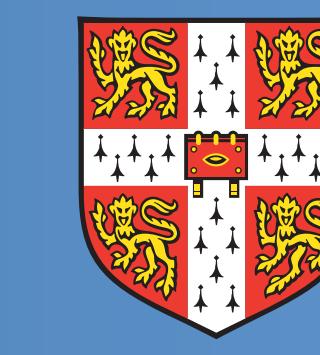


# A Scale-Aware Subgrid Model for Oceanic Quasigeostrophic Turbulence



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## INTRODUCTION

At present, many modelling centers worldwide are poised to enter a new regime of ocean modelling, where mesoscale eddies occur naturally across nearly the whole globe. Indeed, eddy-rich modeling offers potential for more realism and richer dynamics but also introduces difficulties. Notably, eddy parameterizations such as the popular Gent and McWilliams (1990) eddy-induced advection scheme are designed for use in coarsely resolved models where eddies are absent and there is an assumed scale separation between the resolved flow and subgrid-scale turbulence. In reality, current climate-scale ocean models are more akin to large eddy simulations (LES), which do not assume a scale separation between eddy and mean flow at the gridscale. These models generally resolve the largest baroclinic and barotropic eddies near the first baroclinic Rossby deformation radius, or about 10–100 km depending on latitude and stratification. Here, following Fox-Kemper and Menemenlis (2008), models with resolutions fine enough to be mesoscale-eddy-rich will be called Mesoscale Ocean Large Eddy Simulations (MOLES). MOLES require subgrid-scale eddy closures which automatically adjust their effects based on physical principles, the detection of key measures from the resolved flow (flow-awareness), and the ability of the model grid to represent such effects (scale-awareness).

