

Overview of GCMs

David Randall



What is a GCM?

- ◆ **Global Climate Model**
 - ▲ **By definition, aimed at climate.**
 - ▲ **Must include both ocean and atmosphere.**

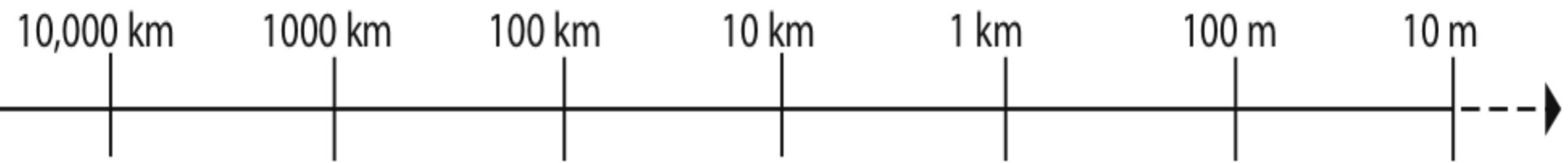
What is a GCM?

- ◆ **Global Climate Model**
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- ◆ **General Circulation Model**
 - ▲ **What does “general” mean, anyway?**
 - ▲ **Can be either ocean or atmosphere, or both.**
 - ▲ **Can be aimed at climate or NWP.**

What is a GCM?

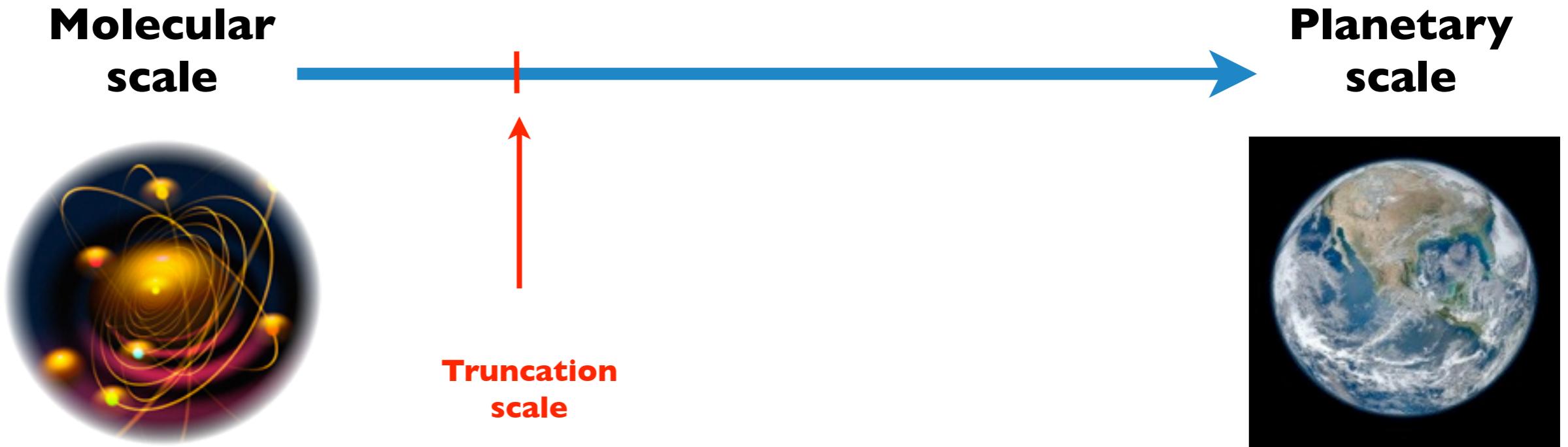
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 - ▲ Can be either ocean or atmosphere, or both.
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- ◆ **Global Circulation Model**
 - ▲ Accurate and flexible.
 - ▲ May be the best use of the acronym.

Range of scales



Planetary waves Extratropical Cyclones Mesoscale Convective Systems Cumulonimbus clouds Cumulus clouds Turbulence =>





The Earth System starts at the molecular scale and works up.

Modelers start at the planetary scale and work down.



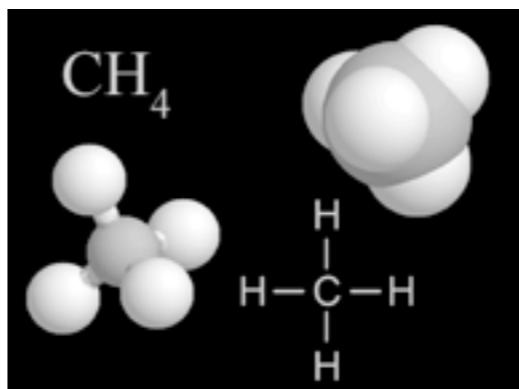
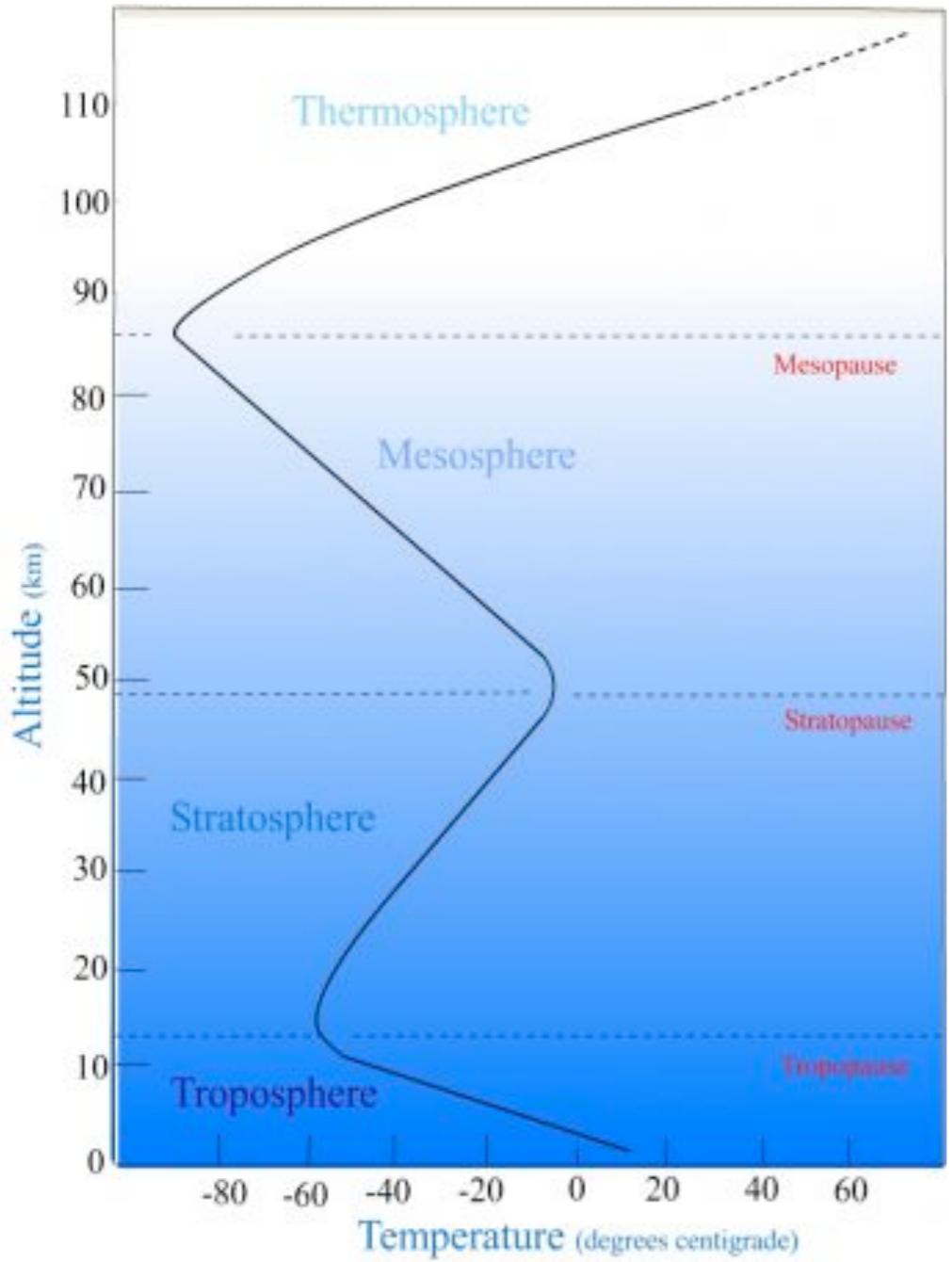
**Building a GCM involves
an ordered sequence of choices.**

Choices I

- ◆ **Scope**
 - ▲ **Stratosphere?**
 - ▲ **Chemistry?**
 - ▲ **Range of scales
(target resolution)?**
- ◆ **Equation set**
- ◆ **Prognostic variables**
- ◆ **Vertical coordinate**



Scope



Choices 2

- ◆ **Grid point or spectral**

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- ◆ **Discretization method**
 - ▲ **Finite volume**
 - ▲ **Spectral element**

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- ◆ **Sign preservation, monotonicity, etc.**

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These choices are coupled, so the order matters.

