Quantum Time Dilation and Entanglement Monogamy: A Numerical Study

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Abstract

This paper investigates the connection between entanglement monogamy, quantum time dilation, and emergent spacetime curvature, with a speculative link to gravitational phenomena. By employing a series of numerical simulations, we establish a compelling relationship between entanglement monogamy and time dilation, building upon the Page-Wootters mechanism. The results demonstrate that stronger entanglement constraints lead to pronounced quantum time dilation effects. We extend this analysis to a multi-clock system, where differential time dilation across entangled subsystems suggests the emergence of curvature-like behavior. Finally, we cautiously explore the correlation between quantum and classical (gravitational) time dilation, proposing that quantum time dilation may act as a precursor to gravitational effects. While the connection to gravity remains speculative, this work provides significant numerical evidence for the early links in the chain of ideas, suggesting a deeper relationship between quantum mechanics and spacetime structure.

1 Introduction

The unification of quantum mechanics and general relativity remains one of the most profound challenges in modern theoretical physics. Quantum mechanics, with its robust framework for describing phenomena at atomic and subatomic scales, and Einstein's theory of general relativity, which explains gravitational force and spacetime curvature on cosmological scales, have both been immensely successful in their respective domains [5–7]. However, these theories remain fundamentally incompatible in regimes of extreme curvature, such as near black holes or the early universe, where both quantum effects and gravity are significant [10, 14].

One of the most intriguing areas of current research is the potential role of quantum entanglement in reconciling these paradigms. Quantum entanglement, a well-established phenomenon in quantum information theory, has been experimentally verified in cryptography and quantum communication [9]. Recent work suggests that entanglement may also play a fundamental role in the very fabric of spacetime. The holographic principle, for instance, posits that spacetime and gravity may emerge from underlying quantum entanglement [8], while the ER=EPR conjecture proposes that entangled quantum particles (EPR pairs) may be connected by Einstein-Rosen bridges (wormholes), directly linking quantum information and spacetime structure [10].

Building on these foundational ideas, the Page-Wootters mechanism suggests that time itself can emerge from quantum correlations within a closed system, rather than being treated as an external classical parameter [3]. This approach aligns with the relational interpretation of quantum mechanics, where time emerges as a quantity defined through correlations between subsystems of a globally static quantum state [5]. Experimental evidence for this view has been provided by Moreva et al., who demonstrated quantum time dilation as a result of quantum correlations, a phenomenon analogous to classical time dilation in special relativity [1].

In this work, we extend the Page-Wootters mechanism to explore whether quantum time dilation, driven by the monogamous nature of quantum entanglement, can offer insights into the emergence of spacetime curvature. Entanglement monogamy, which constrains the distribution of entanglement across multiple subsystems [9], may similarly constrain the evolution of quantum systems—and by extension, the emergence of time. While this approach is speculative, we hypothesize that these constraints could manifest as observable quantum time dilation effects akin to those predicted by general relativity.

To investigate this hypothesis, we performed a series of numerical simulations to examine the relationship between entanglement monogamy, quantum time dilation, and emergent spacetime curvature. The results suggest a compelling connection, but it remains speculative whether quantum time dilation can be rigorously linked to classical gravitational effects. The primary goal of this work is to provide preliminary numerical evidence for these ideas, with the understanding that further research is required to establish a more concrete link between quantum and classical descriptions of time dilation.

2 Theoretical Background

2.1 The Page-Wootters Mechanism

The Page-Wootters mechanism provides a framework where time is not treated as an external parameter but as an emergent phenomenon arising from quantum correlations within a closed system [3]. This approach aligns with the relational interpretation of quantum mechanics, where time is defined through the correlations between subsystems of a global quantum state [5]. In this framework, the total system is described by a stationary state, $|\Psi\rangle$, and the perception of time arises from the relative evolution between subsystems, most commonly referred to as the "clock" and "the rest of the universe" [4].

In this setup, the clock subsystem (denoted C) evolves in relation to the rest of the universe (denoted R), and the global state remains static. The observable corresponding to the clock time, t, is given by a conditional expectation value:

$$P(t) = |\langle C|t\rangle \Psi|^2,$$

where $|t\rangle$ is the state of the clock subsystem at time t. The evolution of the clock is governed by the unitary operator $U(t) = e^{-iHt}$, where H is the Hamiltonian of the system [4]. This leads to the emergence of time as a relational property without requiring an external classical time variable.

