

# **Introduction to Process Coupling**

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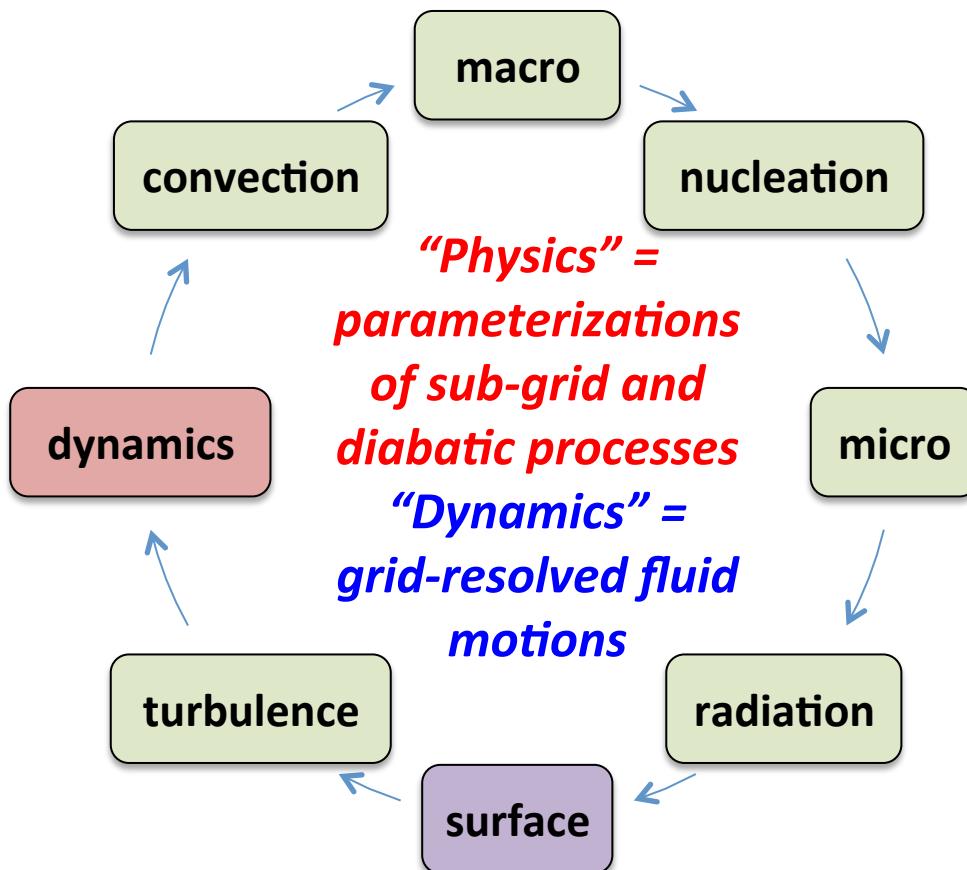


# Outline

1. the Landscape
2. Splitting Methods
3. Considerations
4. Survey: what gets used?
5. Real-World Impact
6. Summary + Discussion

# Opposing Viewpoints

## How a Parameterization Developer sees a GCM:



## How a Fluid Dynamicist sees a GCM:

$$\begin{aligned}
 R_U^{t^*} &= -m_x [\partial_x(Uu) + \partial_y(Vu)] - \partial_\eta(\Omega u) \\
 &\quad - (m_x/m_y)(\alpha/\alpha_d) [\mu_d(\partial_x\phi' + \alpha_d\partial_x p') + \dots] \\
 R_V^{t^*} &= -m_y [\partial_x(Uv) + \partial_y(Vv)] - (m_y/m_x)\partial_\eta(\Omega v) \\
 &\quad - (m_y/m_x)(\alpha/\alpha_d) [\mu_d(\partial_y\phi' + \alpha_d\partial_y p') + \dots] \\
 R_{\mu_d}^{t^*} &= -m_x m_y [\partial_x U + \partial_y V] - m_y \partial_\eta \Omega \\
 R_\Theta^{t^*} &= -m_x m_y [\partial_x(U\theta) + \partial_y(V\theta)] - m_y \partial_\eta(\Omega\theta) + F_\Theta \\
 R_W^{t^*} &= -(m_x m_y / m_y) [\partial_x(Uw) + \partial_y(Vw)] - \partial_\eta(\Omega w) \\
 &\quad + m_y^{-1} g (\alpha/\alpha_d) [\partial_\eta p' - \bar{\mu}_d (q_v + q_c + q_r)] - \dots \\
 R_\phi^{t^*} &= -\mu_d^{-1} [m_x m_y (U \partial_x \phi + V \partial_y \phi) + m_y \Omega \partial_\eta \phi - \dots]
 \end{aligned}$$

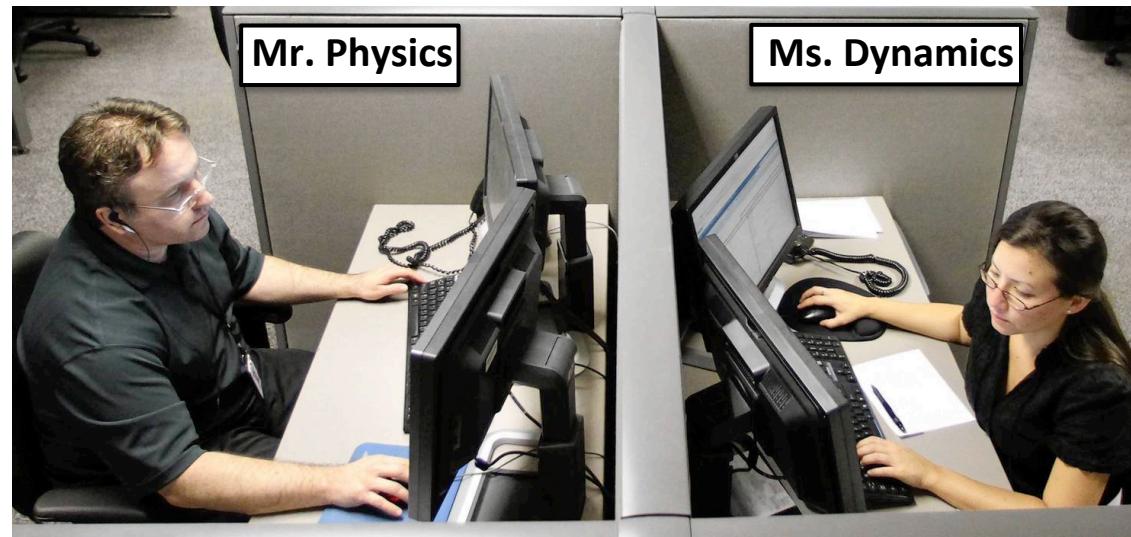
constant forcing term buried in equations  
(example from WRF ARW v3 tech doc)

- *Physics and dynamics researchers think differently*
- *Coupling often falls through the cracks*

# The Benefits of Splitting

Def: Operator Splitting = Breaking a coupled system of equations into a series of smaller sub-problems dependent on each other only through initial conditions

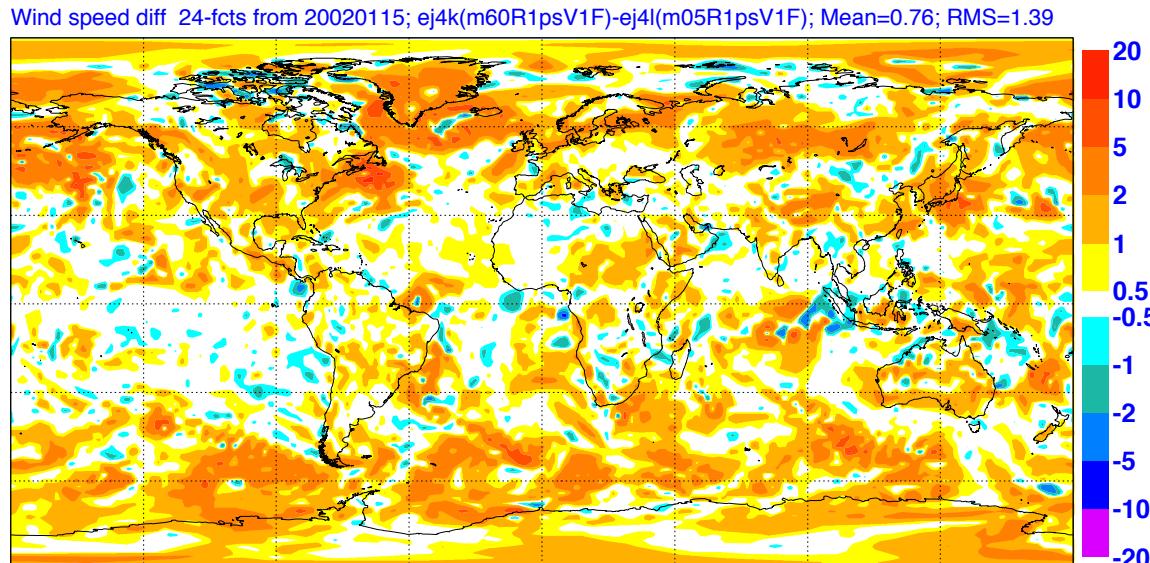
- Splitting breaks the climate system into independent pieces
  - modularity is critical for progress on large group projects
  - not having to solve the coupled equation is essential for tractability
- Splitting allows us to use the time step and integration method most appropriate for each process (Durran, 1999)
  - this is the basis of Implicit/Explicit (IMEX) and multirate approaches