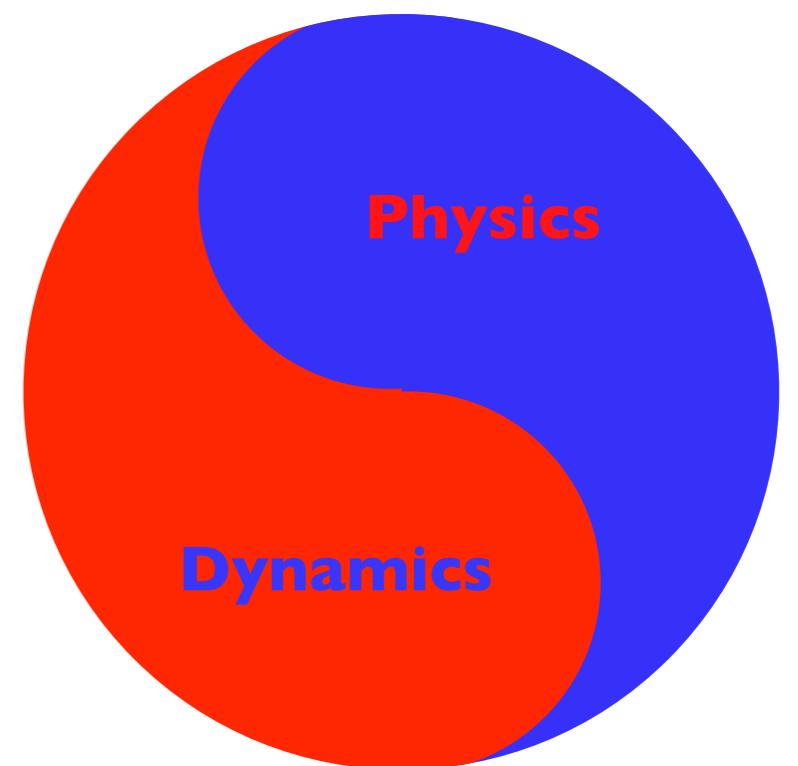
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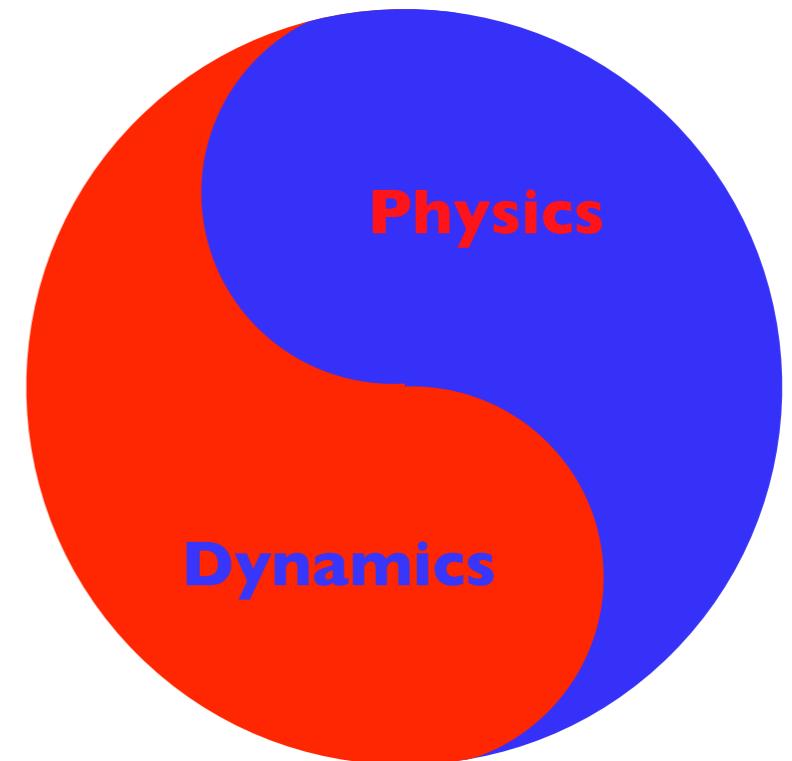
Other speakers will cover the topics listed on this slide.

Physics and dynamics



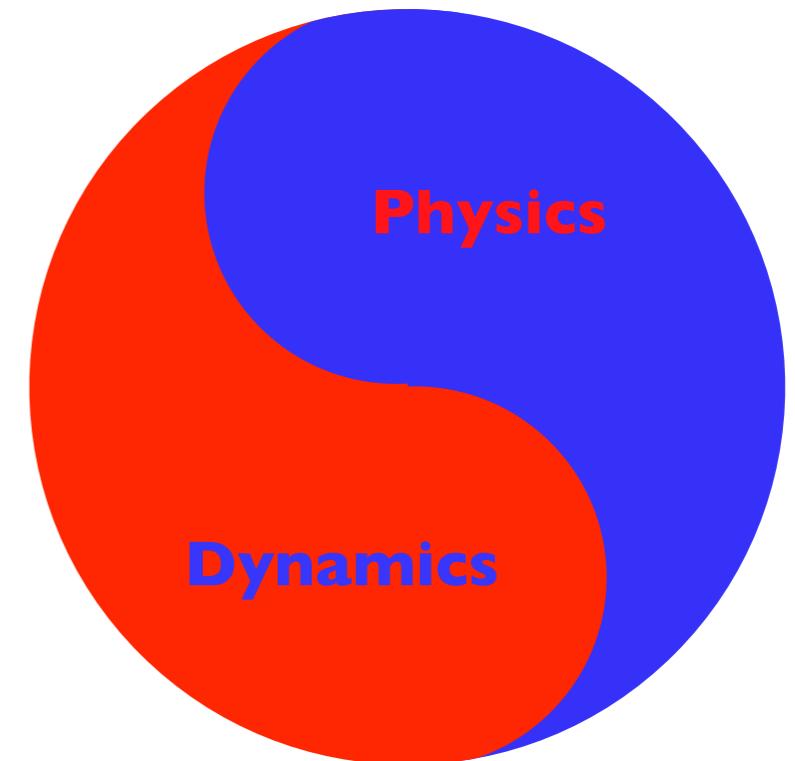
Physics and dynamics

- ◆ “**Physics**” refers to parameterized processes:
 - ▲ **Radiation**
 - ▲ **Vertical fluxes due to turbulence, especially in the boundary layer**
 - ▲ **Cumulus convection**
 - ▲ **Stratiform clouds & microphysics**
 - ▲ **Gravity-wave drag**
 - ▲ **Chemistry, including aerosols**
 - ▲ **Horizontal diffusion due to unresolved scales**



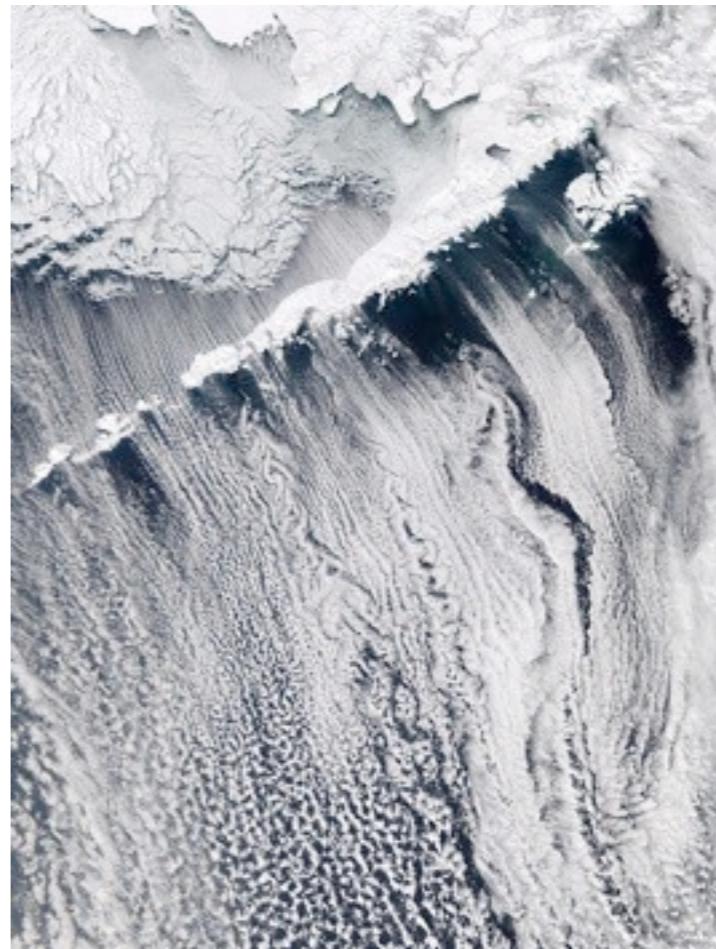
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- ◆ “**Dynamics**” refers to everything else.



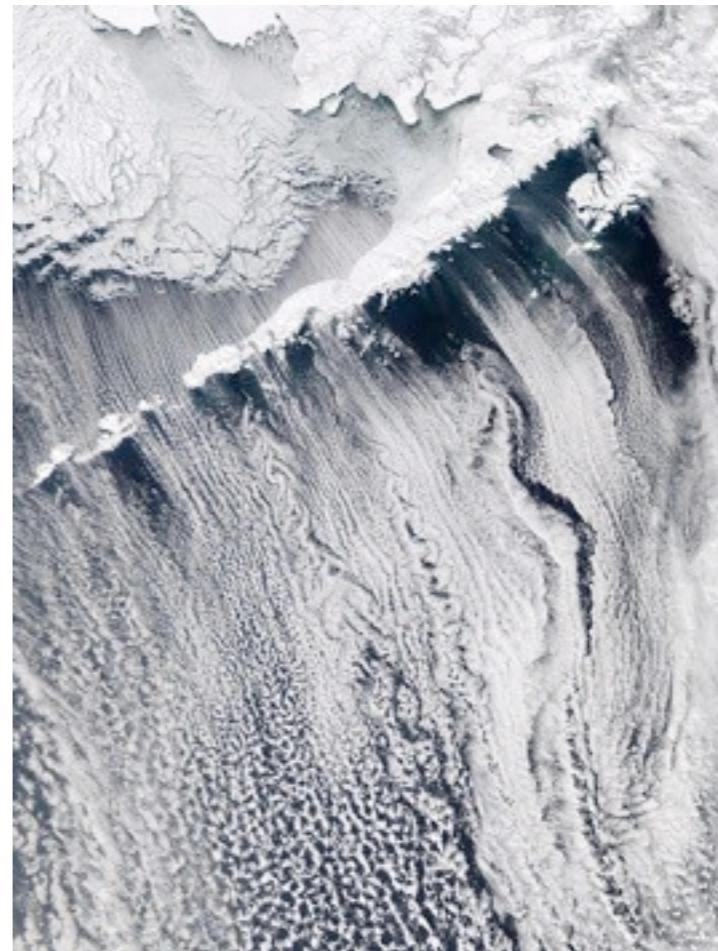
Dry dynamical cores

- ◆ **The equation of motion**
- ◆ **The continuity equation**
- ◆ **The thermodynamic energy equation**
- ◆ **Tracer equations**



Dry dynamical cores

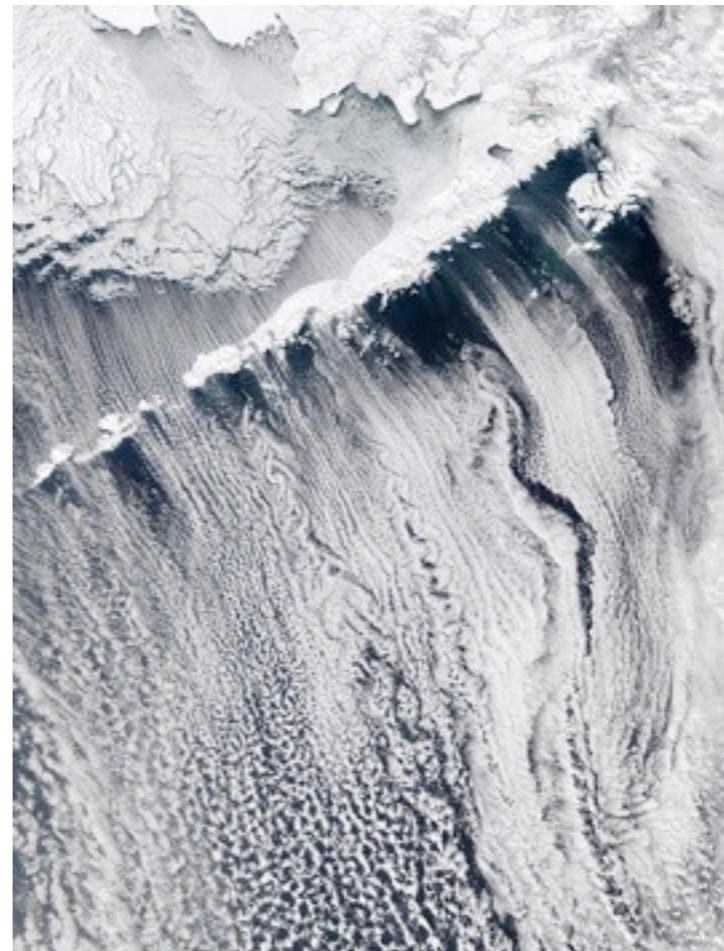
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Can be tested to some extent using the shallow water system.

