DCMIP-2016

Lecture 2

The Components of a General Circulation Model

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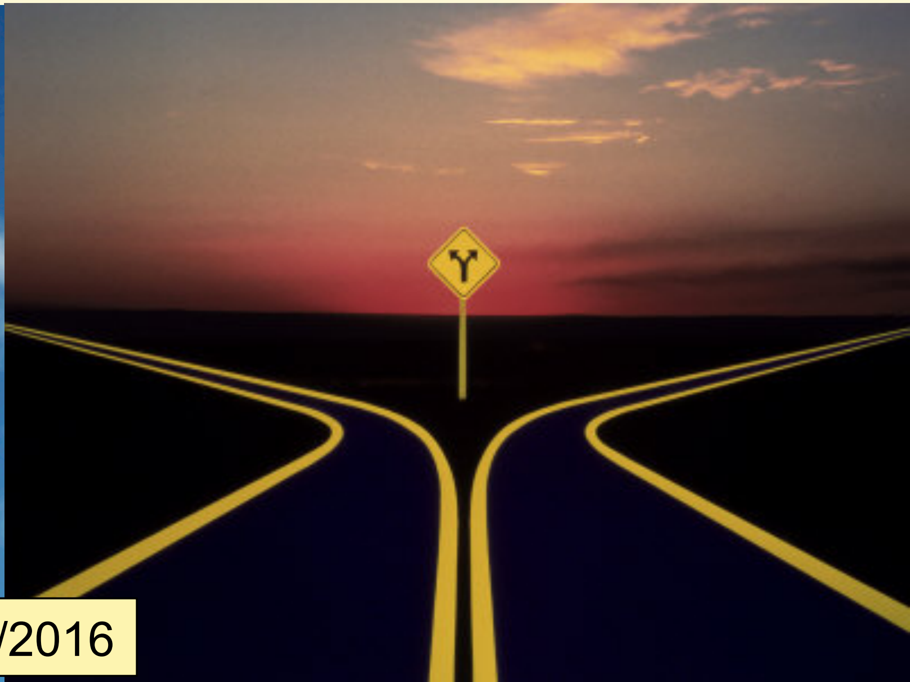


June/6/2016

Atmospheric Numerical Modeling

- Atmospheric numerical modeling is one of several approaches to studying the atmosphere.
- Others are
  - observational studies of the real atmosphere through field measurements and remote sensing
  - laboratory studies
  - theoretical studies
- Each of these four approaches has both strengths and weaknesses. In particular, both numerical modeling and theory involve approximations.
- Most serious weakness of numerical modeling, as a research approach, is that it is possible to run a numerical model built by someone else without having any idea how the model works or what its limitations are.

Atmospheric Numerical Modeling

- Unfortunately, this happens all the time, and the problem is becoming more serious in today's era of "community" models with large user groups.
- One of the purposes of DCMIP is to make it less likely that researchers will use a model without having any understanding of it.
- DCMIP is designed to be a practical, "how to" summer school, which aims at conveying sufficient understanding so that after the two weeks you are able to
  - fully grasp the concept of 'designing a climate model'
  - understand the useful or detrimental properties of numerical schemes and whole models that you may encounter in your future studies
  - know (at least) some limitations of models

Components of a General Circulation Model (GCM)

Aspects to consider and questions to ask (in red this talk)**:**

- What is a General Circulation Model (GCM), weather model, climate model?
- What is a dynamical core of a GCM?
- Scientific and numerical aspects in the design process
- Review of the different forms of the equations and prognostic variables
- Snapshots of numerical methods
- Computational meshes, staggering options, the characteristics and accuracy of numerical discretizations
- Typical horizontal and vertical resolutions of GCMs and their limitations
- Suitable time-stepping schemes and vertical coordinates

Components of a General Circulation Model

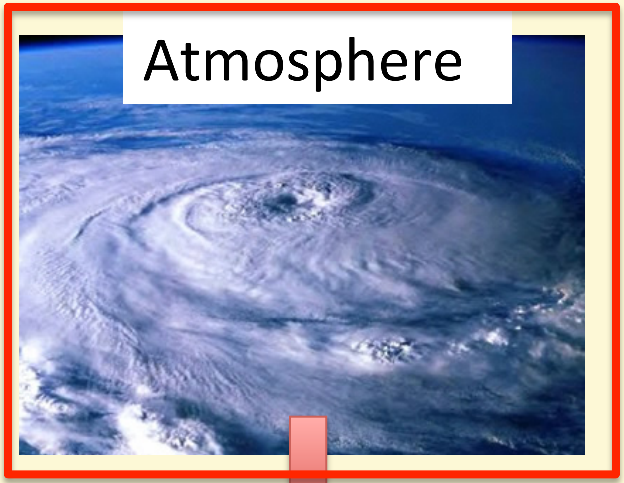
Aspects to consider and questions to ask:

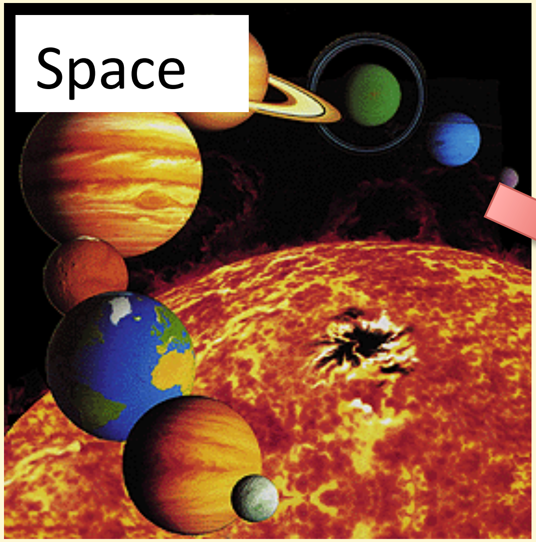
- Physical and computational challenges: conservation, positive tracer advection, stability
- Computational aspects, computer architectures, scalability, efficiency, how it determines science decisions
- Hidden features in the design process: subgrid-scale diffusive and filtering processes
- Evaluation techniques, dynamical core test cases, aqua-planet setups
- Coupling of the dynamical core and the physical parameterizations
- What are physical parameterizations: Review of their basic principles

DCMIP: Overarching goals

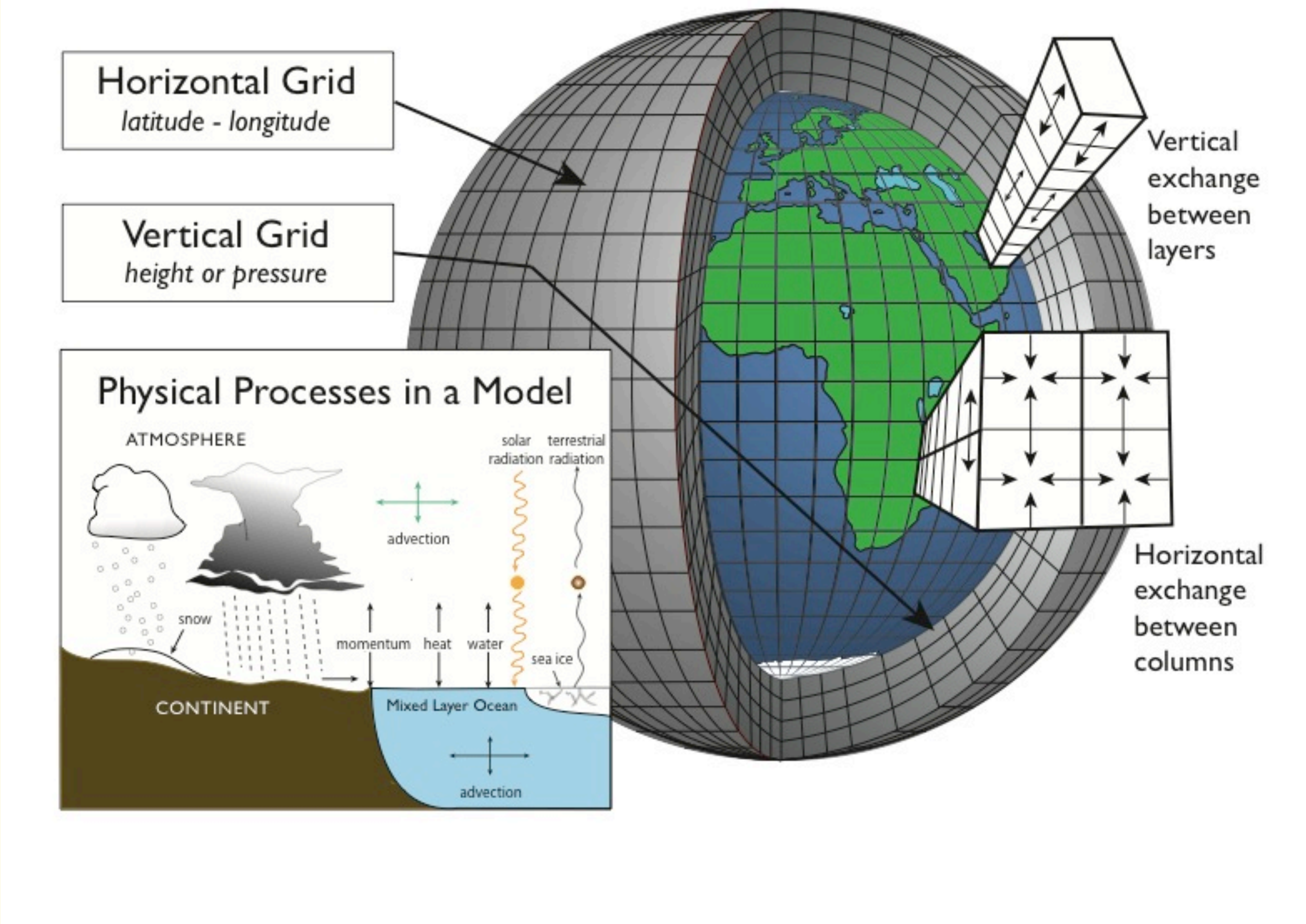
- After DCMIP a GCM should no longer be a black box
- You will be enabled to make informed decisions on how to use GCMs in your research and to be aware of the limitations.
- You will be exposed to real world dynamical cores and GCMs, and software practices in atmospheric science.
- Have a good understanding of the literature and model documentations (where to find information)
- Knowdata portals, especially the Earth System Grid Federation (ESGF) and shared work spaces, www.earthsystemcog.org

What is a Climate Model?





Atmospheric General Circulation Models (AGCMs)

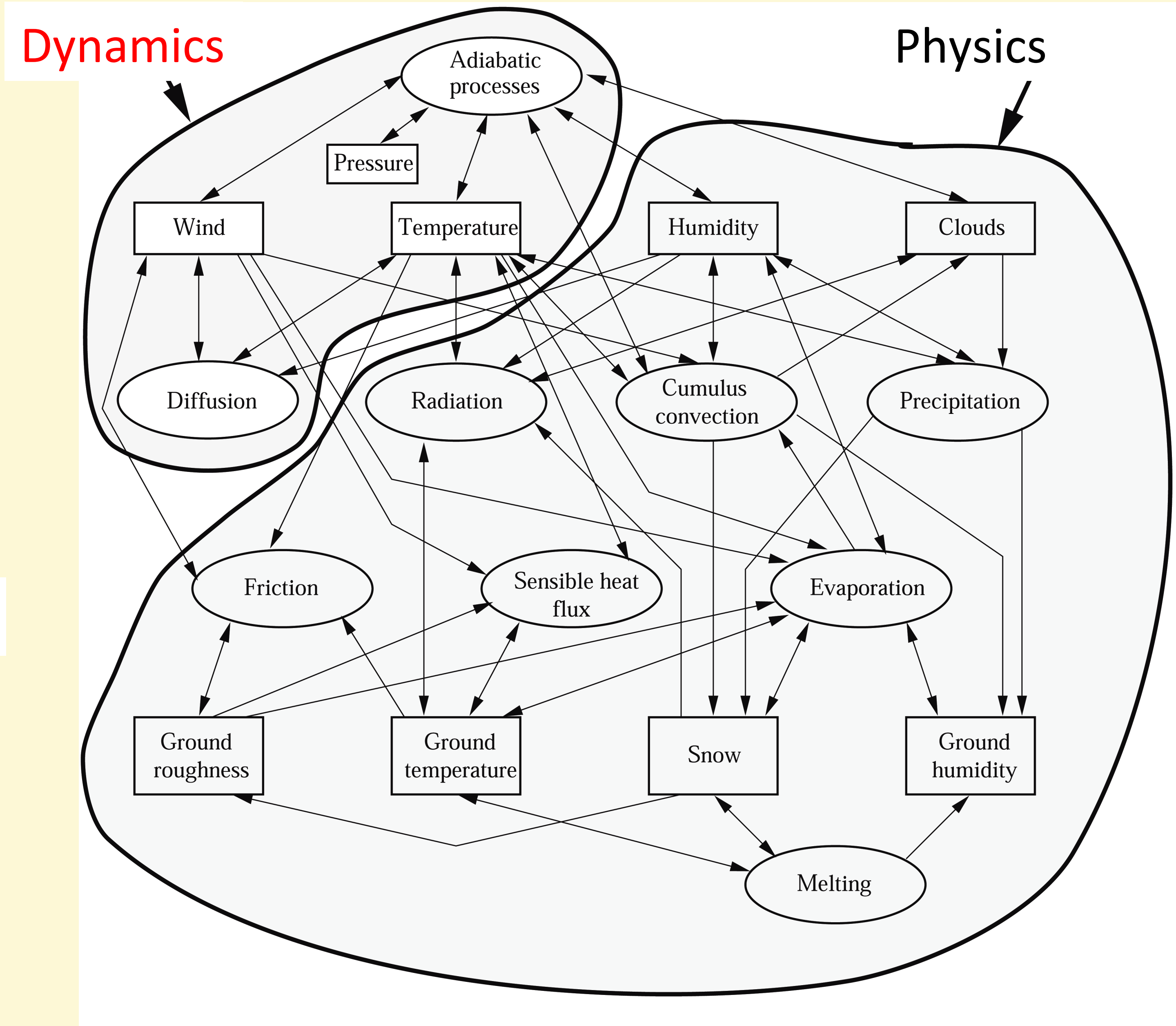


Components of an atmospheric GCM

Process

Variable

Interaction



What is a dynamical core?

- **Fluid dynamics** component of every weather or climate model
- Based on **equations of motion**, they may be approximated
- Describes the **resolved adiabatic motions** on a computational grid
- Contains **filters and diffusion** processes, mostly for numerical purposes, physical justification may be weak
- Determines the choice of the **prognostic** (forecast) variables

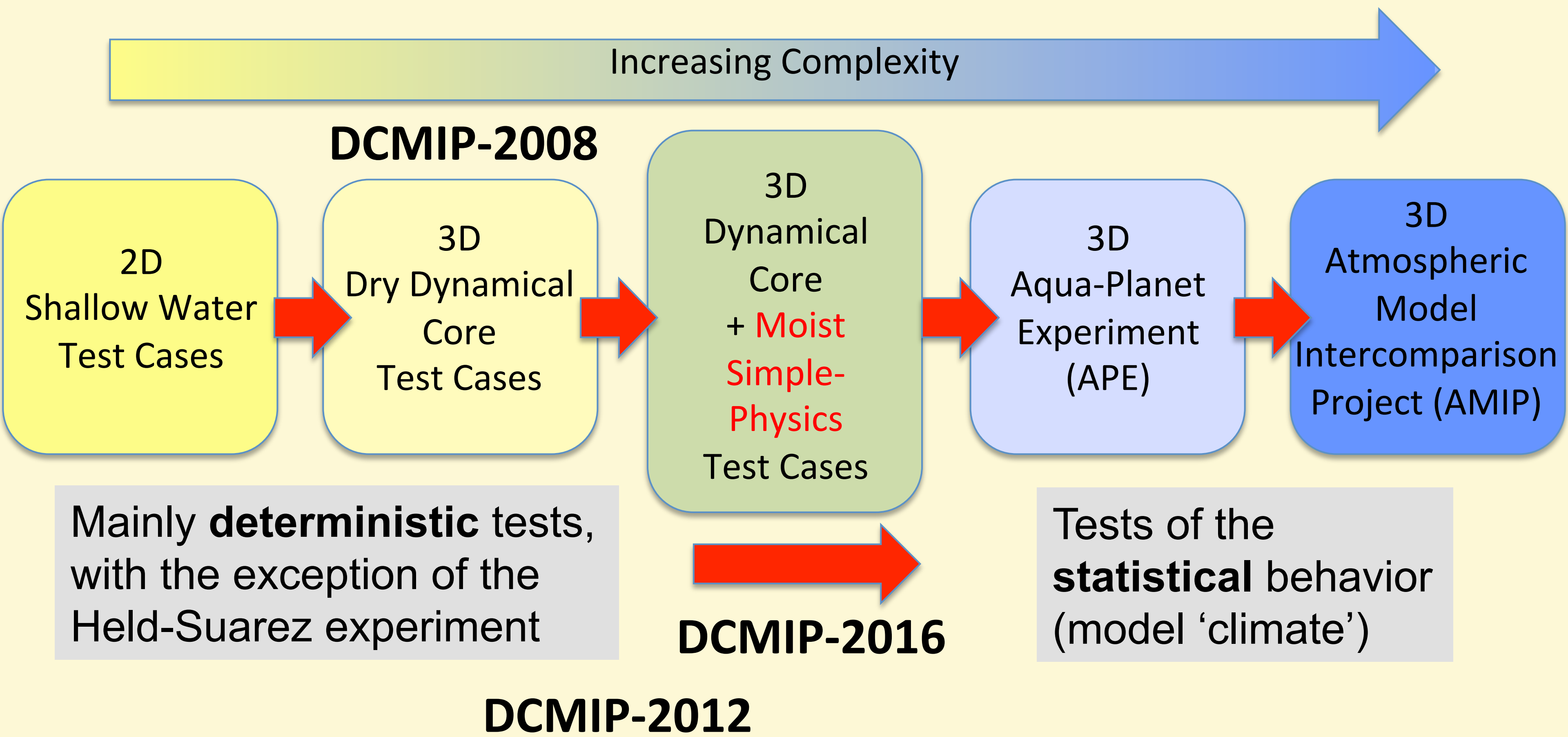
The pursuit of the ‘perfect’ dynamical core: Design aspects