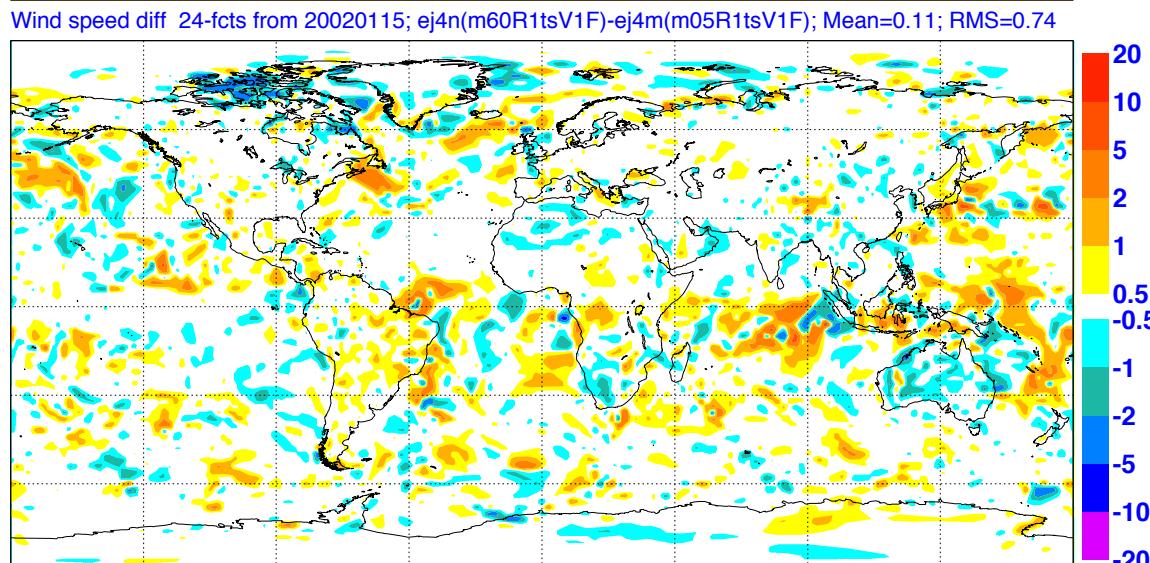
*Fig: Thanks to splitting, physics developers never have to talk to fluid dynamicists!*

# Why Worry about Splitting?

a)



b)

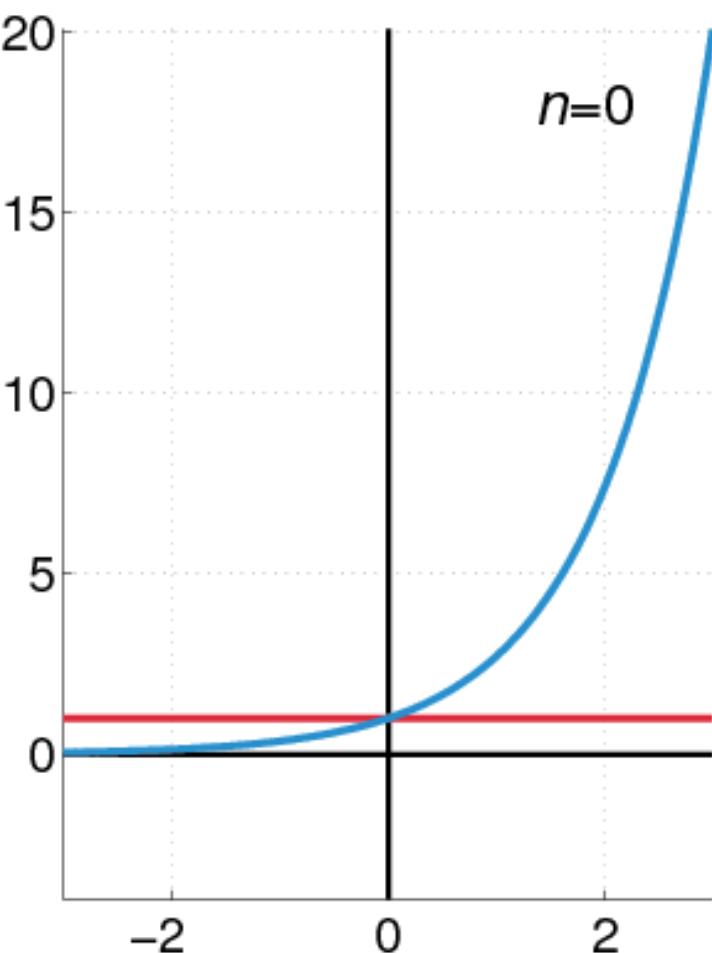


*Splitting improvements may be an easier way to improve model skill than further work on individual components*

Fig: 10m windspeed error for (a) parallel-split and (b) sequential-tendency split versions of the ECMWF model with  $\Delta t=60$  min (using  $\Delta t=5$  min as "truth". From Beljaars et al (2004)

# Def: Order of Accuracy

*Order of accuracy = the exponent of the highest-order term in the Taylor series expansion of the solution which is captured by the approximation*



Suppose we want to solve  $d\psi/dt = A\psi$  using the forward-Euler approx:

$$\psi^{n+1} = \psi^n + A \psi^n \Delta t.$$

The true solution for 1 step is:

$$\psi^{n+1} = \psi^n e^{A \Delta t}$$

which has Taylor expansion

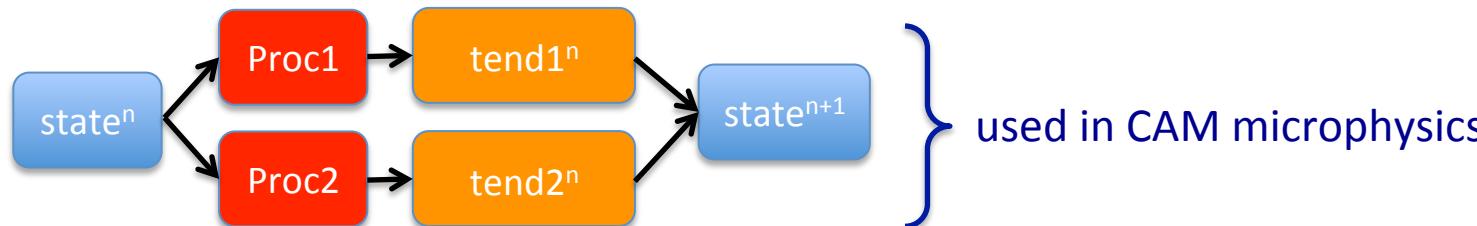
$$\psi^{n+1} = \psi^n + A \Delta t \psi^n + O(\Delta t^2).$$

So the true solution minus our approximation agrees to  $O(\Delta t)$

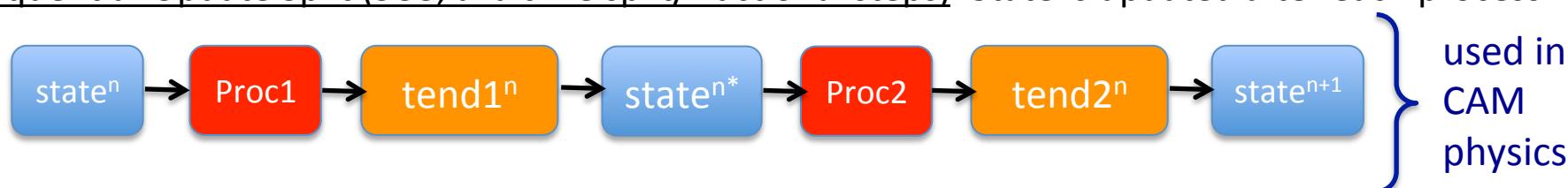
*Fig: Taylor Series of  $\psi(t) = e^t$  centered at zero.*