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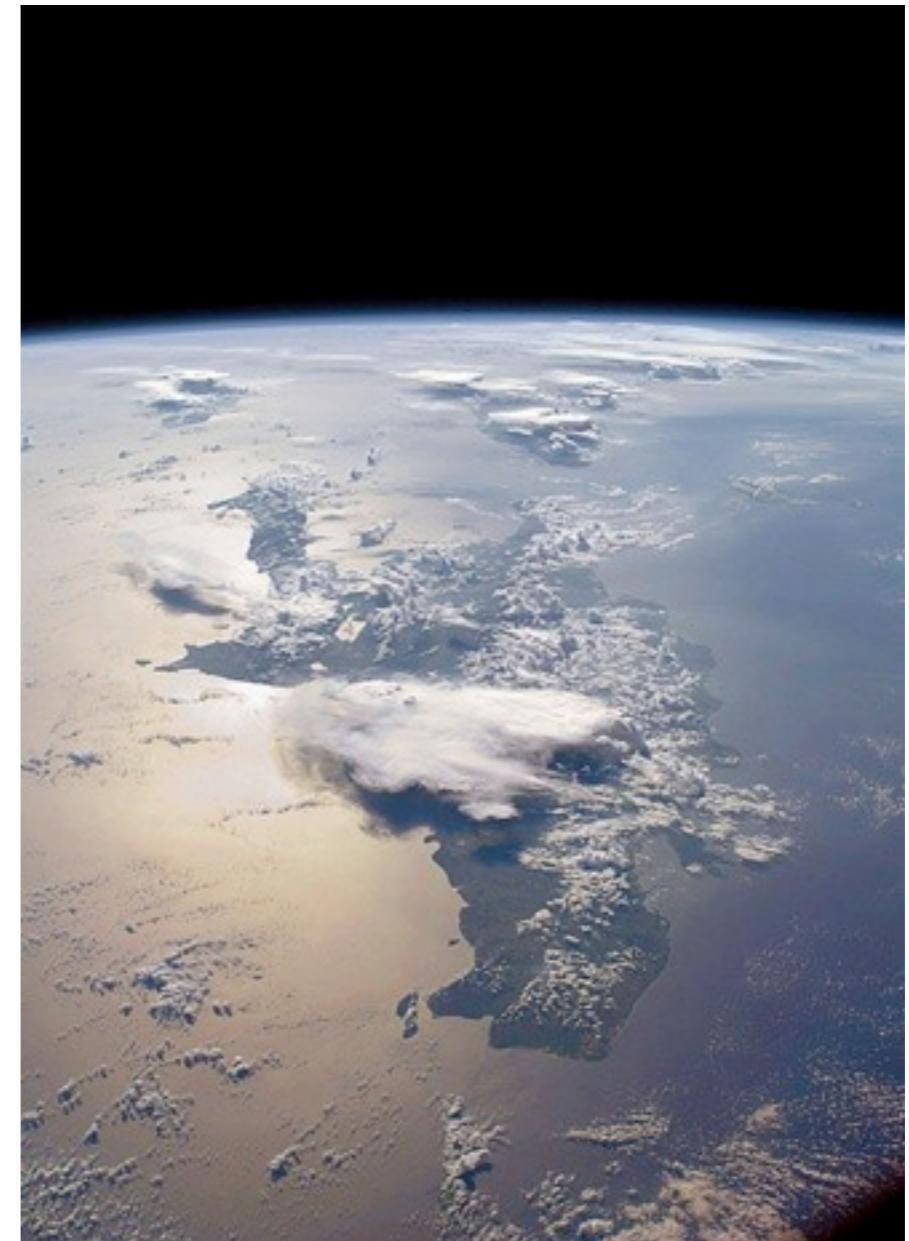


Can be tested to some extent using the shallow water system.

Results cannot be compared with observations, or with exact solutions except for highly idealized special cases.

