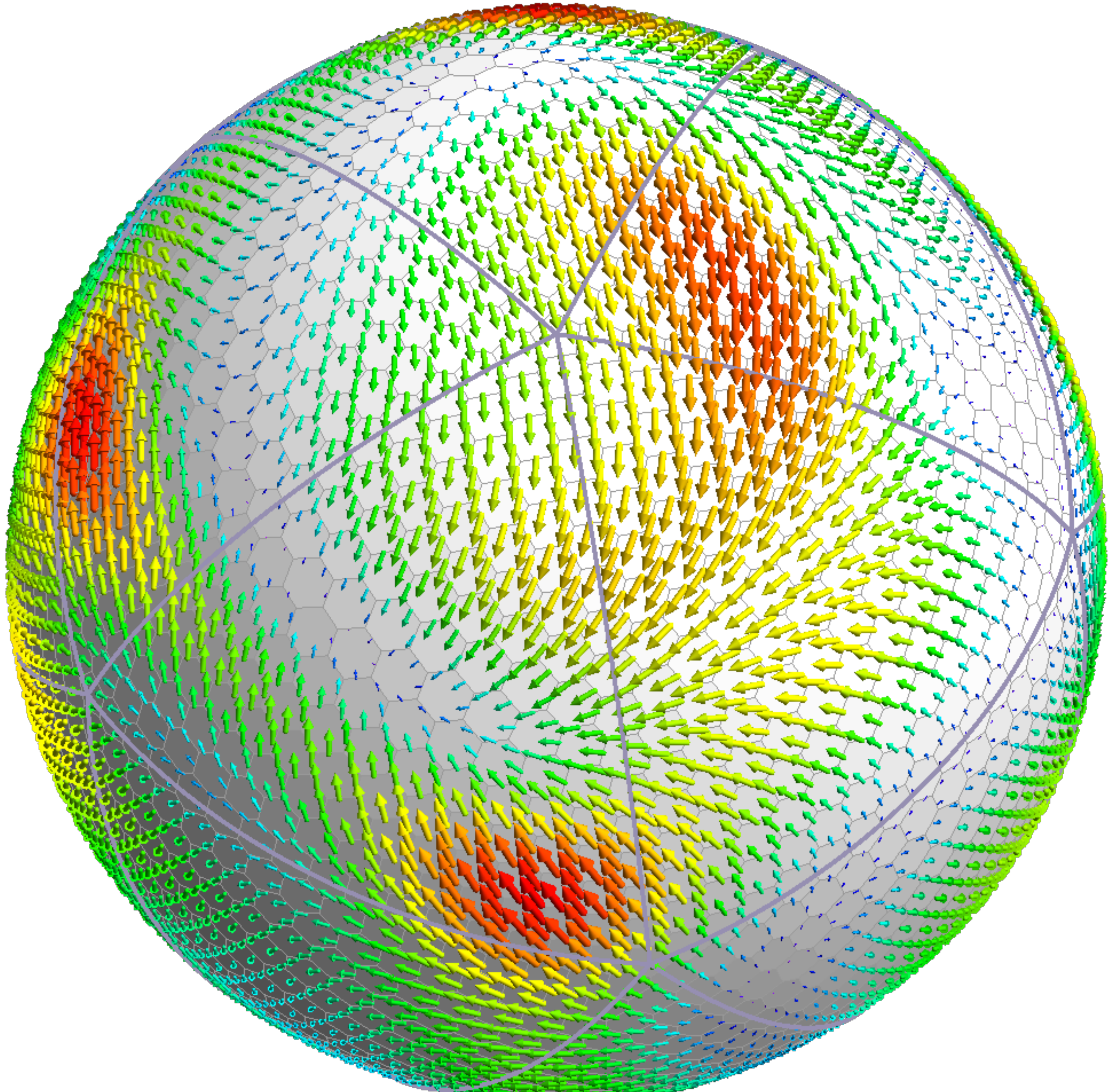
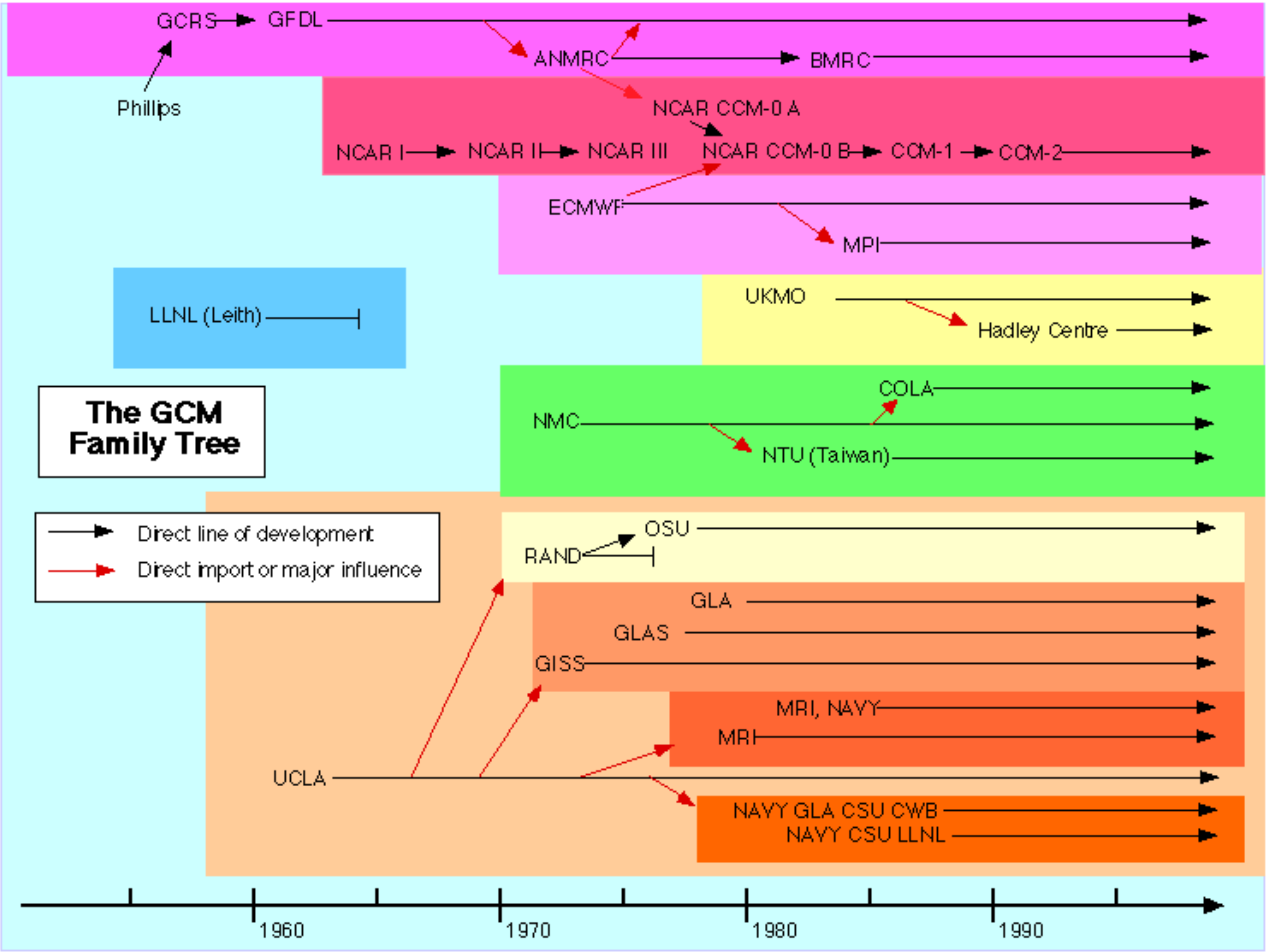
Unified **Z**grid **I**cosahedral **M**odel



