Tempest

A Framework for Experimental Numerical Methods

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Tempest Framework

A comprehensive simulation-to-science infrastructure that tackles the needs of next-generation, high-resolution, data intensive modeling activities.

Tempest Dynamics

- An experimental framework for understanding dynamical discretizations.
- Focus is on explicitly evaluated horizontal discretization and implicitly evaluated vertical discretization (HEVI).
- Built using block-based refinement analogous to Colella and Berger (1989).
- Allows effectively arbitrary (and nested) quadrilateral structures in the horizontal, with current support for Cartesian and Cubed-Sphere grids.

Figure: A cubed-sphere refinement patch over California.

Tempest Dynamics

Currently supports **Spectral Element (SE)**, **Discontinuous Galerkin (DG)** and **Flux Reconstruction (FR)** type methods with arbitrary order-of-accuracy in horizontal and vertical. Goal is to also support upwind and central **Finite Volume (FV)** methods.

- Non-hydrostatic dynamics
- Finite-difference, finite-element and spectral transform based vertical coordinates
- Multiple options for diffusion and filtering
- Multiple options for Implicit-Explicit time integrators
- Parallel scalability

Figure: A cubed-sphere refinement patch over California.

Tempest Dynamics

Non-conservative continuous and discontinuous formulation supporting:

- 2D shallow water on the sphere
- 2D (x-z) and 3D non-hydrostatic dynamics in Cartesian geometry
- 3D non-hydrostatic dynamics on the sphere

Diffusion provided via scalar and vector **hyperviscosity,** requiring one parallel communication per application of the Laplacian.

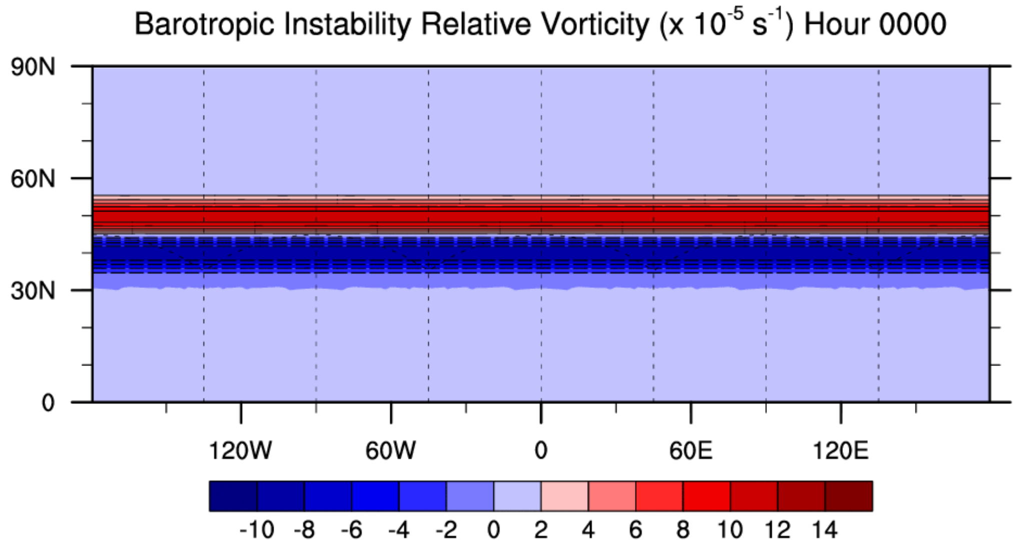

Figure: Relative vorticity for an evolving barotropic instability on the cubed-sphere.

Non-Hydrostatic Dynamics

Continuity

$$\frac{\partial \rho}{\partial t} = -\frac{1}{J}\frac{\partial}{\partial \alpha}(J\rho u^{\alpha}) - \frac{1}{J}\frac{\partial}{\partial \beta}(J\rho u^{\beta}) - \frac{1}{J}\frac{\partial}{\partial \xi}(J\rho u^{\xi})$$

Horizontal Velocity

$$\frac{\partial u_{\alpha}}{\partial t} = -\frac{\partial}{\partial \alpha}(K + \Phi) - \theta\frac{\partial \Pi}{\partial \alpha} + (\boldsymbol{\eta} \times \mathbf{u})_{\alpha}\,,$$

$$\frac{\partial u_{\beta}}{\partial t} = -\frac{\partial}{\partial \beta}(K + \Phi) - \theta\frac{\partial \Pi}{\partial \beta} + (\boldsymbol{\eta} \times \mathbf{u})_{\beta}\,,$$