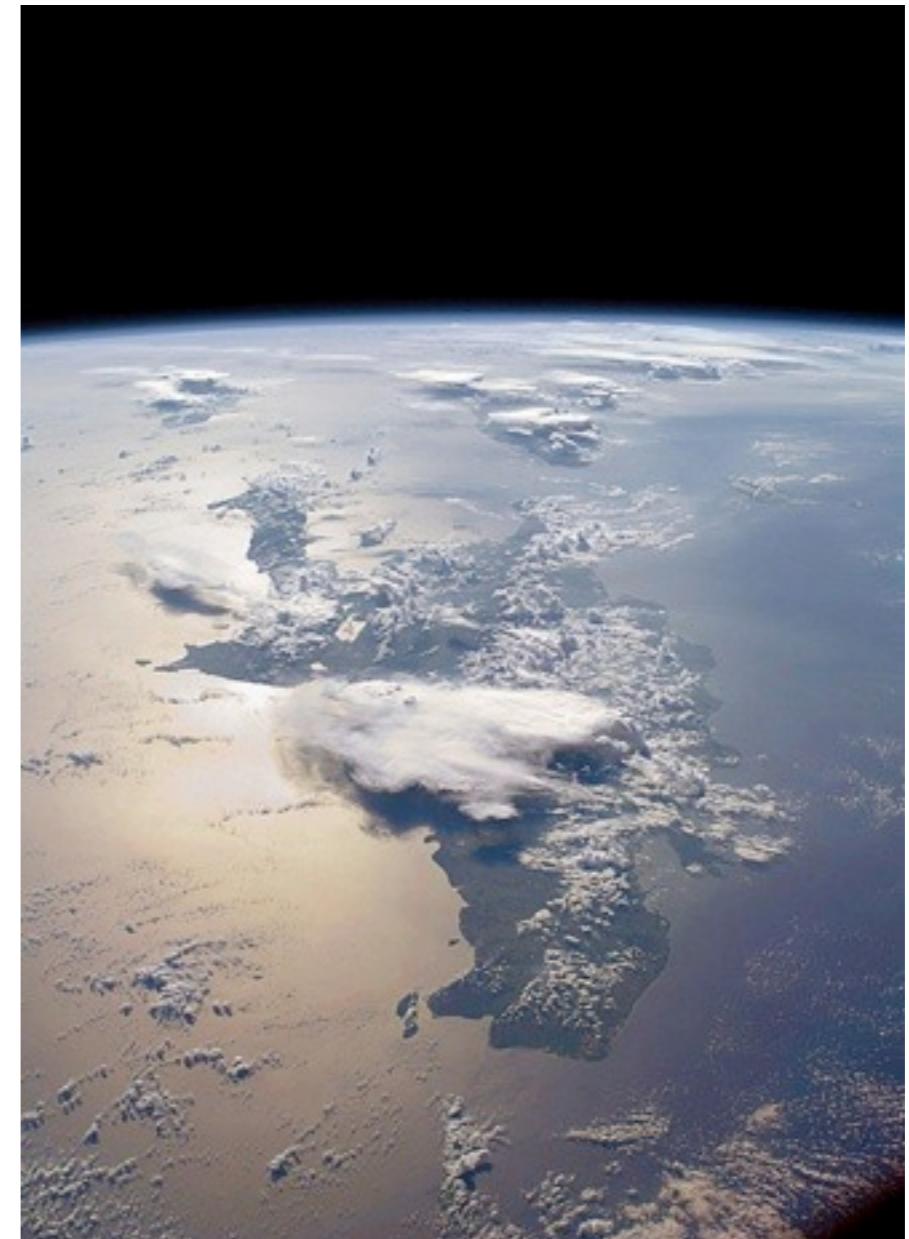
Choice of equation set

- ➊ Fully compressible (“exact”) system of equations



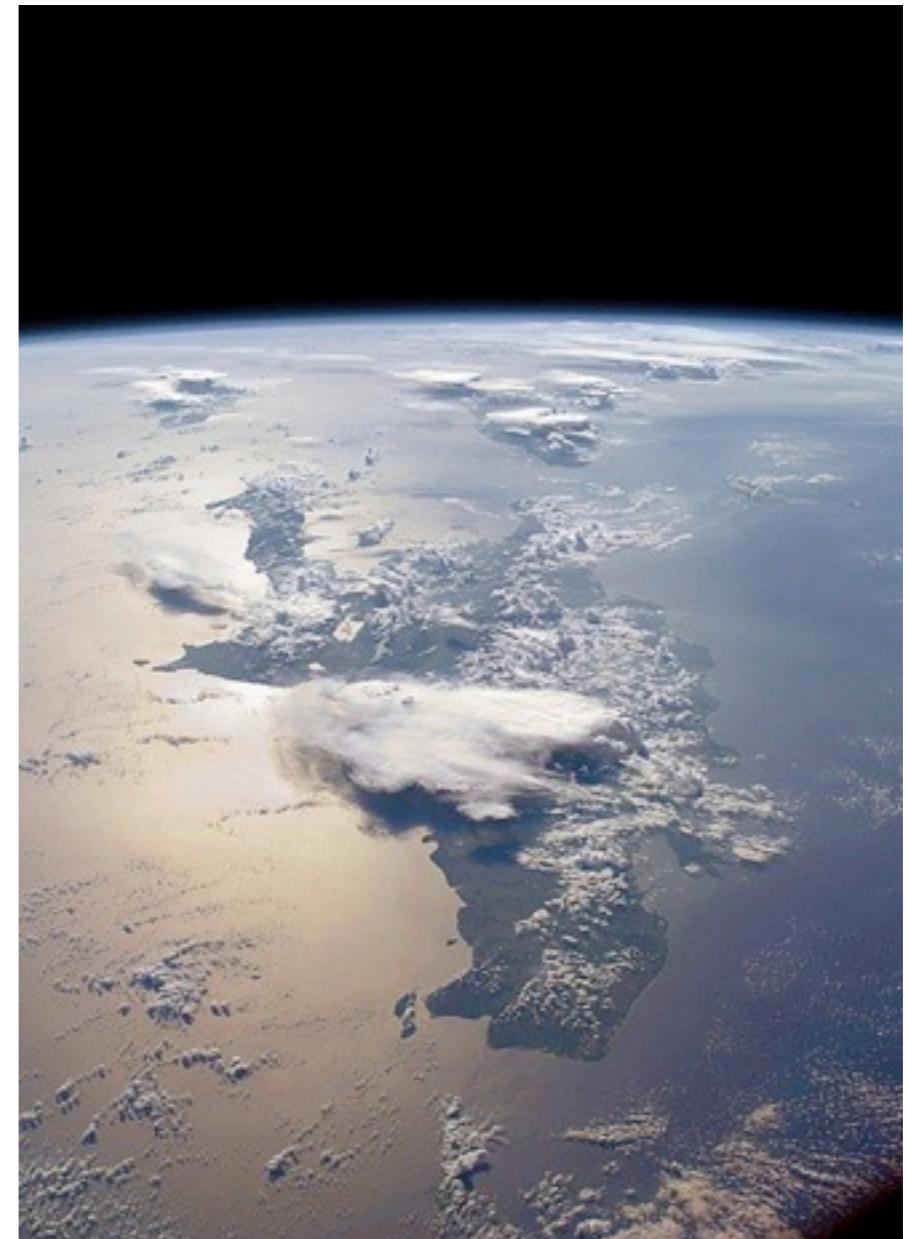
Choice of equation set

- ◆ **Fully compressible (“exact”) system of equations**
- ◆ **Quasi-static system with the “traditional” approximations**



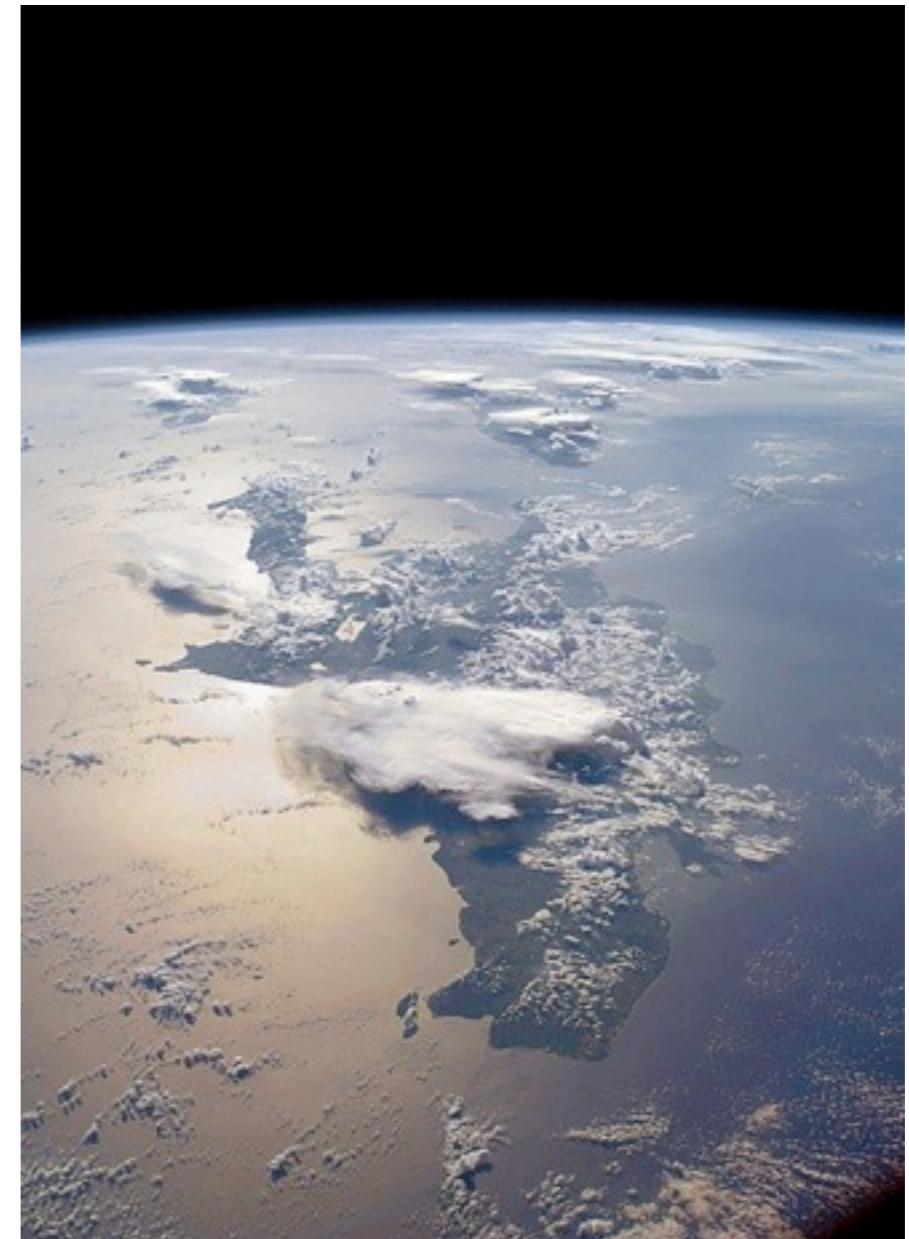
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- ◆ **Various anelastic systems**



Choice of equation set

- ◆ **Fully compressible (“exact”) system of equations**
- ◆ **Quasi-static system with the “traditional” approximations**
- ◆ **Various anelastic systems**
- ◆ **Alternative forms of the pressure-gradient force**



Choice of prognostic variables

- ◆ **Winds**



Choice of prognostic variables

- ◆ **Winds**
 - ▲ **Zonal, meridional, and vertical components**



Choice of prognostic variables

◆ Winds

- ▲ **Zonal, meridional, and vertical components**
- ▲ **Angular momentum instead of the zonal component**



Choice of prognostic variables

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- ▲ **Zonal, meridional, and vertical components**
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- ▲ **Vertical component of the vorticity, and divergence of the horizontal wind vector**



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◆ Thermodynamic energy

- ▲ **Temperature**



Choice of prognostic variables

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- ▲ **Temperature**
- ▲ **Potential temperature**



Choice of prognostic variables

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◆ Moisture

- ▲ **Water vapor mixing ratio**



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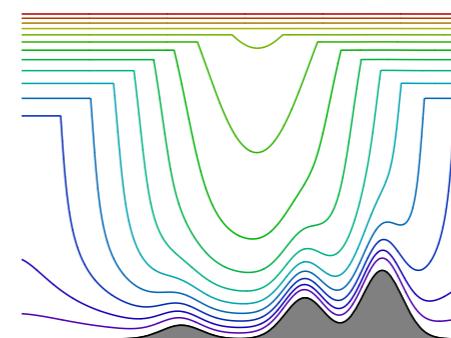
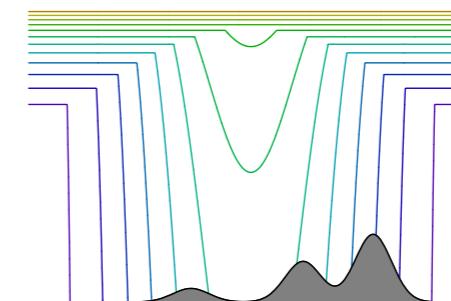
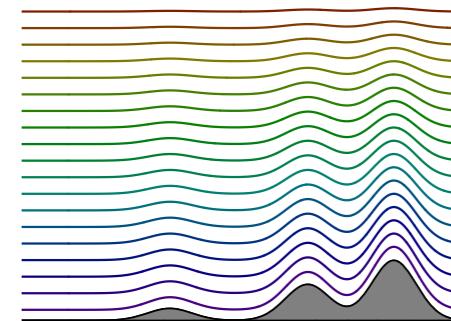
◆ Moisture

- ▲ **Water vapor mixing ratio**
- ▲ **Total water mixing ratio**
- ▲ **Condensed water species**



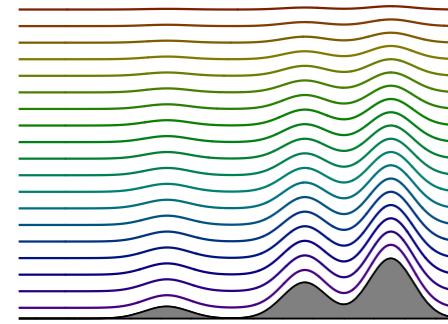
Choice of vertical coordinates

- ◆ Height

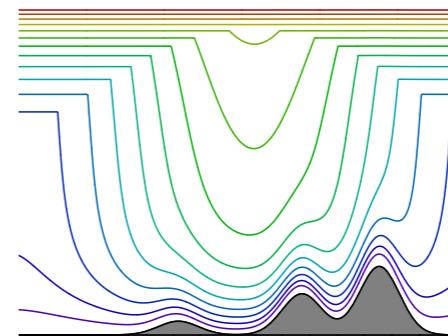
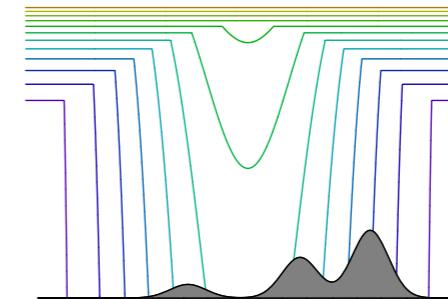


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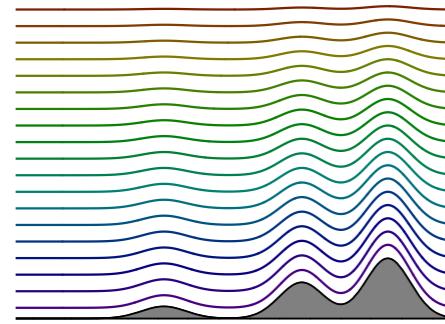