# Simple Coupling Strategies

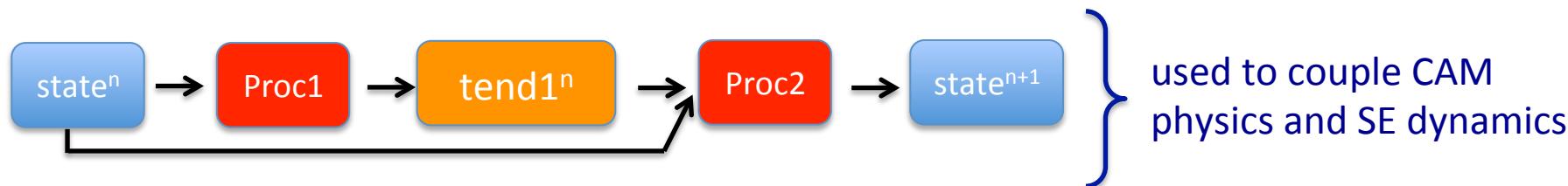
A. Parallel Split (PS, aka process/additive split): All processes are computed from the same state



B. Sequential-Update Split (SUS, aka time split/fractional steps): State is updated after each process



C. Sequential-Tendency Split (STS, aka no step splitting): The tendency from Proc1 is used by Proc2

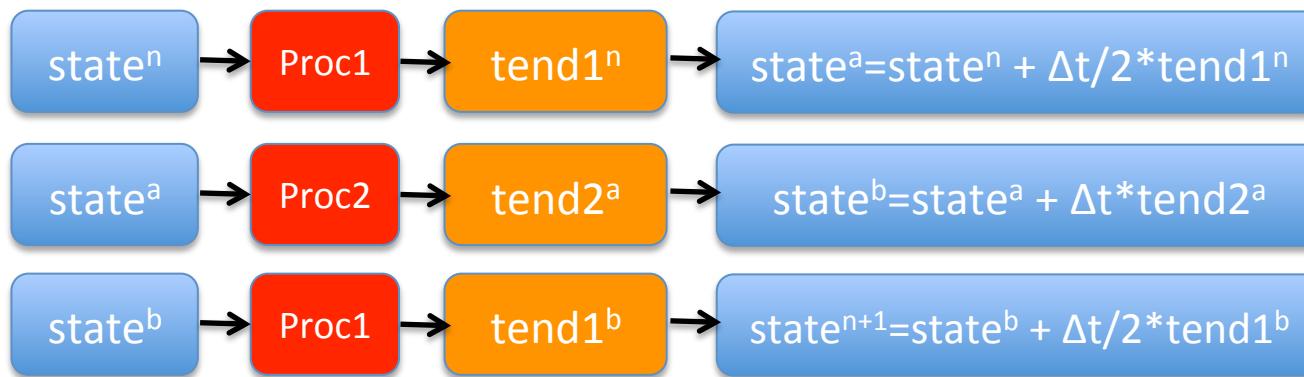


**Except in the unlikely event that processes commute, these approaches are only 1<sup>st</sup> order accurate (Durran, 1999)...**

*What is the point of high-order dynamics if coupling introduces a 1<sup>st</sup> order error?*

# Strang Splitting (SS)

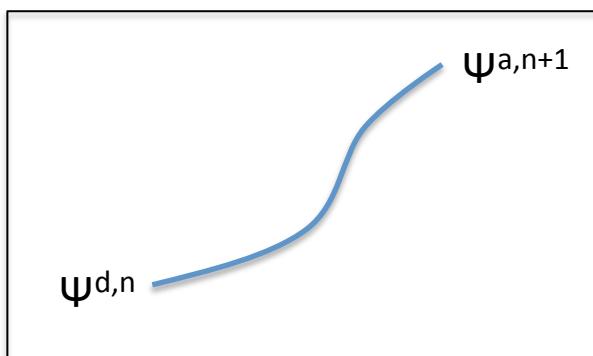
In many cases, using SUS with Proc1 applied for half a timestep before and after the Proc2 update provides a 2<sup>nd</sup> order splitting (Staniforth et al, 2002a, Durran, 1999)



- Expensive since each physics call takes ~50% of run time/step
- Provides incorrect free solution for Caya et al (1998) test case unless trapezoidal timestepping is used (Dubal et al, 2004)
  - though problems diminish with strong damping (Staniforth et al, 2002a)
- Not guaranteed to be 2<sup>nd</sup> order when implicit methods are used (Durran, 1999)
- aka partial splitting, symmetric splitting

# A Note for Semi-Lagrangian Models

- SS is 2<sup>nd</sup> order because it effectively centers derivatives in time. If your control volume is moving, you must also center along Lagrangian trajectory.
  - Failure to initially compute Proc1 at departure point induces an  $O(\Delta t)$  error



# Coupling Tightness

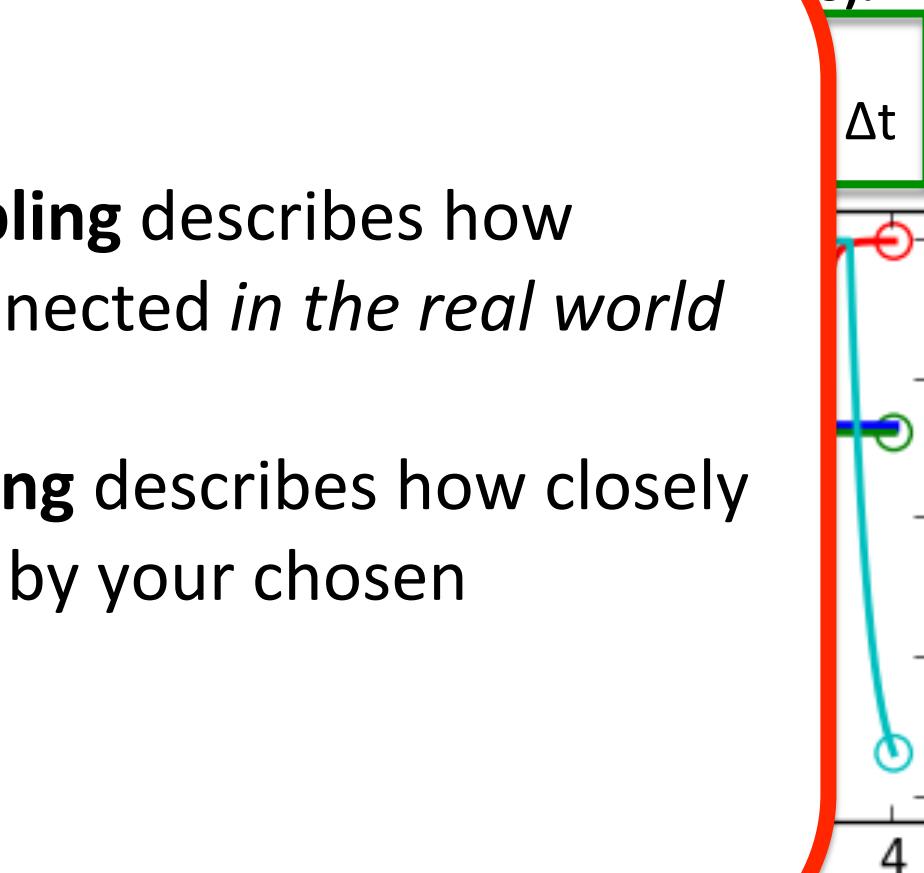
Some methods connect processes better than others. Consider a simple example (TS):

## Disambiguation:

**Strong versus weak coupling** describes how closely processes are connected *in the real world*

**Tight versus loose coupling** describes how closely processes are connected by your chosen *numerical method*

$$\Psi_{n+1} = \Psi_B + 0.5(-5 - \Psi_B/2) \Delta t$$



*Fig. Exact,  $\textcolor{blue}{\circ}$ ,  $\textcolor{red}{\circ}$ , and  $\textcolor{green}{\circ}$  solutions. Circles show solutions after each timestep. Lines show values at each of 100 substes per step.*