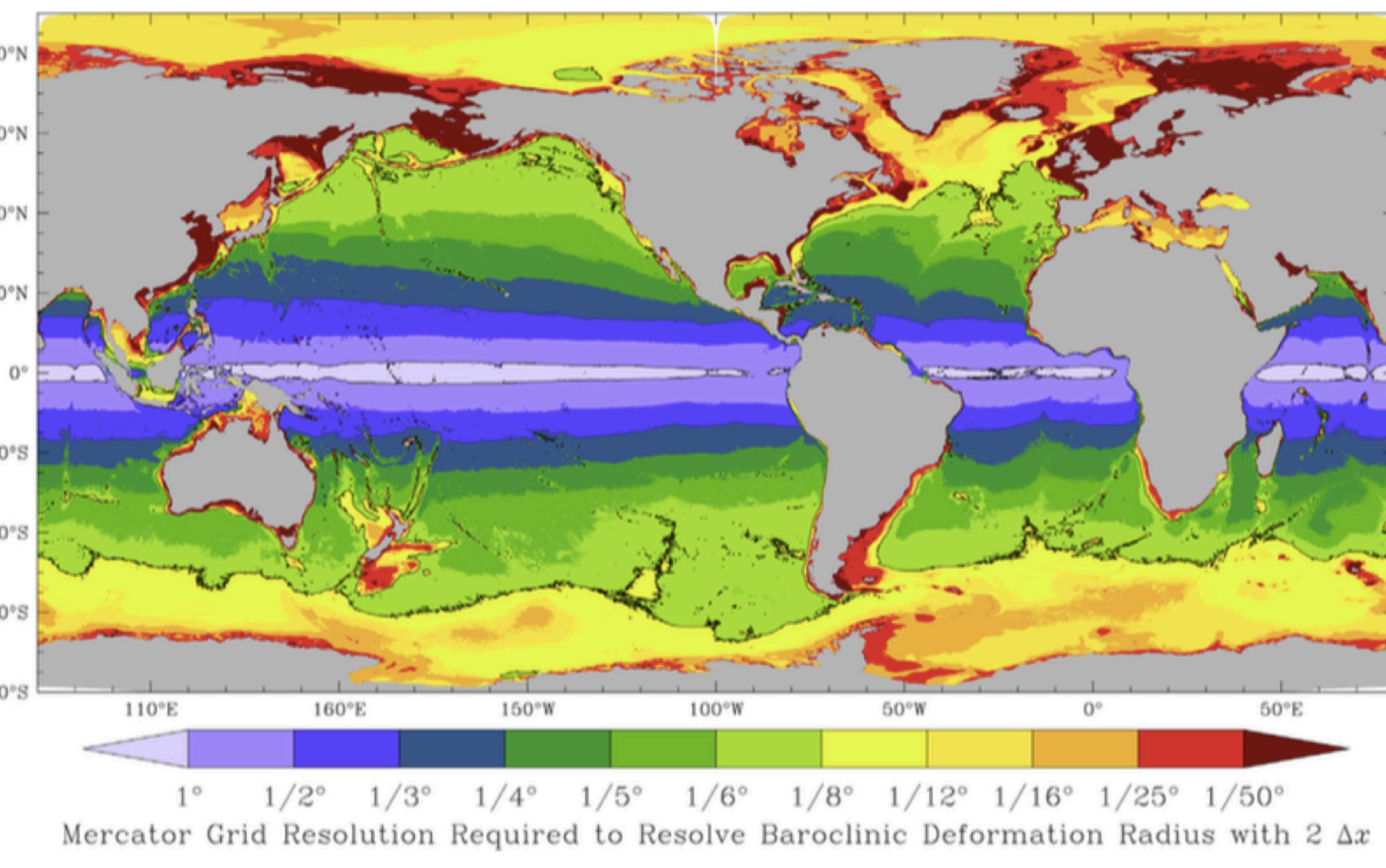


FIGURE 1 FROM HALLBERG (2013)

## A MOLES SUBGRID MODEL

LES subgrid models assume the largest eddies are resolved, while the effects of smallest eddies must be modeled. The grid scale is assumed to lie in the inertial subrange, well away from the forcing and dissipation scales.

For MOLES resolving the QG inertial range, the horizontal dissipation operator is

$$\nu_{QG}^* = \left( \frac{\Delta x}{\pi} \right)^3 |\nabla q_{QG}| \quad q_{2d} = f + \hat{k} \cdot \nabla \times \bar{u}$$

$$q_{QG} = q_{2d} + \left\{ \frac{\partial}{\partial z} \frac{f^2}{N^2} \bar{b} \right\}$$

A variant can also be introduced for MOLES where the dynamics are quasi-2d, wherein the term in curly brackets is set to zero.

Tracer transport is parameterized using the GM along-isopycnal transport scheme. In order to dissipate QG potential vorticity at the correct rate, this requires that the GM transport coefficient be equal to the MOLES viscosity:

$$\frac{Dq_{QG}}{Dt} \approx \nabla \cdot [\nu_{QG}^* \nabla q_{2d} + \kappa_{GM} \nabla (q_{QG} - q_{2d})]$$

$$\rightarrow \kappa_{GM} = \nu_{QG}^*$$

Designing a physically consistent dissipation scheme has the added benefit of specifying a scale-sensitive tracer mixing coefficient as well.