- ◆ Sigma

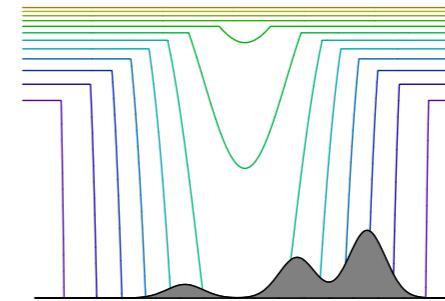


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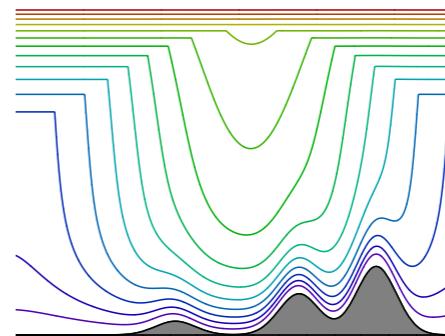
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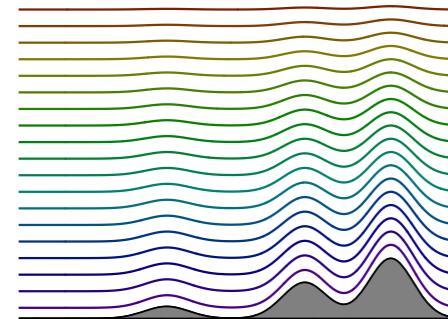


- ◆ Theta

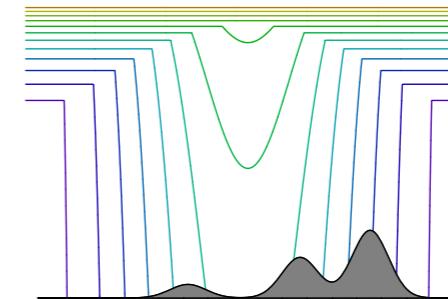


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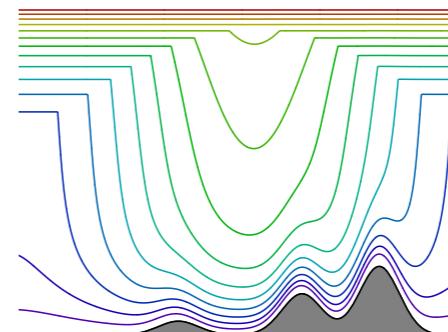
- ◆ Height



- ◆ Sigma



- ◆ Theta



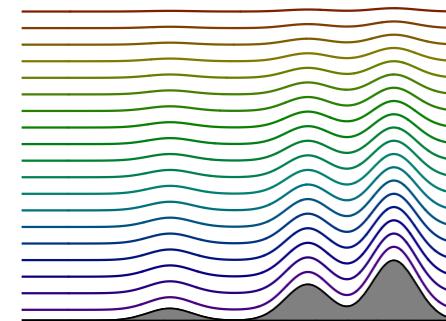
- ◆ Various hybrids

- ▲ Sigma-pressure

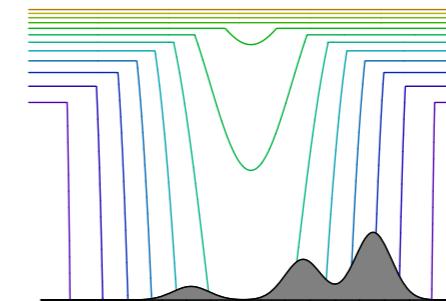
- ▲ Theta-sigma

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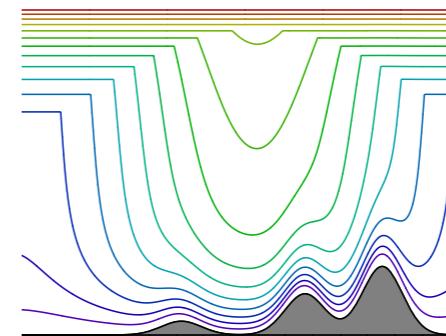
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- ◆ Sigma



- ◆ Theta



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- ▲ Sigma-pressure

- ▲ Theta-sigma

Mike Toy will discuss these topics on Wednesday.

Common approximations

- ◆ **Quasi-static approximation**
- ◆ **Thin atmosphere approximation**
- ◆ **Neglect of some Coriolis terms**
- ◆ **Neglect of the centrifugal acceleration**
- ◆ **Spherical Earth approximation**
- ◆ **Uniform gravity approximation**



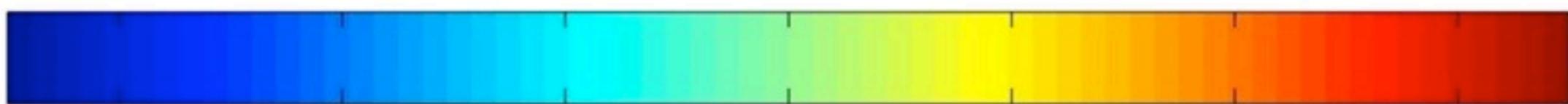
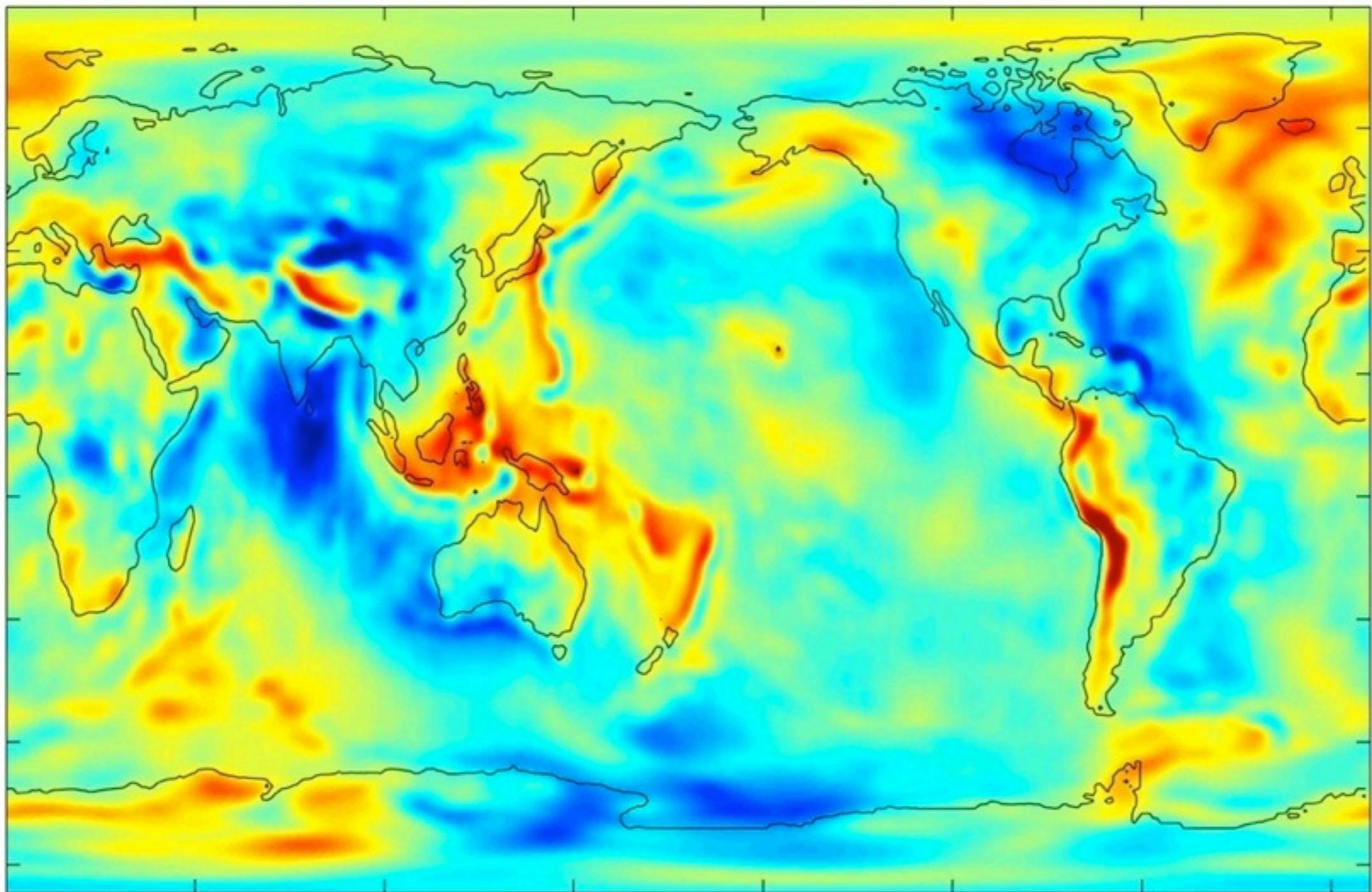
We are gradually eliminating these approximations.

The quasi-static approximation is the most useful, and will be the first to go.





In reality, the Equatorial radius is only about 20 km larger than the polar radius.



Gravity Anomaly (mGal)

The “exact” equations

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0$$

$$\frac{D\mathbf{V}}{Dt} = -2\boldsymbol{\Omega} \times \mathbf{V} - \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}) - \nabla \phi_a - \alpha \nabla p - \alpha \nabla \cdot \mathbf{F}$$

$$\frac{D}{Dt}(c_v T) + p \frac{D\alpha}{Dt} = -\alpha \nabla \cdot (\mathbf{R} + \mathbf{F}_S) + LC + \delta$$

$$\frac{Dq_v}{Dt} = g \frac{\partial F_{q_v}}{\partial p} - C$$

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$$\frac{Dq_v}{Dt} = g \frac{\partial F_{q_v}}{\partial p} - C$$

No coordinate system is used here.