Where we came from

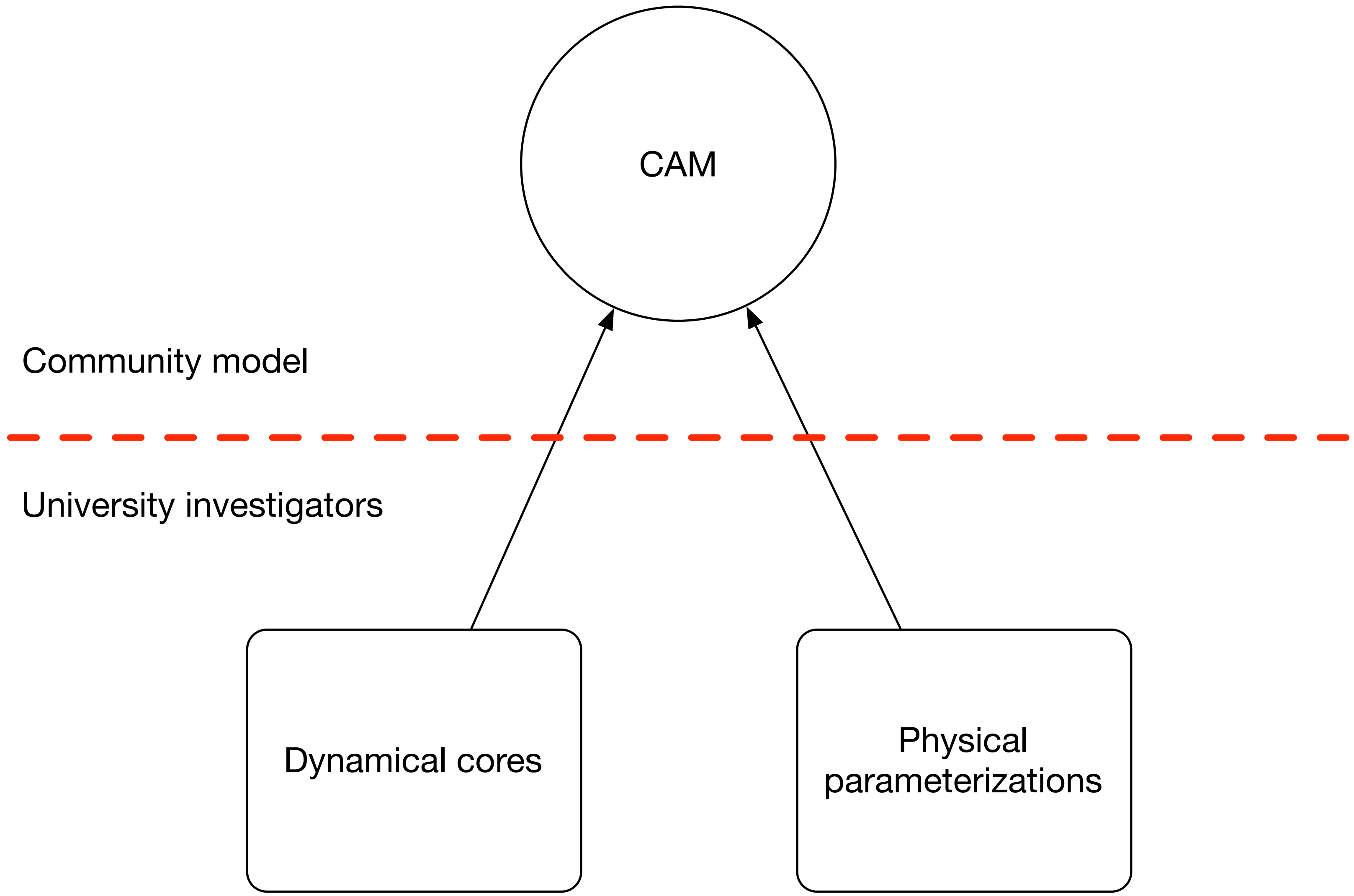
- Our work grew out of the UCLA GCM.
- A lot of our focus is on model development, but we also do “academic applications.”
- We develop both numerics and physics.
- Our university-based effort puts a lot of emphasis on testing risky new ideas.
- The ideas that pass muster are offered for possible use in the CAM.


GCRS
GFDL
Phillips
ANMRC
BMRC
NCAR CCM-0 A
NCAR I
NCAR II
NCAR III
NCAR CCM-0 B
CCM-1
CCM-2
ECMWF
MPI
LLNL (Leith)
UKMO
Hadley Centre
The GCM Family Tree
COLA
NMC
NTU (Taiwan)
Direct line of development
Direct import or major influence
OSU
RAND
GLA
GLAS
GISS
MRI, NAVY
MRI
UCLA
NAVY GLA CSU CWB
NAVY CSU LLNL
1960
1970
1980
1990

The era of "university GCMs" is over.

It's still do-able, but the agencies won't support it.

What we do instead is shown below.



Therefore: There is no "CSU GCM" anymore.

There is, however, a CSU dynamical core, and there are CSU parameterizations.

And occasionally, in the dark of night,
we test the CSU parameterizations with the CSU dynamical core.

Choices

(an ordered list)

- Choice of continuous "system" of equations
- Choice of prognostic variables
- Choice of grid shape
- Choice of where the prognostic variables live on the grid ("staggering")
- Choice of conservation properties (closely related to "mimeticity")
- Choice of discrete operators (order of accuracy, monotonicity, compatability, etc.)

The Unified System™

Unification of the Anelastic and Quasi-Hydrostatic Systems of Equations

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ABSTRACT

A system of equations is presented that unifies the nonhydrostatic anelastic system and the quasi-hydrostatic compressible system for use in global cloud-resolving models. By using a properly defined quasi-hydrostatic density in the continuity equation, the system is fully compressible for quasi-hydrostatic motion and anelastic for purely nonhydrostatic motion. In this way, the system can cover a wide range of horizontal scales from turbulence to planetary waves while filtering vertically propagating sound waves of all scales. The continuity equation is primarily diagnostic because the time derivative of density is calculated from the thermodynamic (and surface pressure tendency) equations as a correction to the anelastic continuity equation. No reference state is used and no approximations are made in the momentum and thermodynamic equations. An equation that governs the time change of total energy is also derived. Normal-mode analysis on an f plane without the quasigeostrophic approximation and on a midlatitude β plane with the quasigeostrophic approximation is performed to compare the unified system with other systems. It is shown that the unified system reduces the westward retrogression speed of the ultra-long barotropic Rossby waves through the inclusion of horizontal divergence due to compressibility.

The Unified System™

Fully compressible system

$$p \equiv p_{qs} + \delta p \qquad \rho \equiv \rho_{qs} + \delta\rho$$
$$\pi \equiv \pi_{qs} + \delta\pi \qquad T \equiv T_{qs} + \delta T$$

Horizontal momentum equation:

$$\frac{D\mathbf{v}}{Dt} + f\mathbf{k} \times \mathbf{v} = -c_p \theta \nabla_H \left(\pi_{qs} + \delta\pi\right) + \mathbf{F}_\mathbf{v}$$

Vertical momentum equation:

$$\frac{Dw}{Dt} = -c_p \theta \frac{\partial \delta\pi}{\partial z} + F_w$$

Thermodynamic equation:

$$\frac{D\theta}{Dt} = \frac{Q}{c_p \pi}$$

Continuity equation:

$$\frac{\partial\left(\rho_{qs} + \delta\rho\right)}{\partial t} = -\nabla_H \cdot \left[\left(\rho_{qs} + \delta\rho\right)\mathbf{v}\right] - \frac{\partial\left[\left(\rho_{qs} + \delta\rho\right)w\right]}{\partial z}$$

Unified System

$$p \equiv p_{qs} + \delta p \qquad \rho \equiv \rho_{qs} + \delta\rho$$
$$\pi \equiv \pi_{qs} + \delta\pi \qquad T \equiv T_{qs} + \delta T$$