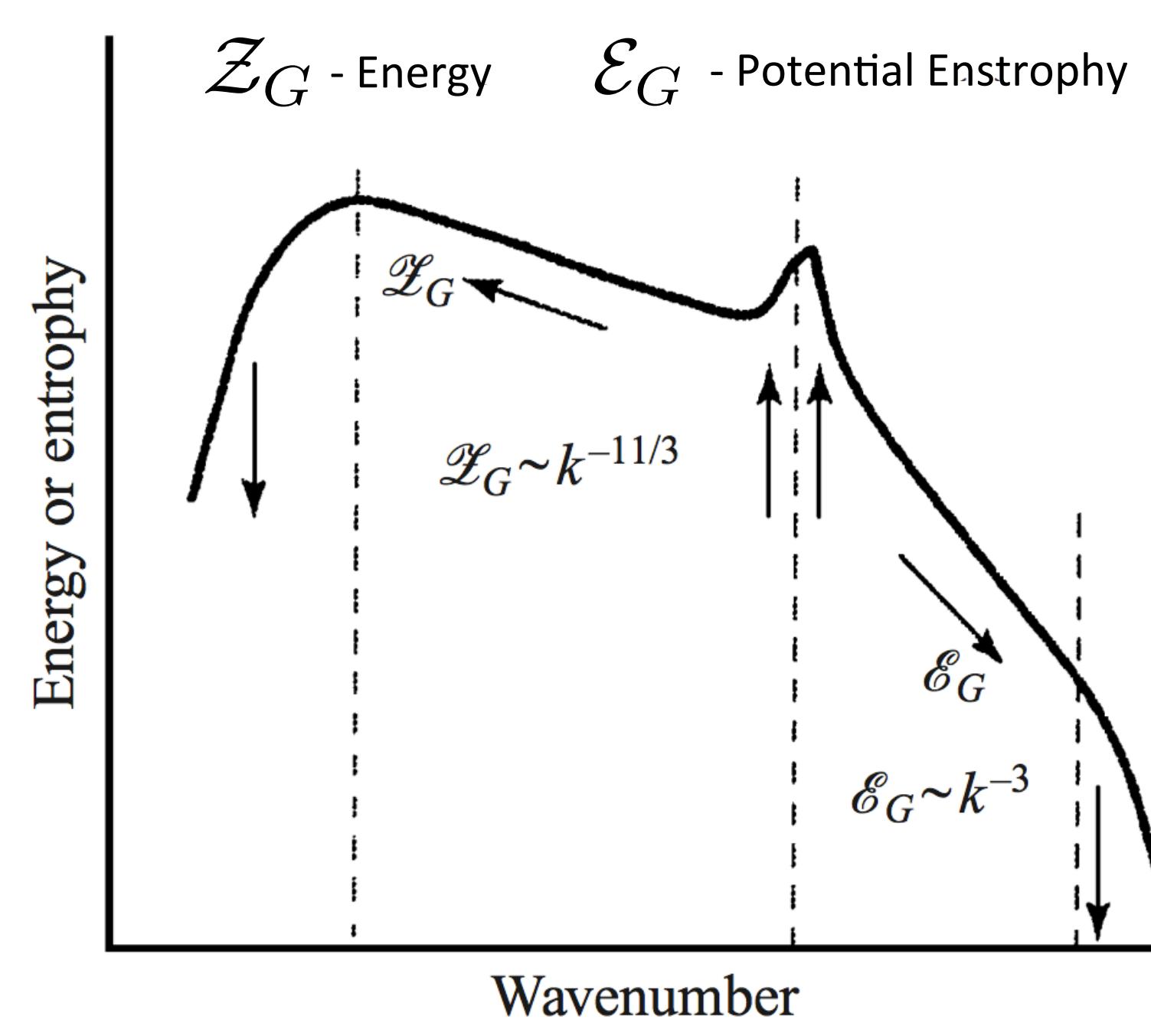