Using spherical coordinates:

$$\frac{\partial \rho}{\partial t} + \frac{1}{r \cos \varphi} \frac{\partial}{\partial \lambda} (\rho u) + \frac{1}{r \cos \varphi} \frac{\partial}{\partial \varphi} (\rho v \cos \varphi) + \frac{1}{r^2} \frac{\partial}{\partial r} (\rho w r^2) = 0$$

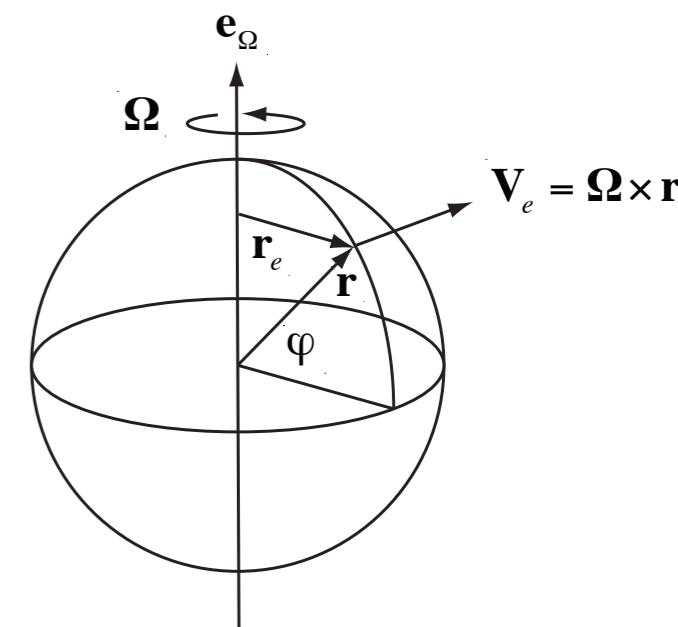
$$\frac{Du}{Dt} - \left(2\Omega + \frac{u}{r \cos \varphi} \right) (v \sin \varphi - w \cos \varphi) = - \frac{\alpha}{r \cos \varphi} \frac{\partial p}{\partial \lambda} - \alpha (\nabla \cdot \mathbf{F})_\lambda$$

$$\frac{Dv}{Dt} + \left(2\Omega + \frac{u}{r \cos \varphi} \right) u \sin \varphi + \frac{vw}{r} + \Omega^2 r \sin \varphi \cos \varphi = - \frac{\alpha}{r} \frac{\partial p}{\partial \varphi} - \alpha (\nabla \cdot \mathbf{F})_\varphi$$

$$\frac{Dw}{Dt} - \left(2\Omega + \frac{u}{r \cos \varphi} \right) u \cos \varphi - \frac{v^2}{r} - \Omega^2 r \cos^2 \varphi + g_a = - \alpha \frac{\partial p}{\partial r} - \alpha (\nabla \cdot \mathbf{F})_r$$

$$\frac{D}{Dt} (c_v T) + p \frac{D\alpha}{Dt} = -\alpha \nabla \cdot (\mathbf{R} + \mathbf{F}_S) + LC + \delta$$

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Using spherical coordinates:

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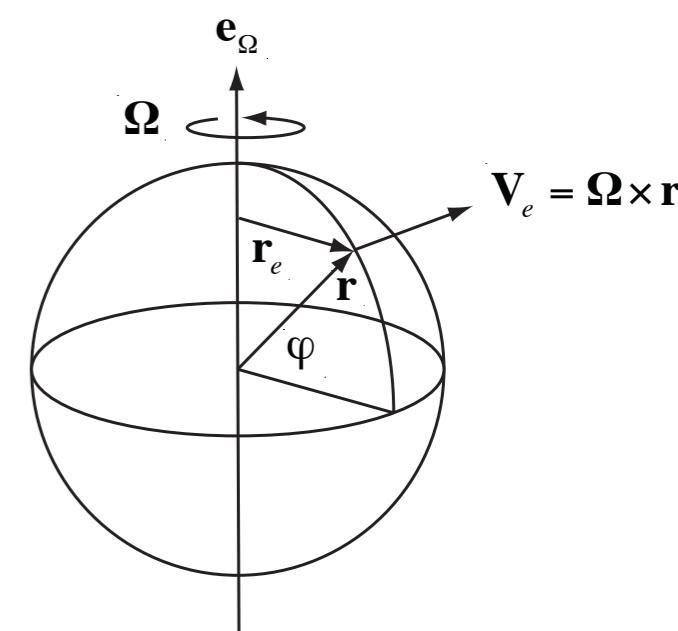
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$$\frac{Dq_v}{Dt} = g \frac{\partial F_{q_v}}{\partial p} - C$$



The components of the equation of motion contain “metric” terms that come from the coordinate system.

Widely used approximations

- ◆ **Thin atmosphere**
- ◆ **Neglect of some Coriolis and metric terms**
- ◆ **Variable gravity and centrifugal acceleration replaced by constant “effective gravity”.**



$$\nabla \cdot \mathbf{H} = \frac{1}{r \cos \varphi} \frac{\partial H_\lambda}{\partial \lambda} + \frac{1}{r \cos \varphi} \frac{\partial}{\partial \varphi} (H_\varphi \cos \varphi) + \frac{1}{r^2} \frac{\partial}{\partial r} (H_r r^2)$$

$$\nabla \cdot \mathbf{H} = \frac{1}{r \cos \varphi} \frac{\partial H_\lambda}{\partial \lambda} + \frac{1}{r \cos \varphi} \frac{\partial}{\partial \varphi} (H_\varphi \cos \varphi) + \frac{\partial H_r}{\partial r} + \cancel{\frac{2 H_r}{r}}$$

$$r \rightarrow a$$

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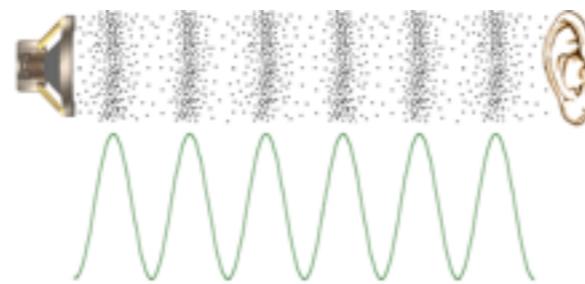
$$\frac{Dv}{Dt} + \left(2\Omega + \frac{u}{a \cos \varphi} \right) u \sin \varphi = - \frac{\alpha}{a} \frac{\partial p}{\partial \varphi} - \alpha (\nabla \cdot \mathbf{F})_{\varphi}$$

$$\frac{Dw}{Dt} + g = - \alpha \frac{\partial p}{\partial r} - \alpha (\nabla \cdot \mathbf{F})_r$$

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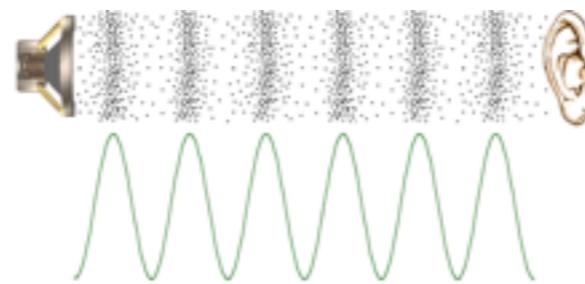
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Vertically propagating sound waves are bad.



In the Earth's atmosphere, the speed of sound is about 300 m/s.

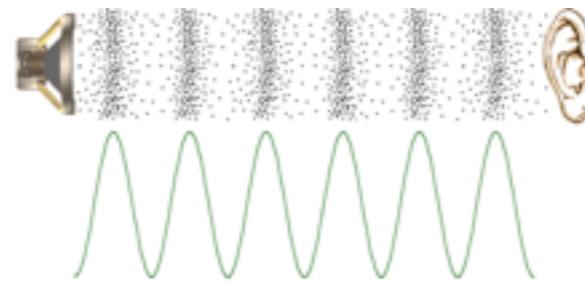
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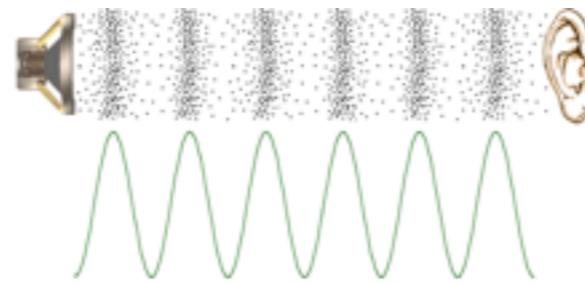


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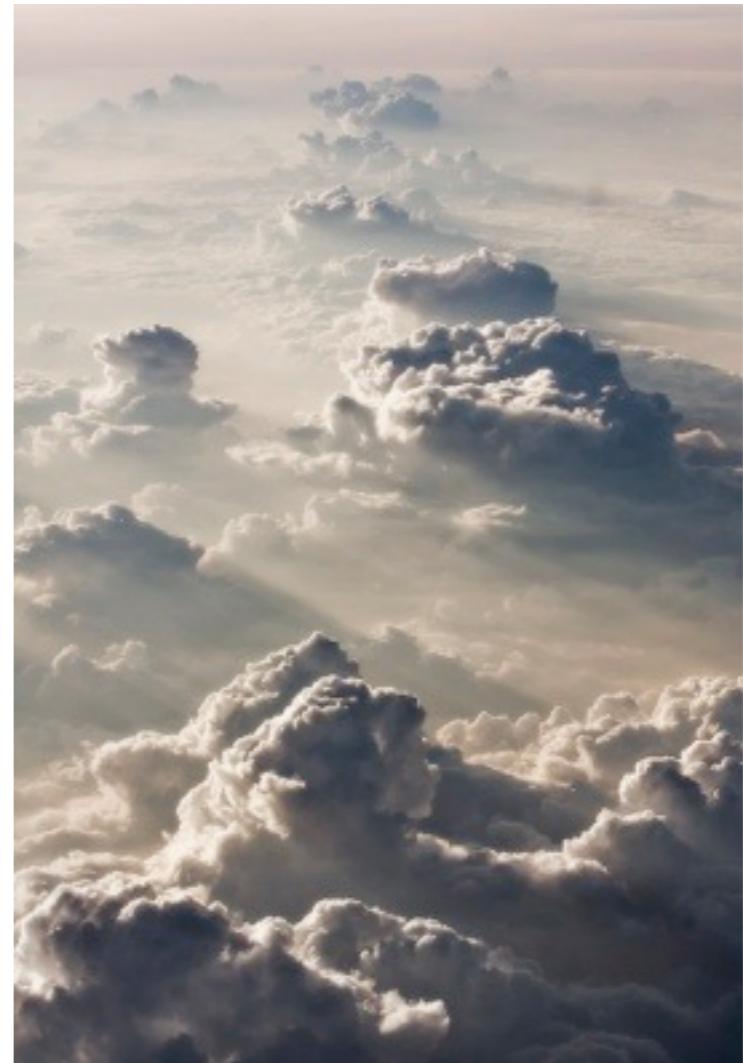
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100 km