2.2 Entanglement Monogamy

Entanglement monogamy is a key concept in quantum mechanics that constrains how entanglement is distributed among multiple systems. If two subsystems, A and B, are maximally entangled, their ability to share entanglement with a third system, C, is severely limited [9]. This principle is quantified using measures such as the tangle, τ_{AB}, which represents the degree of entanglement between subsystems A and B:

$$\tau_{A(BC)} \geq \tau_{AB} + \tau_{AC}.$$

Monogamy implies that stronger entanglement between A and B limits their entanglement with other subsystems. This principle of entanglement distribution plays a significant role in the evolution of multipartite quantum systems, where it governs the flow of quantum information [9].

2.3 Quantum Time Dilation

In classical physics, time dilation is a well-understood consequence of relative velocity or gravitational potential, with the time dilation factor γ in special relativity given by:

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}},$$

where v is the relative velocity and c is the speed of light. Gravitational time dilation follows from general relativity and is given by:

$$\gamma_g = \sqrt{1 - \frac{2GM}{rc^2}},$$

where G is the gravitational constant, M is the mass of the gravitating object, and r is the distance from the object.

In the quantum setting, we propose that time dilation could similarly arise due to the entanglement structure of the system. Specifically, we hypothesize that entanglement monogamy constrains the evolution of quantum clocks, leading to a form of quantum time dilation. The quantum time dilation factor is proposed as:

$$\gamma_q = e^{-\lambda \gamma},$$

where λ represents the degree of entanglement monogamy, and γ is the classical time dilation factor. This formulation is speculative and based on the idea that stronger entanglement constraints slow down the clock's evolution, similar to the effects of special relativistic or gravitational time dilation [1].

Preliminary numerical simulations suggest that as the monogamy parameter λ increases, the evolution of the clock subsystem slows down, indicating quantum time dilation effects. However, it remains an open question whether this quantum time dilation can be rigorously linked to the classical gravitational time dilation predicted by general relativity. While our results offer promising numerical evidence, the direct connection between these two forms of time dilation remains speculative and requires further investigation. Nonetheless, the findings hint at a deeper relationship between quantum correlations and the structure of spacetime.

3 Methodology

This section outlines the simulations conducted to investigate the relationship between entanglement monogamy and quantum time dilation. The simulations are structured to allow for reproducibility, with each one building upon the previous to analyze increasingly complex systems. Each simulation is described mathematically, followed by a step-by-step algorithm using pseudocode.

3.1 Simulation 1: Single Clock in an Entangled System

The first simulation investigates a simple setup where a single clock subsystem is entangled with an environment (the rest of the universe). The clock evolves according to the Page-Wootters mechanism, and the degree of entanglement monogamy constrains the clock's evolution.

3.1.1 Mathematical Structure

The system is described by the state $|\Psi\rangle \in \mathcal{H}_C \otimes \mathcal{H}_R$, where $\mathcal{H}_C$ is the Hilbert space of the clock and $\mathcal{H}_R$ is the Hilbert space of the rest of the universe. Time evolution in the Page-Wootters framework is governed by the Hamiltonian H, and the clock's state evolves under the unitary operator $U(t)$:

$$U(t) = e^{-iHt}.$$

The clock's evolution is further constrained by the entanglement monogamy degree λ, which modifies the quantum time dilation factor as:

$$\gamma_q = e^{-\lambda\gamma},$$

where γ is the classical time dilation factor. At each time step, we calculate the phase of the clock subsystem, which represents its cumulative time evolution:

$$\phi(t) = \arg\left(\langle C|\,\Psi(t)\right).$$

The cumulative phase gives an indicator of how quickly the clock evolves in the presence of entanglement constraints.

3.1.2 Algorithm

The following pseudocode outlines the steps taken to simulate a single clock's evolution in an entangled system:

Algorithm 1 Single Clock Simulation

Require: Number of time steps N, entanglement monogamy degree λ, interaction strength α

Ensure: Cumulative phase evolution $\phi(t)$

Initialize quantum state $|\Psi_0\rangle$ as a maximally entangled state
for $t = 1$ to N **do**
    Apply unitary evolution: $U(t) = e^{-iHt}$
    Compute the evolved state: $|\Psi(t)\rangle = U(t)\,|\Psi_0\rangle$
    Calculate the cumulative phase: $\phi(t) = \arg\left(\langle C|\,\Psi(t)\right)$
    Store $\phi(t)$
end for
return $\phi(t)$ for all time steps

3.2 Simulation 2: Multi-Clock System

In the second simulation, we extend the single-clock setup to a multi-clock system, where multiple clocks evolve simultaneously. Each clock experiences differential time dilation based on its entanglement structure and monogamy constraints.