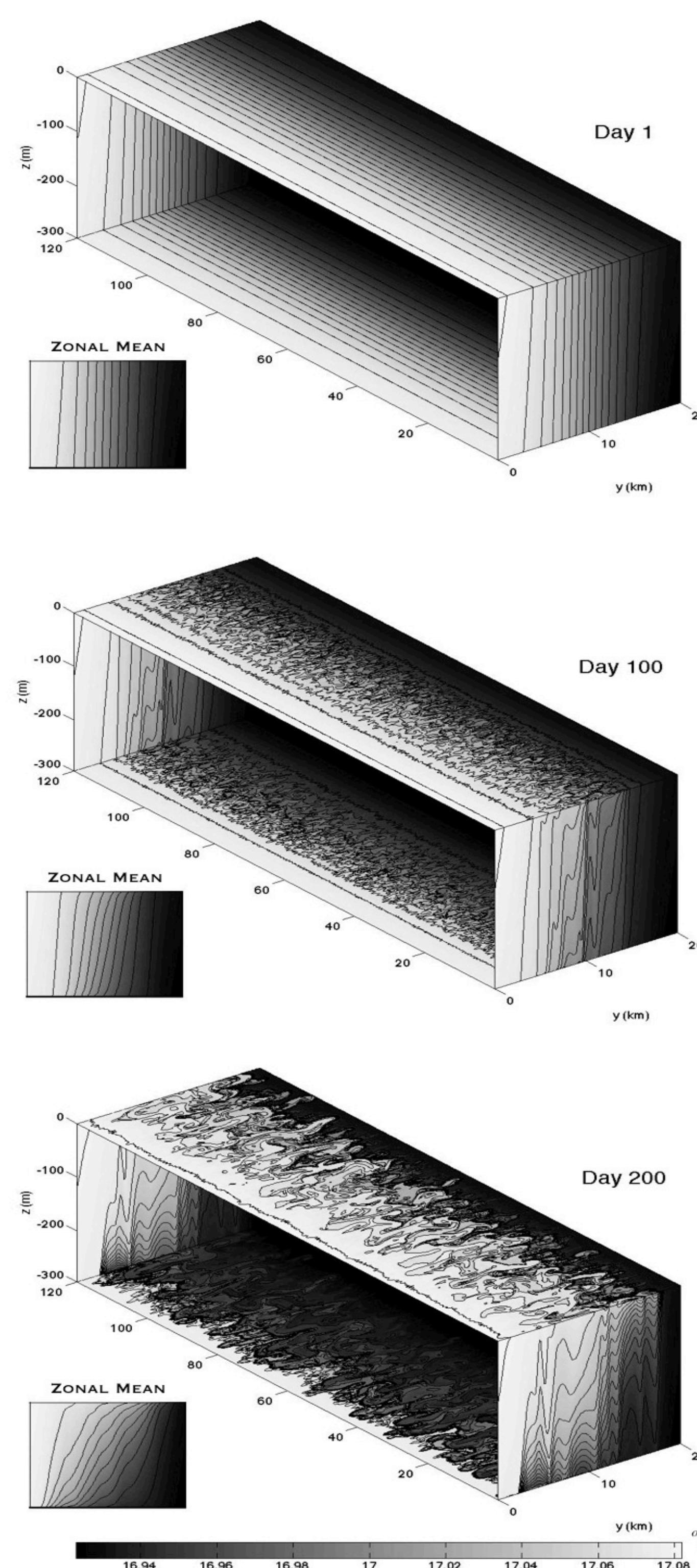