Our scientific and numerical wish list:

- Accurate
- Stable
- Simple
- Computationally efficient
- Obeys physical constraints: conservation properties (which ones?), positive-definite tracer advection
- Truthful representation of the subgrid-scale

GCM (Test) Hierarchy

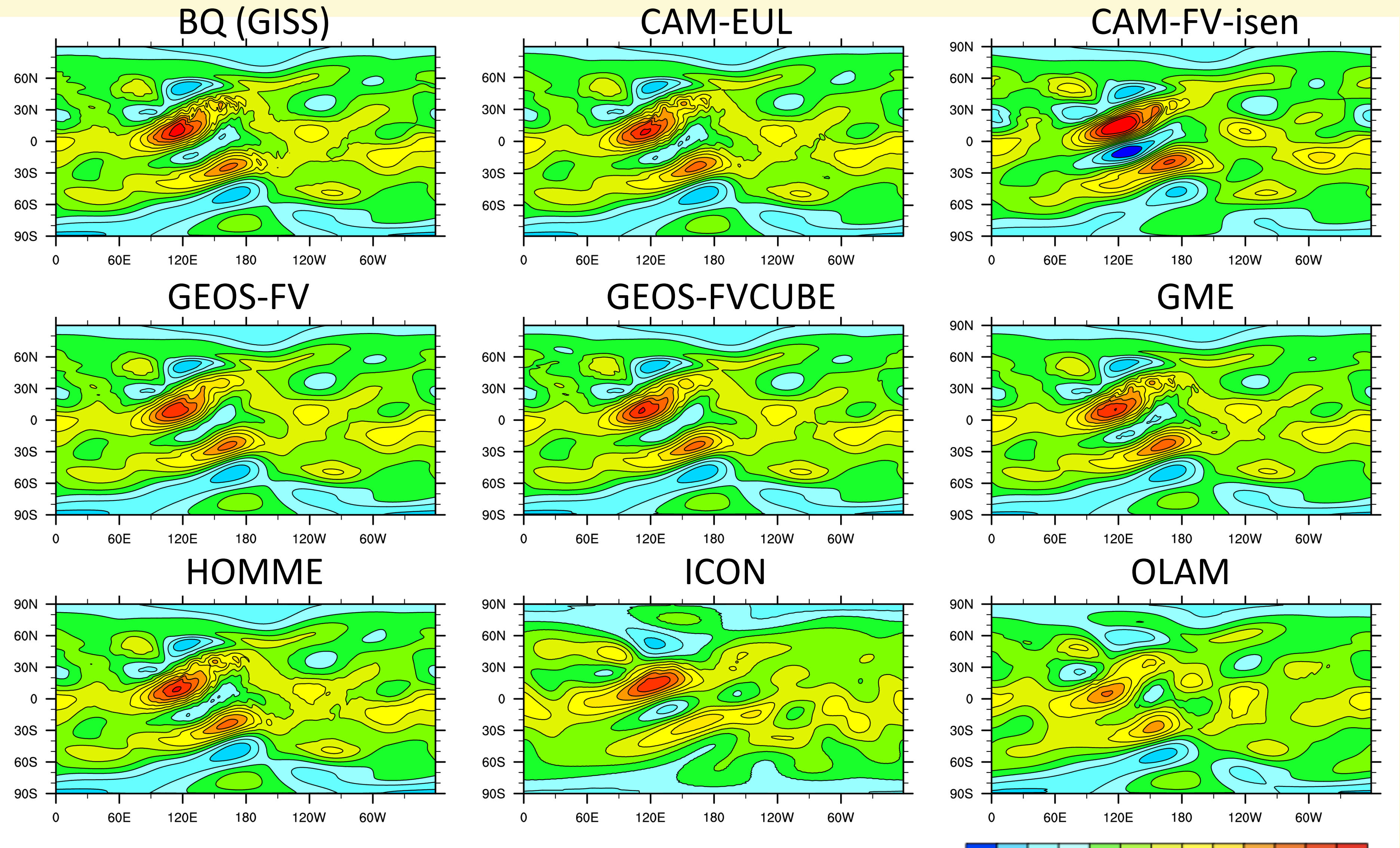
- Typical evaluation hierarchy for Dynamical Core and GCMs assessments

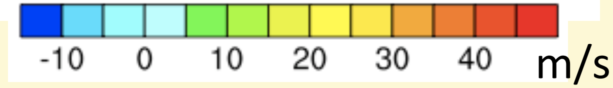


Does the dynamical core matter?

- Provocative: the fluid dynamics problem is solved, physics parameterizations matter most
- Let's take a look at 9 dynamical cores that participated in the DCMIP intercomparison project during the 2008 NCAR Summer Colloquium

Mountain-triggered Rossby waves

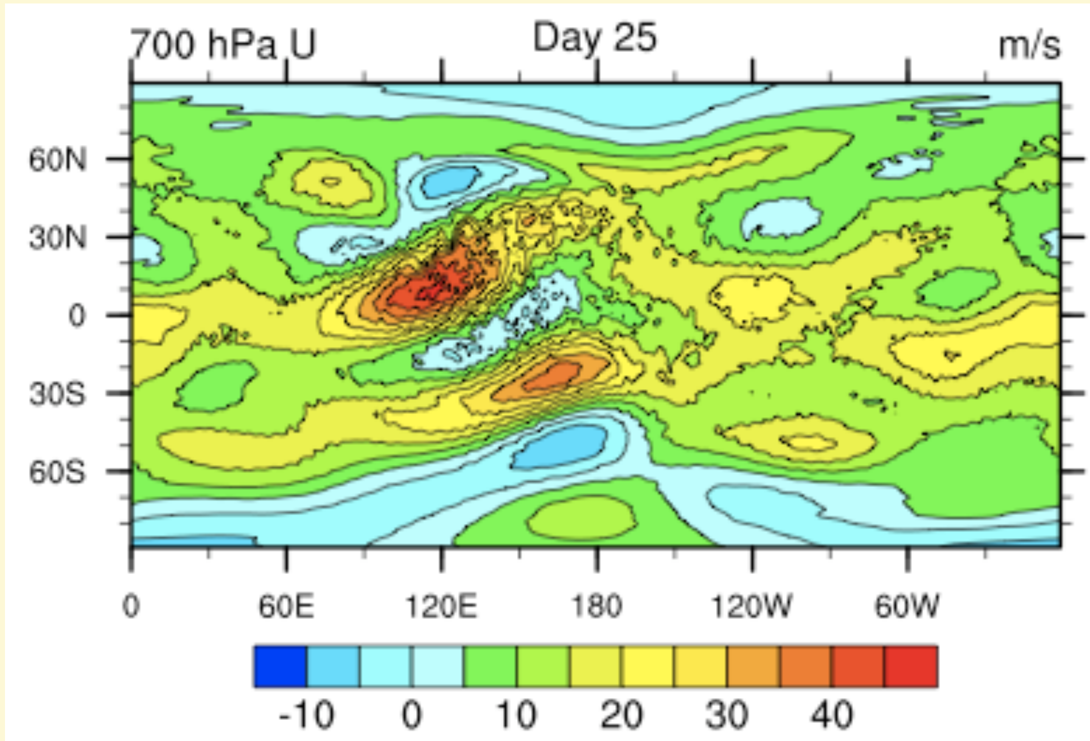




700 hPa zonal wind at day 15 (≈1°×1°L26)

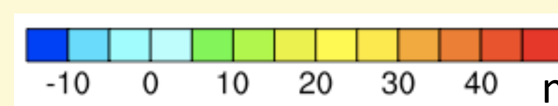
Mountain-triggered Rossby waves

CAM-EUL



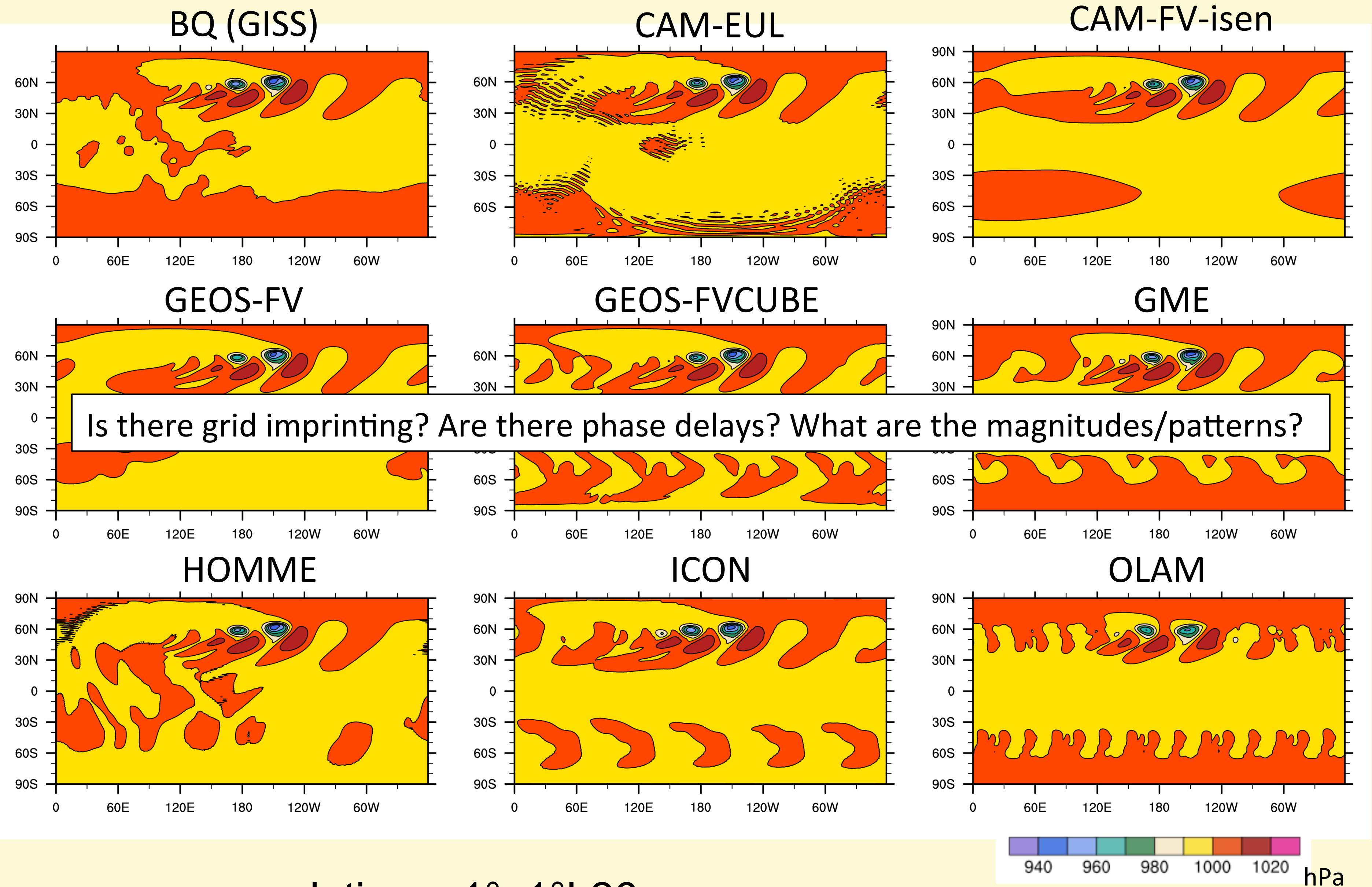
Details matter, diffusion matters

CAM-EUL, 700 hPa zonal wind at day 25 (≈1°×1°L26)



m/s

Baroclinic waves: p_s at Day 9



with α=0°, resolution ≈ 1°×1°L26

DCMIP-2012

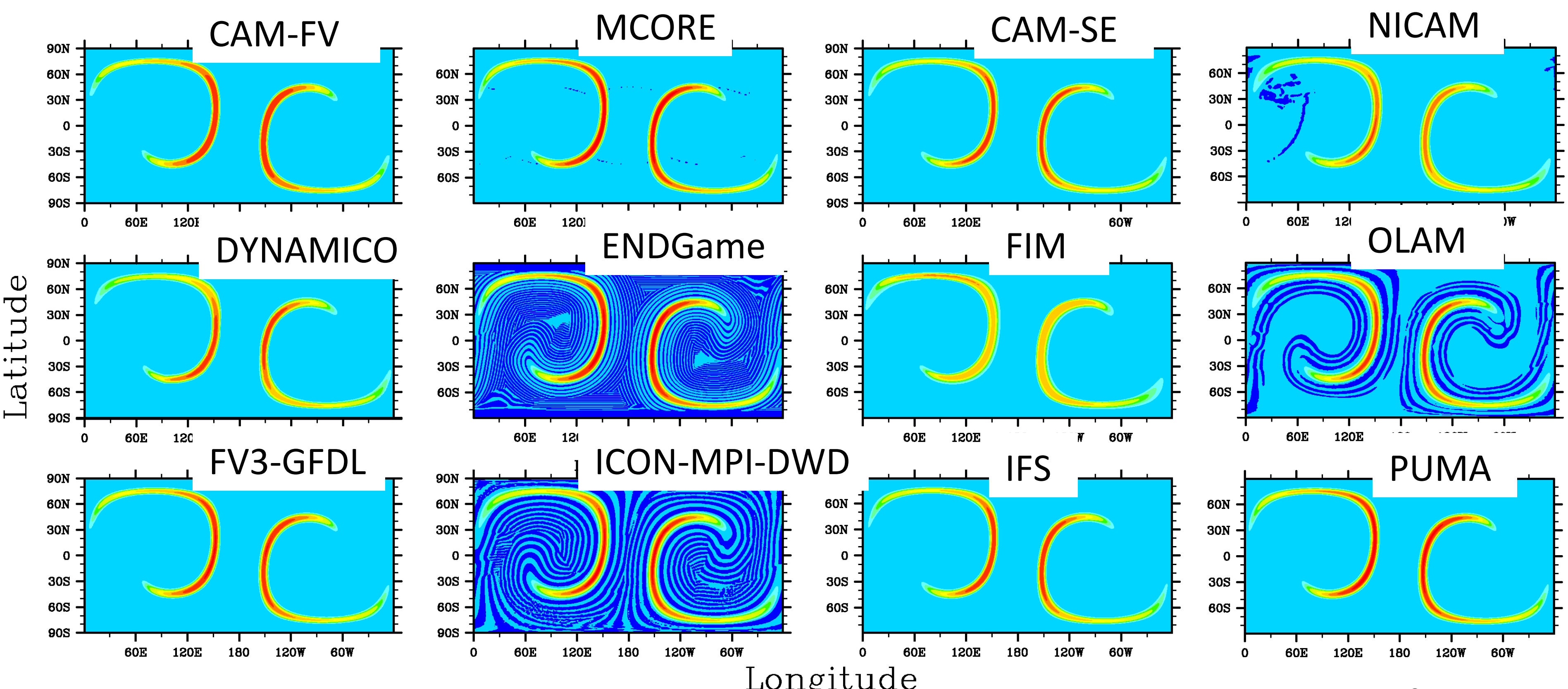
- The Dynamical Core Model Intercomparison Project (DCMIP), held in combination with a 2-week summer school in August 2012
- Assessment of the newest dycores and non-hydrostatic dynamical core developments

http://earthsystemcog.org/projects/dcmip-2012/

DCMIP-2012: 3D Advection in the Deformational Flow Test Case

Test 11: Correlated tracers in a reversing sheared flow (tracer q1 at day 6)

Questions: How diffusive is the method? Are there numerical over- or undershoots?