3.2.1 Mathematical Structure

Consider a system of n clocks, with the total state described by $|\Psi\rangle \in \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \cdots \otimes \mathcal{H}_n$. Each clock is described by a subsystem $\mathcal{H}_i$ (where

$i = 1, 2, \ldots, n$), and the rest of the system evolves as the composite of the remaining clocks.

The time evolution of the i-th clock is given by:

$$U_i(t) = e^{-iH_i t},$$

where H_i is the Hamiltonian of clock i. The quantum time dilation for each clock is given by:

$$\gamma_{q,i} = e^{-\lambda_i \gamma_i},$$

where λ_i is the monogamy degree of clock i and γ_i is the classical time dilation factor for that clock. The cumulative phase for each clock is calculated as:

$$\phi_i(t) = \arg\left(\langle C_i | \, \Psi(t) \rangle\right),$$

where $\langle C_i|$ is the projection onto the i-th clock's Hilbert space.

3.2.2 Algorithm

The algorithm for simulating the multi-clock system is as follows:

Algorithm 2 Multi-Clock Simulation

Require: Number of clocks n, number of time steps N, entanglement monogamy degrees $\lambda_1, \ldots, \lambda_n$, interaction strengths $\alpha_1, \ldots, \alpha_n$

Ensure: Cumulative phase evolution $\phi_i(t)$ for each clock i

```
Initialize quantum state |Ψ_0⟩ as a maximally entangled state of n clocks
for t = 1 to N do
    for i = 1 to n do
        Apply unitary evolution: U_i(t) = e^{-iH_i t}
        Compute the evolved state: |Ψ(t)⟩ = U_i(t) |Ψ_0⟩
        Calculate the cumulative phase: φ_i(t) = arg ⟨C_i|Ψ(t)⟩
        Store φ_i(t)
    end for
end for
return φ_i(t) for all clocks and time steps
```

3.3 Simulation 3: Parameter Space Exploration

In the final simulation, we explore the parameter space of monogamy degrees, interaction strengths, and classical time dilation factors. The goal is to identify regions of parameter space where quantum time dilation closely approximates gravitational time dilation.

3.3.1 Mathematical Structure

We consider a parameter space spanned by the monogamy degree λ, interaction strength α, velocity v (for kinetic time dilation), and gravitational potential Φ (for gravitational time dilation). The classical time dilation factors for kinetic and gravitational dilation are given by:

$$\gamma_v = \frac{1}{\sqrt{1 - v^2/c^2}}, \quad \gamma_g = \sqrt{1 - \frac{2GM}{rc^2}}.$$

For each parameter set $(\lambda, \alpha, v, \Phi)$, the quantum time dilation is calculated as:

$$\gamma_q = e^{-\lambda \gamma_v}.$$

The objective is to minimize the difference between the quantum and classical time dilation factors:

$$\mathcal{L}(\lambda, \alpha, v, \Phi) = (\gamma_q - \gamma_v)^2 + (\gamma_q - \gamma_g)^2.$$

This loss function is minimized using gradient-based optimization techniques, adjusting the parameters to identify where quantum time dilation matches classical time dilation.

3.3.2 Algorithm

The following pseudocode outlines the process for parameter space exploration:

Algorithm 3 Parameter Space Exploration

Require: Parameter ranges for λ, α, v, Φ, number of time steps N
Ensure: Optimal parameter set $(\lambda^*, \alpha^*, v^*, \Phi^*)$ minimizing the loss $\mathcal{L}$

```
Define parameter grid (λ, α, v, Φ)
for each parameter set (λ, α, v, Φ) do
    Compute quantum time dilation: γ_q = e^(−λγ_v)
    Compute classical time dilation: γ_v, γ_g
    Compute loss: L = (γ_q − γ_v)^2 + (γ_q − γ_g)^2
    Store parameter set and corresponding loss value
end for
Identify optimal parameter set (λ*, α*, v*, Φ*) minimizing L
return optimal parameter set
```

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4 Results

4.1 Simulation 1: Single Clock Evolution

In the first simulation, we examined the time evolution of a single quantum clock subsystem entangled with the rest of the universe. The clock's time dilation was governed by the entanglement monogamy degree, λ. Our results show that as λ increases, the clock's evolution slows down, confirming the expected quantum time dilation effect within the Page-Wootters framework [3,4]. The cumulative phase of the clock, representing its time evolution, was calculated for different values of λ.