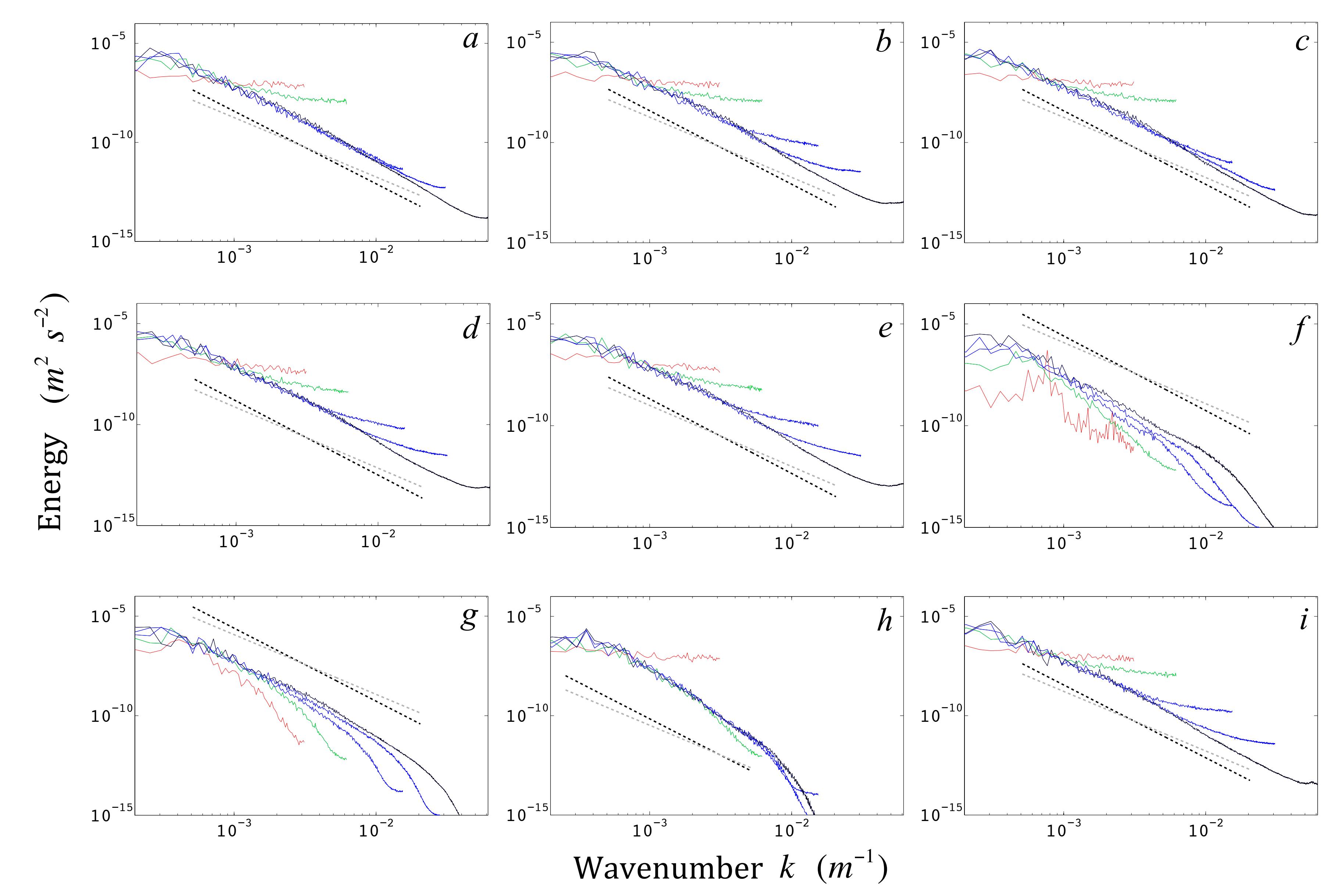