# Coupling Tightness

*Some methods connect processes better than others. Consider a simple example:*

$$\frac{d\psi}{dt} = (\frac{d\psi}{dt})_A + (\frac{d\psi}{dt})_B$$

$$(\frac{d\psi}{dt})_A = -5 - \psi/2$$

$$(\frac{d\psi}{dt})_B = 10 - \psi$$

**Sequential-Update Split (SUS):**

$$\psi_A = \psi_n + (-5 - \psi_n/2) \Delta t$$

$$\psi_{n+1} = \psi_A + (10 - \psi_A) \Delta t$$

**Strang Split (SS)**

$$\psi_A = \psi_n + 0.5(-5 - \psi_n/2) \Delta t$$

$$\psi_B = \psi_A + (10 - \psi_A) \Delta t$$

$$\psi_{n+1} = \psi_B + 0.5(-5 - \psi_B/2) \Delta t$$

**Sequential-Tendency Split (STS):**

$$\frac{d\psi}{dt}_A = -5 - \psi_n/2$$

$$\psi_{n+1} = \psi_n + (\frac{d\psi}{dt}_A + 10 - \psi_n) \Delta t$$

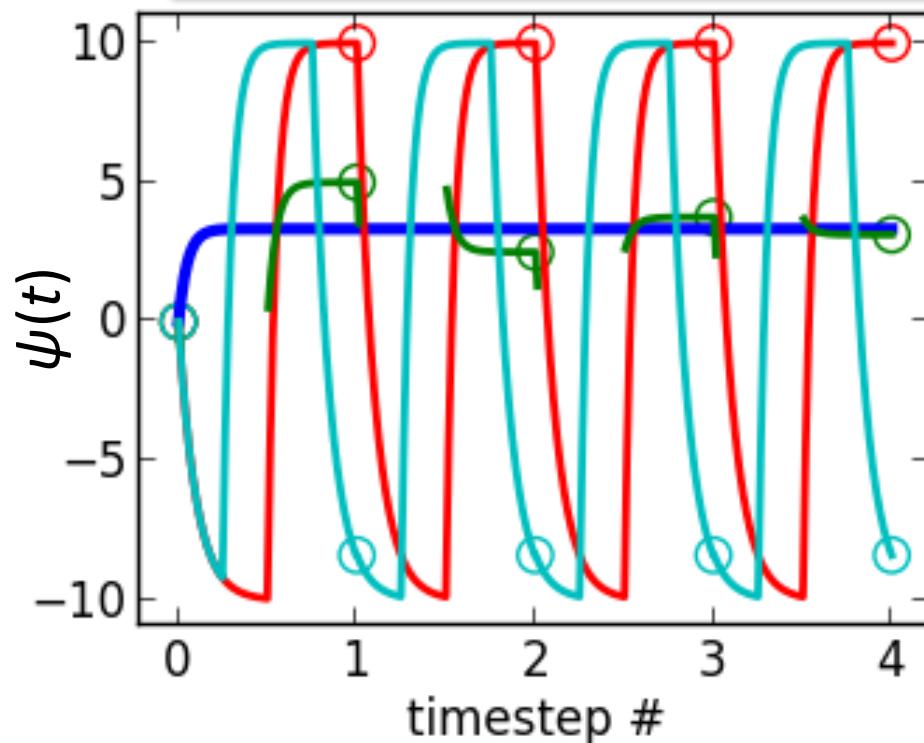


Fig: *Exact*, *SUS*, *STS*, and *SS* solutions. Circles show solutions after each timestep. Lines show values at each of 100 substeps per step.

# Coupling Tightness

- SUS causes model to pinball between unrealistic states
  - Even if individual equations are solved exactly, results will be poor if processes are allowed to operate independently for too long
- STS allows processes to be computed separately yet coupled every substep
  - requires process 1 to be slow compared to process 2
- Explicit PS has no splitting error, but requires small timesteps for stability (so not shown here)
- SS behaves poorly because the timescale for each process's response is shorter than  $\frac{1}{2}\Delta t$

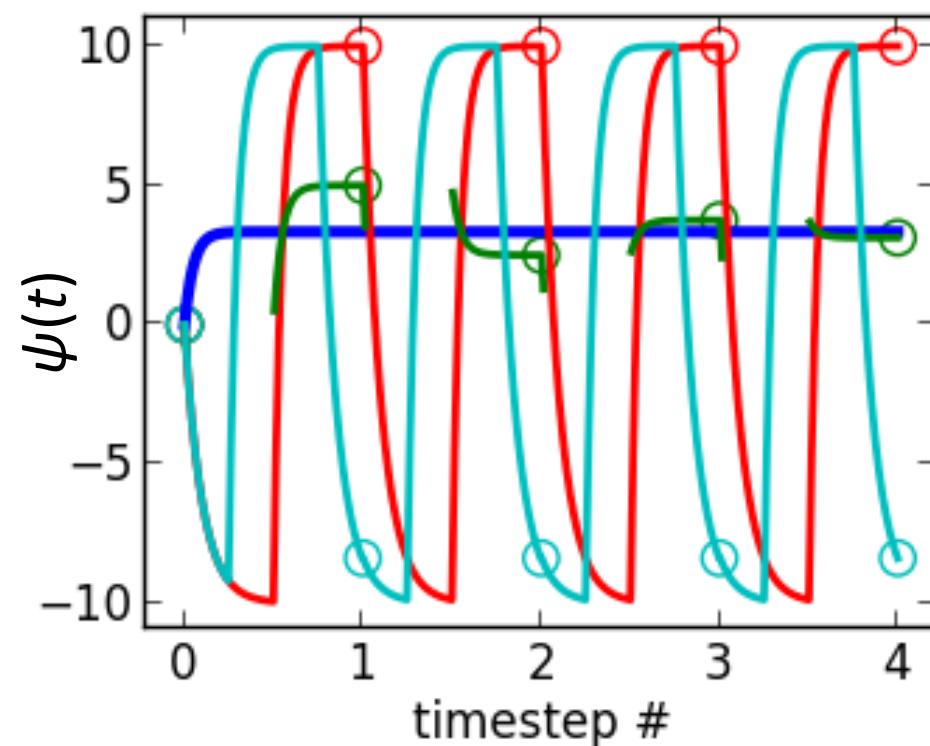
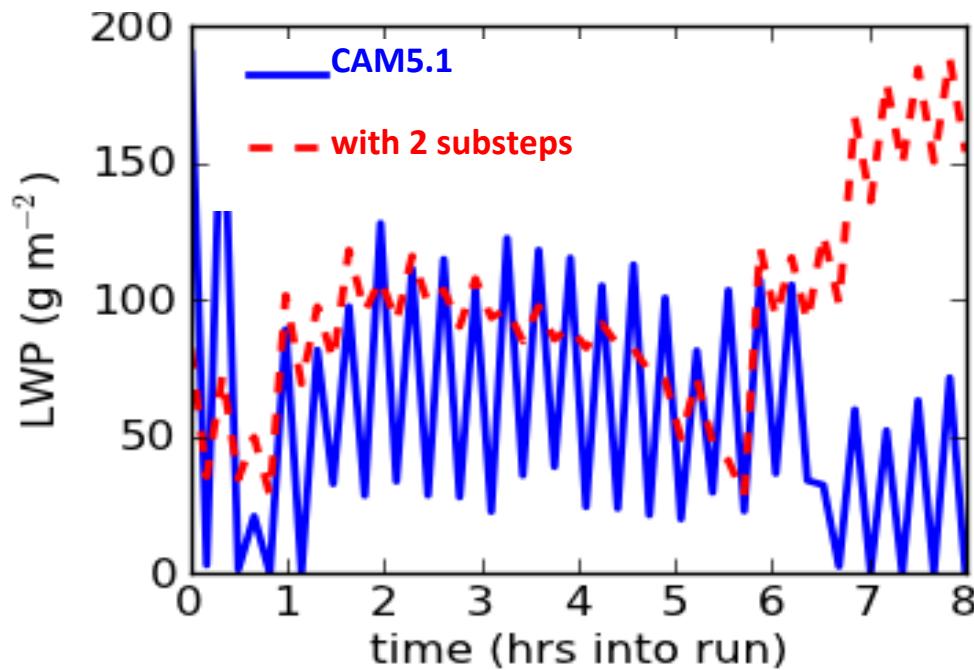


Fig: Exact, SUS, STS, and SS solutions. Circles show solutions after each timestep. Lines show values at each of 100 substeps per step.