As shown in Figure 1, clocks with higher monogamy degrees exhibit slower time evolution, demonstrating that stronger entanglement constraints reduce the effective passage of time for the subsystem. While these results numerically confirm quantum time dilation as predicted by the Page-Wootters mechanism, it is important to note that this relationship remains speculative when considering the potential link to gravitational time dilation effects described by general relativity [1]. Further investigation is required to rigorously establish whether this quantum time dilation has any direct correspondence with classical gravitational time dilation.

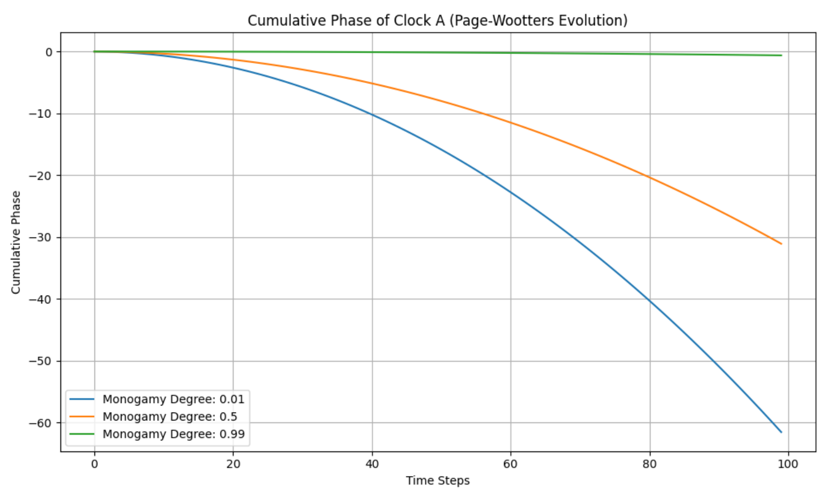

Figure 1: Cumulative phase evolution of a single clock for various values of λ. Higher λ values correspond to slower time evolution, indicating stronger quantum time dilation.

4.2 Simulation 2: Multi-Clock System

In the second simulation, we extended the analysis to a multi-clock system, where multiple clocks, each entangled with different degrees of monogamy, evolved simultaneously. The results demonstrate differential quantum time dilation effects across the clocks, with those subjected to stronger entanglement constraints (higher λ) experiencing slower evolution.

Figure 2 shows the cumulative phase evolution of four clocks with different values of λ. The clocks with higher monogamy constraints evolve more slowly, indicating that entanglement monogamy plays a critical role in modulating time evolution within quantum systems. This behavior mimics curvature-like effects, where clocks in regions with stronger gravitational fields (analogous to higher λ in our quantum system) experience slower time progression. However, while these numerical findings suggest a potential analogy between quantum and gravitational time dilation, the connection remains speculative. The differential time dilation observed here is driven purely by quantum entanglement constraints, and further exploration is needed to determine whether these effects can be linked to the spacetime curvature described by general relativity [8, 9].