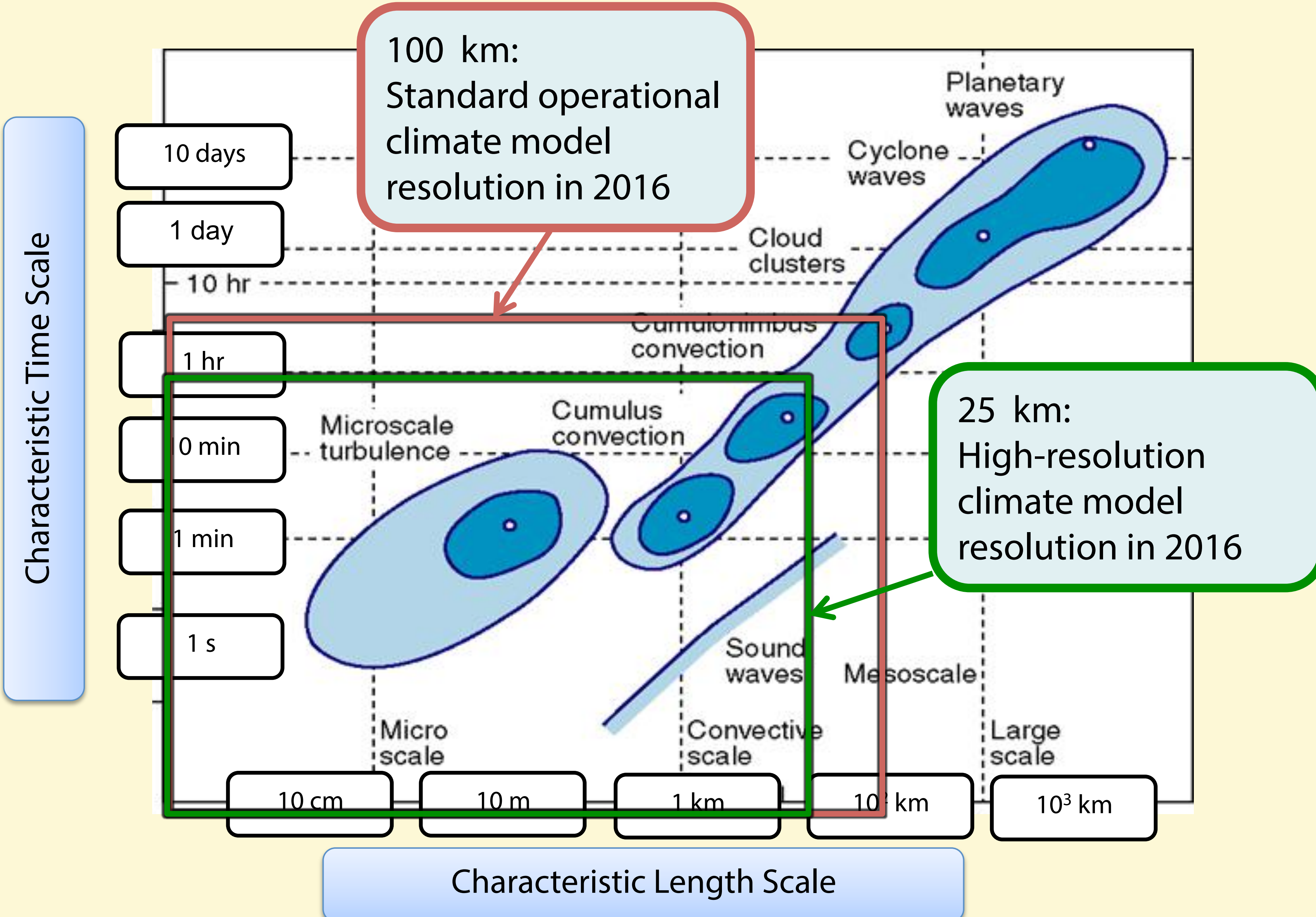
Test 11
dx= 110 km, dz = 200 m

3D version of Lauritzen and Thuburn (QJ, 2012)

Design Choices in Dynamical Cores

- There are many crossroads
  - Equation set and prognostic variables
  - Conservation properties
  - Numerical method
  - Computational grid and grid staggering (horizontal)
  - Vertical coordinate and vertical staggering
  - Diffusion and filtering mechanisms
  - Tuning parameters (e.g. diffusion coefficients)
  - Ways to include moisture
  - Coupling strategy to physics package & length of the physics time step (coupling interval)

Spatial and Temporal Scales: Processes



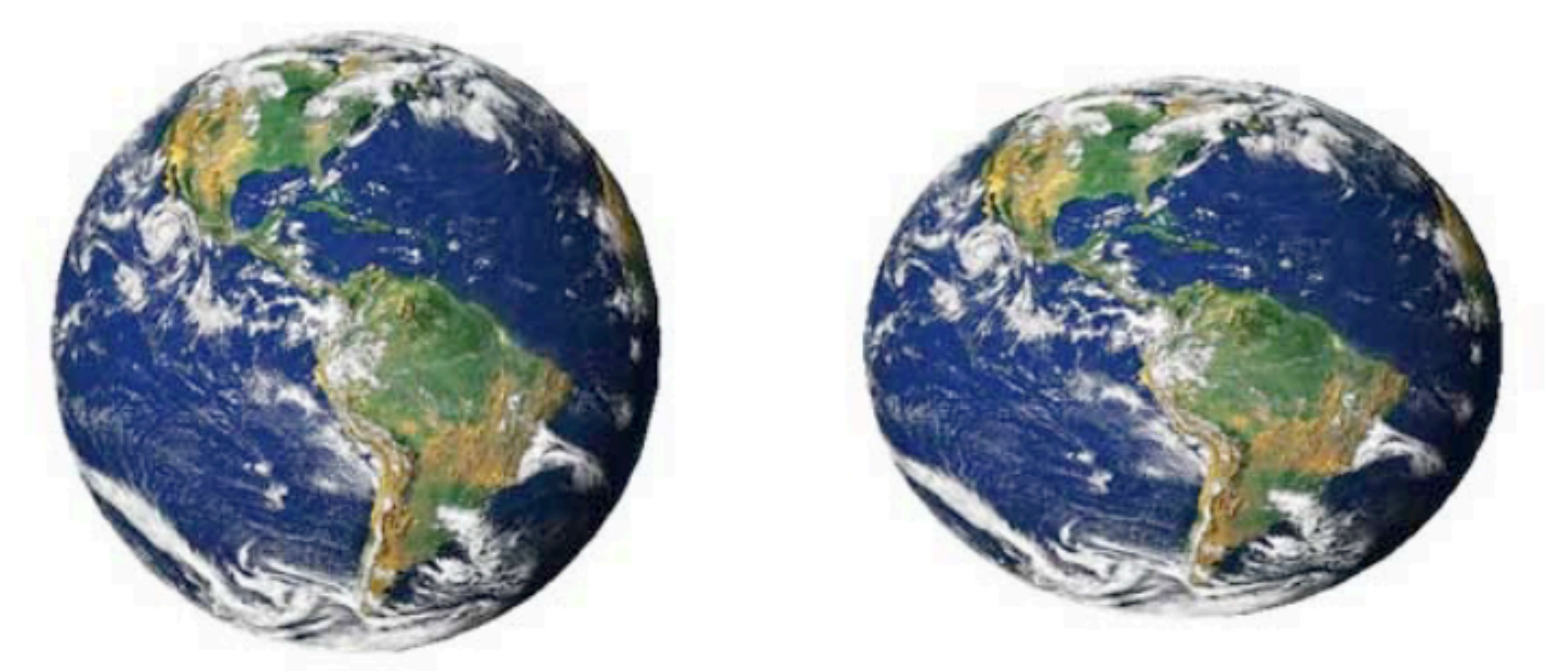
The choice of the equations of motion

- The governing equations are the 3D Euler equations, but we never use them in their original form
- We make **simplifications** (e.g. the Earth is a perfect sphere) and use scaling arguments to simplify the dynamical core design
- The Euler equations contain 6 equations:
  - Three momentum equations
  - Continuity equation (mass conservation)
  - Thermodynamic equation
  - Ideal gas law
- **6 equations, 6 unknowns**: u, v, w, T, p, ρ

Choice of the Equations: Common design decisions

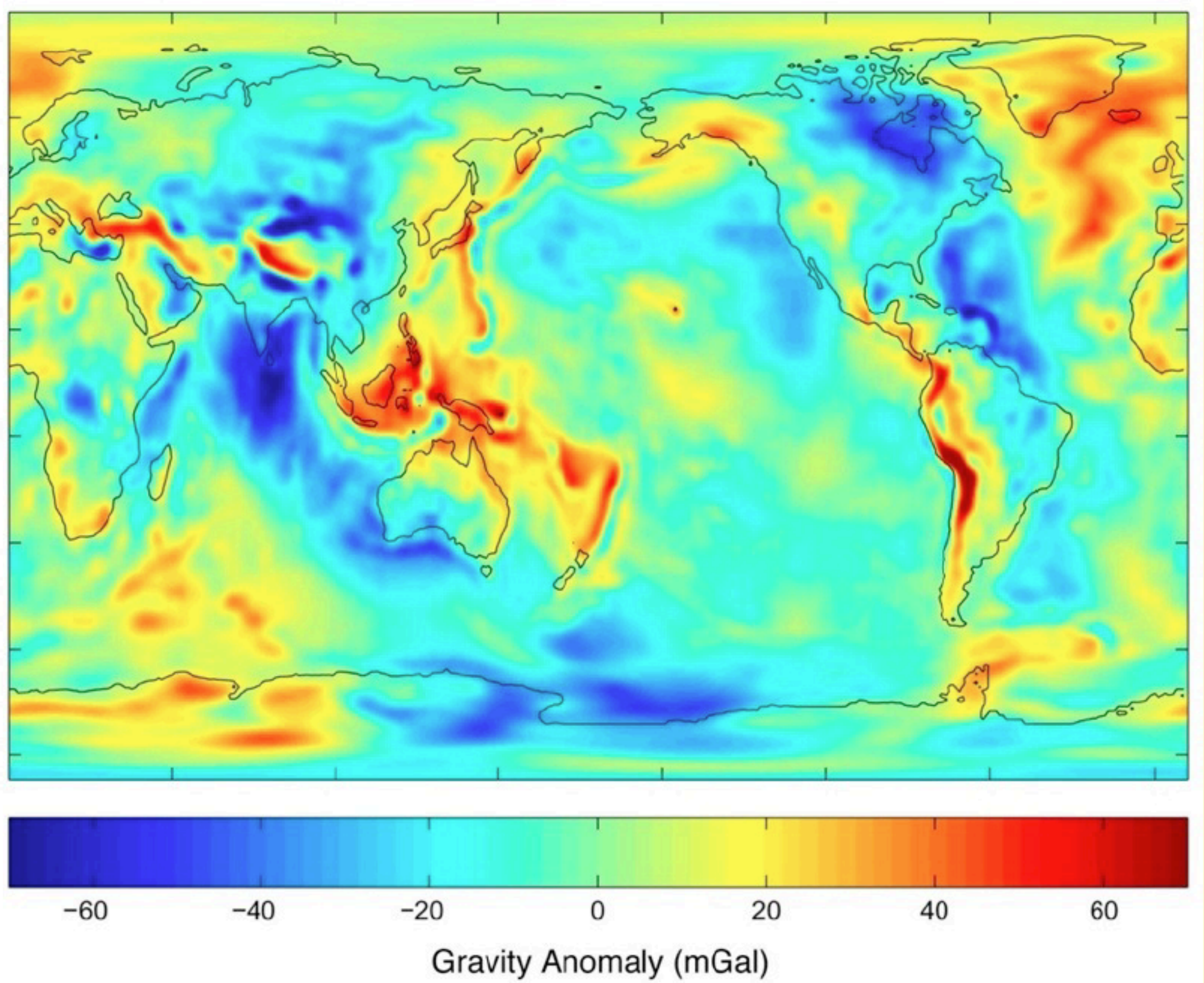
- **Deep or shallow** atmosphere:
  is the distance ‘r’ to the center of the Earth represented as the constant radius ‘a’?
- **Hydrostatic or non-hydrostatic**:
  is forecast equation for the velocity w maintained?
- **Filtered** equations? Anelastic, Boussinesq, pseudo-incompressible, unified
- Which **prognostic variables** are suitable?
- Which **coordinate system** is suitable:
  - Spherical coordinates
  - Local coordinates, Cartesian coordinates

Common Approximation: The Earth is a perfect sphere



- In reality: equatorial radius is about 20 km wider than the polar radius
- Currently: UK Met Office pursues research to eliminate this approximation in their GCM

Another Common Approximation: Gravity is constant or varies only in the vertical



- Measured gravity anomalies

Approximated Equation Sets

Q. J. R. Meteorol. Soc. (2005), **131**, pp. 2081–2107 doi: 10.1256/qj.04.49

Consistent approximate models of the global atmosphere: shallow, deep, hydrostatic, quasi-hydrostatic and non-hydrostatic

By A. A. WHITE[1]*, B. J. HOSKINS[2], I. ROULSTONE[1,3] and A. STANIFORTH[1]

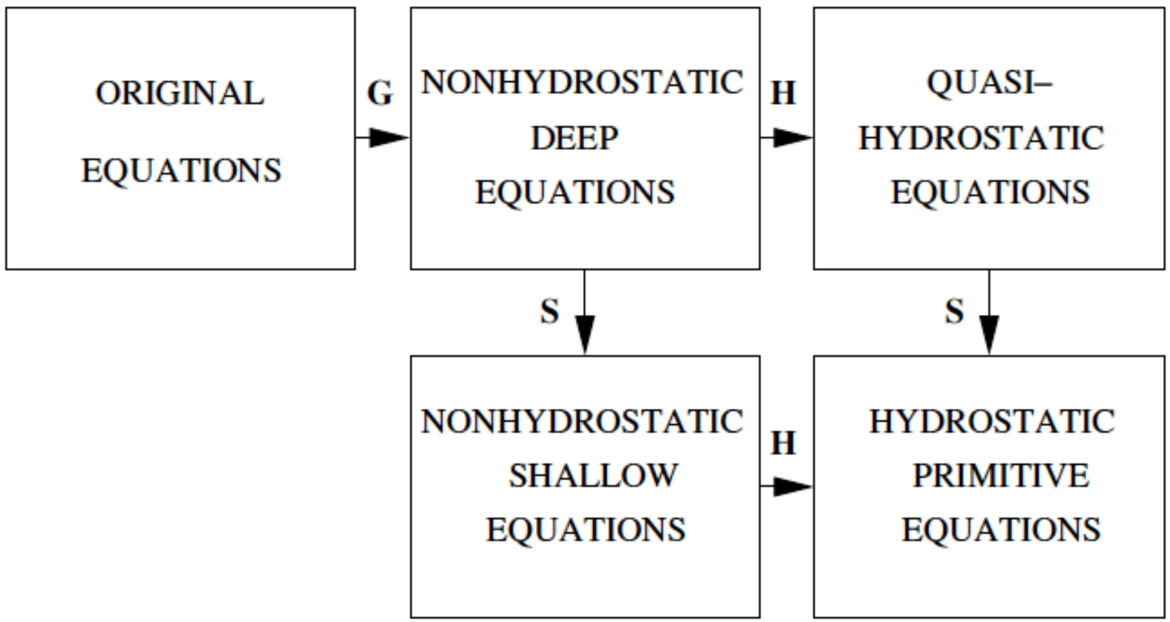


Figure 4. Showing the interrelationships of the four consistent approximate models of the global atmosphere identified in this study (NHD, QHE, NHS and HPE models) and the relationship of the NHD model to the original (unapproximated) equations. **G** denotes the spherical geopotential approximation, **H** the omission of the term $\mathrm{D}w/\mathrm{D}t$ from the vertical component of the momentum equation, and **S** the shallow atmosphere combination of approximations (see text).