Horizontal momentum equation:

$$\frac{D\mathbf{v}}{Dt} + f\mathbf{k} \times \mathbf{v} = -c_p \theta \nabla_H \left(\pi_{qs} + \delta\pi\right) + \mathbf{F}_\mathbf{v}$$

Vertical momentum equation:

$$\frac{Dw}{Dt} = -c_p \theta \frac{\partial \delta\pi}{\partial z} + F_w$$

Thermodynamic equation:

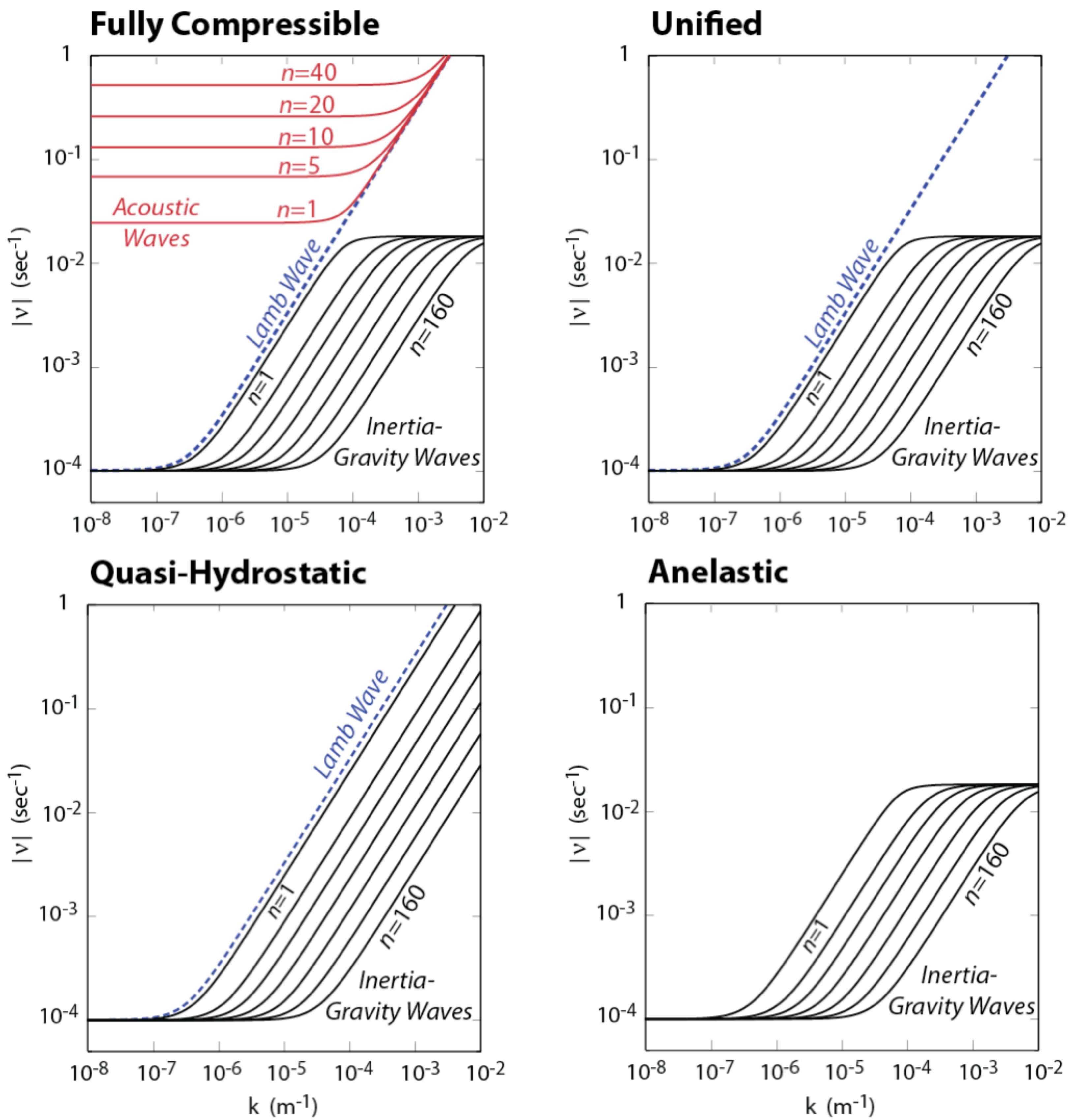
$$\frac{D\theta}{Dt} = \frac{Q}{c_p \pi}$$

Continuity equation:

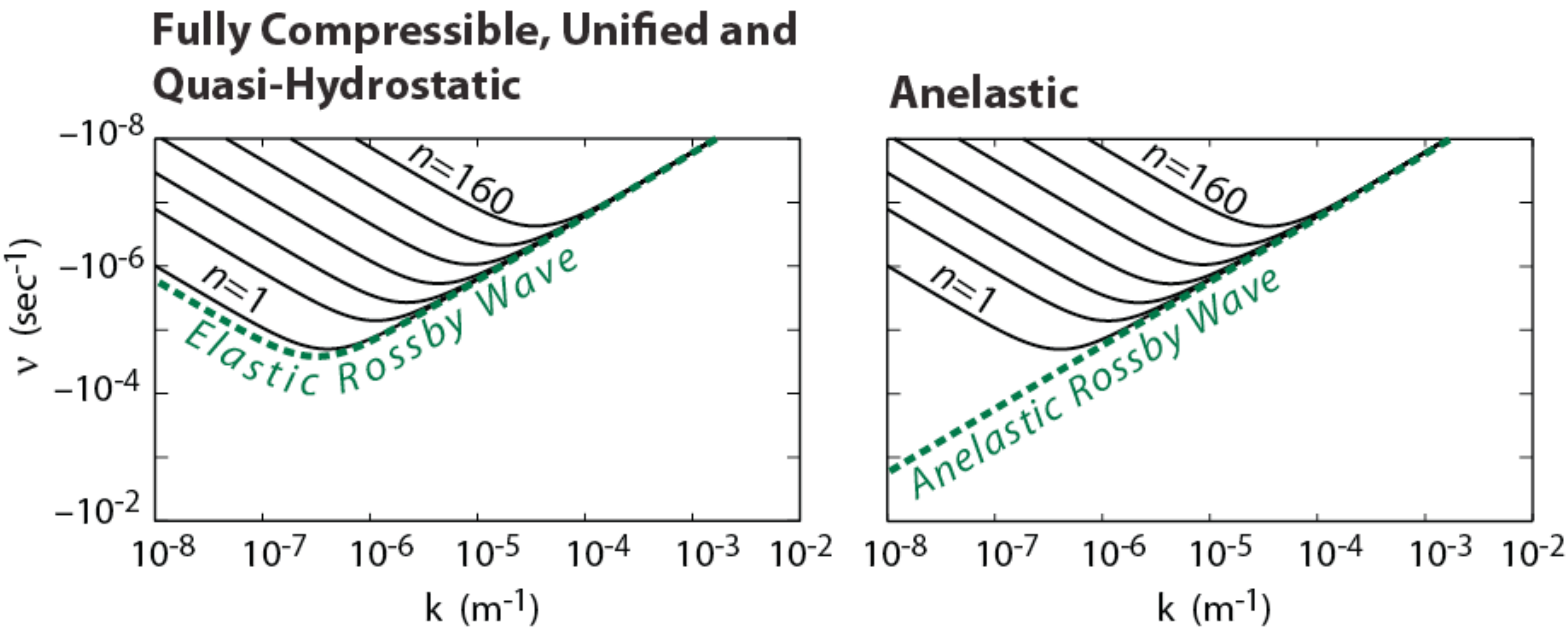
$$\frac{\partial \rho_{qs}}{\partial t} = -\nabla_H \cdot \left(\rho_{qs}\mathbf{v}\right) - \frac{\partial\left(\rho_{qs} w\right)}{\partial z}$$

Like the anelastic system, the Unified System includes a 3D elliptic equation, e.g., for the non hydrostatic part of the pressure.