# Coupling Tightness: Real-World Example

- SUS causes pinballing between macro- and microphysics in CAM:
  - Macrophysics (= large-scale condensation/evaporation) is the main source of condensate in CAM
  - microphysics (rain/freezing/etc) is the main sink
- Using substepping (red line) reduces the problem
  - CAM6 uses substepping to alleviate problems (Gettelman and Morrison, 2015)

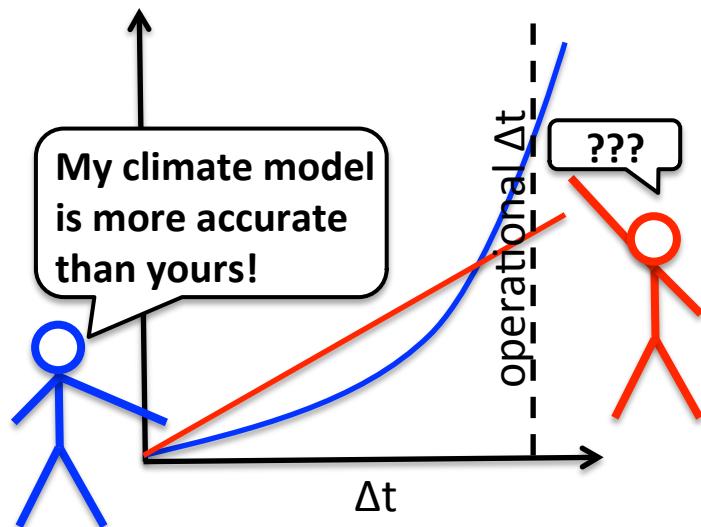


*Fig: Liquid water path before & after micro-physics from a single-column MPACE-B run.*

*Is simply using a shorter coupling timestep the best solution?*

# Accuracy

*On the previous slide, SS wasn't best even though it is formally the most accurate. What does this mean?*



- Accuracy is defined in the limit of small  $\Delta t$ . Physics often runs at very large timesteps (see Fig)
- Physics code is very “iffy” – how do you even define convergence when behavior isn’t smooth?

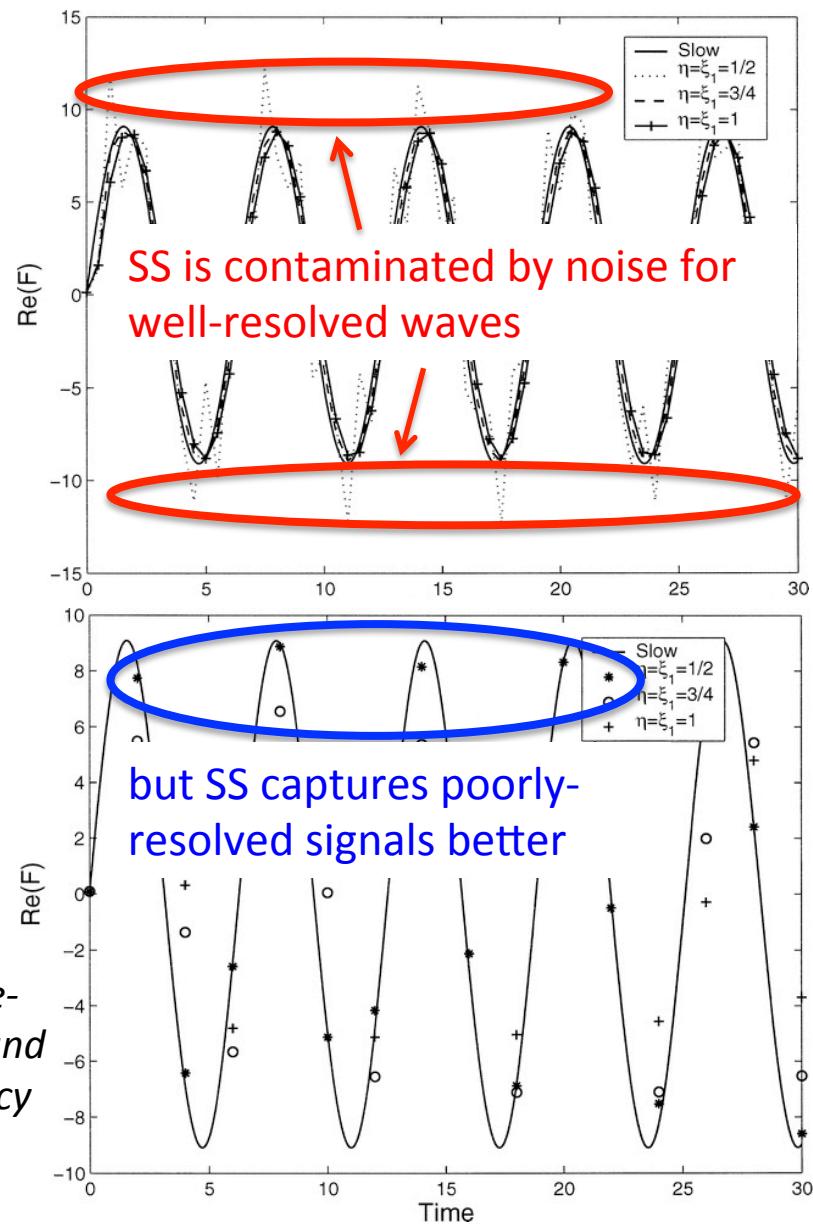
*Is order of accuracy practical when you’re nowhere near the convergence regime? Is this regime a stupid place to be?*

# Accuracy

- Higher-order schemes are less diffusive, which can make them perform worse in the presence of small-scale noise
- But they are better at capturing poorly-resolved features

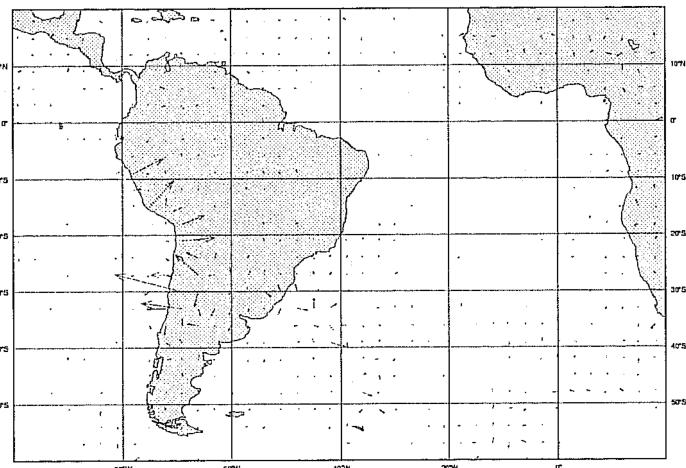
Note: I do think that higher-order schemes are better – I'm just saying there's more to the picture

Fig: Solution of Kreiss (1992) test case  $\varepsilon d\psi/dt = e^{it} - \sigma\psi$  for time-centered SS ( $\eta=1/2$ ), off-centered symmetric splitting ( $\eta=3/4$ ), and pure-implicit SUS ( $\eta=1$ ) using a timestep 16x the signal frequency (top panel) and 4x the signal frequency (bottom panel). From Dubal et al (2004) Fig. 4 and 12.



# Accuracy: It's Complicated...

- When ECMWF tried to implement semi-Lagrangian SS, huge timestep sensitivities showed up in low-level winds near orography (see Fig).
  - To avoid this, their Semi-Lagrangian Averaging over Physical Parameterizations (SLAVE-PP) uses only the arrival-point for vertical diffusion and gravity wave parameterizations and is only 1<sup>st</sup> order accurate as a result
- Important process interdependencies break “one size fits all” coupling. In the ECMWF model:



- Vertical diffusion requires coriolis and pressure gradient terms to produce the Ekman wind profile
- Vertical diffusion requires cloud-top radiative cooling to handle stratocumulus

*Fig: Difference between lowest-level winds from 6-day ECMWF runs using  $\Delta t=300$  s vs  $\Delta t=600$  s with physics averaged across semi-Lagrangian trajectories. From Wedi (1999) Fig. 3.2.1.*