Non-hydrostatic equations of motion (deep atmosphere, spherical coordinates)

$$\frac{Du}{Dt} - \frac{u\mathrm{v}\tan(\phi)}{r} + \frac{uw}{r} = -\frac{1}{\rho\, r\cos\phi}\frac{\partial p}{\partial \lambda} + 2\Omega \mathrm{v}\sin(\phi) - 2\Omega w\cos(\phi) + \nu\nabla^2(u)$$

$$\frac{D\mathrm{v}}{Dt} + \frac{u^2\tan(\phi)}{r} + \frac{\mathrm{v}w}{r} = -\frac{1}{\rho\, r}\frac{\partial p}{\partial \phi} - 2\Omega u\sin(\phi) + \nu\nabla^2(\mathrm{v})$$

$$\frac{Dw}{Dt} - \frac{u^2 + \mathrm{v}^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} - g + 2\Omega u\cos(\phi) + \nu\nabla^2(w) \qquad \text{with variable } g = \frac{d\Phi}{dr} = G\frac{a^2}{r^2}$$

$$\frac{D\rho}{Dt} + \frac{\rho}{r\cos\phi}\left[\frac{\partial u}{\partial \lambda} + \frac{\partial(v\cos\phi)}{\partial \phi}\right] + \frac{\rho}{r^2}\frac{\partial(r^2 w)}{\partial r} = 0$$

$$c_v\frac{DT}{Dt} + p\frac{D}{Dt}\left(\frac{1}{\rho}\right) = J$$

$$p = \rho R T$$

$$\frac{D(\;)}{Dt} = \frac{\partial(\;)}{\partial t} + \frac{u}{r\cos\phi}\frac{\partial(\;)}{\partial \lambda} + \frac{v}{r}\frac{\partial(\;)}{\partial \phi} + w\frac{\partial(\;)}{\partial r}$$

Only approximations: Earth is a perfect sphere, g only varies vertically

Quasi-hydrostatic equations of motion (deep atmosphere, spherical coordinates)

$$\frac{Du}{Dt} - \frac{u\mathrm{v}\tan(\phi)}{r} + \frac{uw}{r} = -\frac{1}{\rho\, r\cos\phi}\frac{\partial p}{\partial \lambda} + 2\Omega \mathrm{v}\sin(\phi) - 2\Omega w\cos(\phi) + \nu\nabla^2(u)$$

$$\frac{D\mathrm{v}}{Dt} + \frac{u^2\tan(\phi)}{r} + \frac{\mathrm{v}w}{r} = -\frac{1}{\rho\, r}\frac{\partial p}{\partial \phi} - 2\Omega u\sin(\phi) + \nu\nabla^2(\mathrm{v})$$

$$\cancel{\frac{Dw}{Dt}} - \frac{u^2+\mathrm{v}^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} - g + 2\Omega u\cos(\phi) + \cancel{\nu\nabla^2(w)} \qquad \text{with variable } g = \frac{d\Phi}{dr} = G\frac{a^2}{r^2}$$

$$\frac{D\rho}{Dt} + \frac{\rho}{r\cos\phi}\left[\frac{\partial u}{\partial \lambda} + \frac{\partial(v\cos\phi)}{\partial \phi}\right] + \frac{\rho}{r^2}\frac{\partial(r^2 w)}{\partial r} = 0$$

$$c_v\frac{DT}{Dt} + p\frac{D}{Dt}\left(\frac{1}{\rho}\right) = J$$

$$p = \rho R T$$

$$\frac{D(\;)}{Dt} = \frac{\partial(\;)}{\partial t} + \frac{u}{r\cos\phi}\frac{\partial(\;)}{\partial \lambda} + \frac{v}{r}\frac{\partial(\;)}{\partial \phi} + w\frac{\partial(\;)}{\partial r}$$

- neglect of *Dw/Dt* has the effect of removing vertically propagating acoustic modes
- puts the vertical momentum balance into a diagnostic form

Shallow atmosphere approximation

- Approximate distance $r = a+z$ to the center of the Earth with the constant mean radius of the Earth a
- Replace r by a and $\partial/\partial r$ by $\partial/\partial z$, where z is height above mean sea level
- Omit all the metric terms not involving tan φ
- Omit those Coriolis terms that vary as the cosine of the latitude
- Neglect all variations of the gravity g *(constant)*
- Neglect the vertical component of the diffusion
- All is necessary to guarantee **energy** and **absolute momentum conservation** on a shallow Earth

Non-hydrostatic equations of motion (shallow atmosphere)

$$\frac{Du}{Dt} - \frac{u\mathrm{v}\tan(\phi)}{a} + \cancel{\frac{uw}{r}} = -\frac{1}{\rho\, a\cos\phi}\frac{\partial p}{\partial\lambda} + 2\Omega \mathrm{v}\sin(\phi) - \cancel{2\Omega w\cos(\phi)} + \nu\nabla^2(u)$$

$$\frac{D\mathrm{v}}{Dt} + \frac{u^2\tan(\phi)}{a} + \cancel{\frac{\mathrm{v}w}{r}} = -\frac{1}{\rho\, a}\frac{\partial p}{\partial\phi} - 2\Omega u\sin(\phi) + \nu\nabla^2(\mathrm{v})$$

$$\frac{Dw}{Dt} - \cancel{\frac{u^2+\mathrm{v}^2}{r}} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g + \cancel{2\Omega u\cos(\phi)} + \cancel{\nu\nabla^2(w)} \qquad \cancel{\text{with variable}}\ g = \cancel{\frac{d\Phi}{dr}} = \cancel{G\frac{a^2}{r^2}}$$

$$\frac{D\rho}{Dt} + \frac{\rho}{a\cos\phi}\left[\frac{\partial u}{\partial\lambda} + \frac{\partial(\mathrm{v}\cos\phi)}{\partial\phi}\right] + \frac{\rho}{\cancel{r^2}}\frac{\partial(\cancel{r^2}w)}{\partial z} = 0$$

$$c_v\frac{DT}{Dt} + p\frac{D}{Dt}\left(\frac{1}{\rho}\right) = J$$

$$p = \rho RT$$

$$\frac{D(\ )}{Dt} = \frac{\partial(\ )}{\partial t} + \frac{u}{a\cos\phi}\frac{\partial(\ )}{\partial\lambda} + \frac{\mathrm{v}}{a}\frac{\partial(\ )}{\partial\phi} + w\frac{\partial(\ )}{\partial z}$$

- Omit terms
- Replace r with a

Non-hydrostatic equations of motion (shallow atmosphere)

$$\frac{Du}{Dt} - \frac{u\mathrm{v}\tan(\phi)}{a} = -\frac{1}{\rho\, a\cos\phi}\frac{\partial p}{\partial\lambda} + 2\Omega \mathrm{v}\sin(\phi) + \nu\nabla^2(u)$$

$$\frac{D\mathrm{v}}{Dt} + \frac{u^2\tan(\phi)}{a} = -\frac{1}{\rho\,\mathrm{a}}\frac{\partial p}{\partial\phi} - 2\Omega u\sin(\phi) + \nu\nabla^2(\mathrm{v})$$

$$\frac{Dw}{Dt} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g \qquad \text{with constant g}$$

$$\frac{D\rho}{Dt} + \frac{\rho}{a\cos\phi}\left[\frac{\partial u}{\partial\lambda} + \frac{\partial(v\cos\phi)}{\partial\phi}\right] + \frac{\partial w}{\partial z} = 0$$

$$c_v\frac{DT}{Dt} + p\frac{D}{Dt}\left(\frac{1}{\rho}\right) = J$$

$$p = \rho RT$$

$$\frac{D(\ )}{Dt} = \frac{\partial(\ )}{\partial t} + \frac{u}{a\cos\phi}\frac{\partial(\ )}{\partial\lambda} + \frac{v}{a}\frac{\partial(\ )}{\partial\phi} + w\frac{\partial(\ )}{\partial z}$$

Hydrostatic equations of motion (shallow atmosphere): **Primitive Equations**

$$\frac{Du}{Dt} - \frac{u\mathrm{v}\tan(\phi)}{a} = -\frac{1}{\rho\, a\cos\phi}\frac{\partial p}{\partial \lambda} + 2\Omega \mathrm{v}\sin(\phi) + \nu\nabla^2(u)$$

$$\frac{D\mathrm{v}}{Dt} + \frac{u^2\tan(\phi)}{a} = -\frac{1}{\rho\,\mathrm{a}}\frac{\partial p}{\partial \phi} - 2\Omega u\sin(\phi) + \nu\nabla^2(\mathrm{v})$$

$$\cancel{\frac{Dw}{Dt}} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g \qquad \text{with constant g}$$

$$\frac{D\rho}{Dt} + \frac{\rho}{a\cos\phi}\left[\frac{\partial u}{\partial \lambda} + \frac{\partial(v\cos\phi)}{\partial \phi}\right] + \frac{\partial w}{\partial z} = 0$$

$$c_v\frac{DT}{Dt} + p\frac{D}{Dt}\left(\frac{1}{\rho}\right) = J$$

$$p = \rho R T$$

$$\frac{D(\ )}{Dt} = \frac{\partial(\ )}{\partial t} + \frac{u}{a\cos\phi}\frac{\partial(\ )}{\partial \lambda} + \frac{v}{a}\frac{\partial(\ )}{\partial \phi} + w\frac{\partial(\ )}{\partial z}$$

More design decisions: The form of the equations

- Lagrangian versus Eulerian form
- Advective form versus flux form
- Model variables
- Vertical coordinate transformations

Lagrangian versus Eulerian framework

- Lagrangian form: The variations are observed following a moving particle, requires the total derivative, e.g. the continuity equation is:

$$\frac{D\rho}{Dt} + \frac{\rho}{a\cos\phi}\left[\frac{\partial u}{\partial \lambda} + \frac{\partial(v\cos\phi)}{\partial \phi}\right] + \frac{\partial w}{\partial z} = 0$$

- Eulerian form: The variations are observed at a fixed location and snapshot in time, requires partial derivatives, e.g. the continuity equation is:

$$\frac{\partial\rho}{\partial t} + \frac{1}{a\cos\phi}\left[\frac{\partial(\rho u)}{\partial \lambda} + \frac{\partial(\rho v\cos\phi)}{\partial \phi}\right] + \frac{\partial(\rho w)}{\partial z} = 0$$

Advective form versus the flux form

- Consider a tracer advection equation for tracer q:

$$\frac{Dq}{Dt} = 0$$

$$\Leftrightarrow \boxed{\frac{\partial q}{\partial t} + \vec{v} \bullet \nabla q = 0}$$

- This is the so-called **advective form**
- The **flux form** can be formed by incorporating the continuity equation: $\boxed{\dfrac{\partial(\rho q)}{\partial t} + \nabla \bullet (\rho q \vec{v}) = 0}$
- The flux form has great advantages concerning mass conservation, especially in finite-volume models

Choice of the model variables

- We can choose (within limits) the model variables
- Hydrostatic models lose the ability to forecast the vertical velocity (becomes diagnostic)
- The choices are also determined by the numerical schemes (e.g. vertical coordinate system)
- A common set is **u, v, w, T, $\mathbf{p_s}$,ρ**
- Another common set is **ζ, δ, T, $\mathbf{p_s}$, ρ** where ζ, δ are the vorticity and divergence
- The thermodynamic variable is sometimes the potential temperature θ instead of T
- Advantage: built-in conservation $\frac{\partial(\rho\theta)}{\partial t} + \nabla \bullet (\rho\theta\vec{v}) = 0$

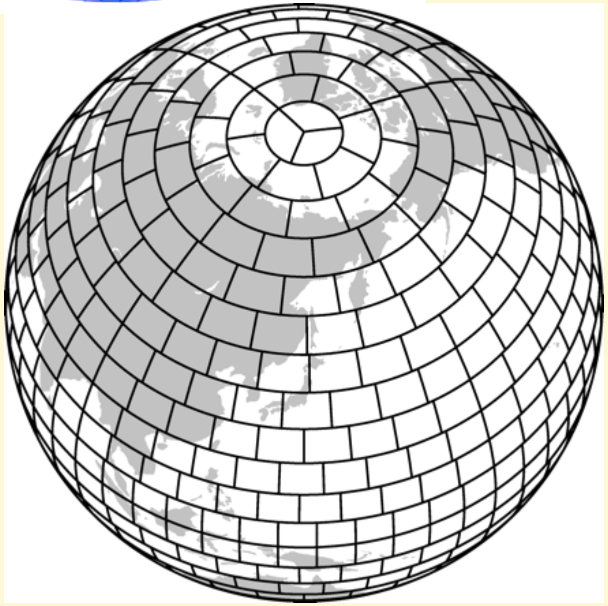
The pursuit of the ‘perfect’ model grid

- How to distribute grid points over the sphere: yet to be solved, no perfect solution
- Possible design criteria:
  - Highly uniform coverage
  - Orthogonal
  - Structured versus unstructured
  - Variable-resolution meshes
  - Conservation characteristics, geostrophic balance

DCMIP Models: Latitude-Longitude

Latitude-longitude or Gaussian grids

- ENDGame (U.K. Met Office)

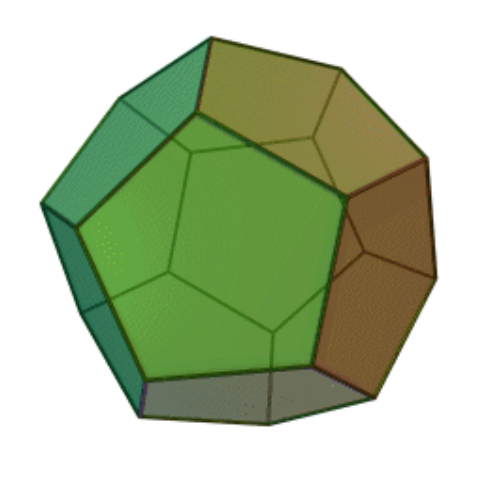


Reduced Gaussian grid

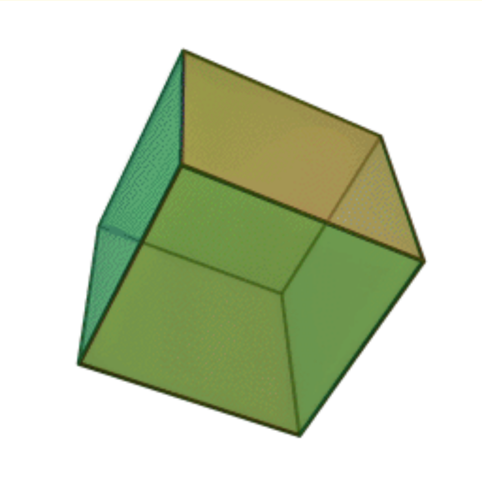
- FVM (ECMWF)

Platonic solids - Regular grid structures

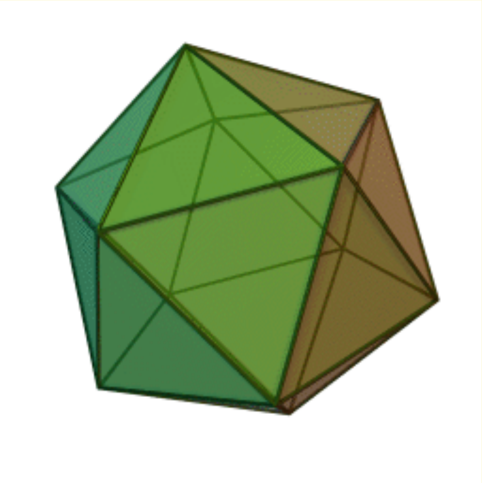
Dodecahedron (12 pentagons)



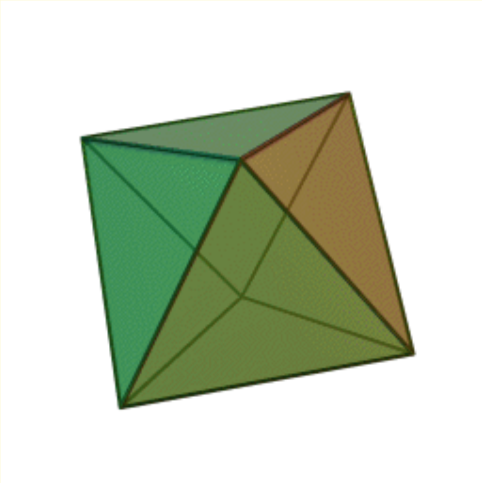
Cube (Hexahedron, 6 faces)



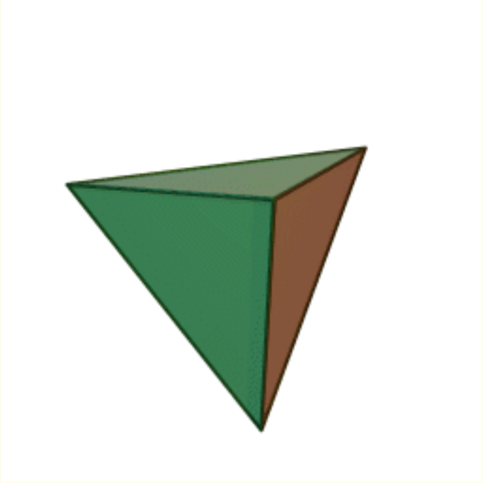
Icosahedron (20 triangles)



Octahedron (8 triangles)



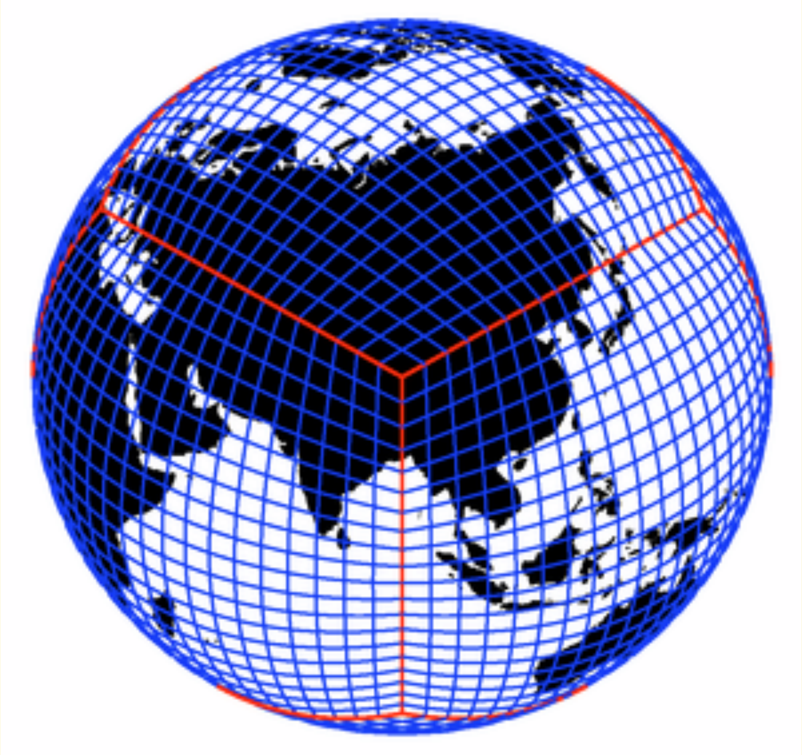
Tetrahedron (4 triangles)



- Platonic solids can be enclosed in a sphere

http://en.wikipedia.org/wiki/Platonic_solid

DCMIP Models: Cubed-Sphere Grids



Cubed-sphere grids

- CAM-SE
  (NCAR & Sandia Labs)
- FV3-GFDL
  (GFDL & NASA)
- NEPTUNE
  (NRL)
- Tempest
  (UC Davis)
- Chombo
  (LBNL, Uni. Michigan)

Cubed-Sphere Grids

There are many cubed-sphere projections, e.g.