Dispersion of Inertia-Gravity Waves



Dispersion of Rossby Waves



Predicting the winds with the Unified System

With the Unified System, there are only two prognostic degrees of freedom in the wind field.

For example, if the horizontal wind vector is predicted, then the vertical velocity can be diagnosed from the continuity equation.

Strengths & Weaknesses of the Unified System

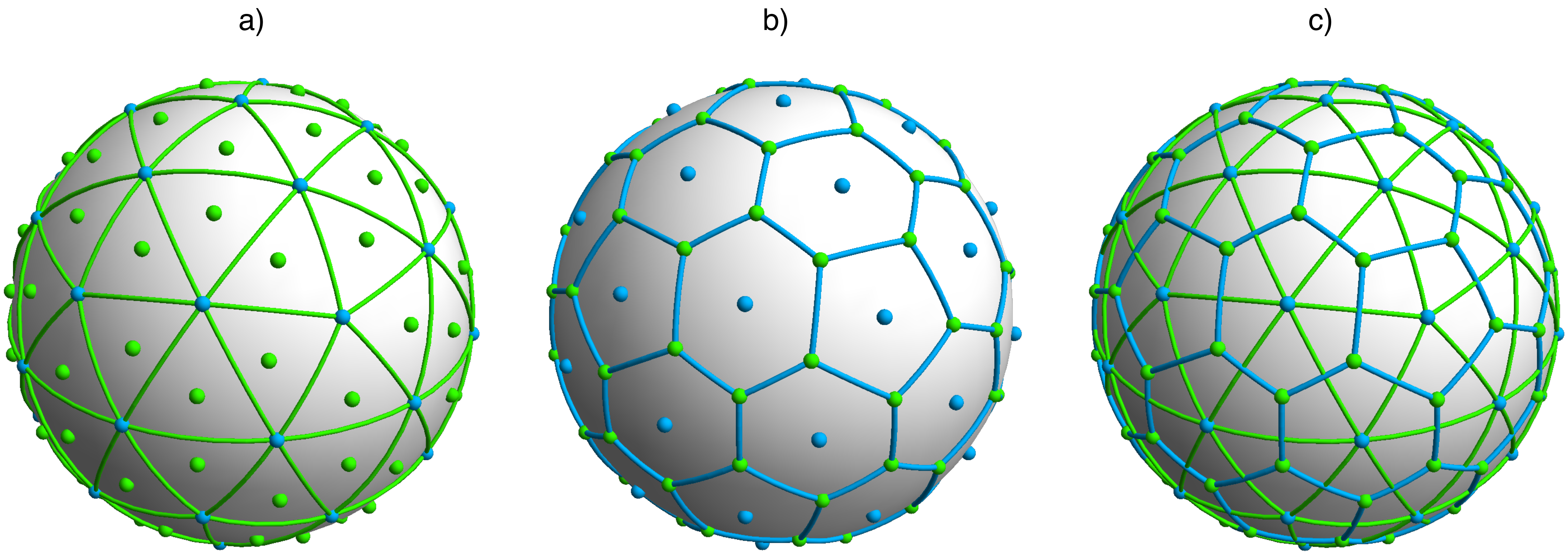
Strengths:

- Filters vertically propagating sound waves, allowing longer time steps without numerical tricks
- Allows the Lamb wave
- Allows compressibility that slows down long Rossby waves
- Does not need a basic or reference or mean state
- Is as accurate as the fully compressible system for non-acoustic modes
- Can easily be "switched" to the quasi-static system
- Conserves energy

Weaknesses:

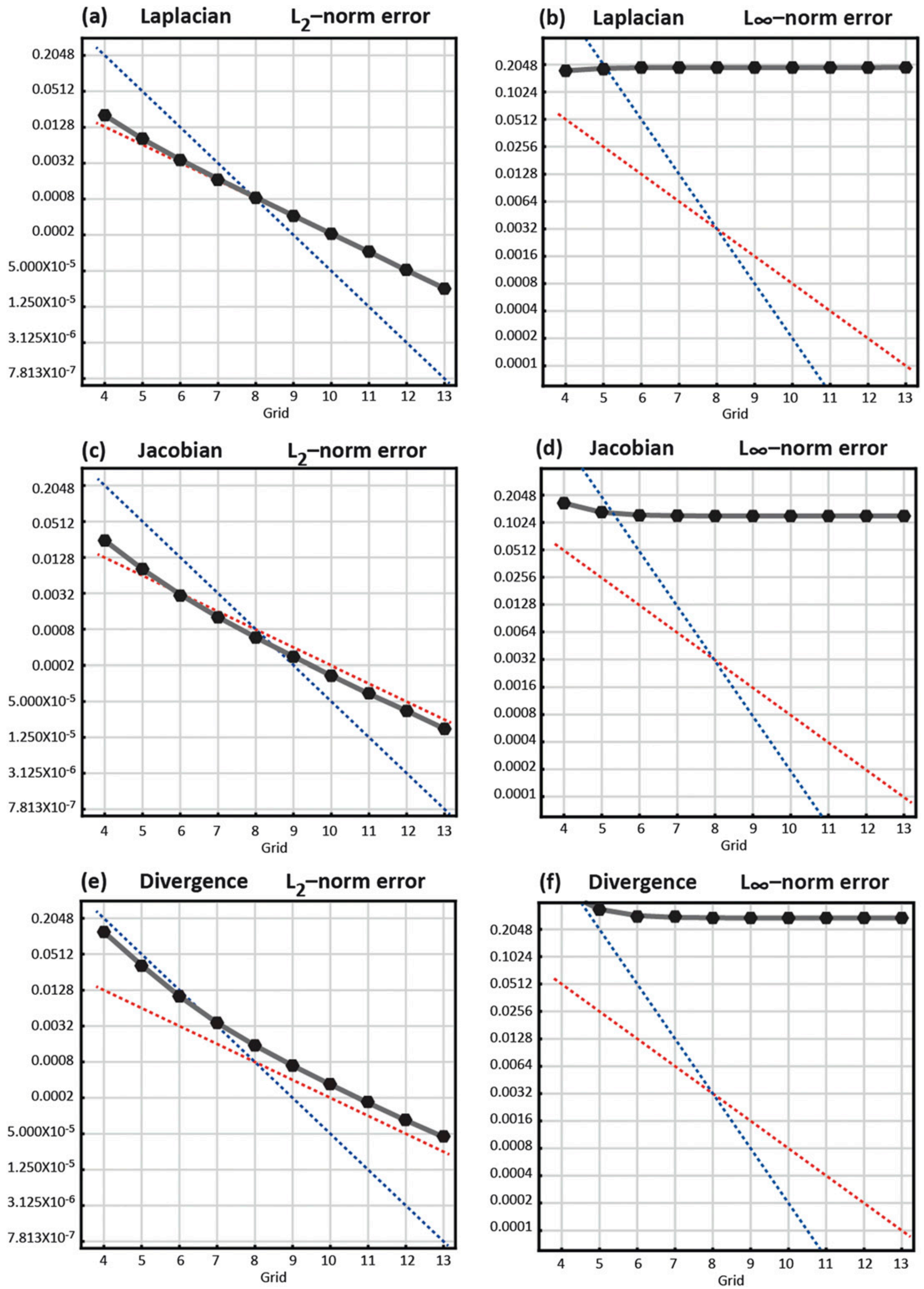
- Requires solution of a three-dimensional elliptic system