FIGURE 1C ADAPTED FROM SMITH ET AL. (2002)

## IDEALIZED TEST SIMULATIONS



A series of idealized frontal spindown models are used to test the MOLES closures. The domain is zonally periodic, and is sufficiently long to permit accurate energy and enstrophy spectra to be diagnosed. The initial state features stratification as in the Eady problem.



Vertically-averaged energy spectra diagnosed from the Eady channel models. Each model is run at five different resolutions:

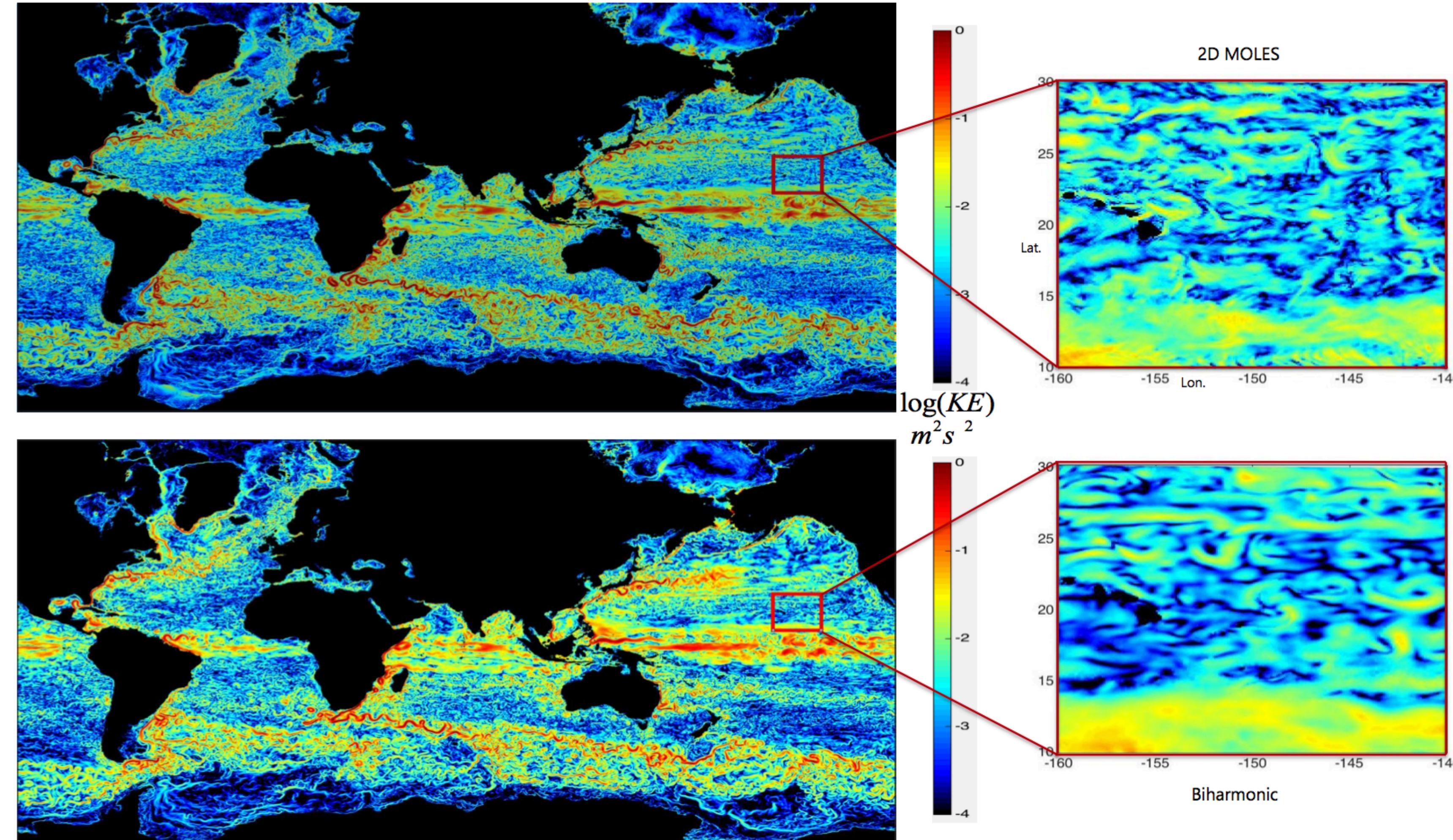
$$\Delta x = 2 L_d \quad \Delta x = L_d \quad \Delta x = \frac{2}{5} L_d \quad \Delta x = \frac{1}{5} L_d \quad \Delta x = \frac{1}{10} L_d,$$

