2}$$

Current line is bent towards to the normal if the resistivity of the second medium ρ_2 is larger than the one of the first medium ρ_1.

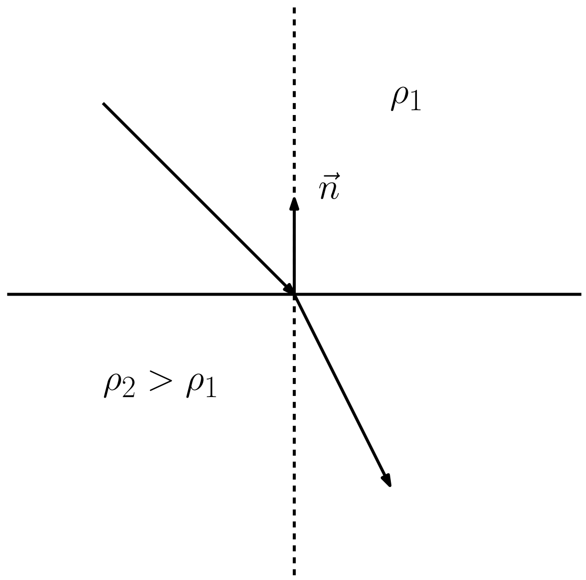

Figure 3.6: asdf1

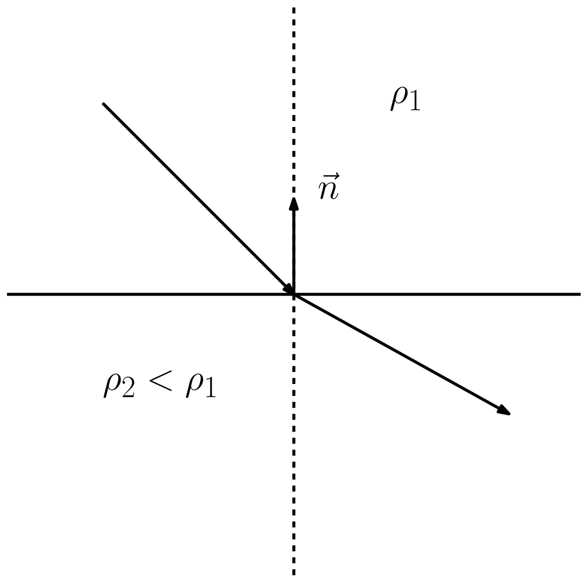

Figure 3.7: asdf2

3.1.2 Potential distribution at the surface of a horizontally stratified earth (Solution of the Laplace equation (3.6))

Starting with a *model*:

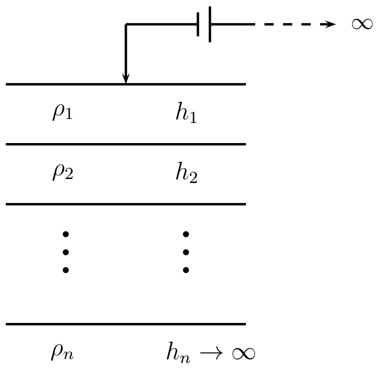

Figure 3.8: Model of n layer structure

The subsurface consists of finite number of layers with the last layer having infinite layer thickness ($h_n \to \infty$). We assume that ρ_i is isotropic (no dependence of the direction of measurement). The field is generated by a point source with the current I is a direct current.

Starting from the Laplace equation with potential V:

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0 \tag{3.11}$$

In cylindrical coordinates (r, θ, z):

$$\frac{\partial^2 V}{\partial r^2} + \frac{1}{r}\frac{\partial V}{\partial r} + \frac{\partial^V}{\partial z^2} + \frac{1}{r^2}\frac{\partial^2 V}{\partial \theta^2} = 0 \tag{3.12}$$

The solution is symmetrical to the vertical axis, so $\frac{\partial V}{\partial \theta} = \frac{\partial^2 V}{\partial \theta^2} = 0$, so $V(r, \theta, z) = V(r, z)$. So the Laplace equation to be solved reduces to:

$$\frac{\partial^2 V}{\partial r^2} + \frac{1}{r}\frac{\partial V}{\partial r} + \frac{\partial^V}{\partial z^2} = 0 \tag{3.13}$$