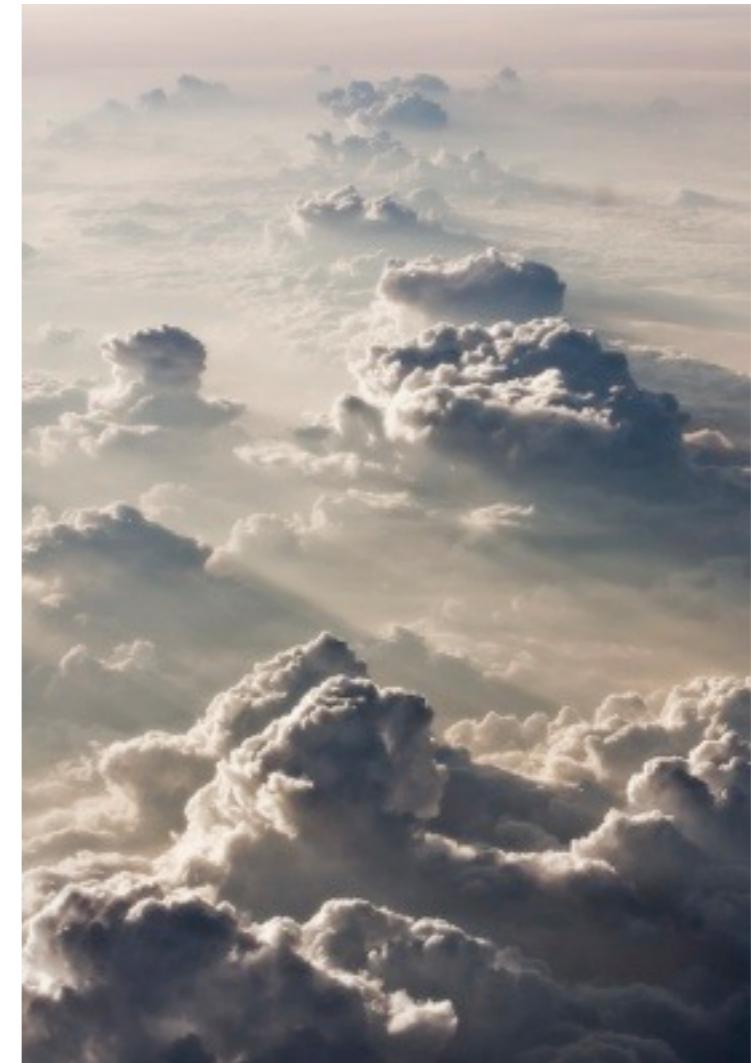
Four ways to deal with vertically propagating sound waves

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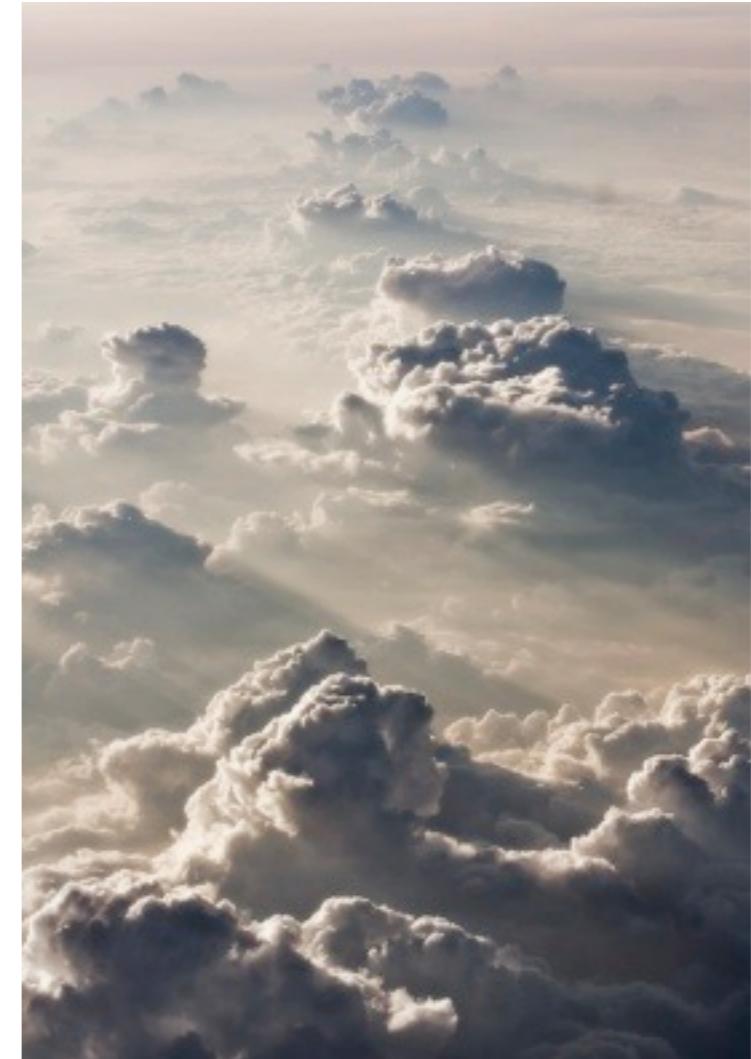
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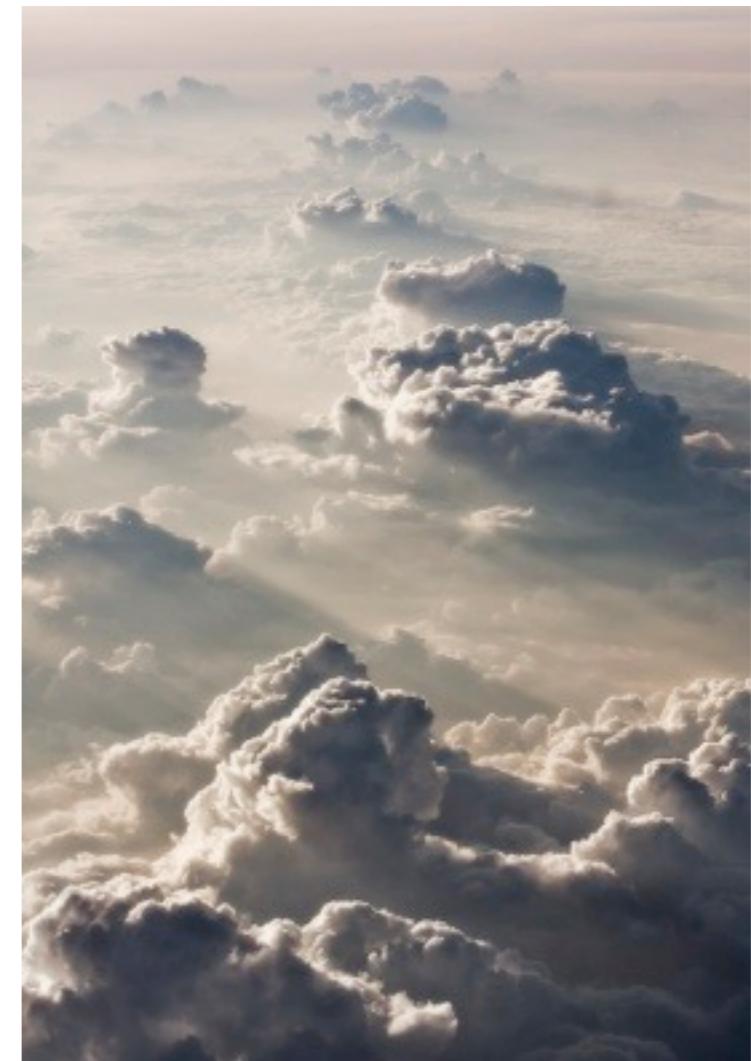
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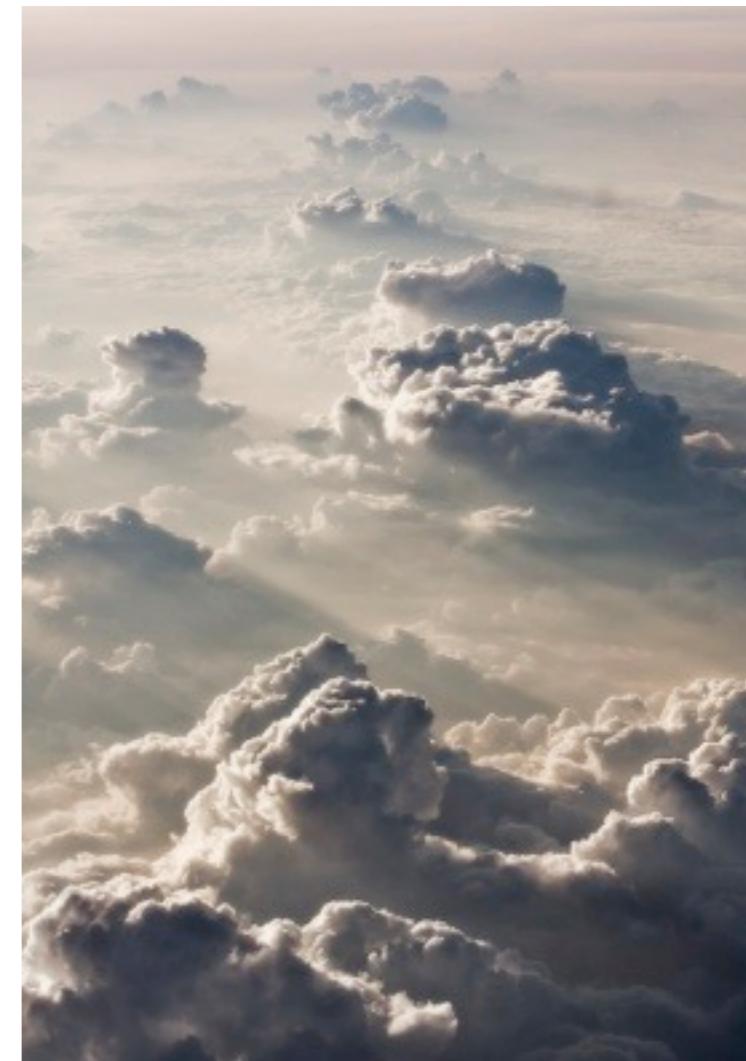
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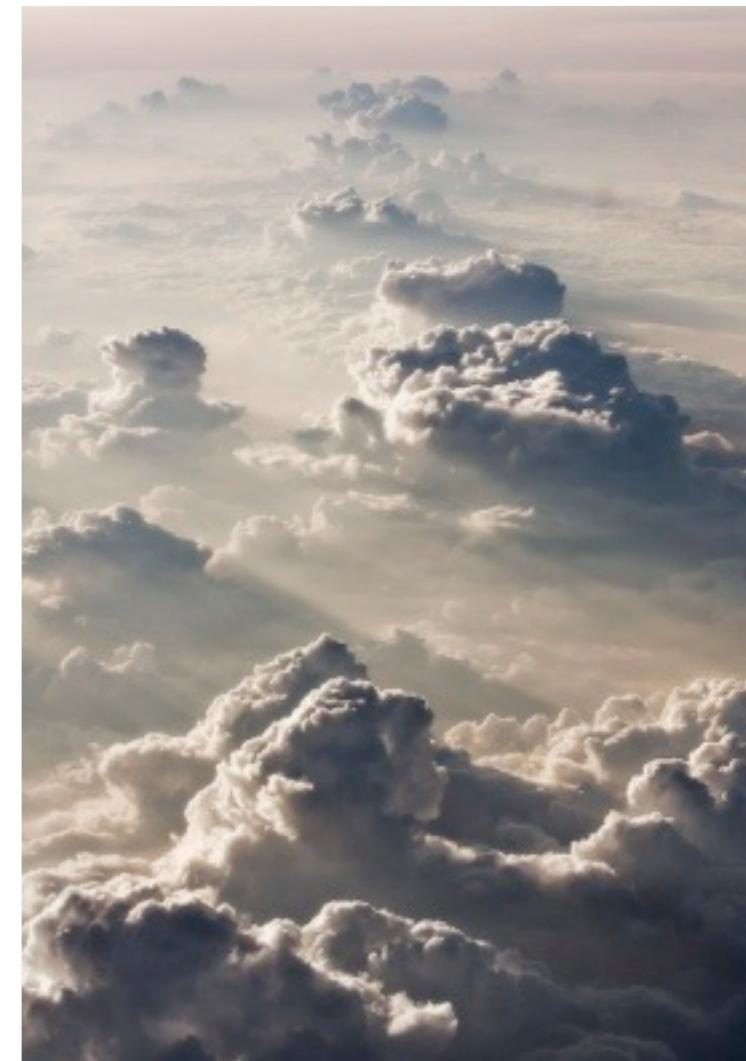
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The first three approaches are numerical.