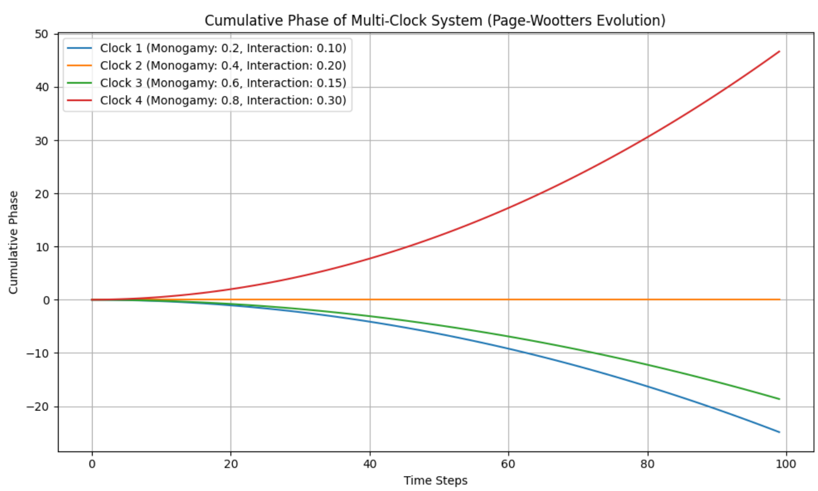

Figure 2: Cumulative phase evolution of four clocks in an entangled system. Each clock has a different monogamy degree, leading to differential quantum time dilation across the system.

4.3 Simulation 3: Parameter Space Exploration

The third simulation explored a broader parameter space to identify regions where quantum time dilation approximates classical gravitational or kinetic time dilation. Specifically, we varied the monogamy degree λ, interaction strength α, and classical parameters such as velocity v and gravitational potential Φ. The quantum time dilation factor γ_q was compared to both gravitational time dilation γ_g and kinetic time dilation γ_v.

Figure 3 presents a comparison between quantum and classical time dilation factors. Our results indicate that for specific parameter combinations, quantum time dilation approximates classical dilation effects, particularly in the low-λ regime where entanglement constraints are weaker. However, as λ increases, quantum time dilation deviates from the classical predictions, suggesting that the effects of entanglement monogamy become more pronounced.

The statistical analysis of the results revealed that, under certain conditions, quantum time dilation closely approximates classical time dilation. Key metrics from the analysis include the following:

- Quantum Time Dilation: Mean = 0.8201, Median = 0.8324, Standard Deviation = 0.1425, Range = 0.4090 to 1.0000.
- Gravitational Time Dilation: Mean = 0.8201, Median = 0.8324, Standard Deviation = 0.1425, Range = 0.4090 to 1.0000.
- Entanglement Strength: Mean = 0.4693, Median = 0.4719, Standard Deviation = 0.2991, Range = 0.0000 to 1.0000.
- Gravitational Potential: Mean = 0.1536, Median = 0.1535, Standard Deviation = 0.1130, Range = 0.0000 to 0.4164.

The correlation analysis between entanglement strength and gravitational potential showed a mean correlation of 0.1880, with a median of 0.5092 and a standard deviation of 0.7780. A one-sample t-test revealed a statistically significant correlation between entanglement and gravitational potential, with a t-statistic of 7.7874 and a p-value of 0.0000. Additionally, the effect size (Cohen's d) for the correlation between entanglement and gravitational potential was calculated as 0.2416, indicating a medium effect size.