Conformal
(almost orthogonal)

Gnomonic, equi-angular
(**most popular, non-orthogonal**)

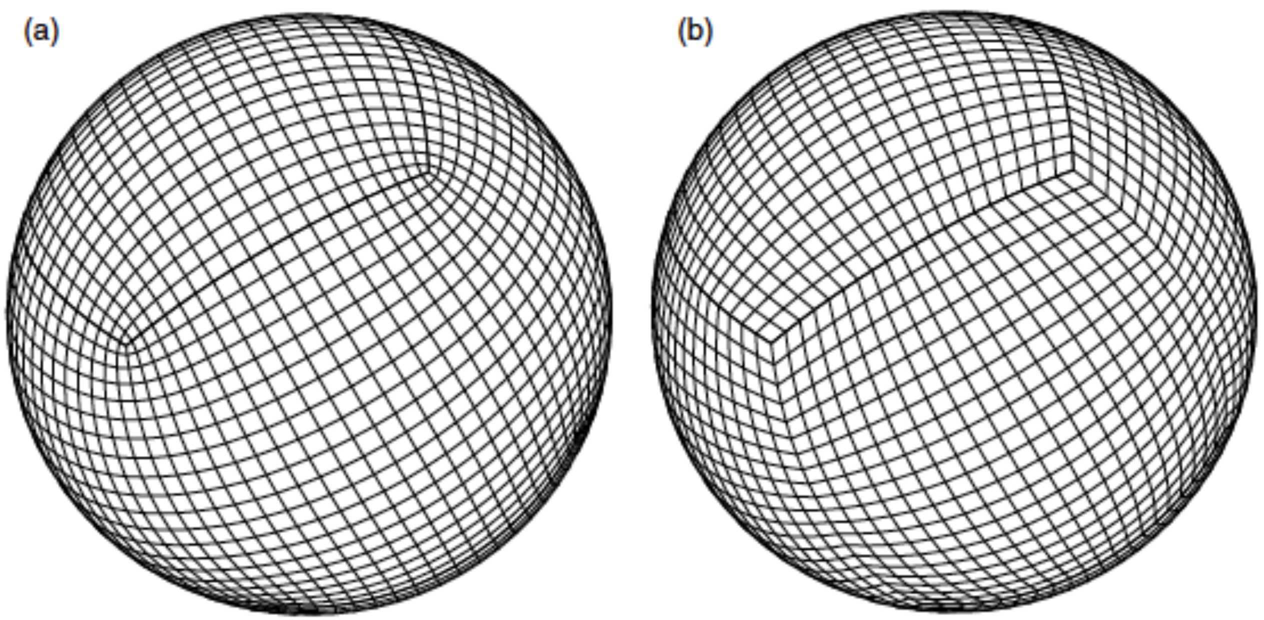


Figure 5. Cubed sphere: (a) conformally projected with isolines equally spaced in map coordinates, and (b) gnomonically projected with equal angles at an axis of the sphere.

Grid spacing varies by several factors,
can be factor 10 at very high resolutions (10 km)

Almost uniform, grid spacing varies
by factor 1.3 (at most)

DCMIP Models: Triangular Grids

Spherical geodesic or icosahedral-based grids

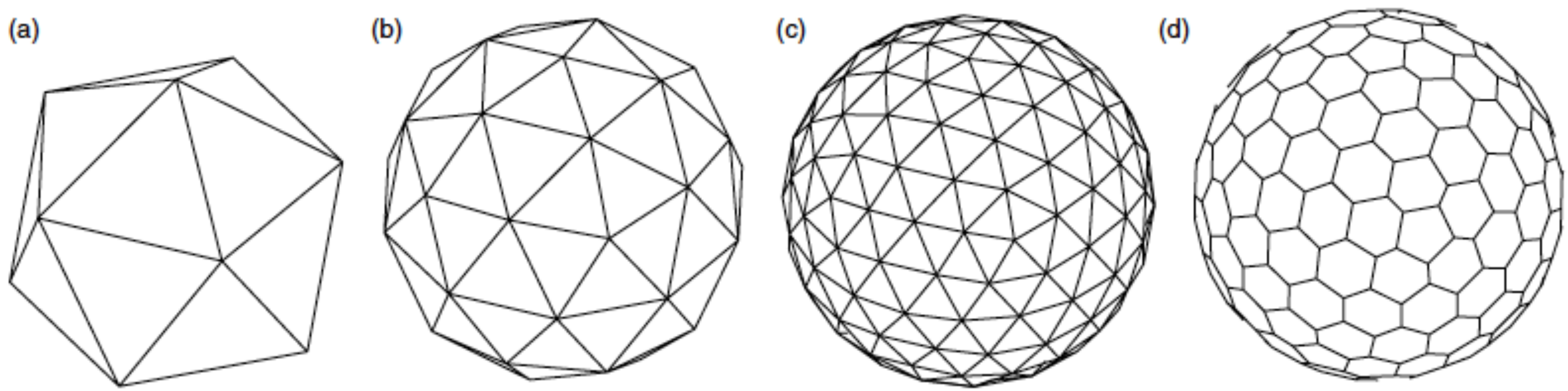
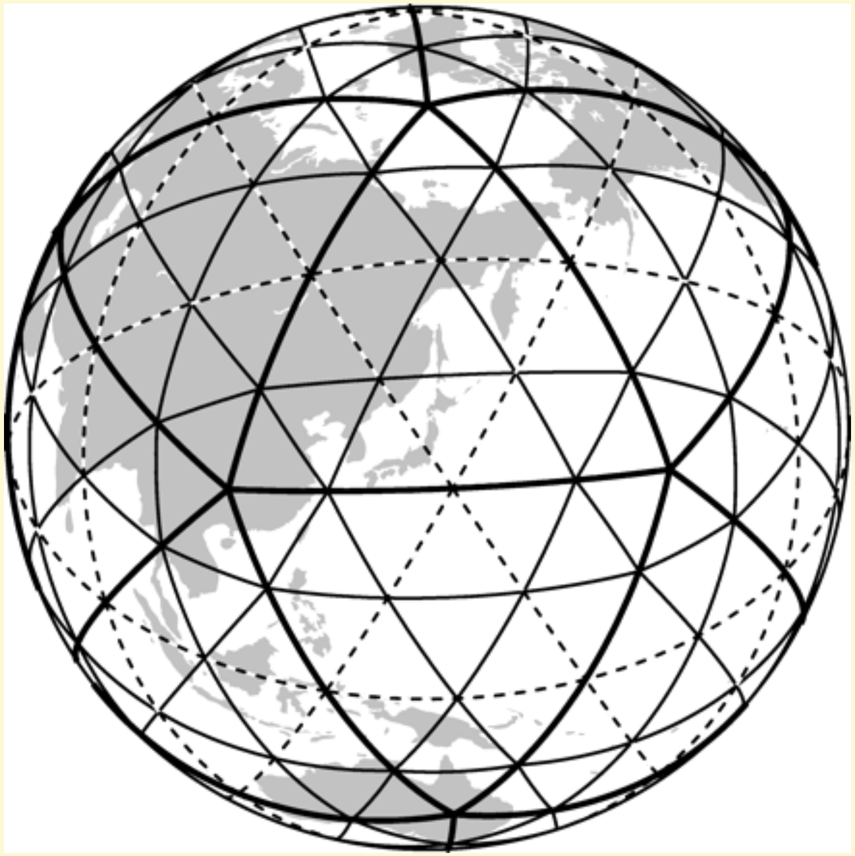


Figure 12. Icosahedral grid construction: (a) regular icosahedron, (b) after division of each triangular face into four subtriangles, (c) after decomposition into 4^2 subtriangles, and (d) the dual pentagonal-hexagonal grid of (c).

- Initial icosahedron consists of 20 triangles
- 12 special points where the initial icosahedron meets the sphere: Dual grid is a mix of hexagons and 12 pentagons

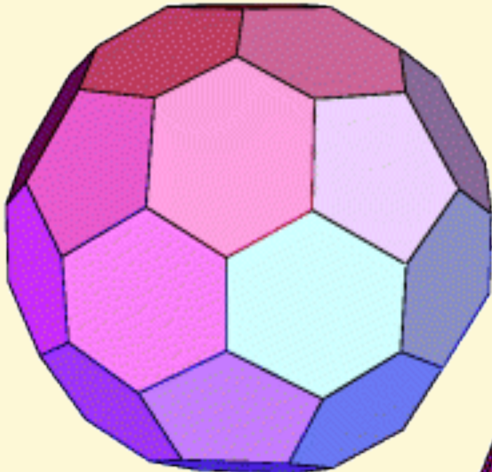
DCMIP Models: Triangular Grids

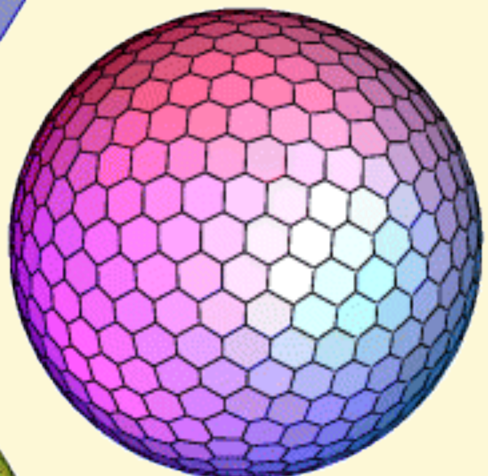


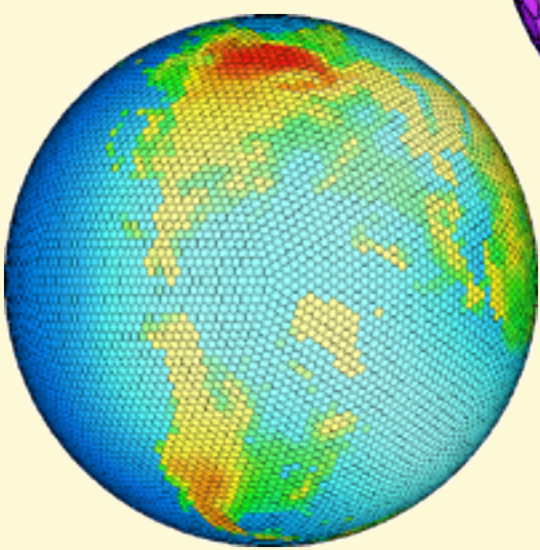
Spherical geodesic or icosahedral-based grids

- ICON
  (Max-Planck Institute, German Weather Service)
- DYNAMICO
  (IPSL, Paris)

DCMIP Models: Hexagonal Grids



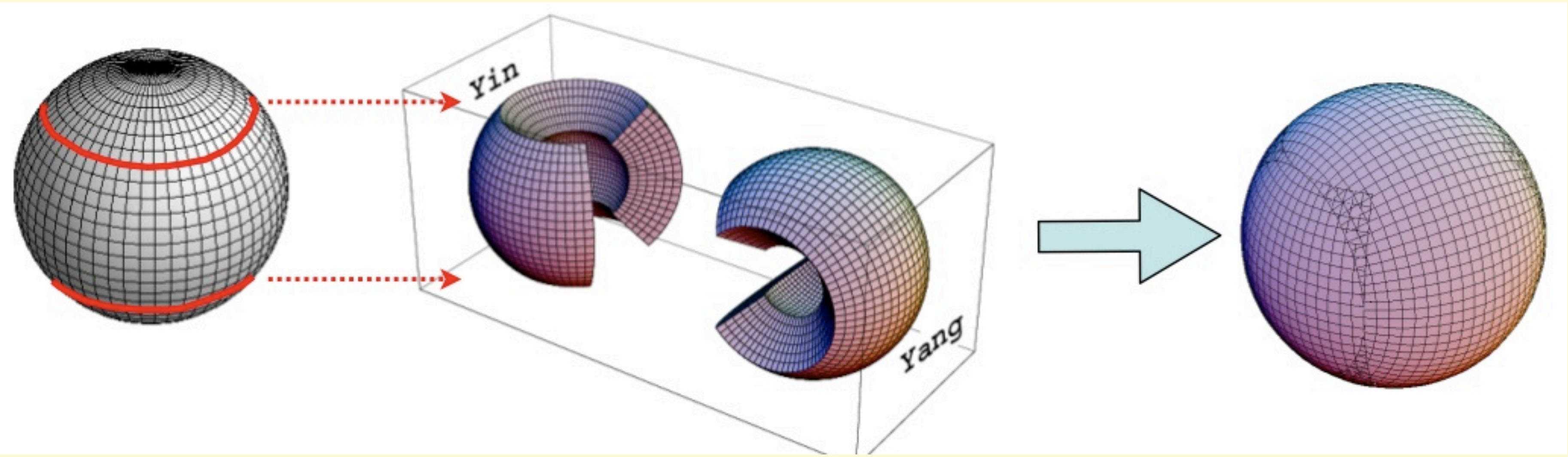




Hexagonal or spherical Voronoi grids

- MPAS (NCAR)
- OLAM (University of Miami)
- NICAM (RIKEN, JAMSTEC)
- UZIM (Colorado State University)

DCMIP Models: Yin-Yang Grid



- Yin-Yang grid is a composite grid, e.g. used at Environment Canada in the newest version of the model GEM
- Main issue: how do you glue the seam together, so that the model does not generate numerical noise, does not degrade to first-order accuracy or loses its mass conservation properties

Other Composite Meshes

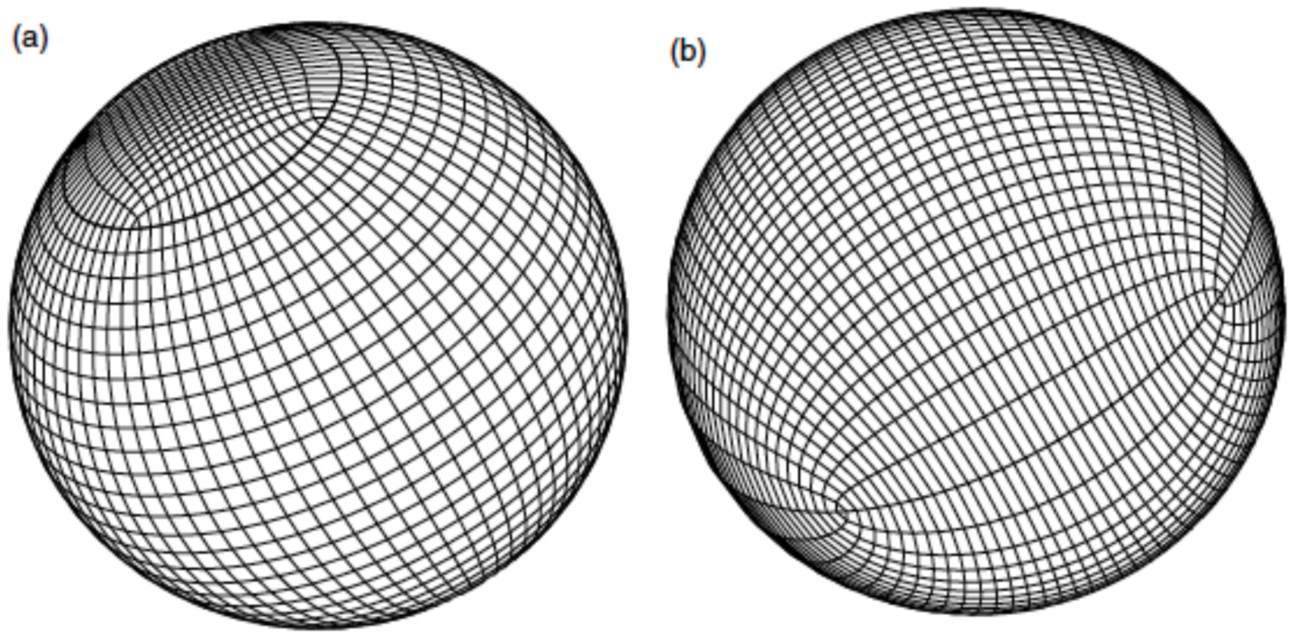
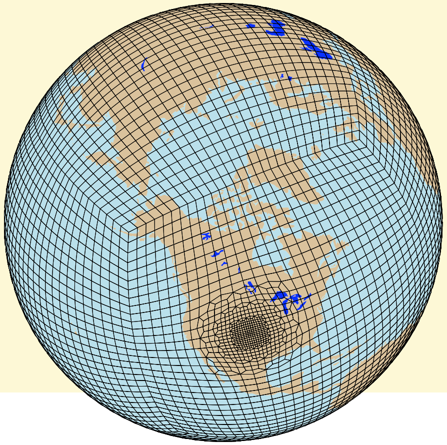
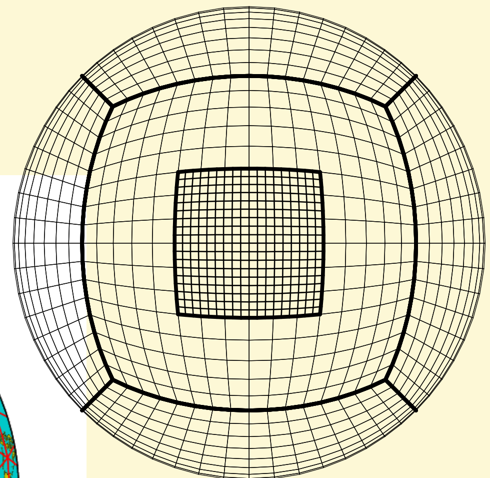


Figure 7. Composite meshes composed of a lat–lon tropical belt and twin polar-cap meshes: (a) where the meshes are joined at latitudes $\pm 60°$, and (b) the limiting case where the lat–lon tropical belt shrinks to zero, and the twin polar-cap meshes are hemispheric and join at the Equator. The polar-cap meshes are obtained by conformally mapping a square Cartesian mesh to a circle before stereographically projecting onto the polar caps.