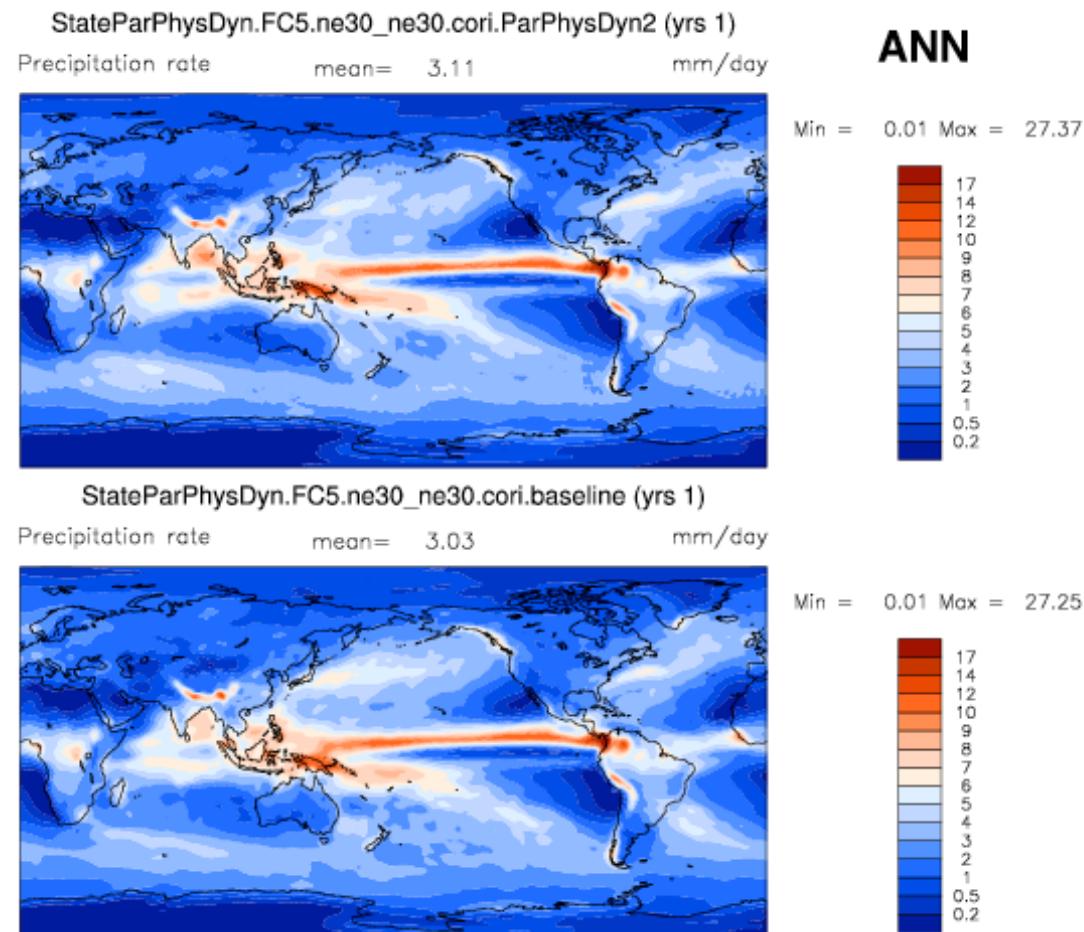


# Efficiency: Stability

- PS generally produces biased solutions to toy problems unless it is explicit, which severely limits the timestep
- For SUS, if each step is stable the coupling will be stable (Durran, 1999)
- For efficiency with SUS, it makes sense to do slow processes explicitly 1<sup>st</sup>, faster processes implicitly last
- Stability is not guaranteed for STS (Durran, 1999)

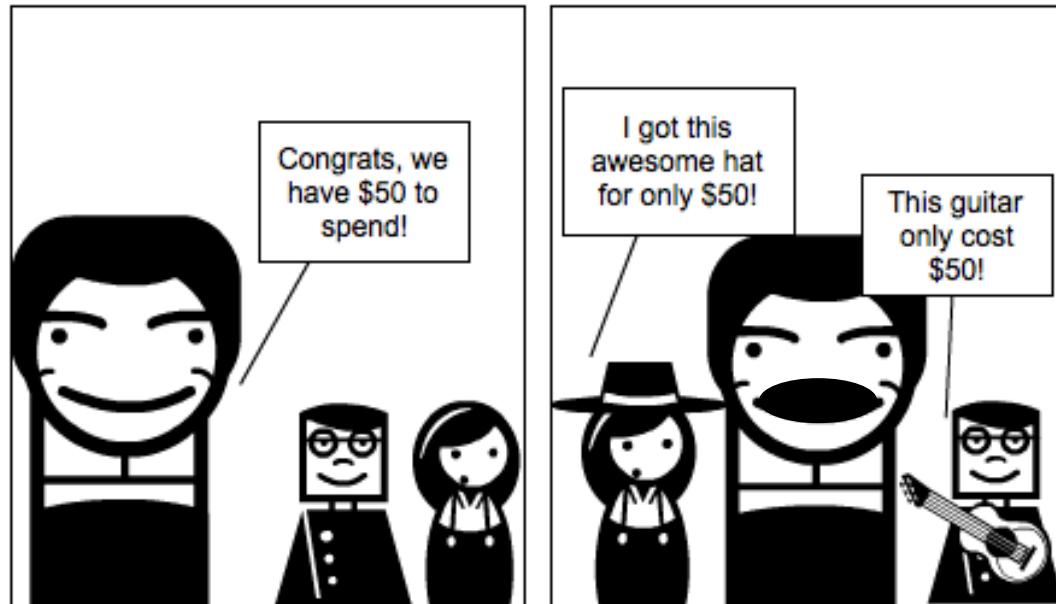
# Efficiency: Parallel Splitting can be Done... in Parallel!

- Running physics and dynamics at the same time could speed the model up by 2x!
  - neglecting increased communication costs
- Physics is embarrassingly parallel
  - physics complexity can be increased without increased wall time by using more cores!
- Parallel splitting in CAM does not seem to degrade solutions (see Fig)
  - echoes results of Williamson et al (2002)



*Fig: precipitation from 1 yr 1° 20 C CAM5-SE runs with STS (top) and PS (bottom).*

# Overconsumption



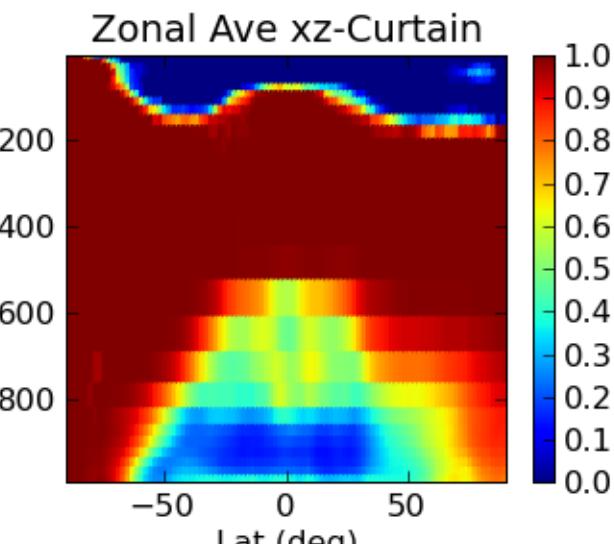
“Doing things in parallel creates opportunities for overconsumption”

- Impact of overconsumption by PS:
  - Negative water, energy, or aerosol species. Fixing this often violates conservation
  - Overstabilized atmosphere due to competition between resolved and parameterized convection
- STS + substepping can also overconsume:
  - process 1 + 2 deplete quantity in initial substeps
  - once quantity is depleted, constant forcing from process 1 drives quantity negative in final substeps
  - This happens in CAM-SE
- **SUS avoids overconsumption issues by having 1 process operate at a time!**

# Overconsumption: Does it Really Happen?

- Yes. Using PS phys/dyn greatly increases the frequency of replacing negative quantities with zero (“QNEG3 errors in CAM; see background)
- Processes in microphysics are parallel-split. Overconsumption is a big problem (see Fig)

*Is there a good method to deal with overconsumption? Just use a smaller timestep?*



*Fig: Frac of time micro starts with significant cloud water and depletes it within a timestep (from 5 yr 20<sup>th</sup> C CAM5 run at  $\Delta x=2^{\circ}$ )*

# Sensitivity to Arbitrary Choices

- For explicit SUS (like CAM physics), it is unclear which process should come first
- process ordering makes a big difference (see Fig)
  - In CAM5, switching order makes a 20% difference in important fields!
  - The default ordering is not best!