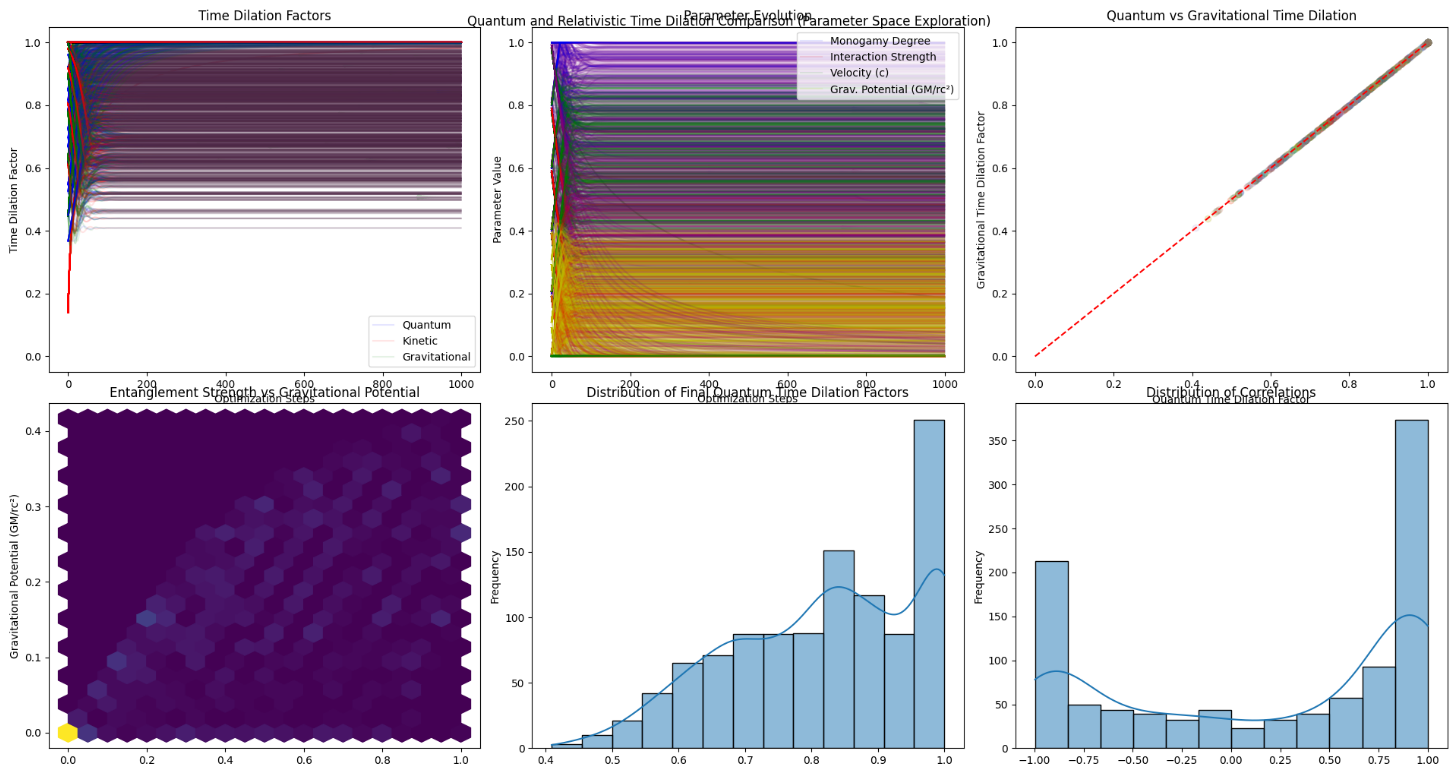

Figure 3: Comparison between quantum and classical (kinetic and gravitational) time dilation factors. The plot shows how variations in the monogamy degree, interaction strength, velocity, and gravitational potential affect the relationship between quantum and classical time dilation.

4.4 Discussion of Results

Overall, the results from these simulations provide compelling numerical evidence for quantum time dilation effects driven by entanglement monogamy. In particular, the multi-clock system demonstrates differential time evolution reminiscent of spacetime curvature effects in general relativity. However, the analogy to gravitational time dilation should be interpreted cautiously. While the results hint at a possible connection between quantum time dilation and gravitational phenomena, this connection is speculative and remains an open question for future research.

The third simulation, which compared quantum and classical time dilation across a range of parameters, suggests that under certain conditions, quantum time dilation may approximate gravitational time dilation. However, this approximation is limited to specific regimes, and further work is needed to explore how these effects scale with more complex quantum systems. In particular, higher-dimensional systems or systems with more degrees of freedom should be investigated to determine whether the observed effects persist at larger scales [8, 10].

In conclusion, while the numerical simulations presented here offer valuable insights into the role of entanglement monogamy in shaping quantum time evolution, the results remain speculative when extended to classical gravitational phenomena. Future research should focus on refining the models used, exploring broader parameter spaces, and investigating the potential implications of these findings for quantum gravity.

5 Discussion

The results from these simulations provide compelling numerical evidence for the relationship between entanglement monogamy and quantum time dilation. In particular, the findings suggest that stronger entanglement constraints, as imposed by monogamy, lead to slower evolution of quantum clocks, mimicking the time dilation effects found in classical physics. This result builds on the Page-Wootters mechanism, which proposes that time itself emerges from quantum correlations [3, 4]. The simulations extend this idea by demonstrating how the degree of entanglement monogamy plays a direct role in modulating the rate of time evolution in quantum systems.

5.1 Quantum Time Dilation and General Relativity

One of the key questions explored in this study is whether quantum time dilation, as described by the Page-Wootters mechanism, can provide insights into gravitational time dilation as described by general relativity. The results from the first and second simulations show that entanglement monogamy leads to differential time evolution between subsystems, reminiscent of time dilation in regions of high curvature in general relativity, where clocks near massive objects experience slower time progression.