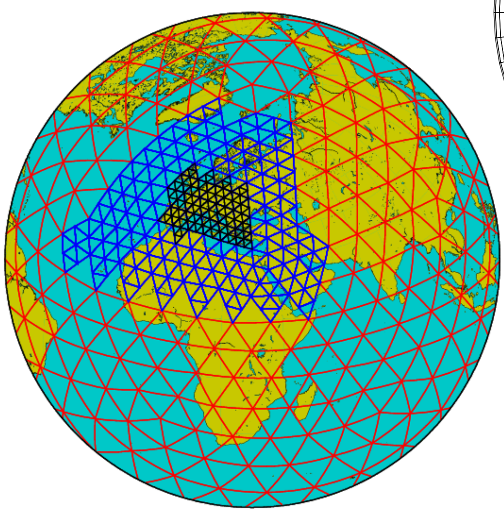
More information on map projections: http://en.wikipedia.org/wiki/Map_projection

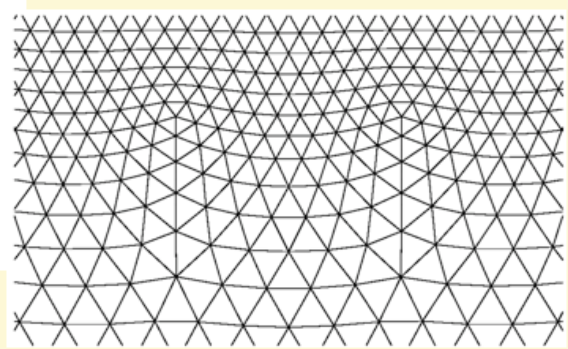


DCMIP Models: Nested Grids

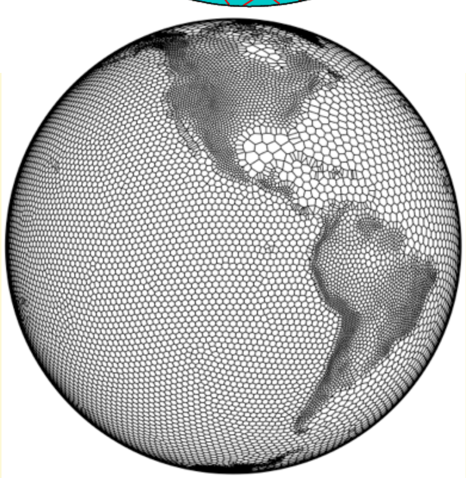


Models with optional variable-resolution-grid via embedded high-resolution regions

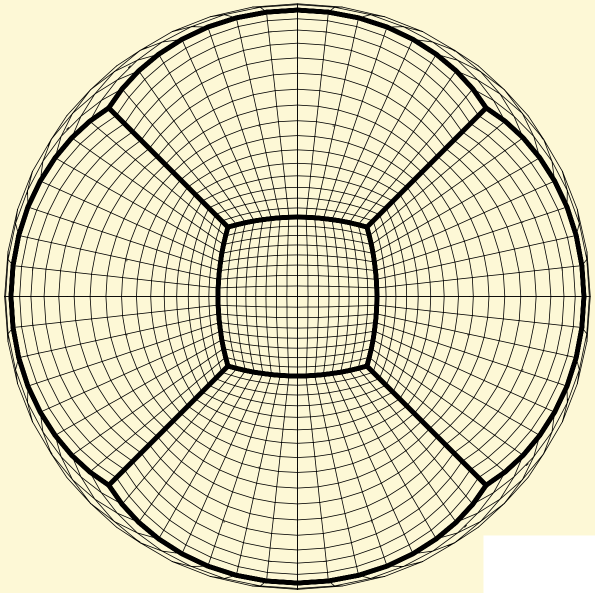




- CAM-SE
- FV3-GFDL
- ICON
- OLAM
- MPAS
- Chombo (moving nests: AMR)

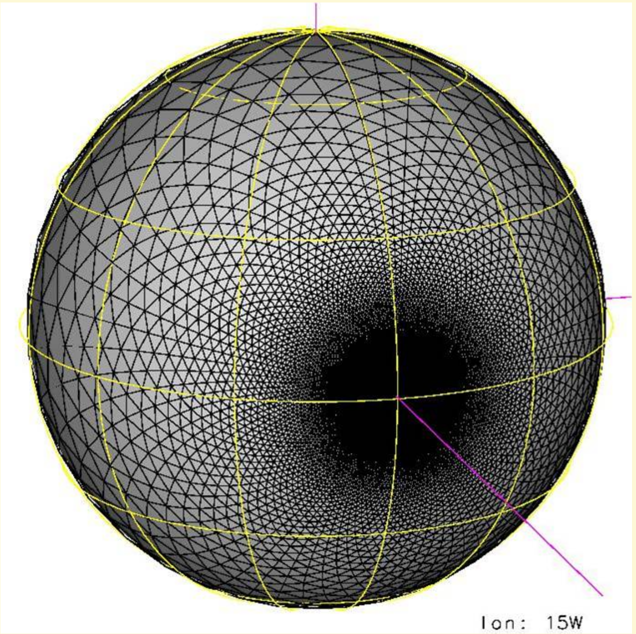


DCMIP Models: Stretched Grids



Models with optional variable-resolution grids via **stretched grids**

- FV3-GFDL
- NICAM



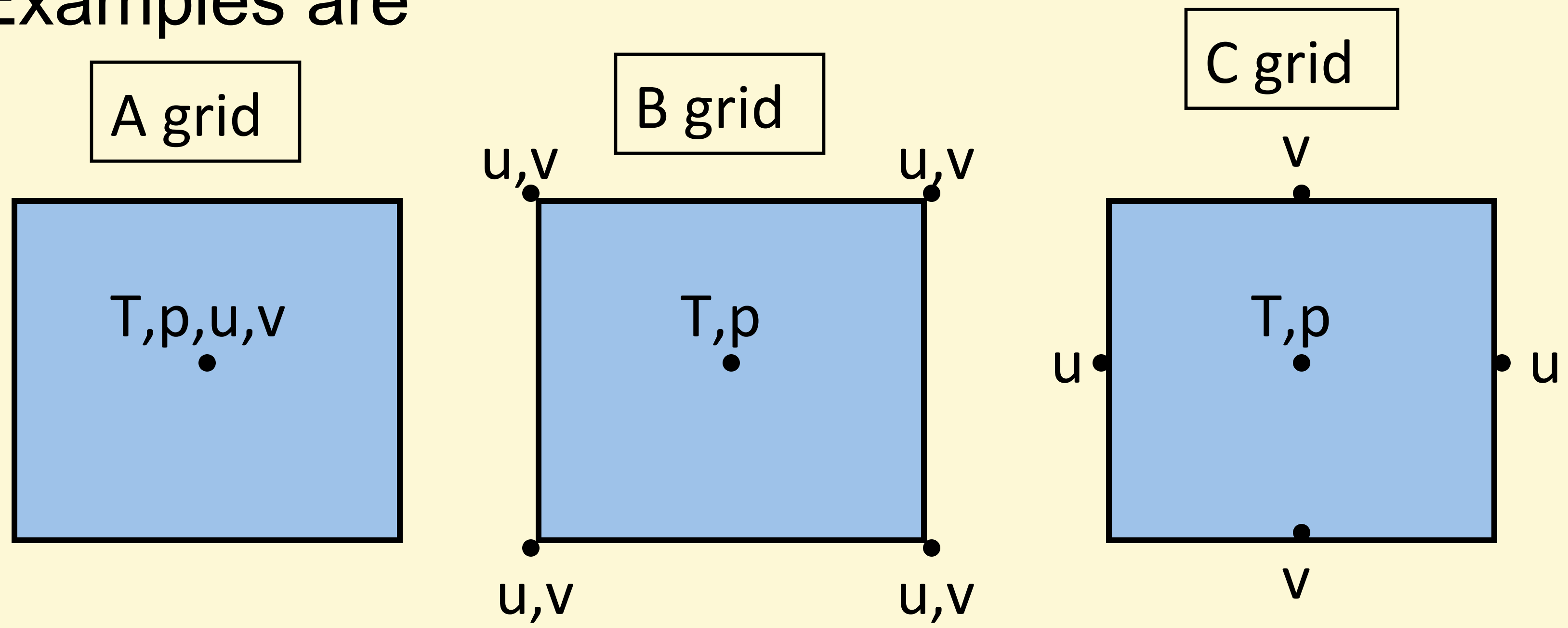
Popular stretching method is the Schmidt transformation

The pursuit of the ‘perfect’ positions of variables in the discrete system

- Having decided on the basic distribution of grid points, a choice has to be made as to how to arrange the different prognostic variables on the grid
- Most obvious choice of representing all variables at the same point has disadvantages (the so-called A grid)
- There are many choices, called:
  A, B, C, D, E, Z or ZM grid (the first five are based on a classification by Akio Arakawa (UCLA))

Example: Grid staggerings

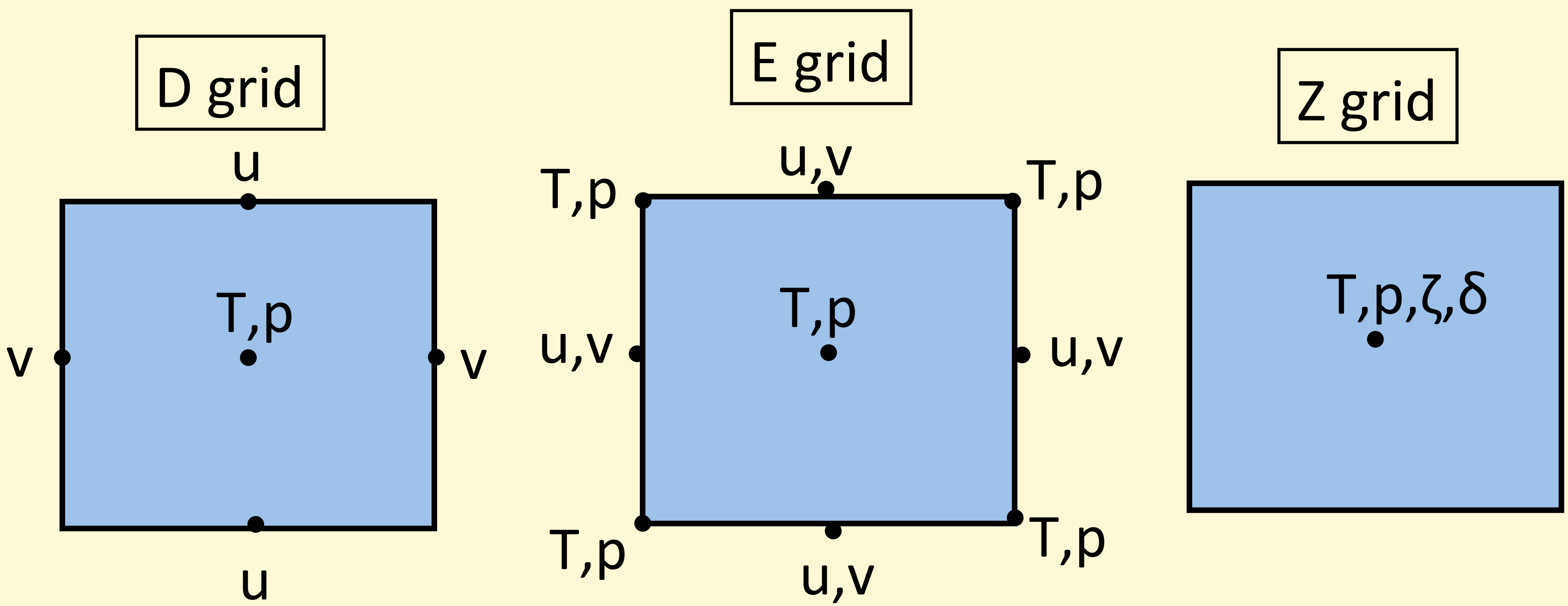
- Many choices how to place scalars and vector winds in the computational grid
- Examples are



- Staggerings determine properties of the numerical schemes: dispersion and diffusion properties
- Additional staggering options in the vertical

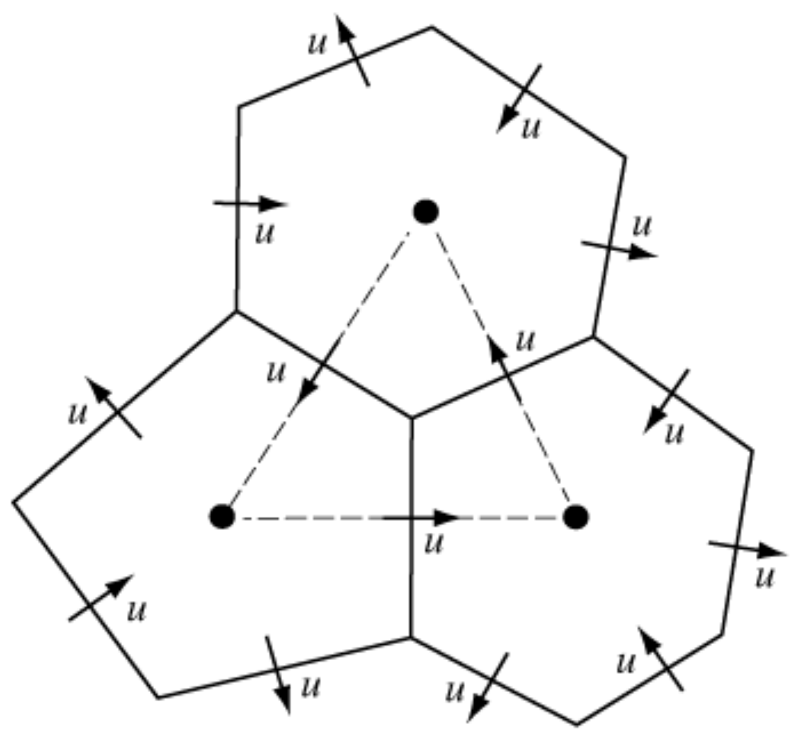
Example: Grid staggerings

- Many choices how to place scalars and vector winds in the computational grid
- More examples are