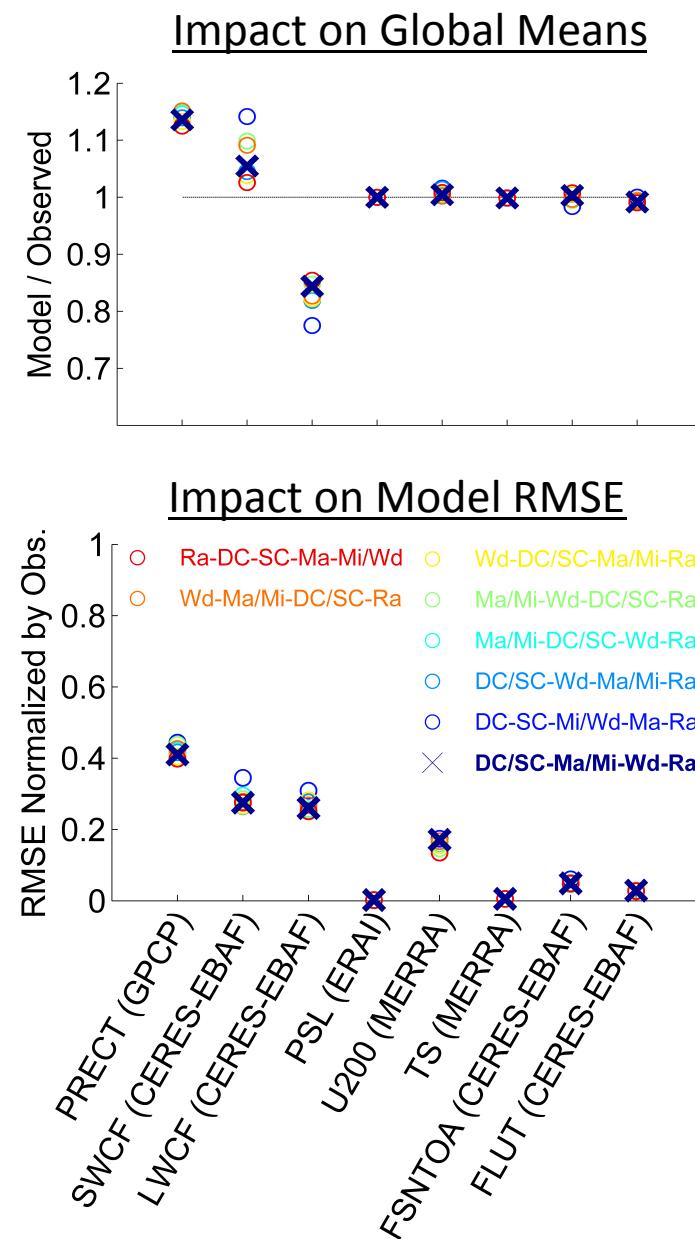
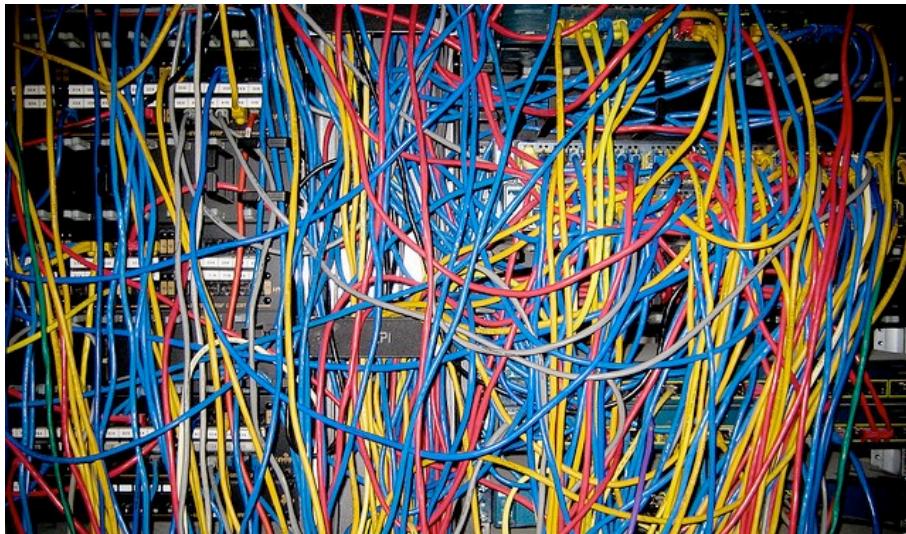


Fig: mean (top) and RMSE (bottom) for climatologically important variables from 1-yr CAM5 simulations with altered process ordering. Courtesy Aaron Donahue.

# Ease of Implementation



*Fig: Coupling code in an atmospheric model is complicated. Changing it opens a box of worms!*

- SUS and PS are easy to implement
- STS can be very invasive
  - Passing phys tendencies to dyn could be easy
    - is assuming phys is slow compared to dynamics reasonable?
  - Passing tendencies to physics schemes is often really hard!

# Designer Splittings

Consider the Kreiss (1992) eq:  $\epsilon dF/dt = e^{it} - \sigma F$ .

Implicit and symmetric SUS yields:

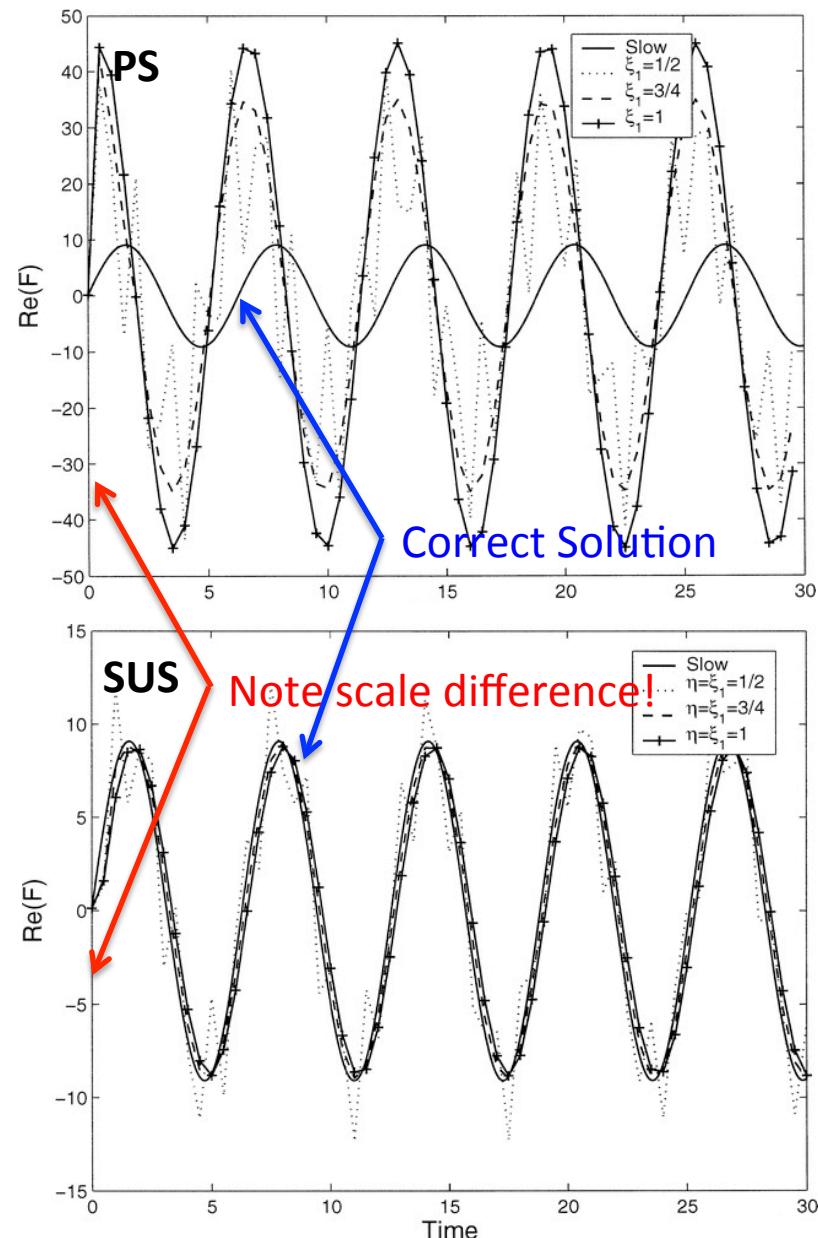
$$\frac{F^* - F(t)}{\Delta t} = \frac{\eta}{\epsilon} [\xi_2 e^{i(t+\Delta t)} + (1 - \xi_2) e^{it}], \quad (5.1)$$

$$\frac{F^{**} - F^*}{\Delta t} = -\frac{\sigma}{\epsilon} [\xi_1 F^{**} + (1 - \xi_1) F^*], \quad (5.2)$$

$$\frac{F(t + \Delta t) - F^{**}}{\Delta t} = \frac{(1 - \eta)}{\epsilon} [\xi_3 e^{i(t+\Delta t)} + (1 - \xi_3) e^{it}].$$

- by choosing  $\xi$  and  $\eta$ , you can *design* implicit-SUS to minimize errors
- Parallel splitting lacks these knobs, so has bigger error
- Are designer tunings practical for complex codes?