Grid generation

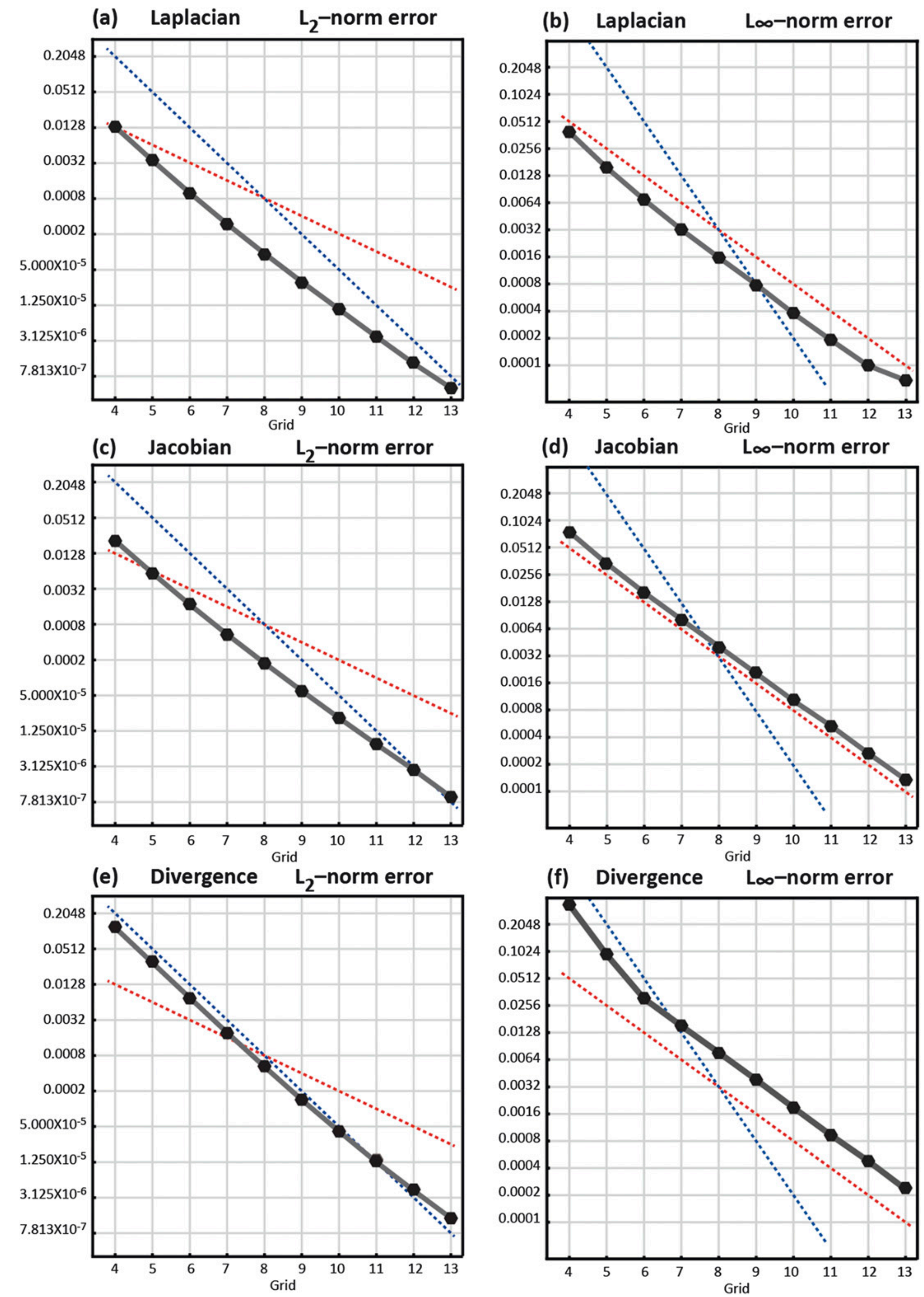


The grid is optimized using a variational method. The goal of the optimization is to minimize the distance between the mid-point of the cell wall and the point where the grid segment intersects the cell wall, by displacing the cell centers relative to those of the raw grid.

Optimization improves the accuracy.

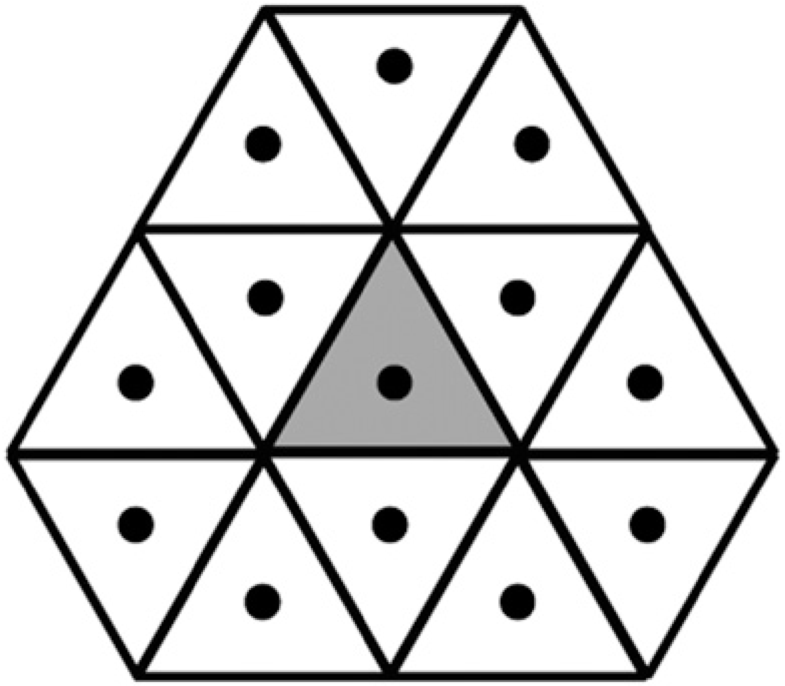


Raw grid

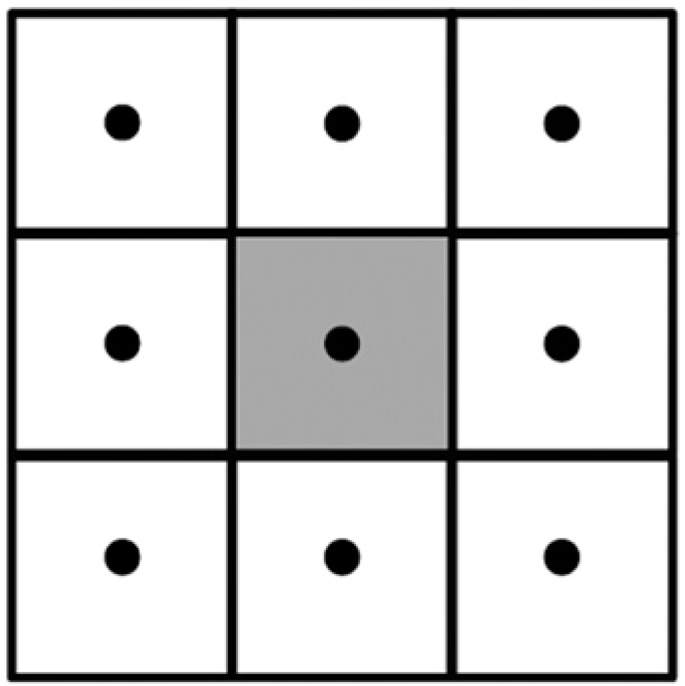


Optimized grid

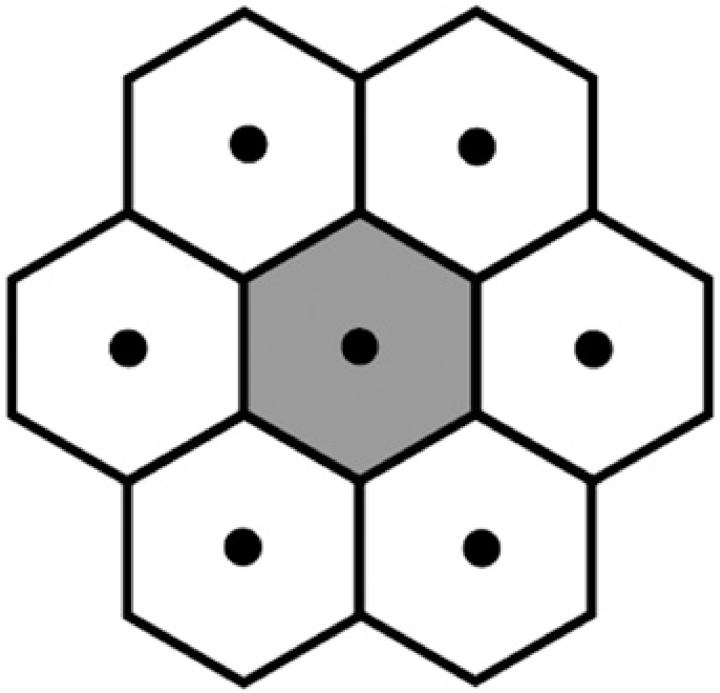
Triangles or Hexagons?