Grid staggerings on non-quadrilateral grids

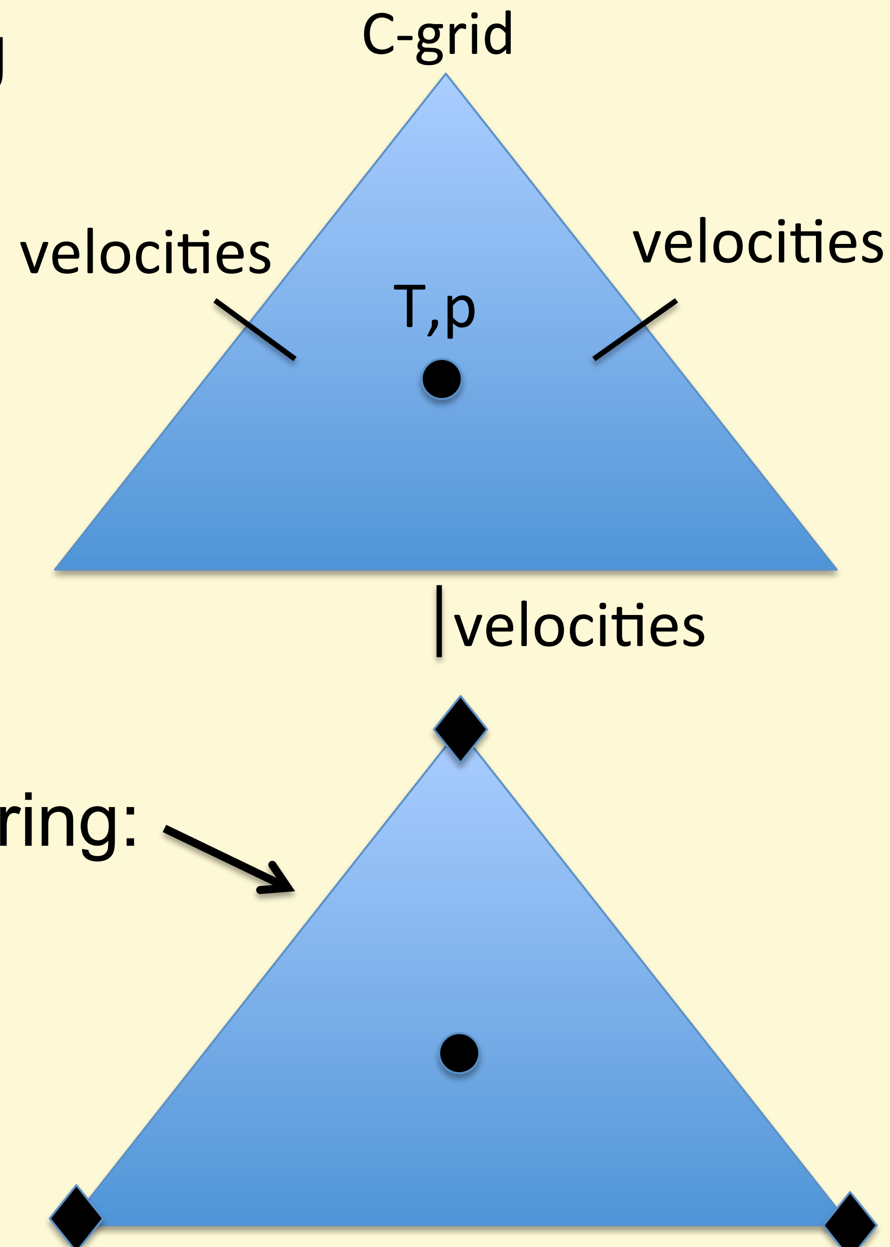
- It is more complicated to think about staggered grids for non-quadrilateral grids
- Example: C-grid staggering on a Voronoi mesh (here equivalent to a hexagonal grid with 12 embedded pentagons)
- Scalars are defined at midpoints (filled circles)
- The tangential and normal velocity components are defined at the edges



C-grid staggered variables on the horizontal Voronoi mesh. Normal velocities are defined on the cell faces and all other scalar variables are defined at the cell centers. Vertical vorticity is defined at the cell vertices.

Grid staggerings on non-quadrilateral grids

- Example: C-grid staggering on a triangular grid
- Scalars are defined at midpoints (filled circles)
- The tangential and normal velocity components are defined at the edges (line)
- Decisions for A-grid staggering: Do you place all variables at the midpoint (circle) or corner points, leads to different grid distances



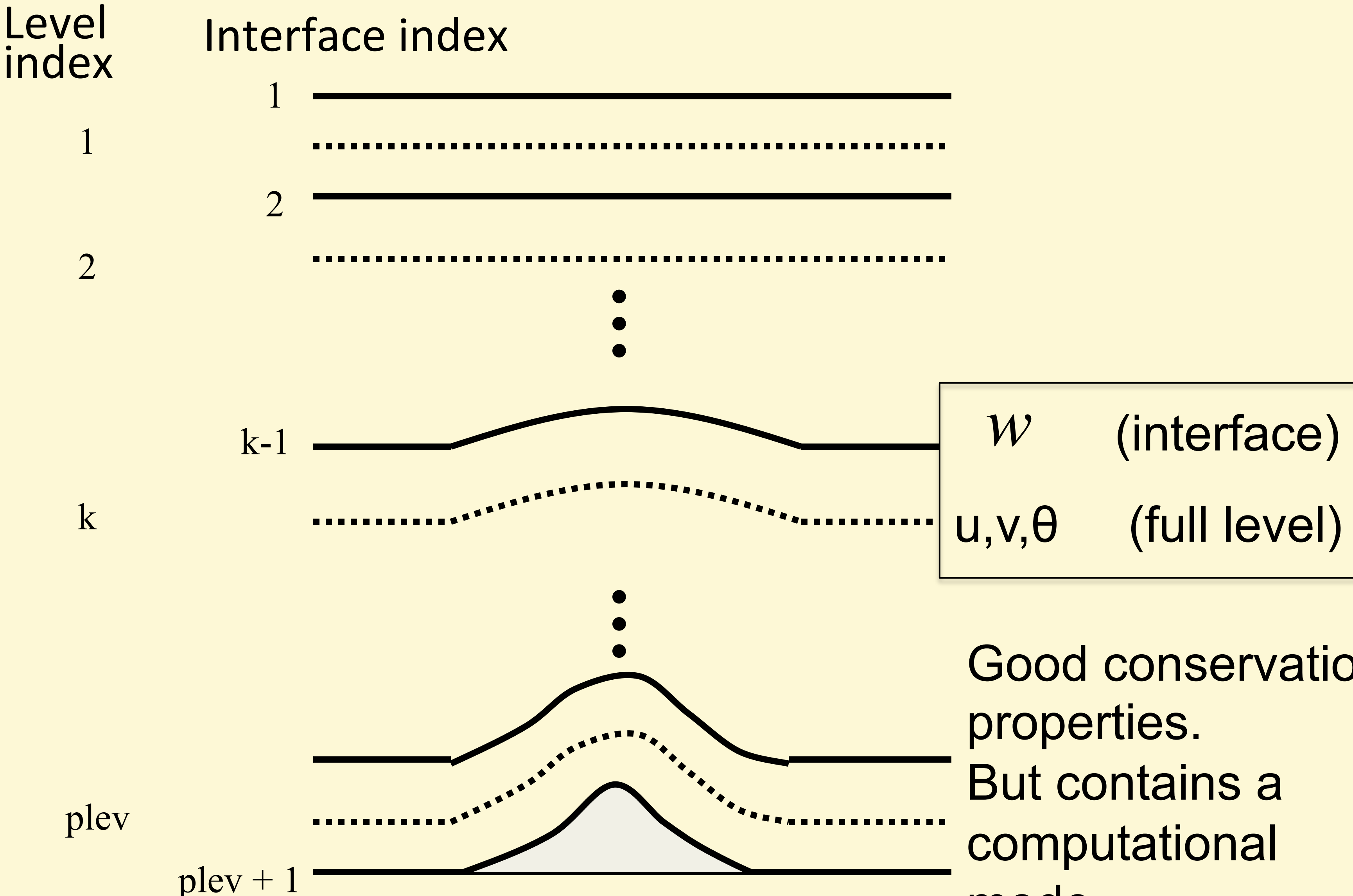
Choice of the vertical coordinate

The choices for the vertical coordinate are:

- Height
- Pressure
- Hydrostatic mass
- Isentropes
- Hybrids of the coordinates listed above
- Floating Lagrangian coordinate
- Topography-following variants
- Shaved cells, step coordinate

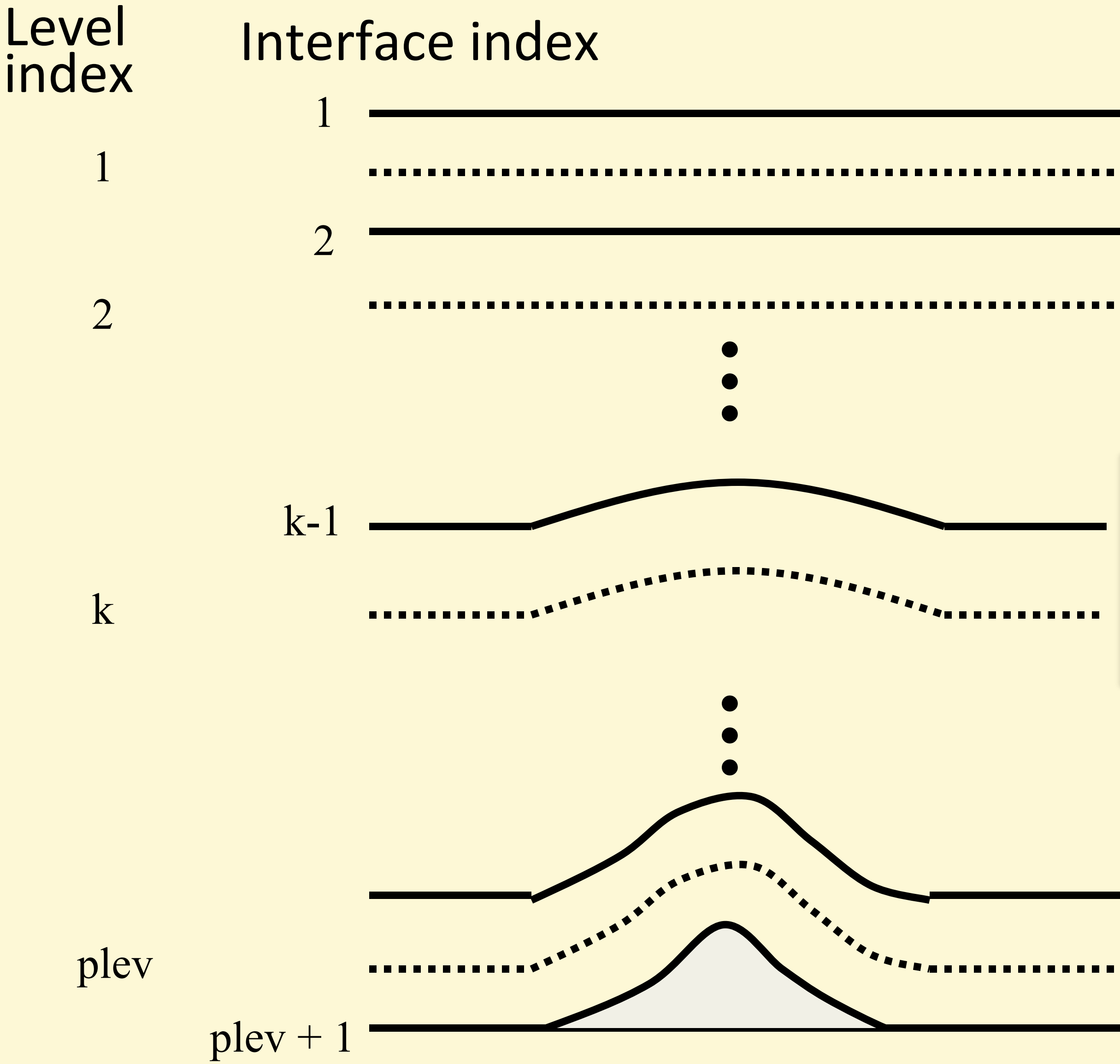
- **Requirement:** the new vertical coordinate needs to be **monotonic**
- Whatever we choose it requires a coordinate transformation and modifies the equations of motion

Vertical grid staggerings: **Lorenz**



Good conservation properties.
But contains a computational mode

Vertical grid staggerings: **Charney-Phillips**



w, θ (interface)

u,v (full level)

The pursuit of the ‘perfect’ numerical scheme

- We want: high order of accuracy, but computationally cheap method
- We need: discretizations in space x,y,z and time t

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} + \frac{\partial (\rho w)}{\partial z} = 0$$

- Many space discretization philosophies:
  – Finite difference methods (FD)
  – Finite volume methods (FV)
  – Finite (spectral) element methods (FE, SE)
  – Spectral methods (spectral transform or double Fourier methods)
- Often: different spatial methods are used in the horizontal and vertical directions

The pursuit of the ‘perfect’ numerical scheme

- Phase errors and damping should be small (often a compromise)
- Explicit time-stepping scheme is ‘easy’ to program, but it will only be conditionally stable and so the choice of time step is limited
- Implicit time-stepping schemes are absolutely stable; however at every time step a system of simultaneous equations has to be solved
- More than two time levels (current and future time)? If yes, we get additional computational modes and possibly separation of the solution at odd and even time steps. Higher storage (memory) requirements.

Abstract View of a GCM

Time tendency from the dynamical core (adiabatic)

Time tendency from physical parameterizations (diabatic)

$$\frac{\partial \psi}{\partial t} = Dyn(\psi) + Phys(\psi) + \boxed{F_\psi}$$

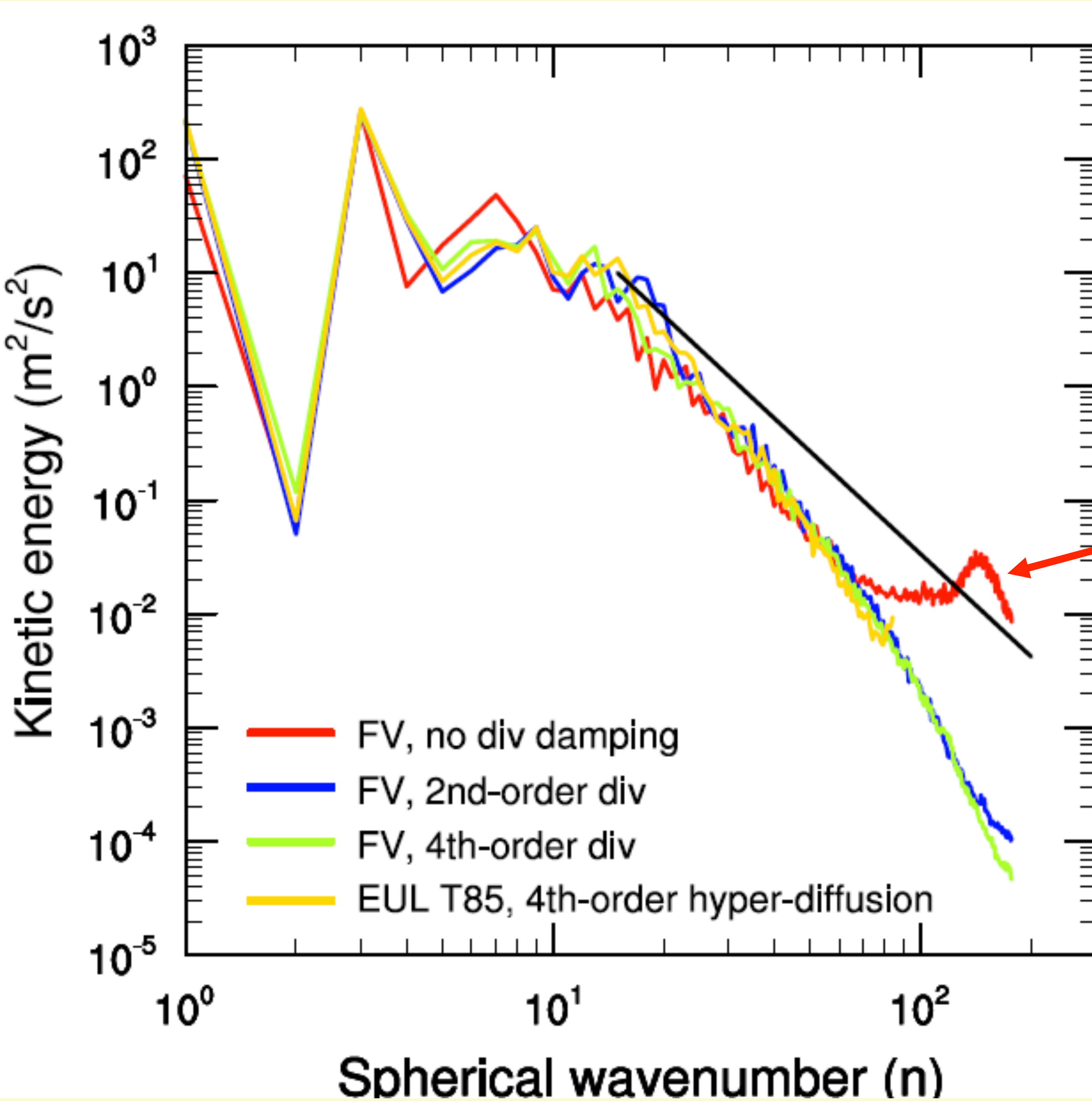
Time tendency of the forecast variable ψ

Time tendency from dissipative mechanisms (mostly considered part of the dynamical core)

Diffusion/Dissipation/Filters in Dynamical Cores

- All dynamical cores need some form of dissipation, either explicitly added or implicitly included via the choice of the numerical scheme
- This is due to the truncation of the spatial scales:
  - Dissipation is needed to prevent an accumulation of energy at the smallest grid scales
  - This prevents numerical instabilities
- Dissipation mechanisms
  - are often hidden in the dynamical cores
  - are rarely fully documented in publications (maybe in technical reports), ask your mentor
  - and their coefficients are often empirically determined and resolution-dependent (‘tuning’ knobs in the dynamical cores), no physical basis