The fourth method is based on a physical approximation.

The quasi-static approximation

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$$g = -\alpha \frac{\partial p}{\partial r}$$

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This is justified when the vertical velocity is weak and the weather system is much wider than it is tall.

It is *not justified* for vigorous small-scale weather systems.

The quasi-static system with the traditional approximations

$$\frac{\partial \rho}{\partial t} + \frac{1}{a \cos \varphi} \frac{\partial}{\partial \lambda} (\rho u) + \frac{1}{a \cos \varphi} \frac{\partial}{\partial \varphi} (\rho v \cos \varphi) + \frac{\partial}{\partial r} (\rho w) = 0$$

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With the quasi-static system, we cannot use the equation of vertical motion to determine the vertical velocity.

The method used to determine the vertical velocity depends on the choice of vertical coordinate system.

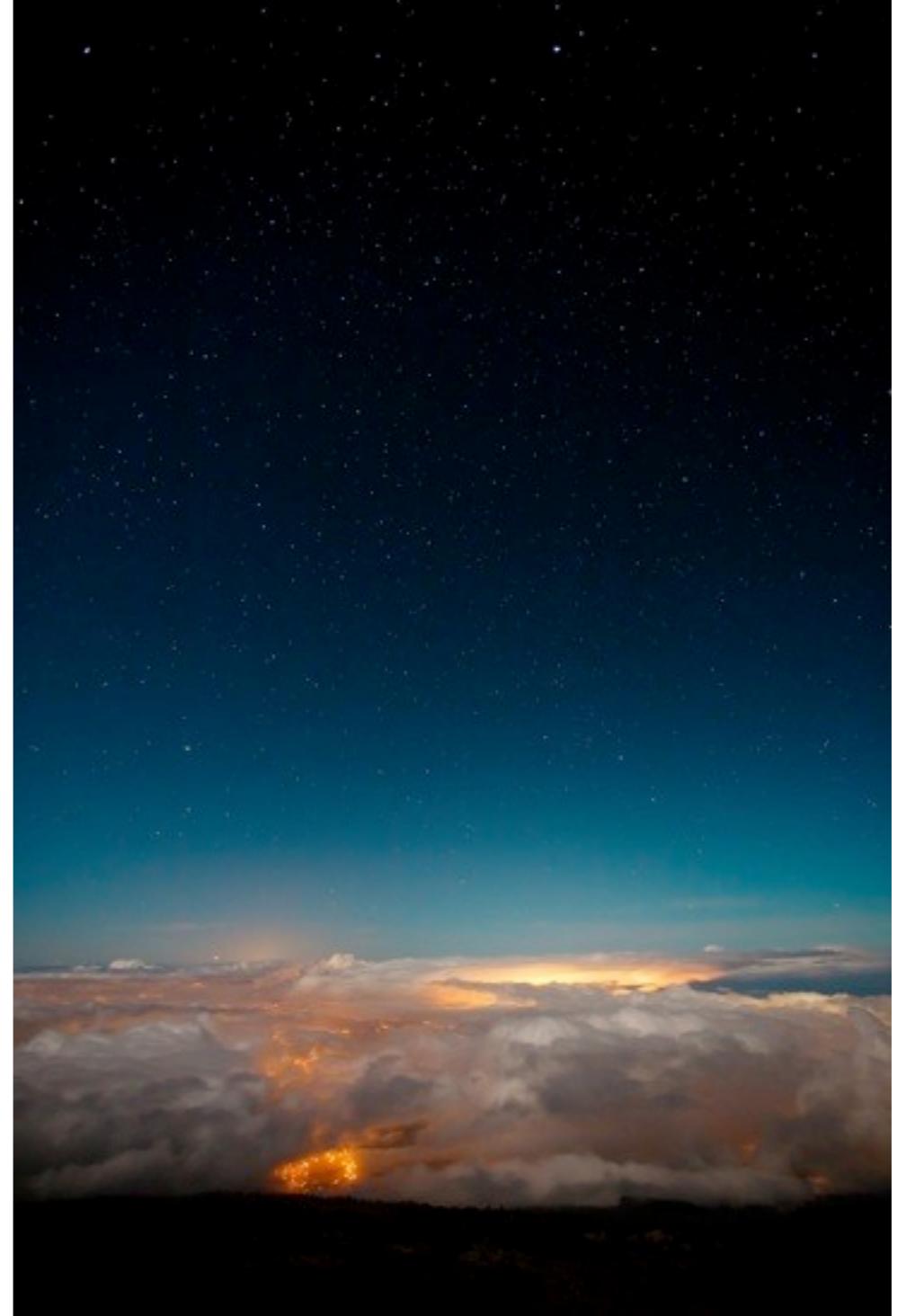
The trend to non-hydrostatic models

Faster, massively parallel computers are allowing us to use finer grids.

Finer grids can resolve weather systems, e.g. thunderstorms, that are not quasi-static.

For this reason, we are now building GCMs that do not use the quasi-static approximation.

These new high-resolution GCMs can borrow ideas from the well established mesoscale modeling community.



Filtering vertically propagating sound waves without using the quasi-static approximation



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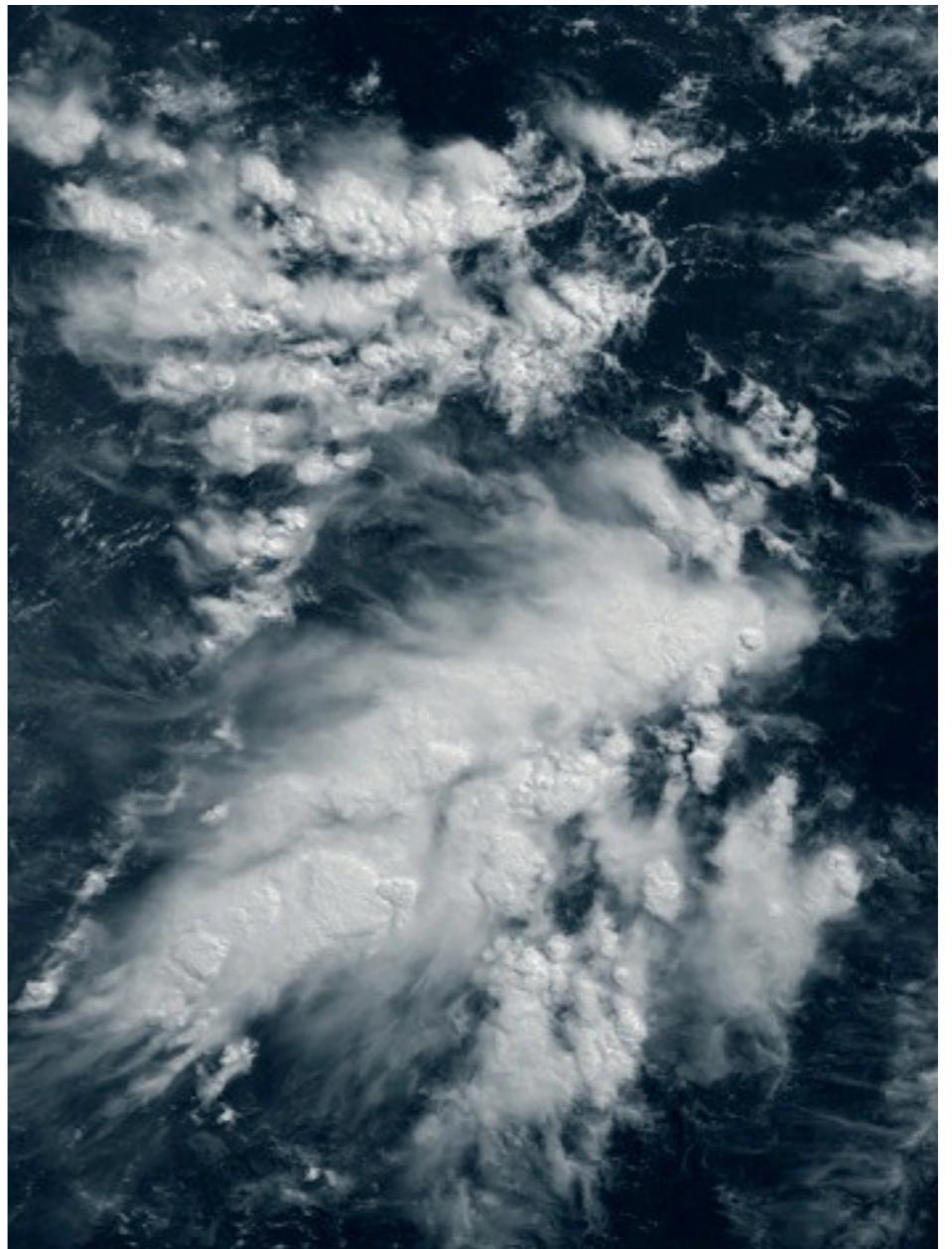
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The main issues are accuracy and conservation properties.

I will discuss this further later in the week.

High-resolution physics

- **The nature of the “sub-grid” physical processes depends on the grid size.**
- **Physics appropriate for low-resolution models is not appropriate for high-resolution models, and vice-versa.**
- **Can we design intelligent physical parameterizations that can adapt to a wide range of grid spacings?**



Conclusions

- ◆ **GCM design involves an ordered sequence of choices.**
- ◆ **The choice of equation set is determined partly by the intended resolution of the model.**
- ◆ **The new generation of high-resolution models will have to give up the venerable quasi-static approximation.**
- ◆ **The emerging 21st century models may also spurn various other approximations that have been widely used in the past.**
- ◆ **The creation of a new generation of models is an opportunity to re-examine our traditional approaches to physical parameterization.**