12 neighbors
3 wall neighbors



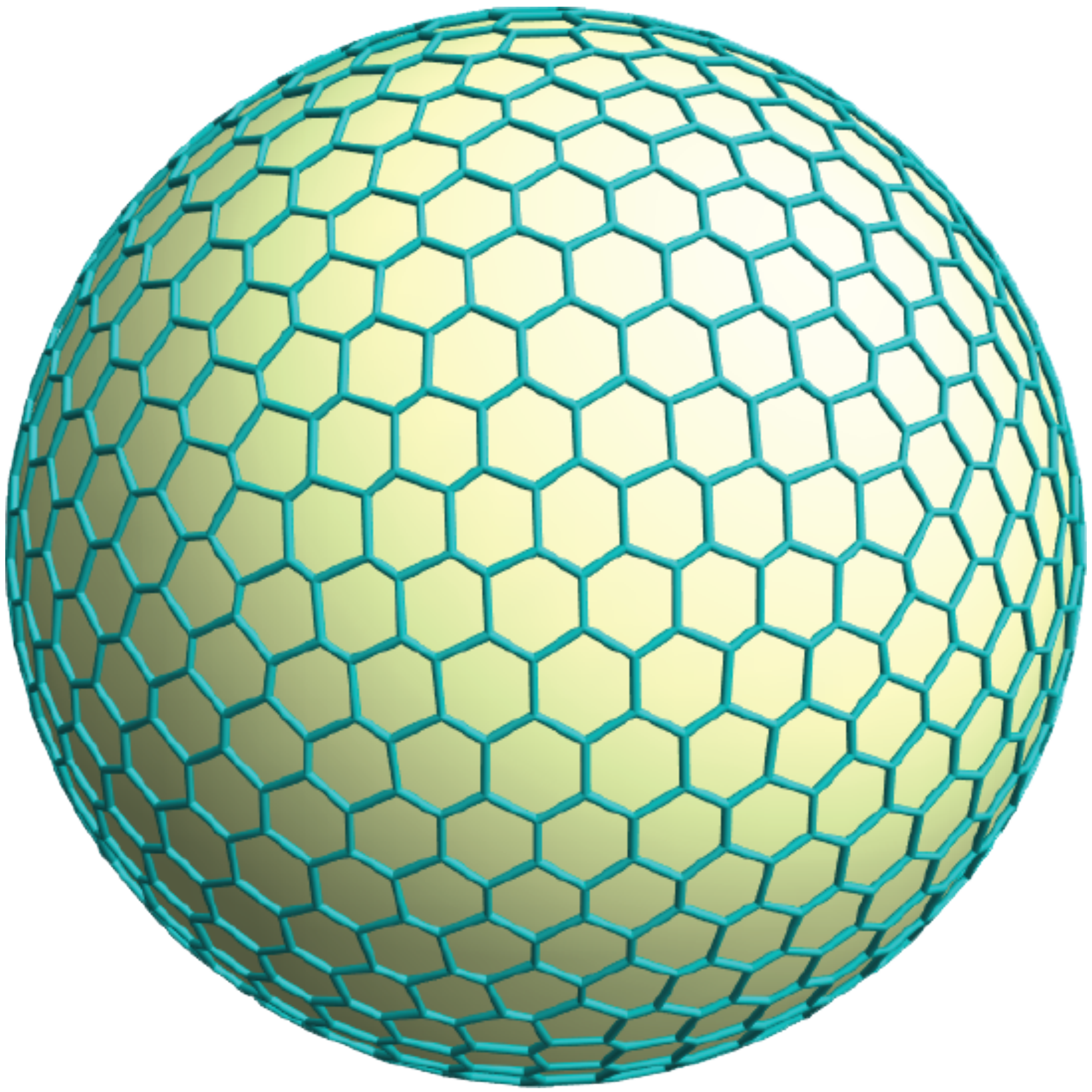
8 neighbors
4 wall neighbors



6 neighbors
6 wall neighbors

We choose the hexagonal grid because of its "isotropy."

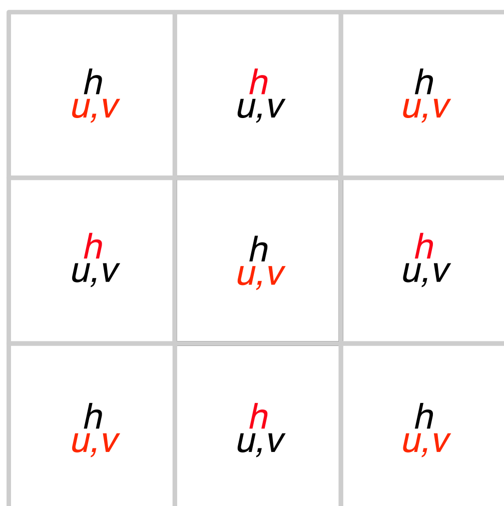
Various resolutions



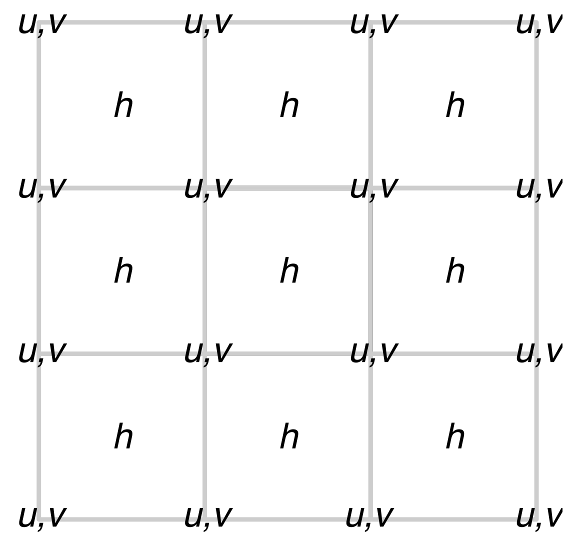
Grid	No. of grid points N	Avg grid distance ℓ (km)
G0	12	6699.1
G1	42	3709.8
G2	162	1908.8
G3	642	961.4
G4	2562	481.6
G5	10 242	240.9
G6	40 962	120.4
G7	163 842	60.2
G8	655 362	30.1
G9	2 621 442	15.0
G10	10 485 762	7.53
G11	41 943 042	3.76
G12	167 772 162	1.88
G13	671 088 642	0.94