*Fig: Solution of Kreiss (1992) test case for trapezoidal ( $\xi_1=1/2$ ), mostly-implicit ( $\xi_1=3/4$ ), and purely implicit ( $\xi_1=1$ ) solutions with PS and SUS schemes. For SUS, optimal symmetrization ( $\eta$ ) factors were chosen. From Dubal et al (2004) Fig. 2 and 4.*



# Summary: Benefits and Drawbacks

Consideration:	Importance:	Assessment:
Coupling Strength	high	Explicit PS has no splitting error (but requires small steps=frequent coupling). If one process is slow, STS works well. SUS is bad
Overconsumption	high for climate	SUS and SS inherit conservation properties of individual steps. PS and (to a lesser degree) STS have conservation problems.
Efficiency	high	PS can be run in parallel. SS and implicit steps require additional calculations. SUS and SS inherit stability of individual steps, so can use a longer timestep. PS may be timestep limited
Ease of Implementation	high	PS and SUS are straightforward to implement. SS is slightly more difficult and STS is hard.
Accuracy	moderate?	SS has highest formal accuracy but benefit is unclear
Arbitrary Choices	moderate	PS avoids all choices about process ordering. There is some logic to STS and SS ordering. SUS is fairly arbitrary.
Tunability	dubious	SUS best; SS is an example of judicious SUS tuning. STS may be good. PS is bad

- There are benefits and drawbacks to all methods
- Switching from one method to another is hard

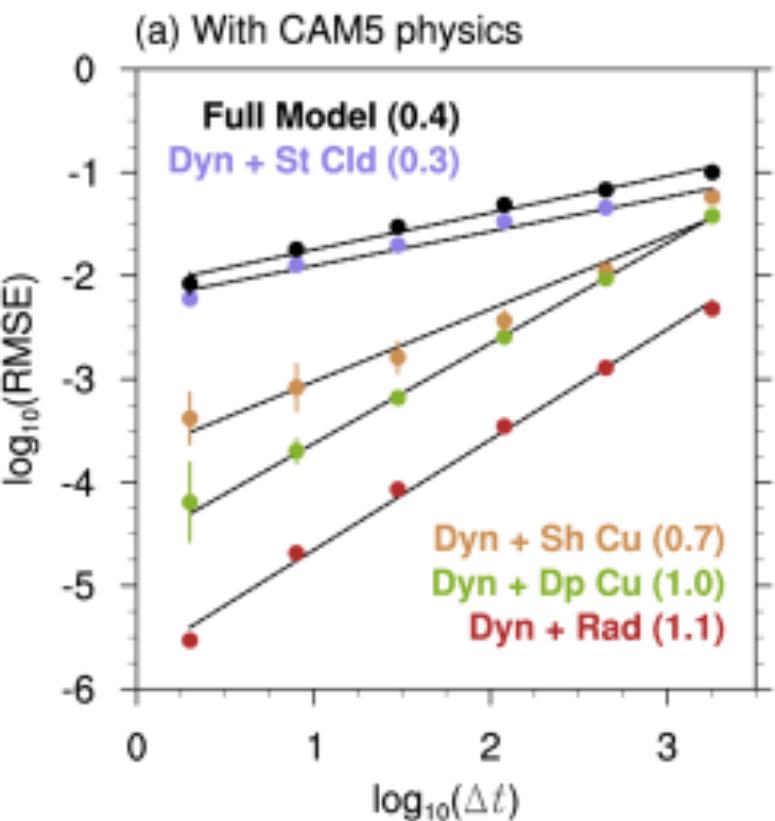
# What is Used In Practice?

Model:	Approach:
CAM5	phys/dyn = SUS for FV, STS for SE. Within phys = SUS
GFDL CM3	phys/dyn = SUS. Within phys = SUS except deep/shallow convection are PS
ECHAM5	phys/dyn = STS. Radiation and turbulence are PS, other phys are SUS
WRF	phys/dyn = STS (phys tendencies are dripped into acoustic substeps) except that microphysics is SUS from rest of terms. Other phys terms are PS.
ECMWF	phys/dyn = SLAVE-PP $\approx$ Semi-Lagrangian SS
UKMO	mixture of PS and SUS

SU= sequential update split, ST = sequential tendency split, P = parallel split

# Whole-Model Convergence

*Forget 2<sup>nd</sup> order convergence - cloud processes aren't even 1<sup>st</sup> order accurate!*



- Results are for CAM5 – UKMO and ECMWF models are probably better
- Issue is related to assumptions in macrophysics and microphysics schemes
- Punchline: physics developers don't think about convergence.

Fig: Temporal convergence of 3D temperature RMSE in hour-long CAM5 runs with only certain processes turned on. Convergence rates are given in parentheses after the description of each run. From Wan et al (2015) Fig. 3.

# Why is Physics so Crude?

- Cost
- Unlike dynamics, the correct equations for physics are often unknown or impractical
  - Capturing spectral dispersion due to big drops falling faster is prohibitively expensive
  - Most climate models use 2-moment schemes which are fundamentally unable to capture this aspect of rainfall

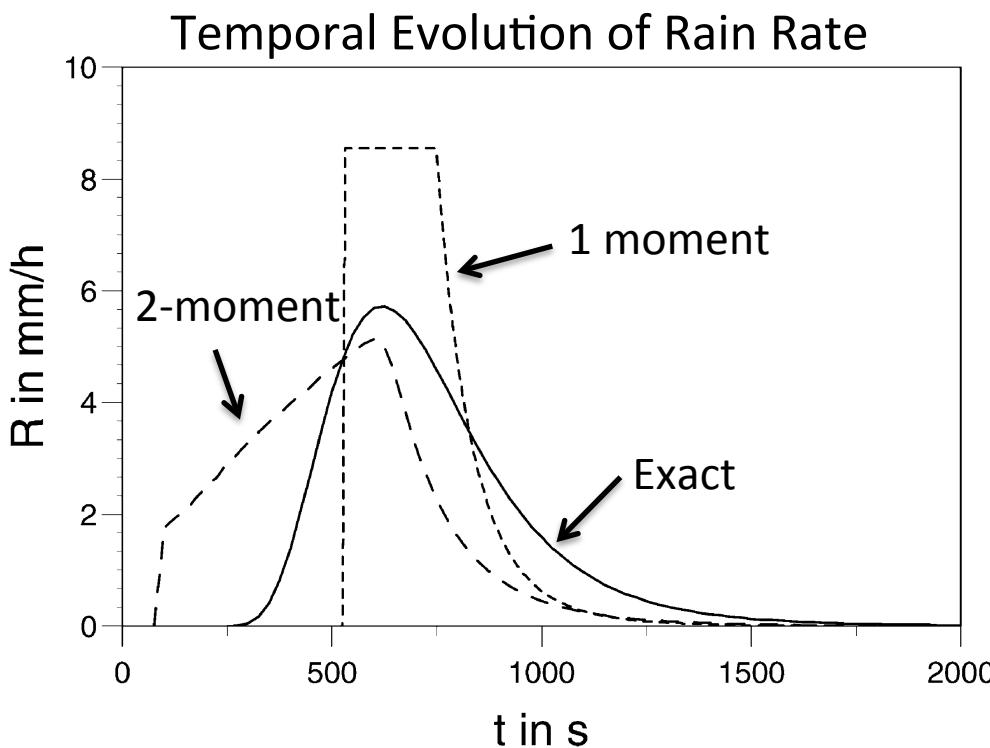


Fig: Analytic and/or accurate solutions to idealized rain sedimentation test problem for 1-moment (constant size distn) and 2-moment (liq. water content and drop # predicted). From Wacker & Seifert (2001; Atm Research)

*What's the point of accurately solving incorrect equations?*

# Simple Test Cases

- Caya et al (1998):  $\frac{d\Psi}{dt} + \beta\Psi = G$ 
  - $\beta$  represents a damping or oscillatory term (dynamics and/or vertical diffusion) and  $G$  represents a constant forcing like diabatic heating
  - Also used (with extension) by Staniforth et al (2002a,b) and Dubal et al. (2004, 2005, 2006)
- Kreiss et al (1992):  $\varepsilon \frac{d\Psi}{dt} = e^{it} - \sigma\Psi$ 
  - simple, stiff system
  - Also used by Dubal et al (2004)
- Advection/Diffusion:  $\frac{\partial\Psi}{\partial x} + c \frac{\partial\Psi}{\partial x} = d \frac{\partial^2\Psi}{\partial x^2}$ 
  - $c$  is a constant advection velocity and  $d$  is diffusive strength
  - Used by many

My new scheme is A-stable and 3<sup>rd</sup> order accurate!



Idealized test case

Then why is it making my turbulence scheme blow up?



Operational Model

How do we cross this valley of death?

# Lessons

- Most models use very simple coupling strategies
- Crude physics and coupling wreck high-order accuracy from dynamics!
- Solving physics accurately doesn't necessarily make sense because the underlying equations aren't clear
- Tests using simple equations are important but not sufficient
- Short timesteps solve everything (but are impractical)
- There are benefits and drawbacks to every coupling approach
  - SUS can cause pinballing between unrealistic states
  - PS and STS have conservation issues. SUS and SS don't
  - PS can be run in parallel

# Discussion Items:

- What is the benefit of high-order dynamics if coupling introduces a (sub)-1st order error?
- What's the point of accurately solving incorrect equations?
- Is order of accuracy the right measure when operational timesteps are far from converged?
- Is simply using a shorter timestep more cost effective than using a more sophisticated (and hence expensive) coupling scheme?
- Is lack of local conservation (e.g. by PS) ok?
- How do we move from improvement in idealized settings to improved skill in the full model?

That's all Folks!