Vertical Velocity

$$\left(\frac{\partial r}{\partial \xi}\right)\frac{\partial w}{\partial t} = -\frac{\partial}{\partial \xi}(K + \Phi) - \theta\frac{\partial \Pi}{\partial \xi} + (\boldsymbol{\eta} \times \mathbf{u})_{\xi}$$

Thermodynamics

$$\frac{\partial \theta}{\partial t} = -u^{\alpha}\frac{\partial \theta}{\partial \alpha} - u^{\beta}\frac{\partial \theta}{\partial \beta} - u^{\xi}\frac{\partial \theta}{\partial \xi}$$

Five basic governing equations for resolved-scale atmospheric dynamics. **Clark form** used to preserve desirable mimetic properties, such as energy conservation.

Non-Hydrostatic Dynamics

Continuity

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Vertical Velocity

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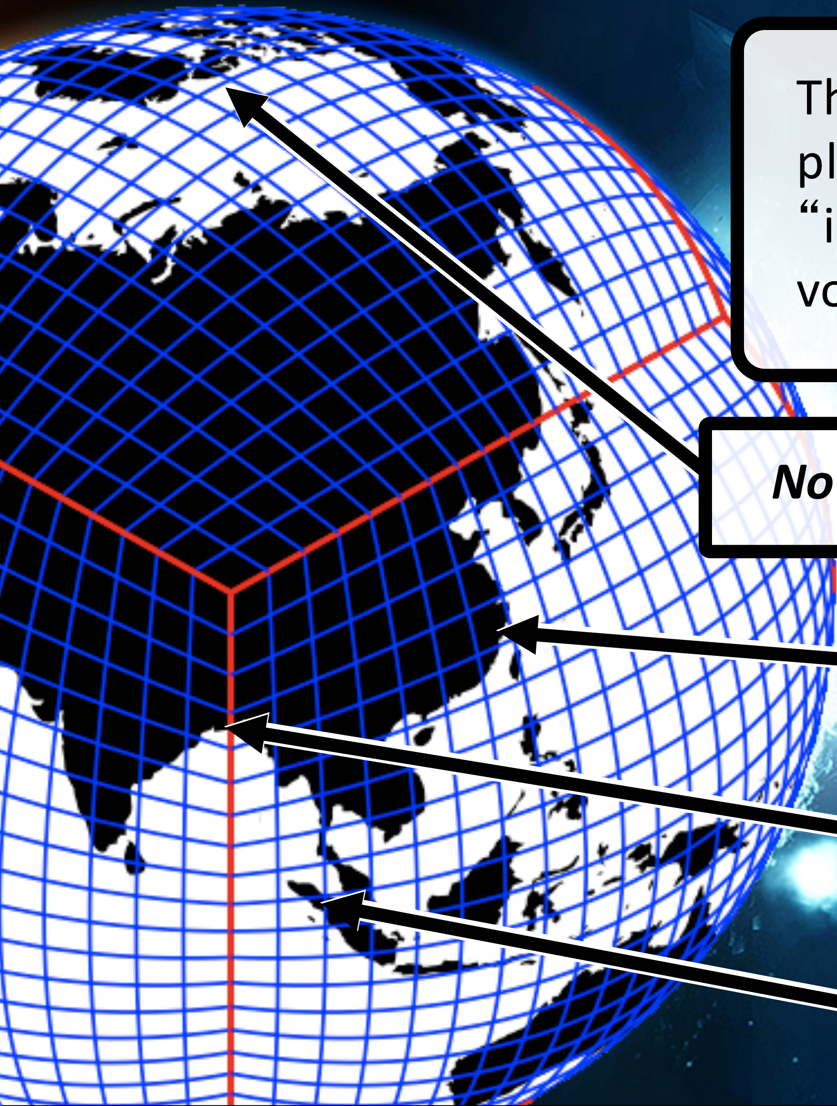
Thermodynamics

$$\boxed{\frac{\partial \theta}{\partial t}} = -u^{\alpha}\frac{\partial \theta}{\partial \alpha} - u^{\beta}\frac{\partial \theta}{\partial \beta} \boxed{-u^{\xi}\frac{\partial \theta}{\partial \xi}}$$

When high aspect ratios are present, geometrically stiff terms in the vertical direction can be integrated **implicitly**. This leads to a Horizontally Explicit, Vertically Implicit (HEVI) formulation.

The Cubed-Sphere



The cubed-sphere grid is obtained by placing a cube inside a sphere and "inflating" it to occupy the total volume of the sphere.