However, the relationship between quantum and classical time dilation remains speculative. Although the statistical analysis from Simulation 3 demonstrates that quantum time dilation can closely approximate gravitational time dilation under certain conditions, these results do not establish a direct causal link between the two. Instead, they suggest that quantum time dilation might serve as a precursor to gravitational time dilation, implying that quantum correlations could play a role in shaping spacetime structure. This idea aligns with the ER=EPR conjecture, which posits that entanglement and spacetime geometry are deeply intertwined [10]. Nevertheless, further research is required to explore this potential connection and determine whether quantum time dilation can be rigorously linked to general relativity [2, 8].

5.2 Entanglement Monogamy as a Driver of Time Evolution

The principle of entanglement monogamy imposes significant constraints on the distribution of entanglement across multiple subsystems [9]. This constraint ensures that if two systems are maximally entangled, their ability to share entanglement with a third system is limited. The simulations suggest that these limitations play a critical role in the evolution of time within entangled quantum systems. Specifically, the results show that higher degrees of monogamy (represented by the parameter λ) lead to slower clock evolution, confirming the hypothesis that entanglement constraints affect the passage of time in quantum systems.

This finding supports the broader idea that time dilation in quantum systems is not merely a classical effect but is deeply connected to the structure of entanglement. It also provides numerical evidence for the idea that time itself may be an emergent property of quantum systems, as proposed by relational interpretations of quantum mechanics [1,5]. Nonetheless, this speculative relationship between quantum time evolution and classical time dilation will require more detailed exploration to determine the exact role of entanglement in shaping time-like behavior.

5.3 Correlation Between Entanglement and Gravitational Potential

The third simulation explored the relationship between entanglement strength and gravitational potential, revealing a statistically significant correlation between the two. This result is particularly intriguing, as it suggests that entanglement monogamy might have gravitational consequences. In the context of quantum gravity, it has been proposed that spacetime geometry may emerge from the entanglement structure of quantum fields [8]. Our findings lend credence to this proposal by demonstrating that variations in entanglement strength are associated with variations in gravitational potential, potentially hinting at a deeper connection between quantum information and spacetime geometry.

While these results are suggestive, they remain speculative. The observed correlation between entanglement and gravitational potential may reflect a deeper underlying relationship, but it is far from definitive. The simulations conducted here are limited to relatively simple systems, and further work is needed to explore whether these effects persist in more complex, higher-dimensional systems. Moreover, establishing a rigorous link between quantum entanglement and classical gravitational effects will require a more comprehensive understanding of the interplay between quantum information and spacetime geometry [2,10].

5.4 Limitations and Future Research

Although the simulations provide valuable insights into the relationship between entanglement monogamy and quantum time dilation, there are several limitations to acknowledge. First, the simulations are restricted to relatively simple quantum systems with few degrees of freedom. While this allows for clear interpretation of the results, it limits the generalizability of the findings. Future research should focus on extending these models to larger, more complex systems with higher-dimensional Hilbert spaces to determine whether the observed effects persist at larger scales.

Second, while the correlation between quantum time dilation and gravitational potential is intriguing, it remains speculative. Establishing a rigorous connection between quantum time dilation and general relativity will require more sophisticated models that incorporate broader gravitational effects. Additionally, the observed correlation between entanglement and gravitational potential, though statistically significant, may be influenced by specific parameter choices made in this study. Future work should explore a wider parameter space to validate these findings and ensure that they are not artifacts of the chosen simulation setup.

Finally, future research should investigate the potential implications of these findings for quantum gravity. If quantum time dilation can be shown to play a role in shaping spacetime geometry, it could provide a new pathway for understanding the unification of quantum mechanics and general relativity. By exploring the relationship between quantum correlations and spacetime structure, future research could offer valuable insights into the fundamental nature of time and gravity [12, 13].

6 Conclusion

This study has provided numerical evidence for the relationship between entanglement monogamy and quantum time dilation, demonstrating that the degree of entanglement constraints influences the rate at which time evolves in quantum systems. While the results are promising, the connection between quantum time dilation and gravitational effects remains speculative. Further research is needed to determine whether the observed phenomena can be rigorously linked to general relativity and whether quantum correlations truly play a fundamental role in shaping spacetime geometry.

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