Example Analysis Technique: Kinetic energy spectra



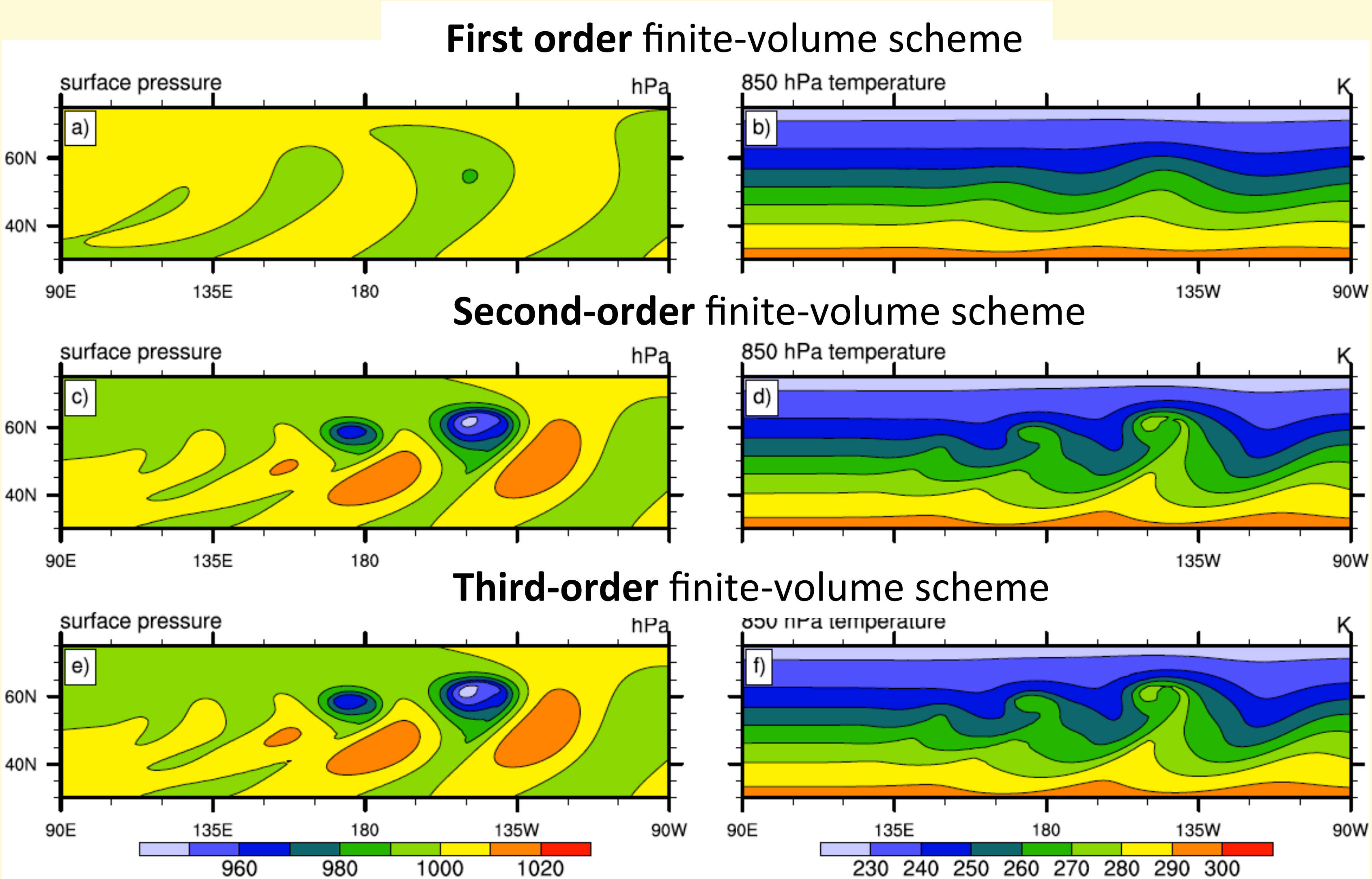
Very harmful: Accumulation of energy at small scales in this simulation

700 hPa KE spectra from CAM-FV and CAM-EUL baroclinic wave simulations at 1°x1° and T85

Broad Spectrum of Dissipation Techniques Used

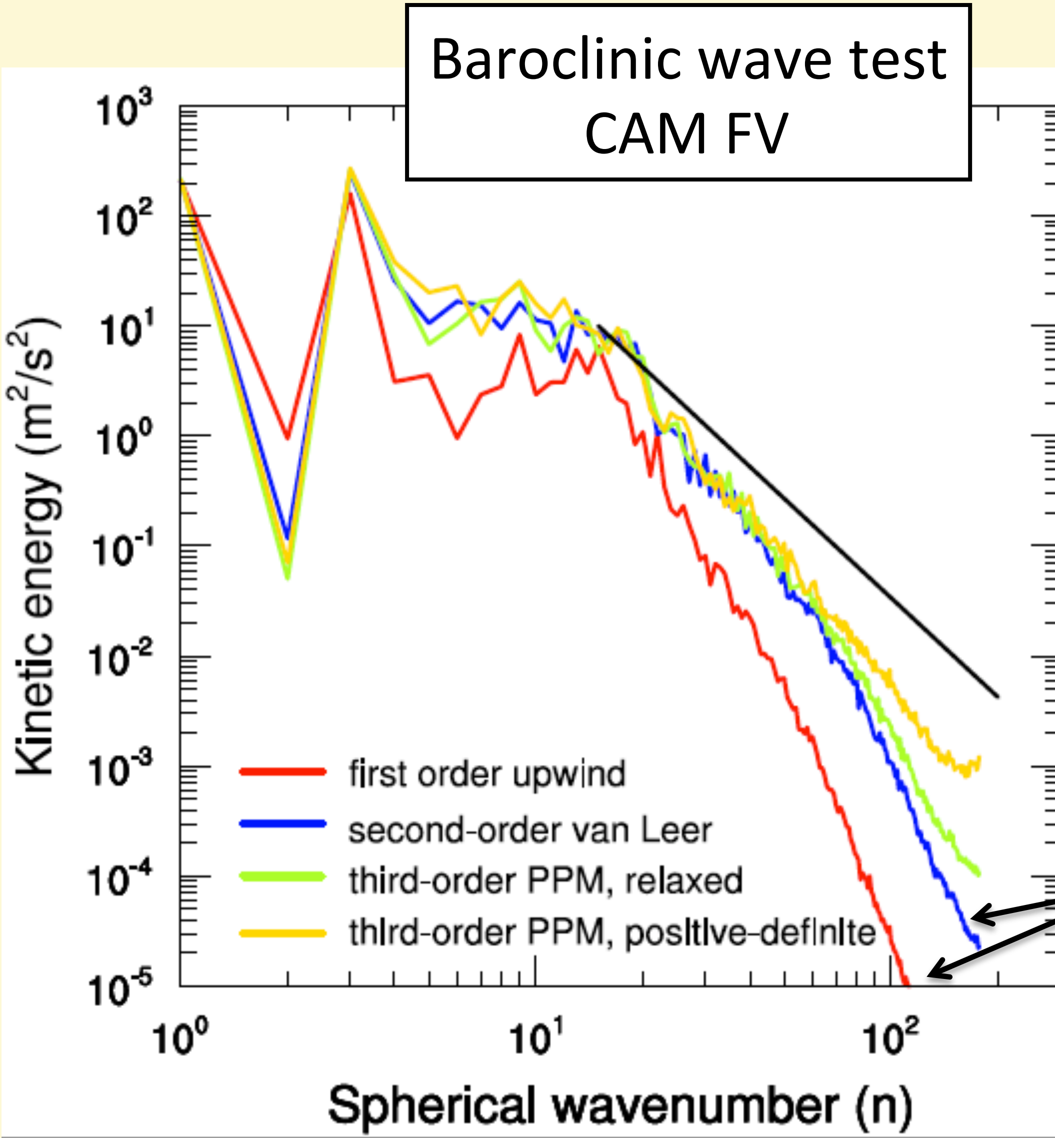
- **Explicitly added** dissipation mechanisms
  - Horizontal diffusion or hyper-diffusion
  - Divergence damping
  - Vorticity damping
  - External mode damping
  - Rayleigh friction and sponge layers near the model top
- **Implicit numerical** dissipation
  - Variation of the order of accuracy
  - Off-centering
  - Monotonicity constraints and flux limiters
  - Damping by interpolations in semi-Lagrangian schemes
- **Filters:**
  - Spectral Fast-Fourier-Transform (FFT) filters
  - Digital filters: e.g. Shapiro filters
  - Time filters: e.g. Asselin-filter
- ***A posteriori* Fixers:** Mass, tracer mass, total energy

Example of Implicit Diffusion: Order of Accuracy



JW (2006) Baroclinic wave test: CAM FV 1°x 1° L26 $T_{850\ hPa}$ at day 9

Measuring the Impact of Implicit Diffusion via KE Spectra: Order of Accuracy

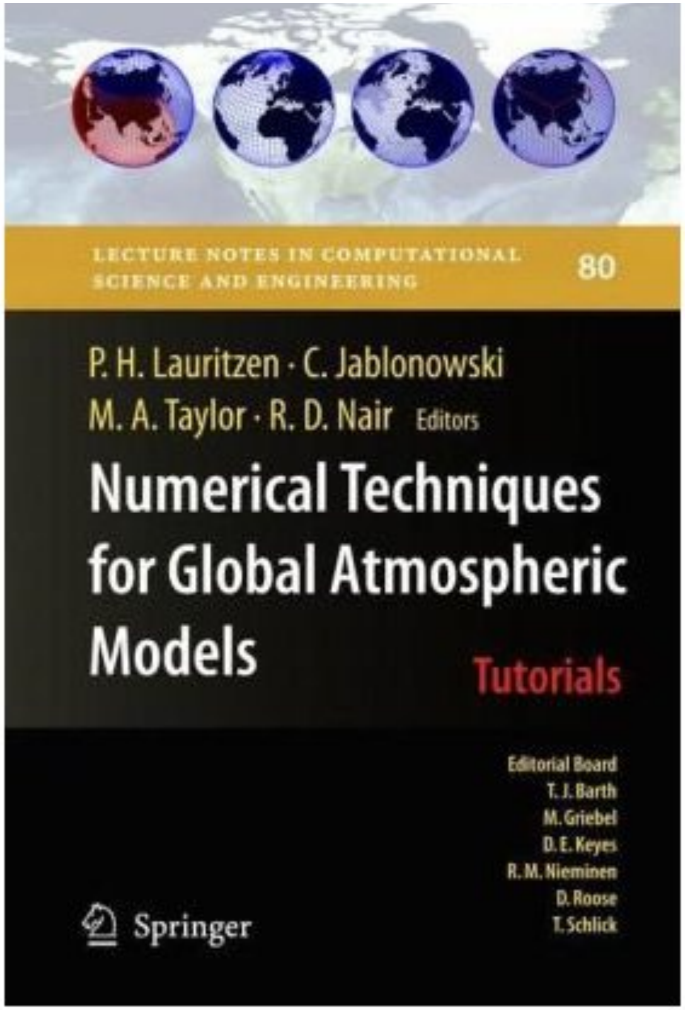


- 700 hPa kinetic energy spectrum (day 30) at 1° horizontal resolution
- Third-order (PPM)
- Second-order (van Leer scheme)
- First-order (upwind scheme)
- Tails of 1st -order and 2nd-order schemes drop faster due to damping effects of the implicit diffucion

Overview of Most Diffusion/Dissipation Techniques

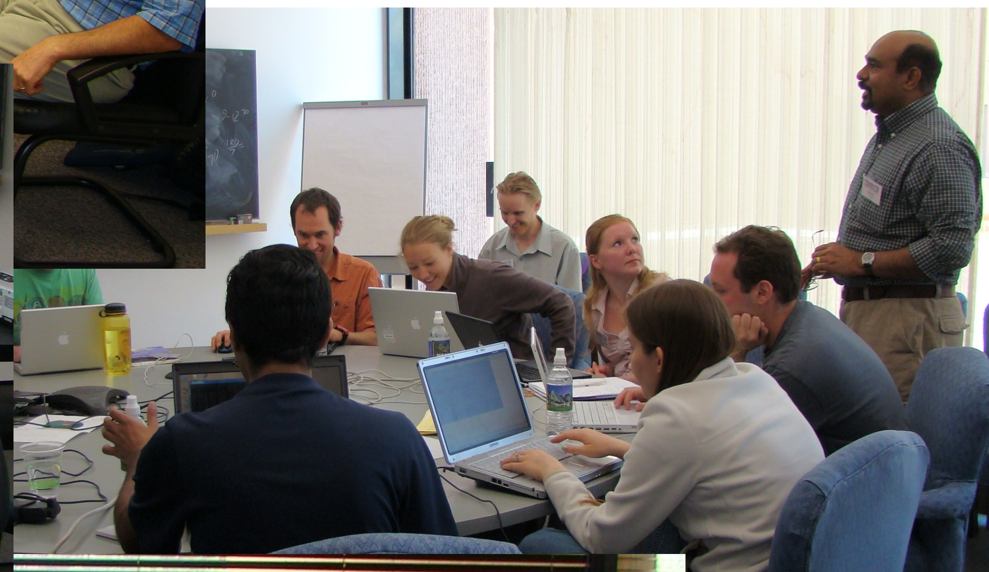
- Springer book based on the 2008 NCAR Dynamical Core Summer School
- 16 chapters with contributions from about 25 authors (> 550 pages)
- Chapter 13 by Christiane Jablonowski and David L. Williamson: **The Pros and Cons of Diffusion, Filters and Fixers in Atmospheric General Circulation Models,** pp. 381- 493
- Chapter 14 by William Skamarock: **Kinetic Energy Spectra and Model Filters**, pp. 495 - 512

published in 2011
also available as a Springer e-book online



http://www.springer.com/mathematics/computational+science+%26+engineering/book/978-3-642-11639-1

What to expect?
Snapshots of a typical DCMIP afternoon (in 2008)



And finally ….

- Have fun
- Make friends
- Meet your future colleagues or employers
- Build up your scientific network that fosters your future career
- Get help from your mentor and the organizing team if you get stuck
- Get help from NCAR's computing support in case of technical difficulties
- Enjoy the time in Boulder and learn something about dynamical cores in a unique framework

Modeling Resources

- There is no single resource that covers everything, but here are some examples for follow-up studies

  - Dave Randall, Colorado State University: Lecture Notes 2016
    An Introduction to Atmospheric Modeling
    http://kiwi.atmos.colostate.edu/group/dave/at604.html

  - Comprehensive UK Met Office Model Documentation, written by A. Staniforth and A. White and N. Wood and J. Thuburn and M. Zerroukat and E. Cordero and T. Davies and M. Diamantakis:
    Joy of U.M. 6.3 - Model Formulation
    http://research.metoffice.gov.uk/research/nwp/publications/papers/unified_model/

  - Peter. H. Lauritzen, Christiane Jablonowski, Mark. A. Taylor, Ram Nair (Eds.), Springer book, 2011:
    Numerical Techniques for Global Atmospheric Models
    also available online as a Springer ebook:
    http://link.springer.com/book/10.1007/978-3-642-11640-7/page/1

Resources: Selected Books

- Dale Durran
  Numerical Methods for Fluid Dynamics with Applications to Geophysics, 2nd edition, Springer, 2010 or 1st edition (1999), available online as a Springer ebook:
  http://link.springer.com/book/10.1007/978-1-4419-6412-0/page/1
- Warren M. Washington and Claire L. Parkinson
  An Introduction to Three-Dimensional Climate Modeling
  2nd edition, University Science Books, 2005
- Masaki Satoh
  Atmospheric Circulation Dynamics and General Circulation Models, Springer Praxis, 2004, or newer 2014 edition
  available online as a Springer ebook:
  http://link.springer.com/book/10.1007%2F978-3-642-13574-3
- Philip Mote and Alan O'Neill (Eds.)
  Numerical Modeling of the Global Atmosphere in the Climate System,
  NATO Science Series, Vol. 550, 2000

Resources: Selected Books

- Leo Donner, Wayne Schubert, Richard Somerville (Eds), 2010: Collection of model development and model analysis chapters. ***The Development of Atmospheric General Circulation Models Complexity, Synthesis and Computation*** http://www.cambridge.org/us/academic/subjects/earth-and-environmental-science/climatology-and-climate-change/development-atmospheric-general-circulation-models-complexity-synthesis-and-computation

- Kendal McGuffie, Ann Henderson-Sellers ***Climate Modeling Primer***, 3rd edition, Wiley (2005), easy to read, freely available online: http://onlinelibrary.wiley.com/book/10.1002/0470857617;jsessionid=8989D2C3EE2D6259D03EED3515F64D6D.d02t03

- DCMIP-2008, DCMIP-2012 and DCMIP-2016 web pages: https://www.earthsystemcog.org/projects/dcmip-2016/