Non-hydrostatic regime

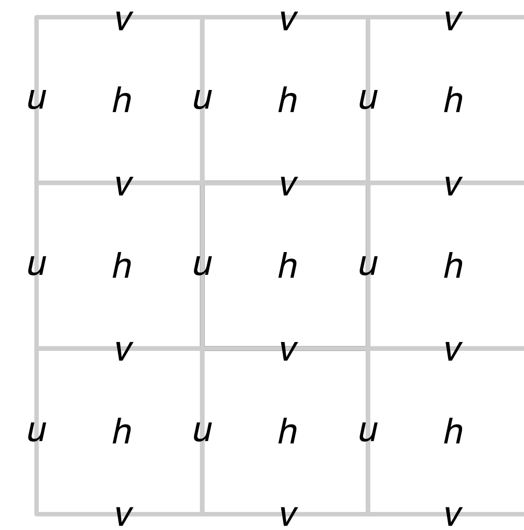
Arranging the variables on square grids



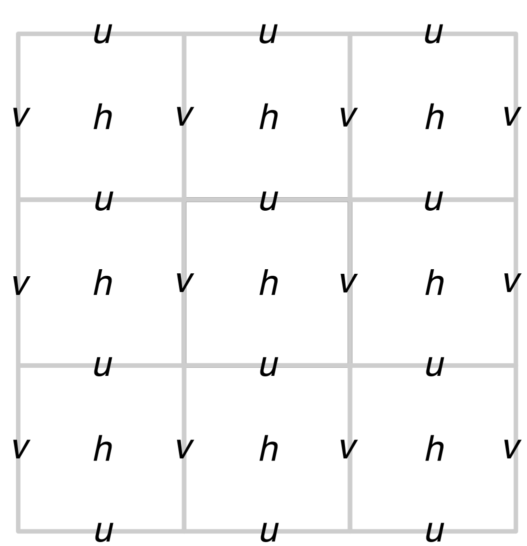
A grid



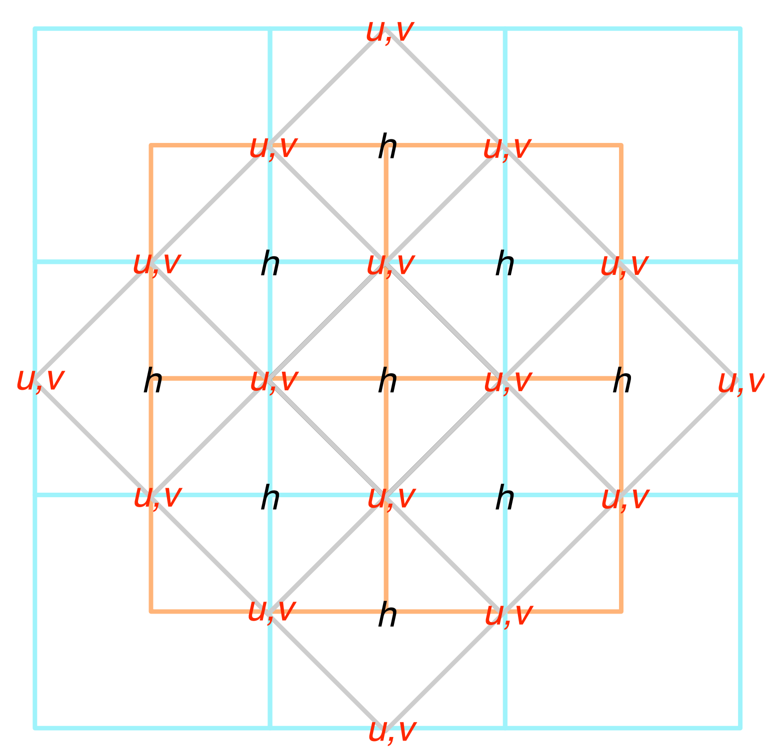
B grid



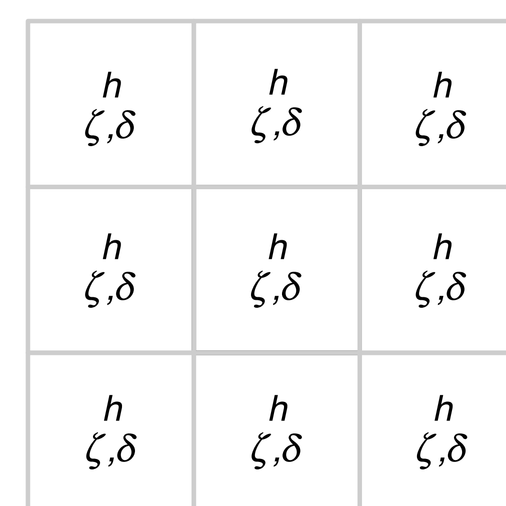
C grid



D grid

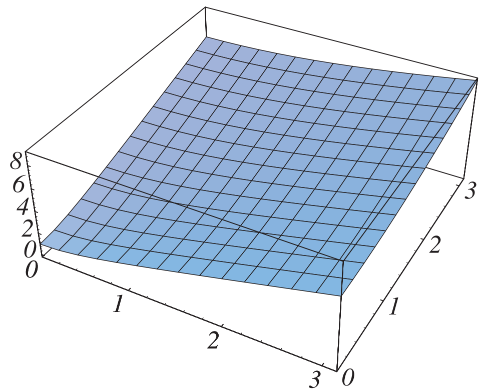


E grid



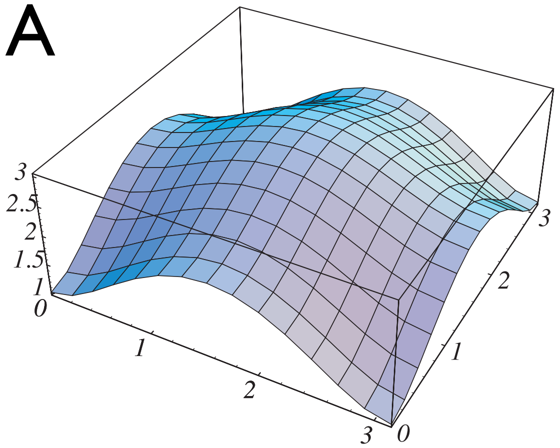
Z grid

Right answer

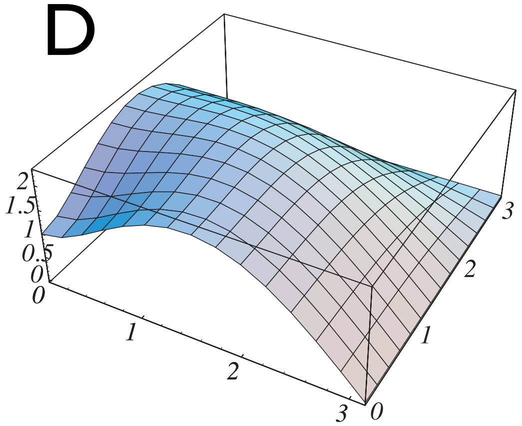


These dispersion plots show how well (or how badly) geostrophic adjustment works on the various grids.