where  $L_d = NH/f$  is the external deformation radius. The spectral slopes are then compared against the black lines, whose slopes (dashed: -3, dotted: -11/3) are predicted by theory (Smith et al., 2002). Several different viscous closures are tried (details about the dynamical and biharmonic variants can be found in Fox-Kemper and Menemenlis (2008)):

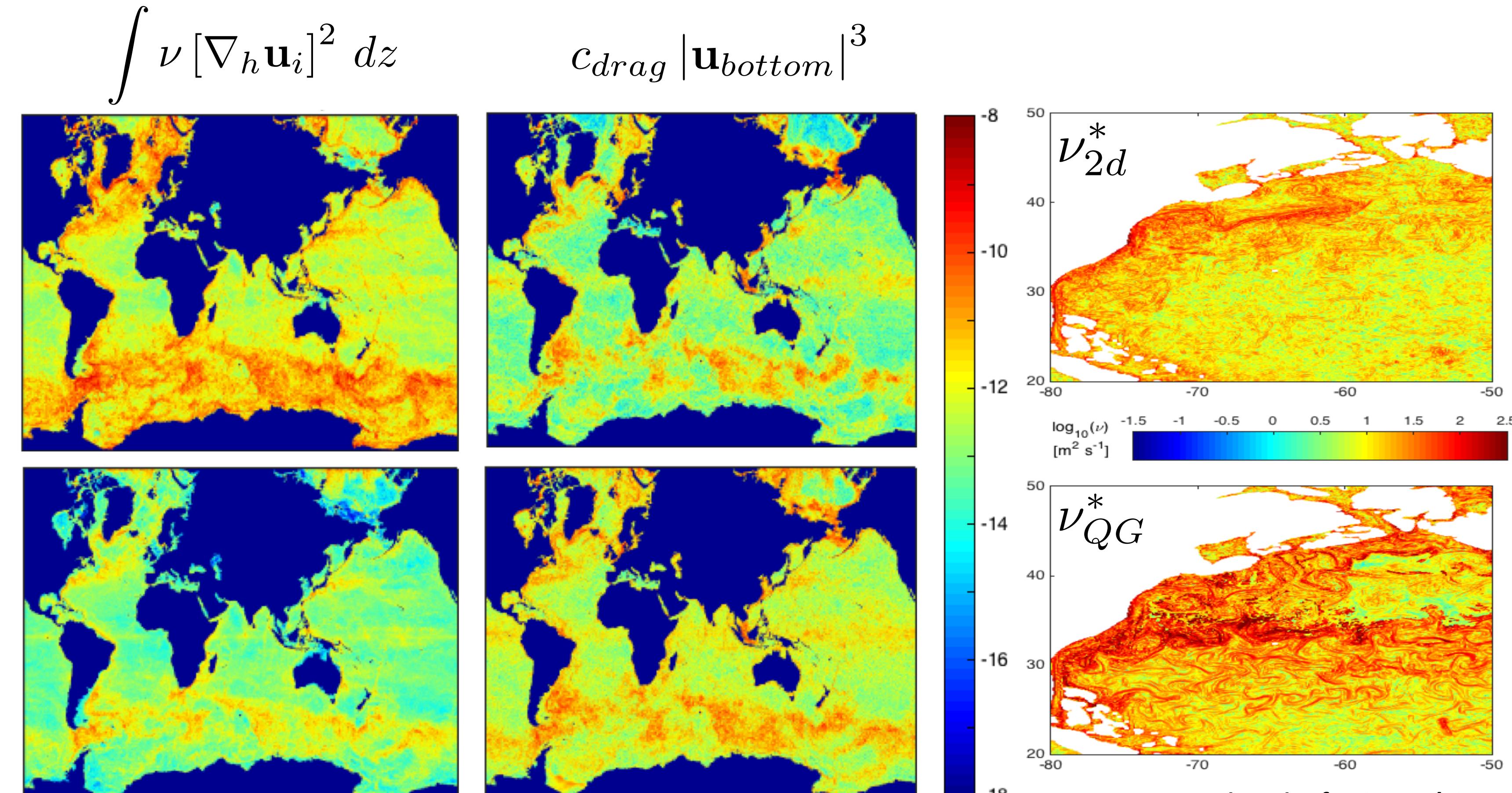
- a) QG MOLES
- b) QG MOLES, dynamical coefficient (test filter width:  $2\Delta x$ )
- c) QG MOLES, dynamical coefficient (test filter width:  $8\Delta x$ )
- d) 2d MOLES, harmonic
- e) 2d MOLES, biharmonic
- f) Smagorinsky, harmonic
- g) Smagorinsky, biharmonic
- h) Tuned harmonic
- i) Tuned biharmonic

The QG MOLES scheme produces the least amount of spectral roll-off as the grid scale is approached. Dynamical variants (panels b and c) do not show improvement over simply using a constant coefficient (panel a). Similar plots can be produced for the enstrophy.

## POP (NCAR) TEST SIMULATIONS



Global (left) and regional (right) maps of surface kinetic energy from POP using 2D MOLES (top) and biharmonic (bottom) subgrid schemes. The regional maps show the area enclosed by red lines in the global map (10°N–30°N and 160°W–140°W). The globally integrated kinetic energy in the 2D MOLES simulation is 20% higher than in the biharmonic simulation. The difference in energies is concentrated at the small scales.



Energy dissipation through horizontal viscosity (left panels) and bottom drag (right panels), for standard biharmonic viscosity (top panels) and the 2D MOLES scheme (bottom panels). Viscous dissipation by 2D MOLES is 10x less than that of biharmonic.

## SUMMARY

Current eddy-permitting ocean models can be classified as MOLES, requiring a paradigm shift in the conceptualization and development of subgrid models. Physically-based, scale-sensitive, flow-adaptive eddy closures are not only desired, but necessary.

A subgrid closure has been developed for QG turbulence, which dissipates QG PV at the correct rate.

This closure also specifies a scale-sensitive mixing coefficient for the Gent-McWilliams parameterization.

The QG scheme better reproduces the energy (and enstrophy, not shown) spectra in an idealized channel model.

2D MOLES in a global ocean model allows for small scale features, which contain substantial kinetic energy, and smooth statistics to the grid scale. The choice of sub-grid scheme also affects larger-scale phenomena, such as mass transports.

The 2D MOLES closure is 10x less dissipative than biharmonic viscosity.

Gridscale Re and Pe are O(1) in MOLES, implying minimal spurious diapycnal mixing.

## REFERENCES

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