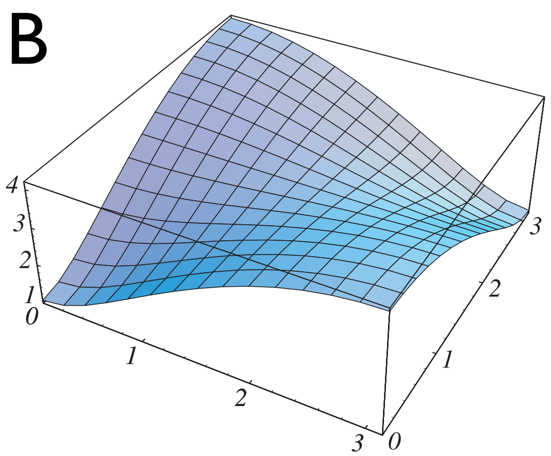
A



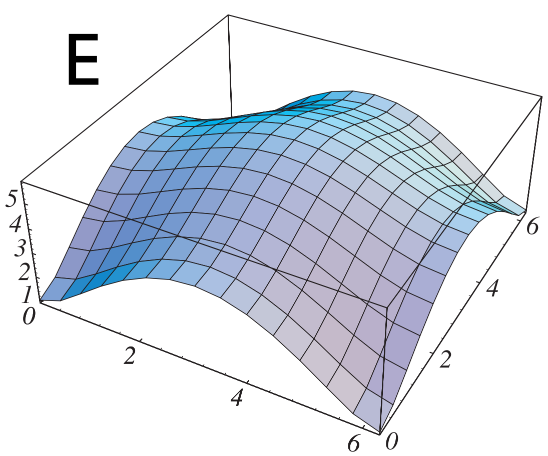
D



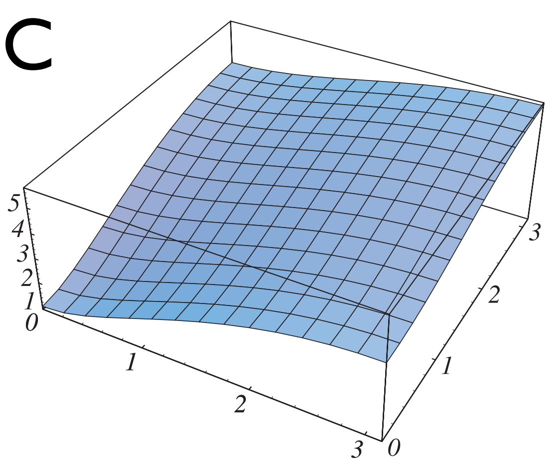
B



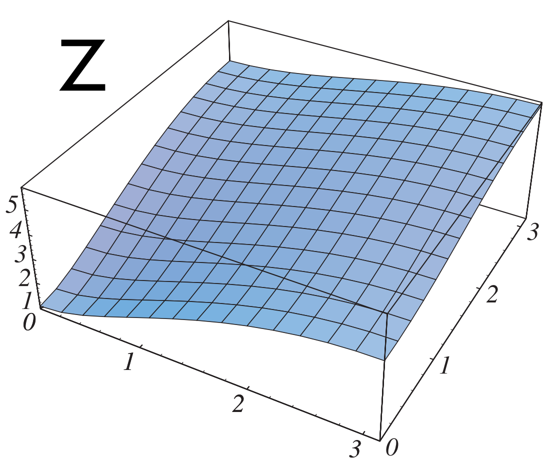
E



C

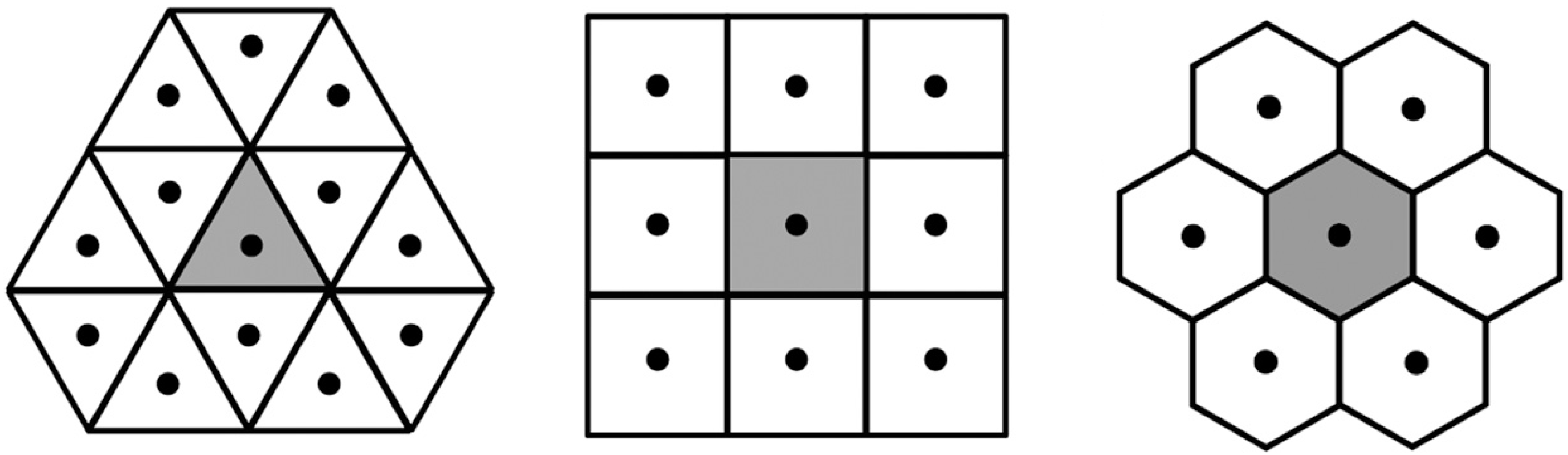


Z



The two best choices are C and Z.

C or Z?



With the C-grid, a shallow-water model predicts the normal component of the wind on each cell wall.

With a quadrilateral grid, there are 2 *wind components per mass point.* That's the right number.

With a triangular grid, there are *1.5 wind components per mass point.*

With a hexagonal grid, there are *3 wind components per mass point.*

If the number of wind components per mass point is not 2, the model will have computational modes.

Conclusion: If you want to use the C-grid, you should use quadrilateral grid cells.

Z

$$\frac{\partial \delta}{\partial t} - f\zeta + g\left(\frac{\partial^2}{\partial x^2}h + \frac{\partial^2}{\partial y^2}h\right) = 0$$

$$\frac{\partial \zeta}{\partial t} + f\delta = 0$$

$$\frac{\partial h}{\partial t} + H\delta = 0$$

$$\delta = \nabla^2 \chi$$

$$\zeta = \nabla^2 \psi$$

Multigrid solver

Strengths & Weaknesses of the Z-grid

Strengths:

- No computational modes
- Excellent dispersion properties for inertia-gravity waves
- Direct prediction of the vertical component of the vorticity
- Prognostic pseudo-scalars, rather than vectors

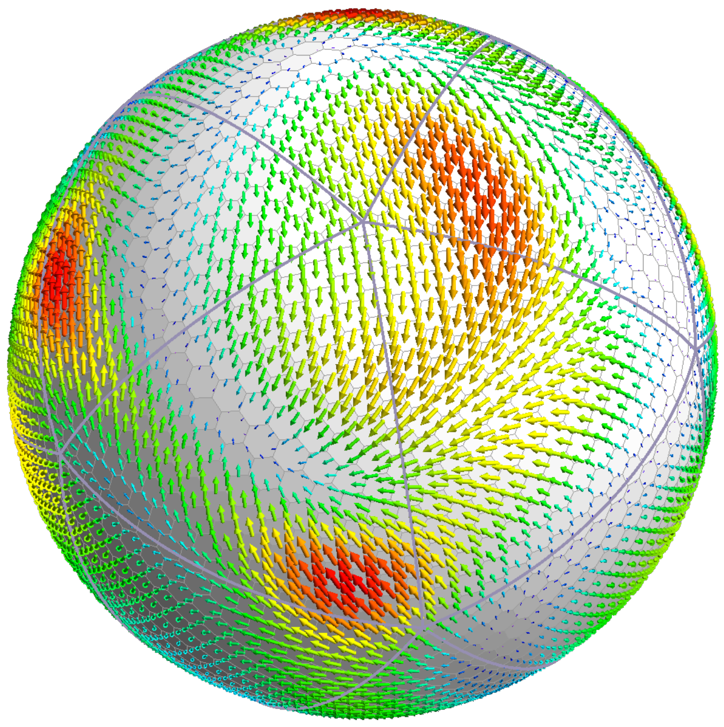
Weaknesses:

- Requires solution of a pair of two-dimensional elliptic equations at each level on each time step

Summary

- We use the **U**nified System to filter vertically propagating sound waves.
- We use the **Z** grid because it simulates geostrophic adjustment well and has no computational modes.
- We use a hexagonal-pentagonal grid derived from the **I**cosahedron because of its homogeneity and isotropy.

U*nified* **Z***grid* **I***cosahedral* **M***odel*



Where we are going

We are testing a new version of the dynamical core that *predicts the curl of the horizontal vorticity vector instead of the divergence of the horizontal wind.*

We are calling this dynamical core "Curl Curl."

I know that predicting the curl of the horizontal vorticity vector sounds crazy and complicated, but it's not.

The motivations are compelling, and the governing equation turns out to be pleasingly simple.

Stay tuned.