No polar singularities

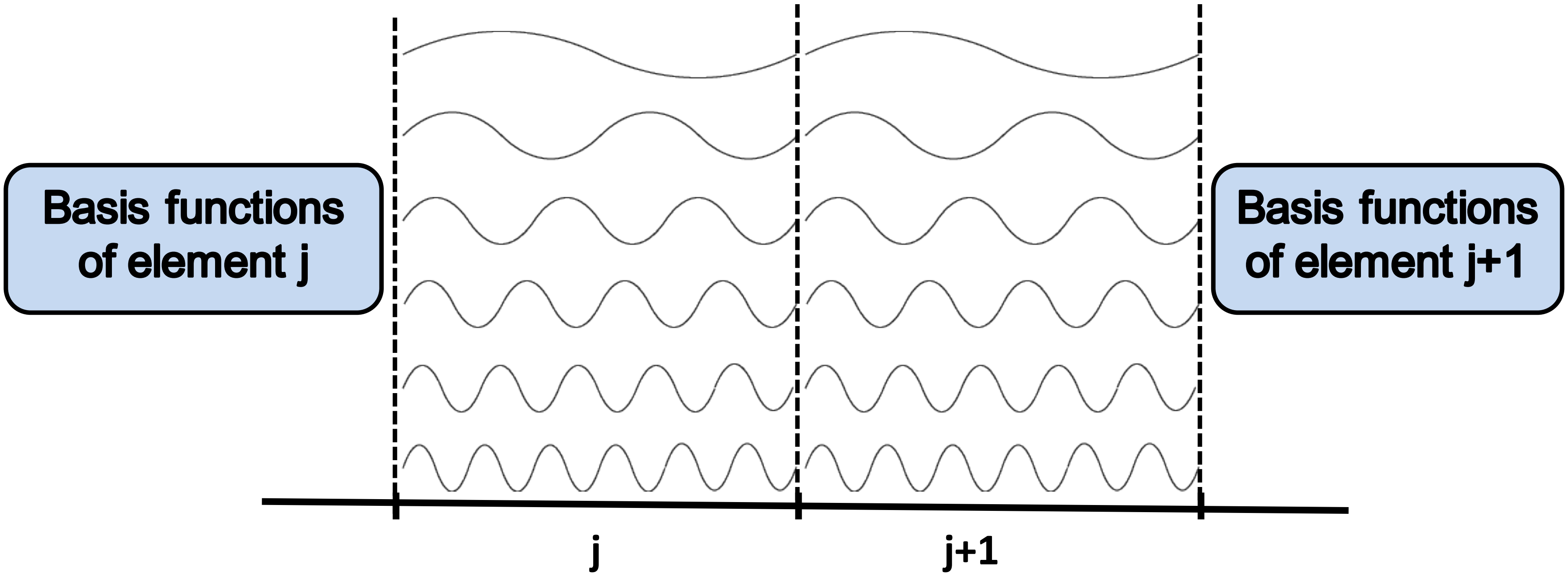
Grid faces individually Cartesian

Some difficulty at panel edges

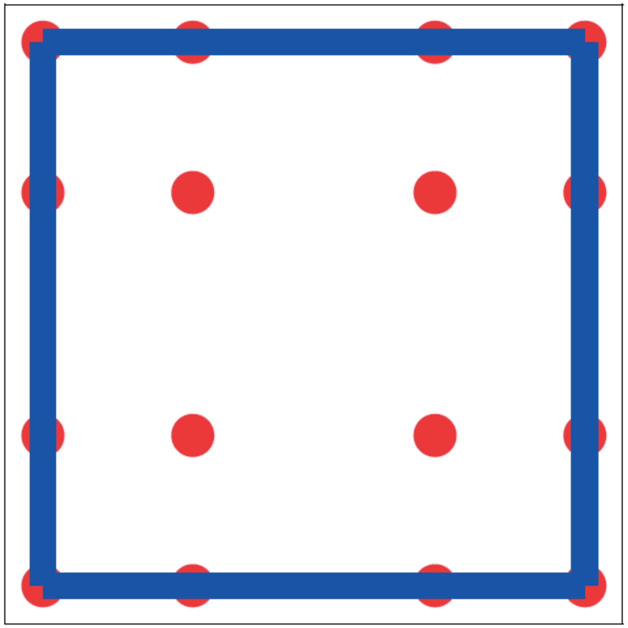
Non-orthogonal coordinate lines

Finite Element Methods

- Finite element methods take the benefits of the spectral transform method with the locality principal of finite-volume methods.
- Can be thought of as spectral transform "in an element"



Nodal Finite Element Method



Fourth order GLL nodes

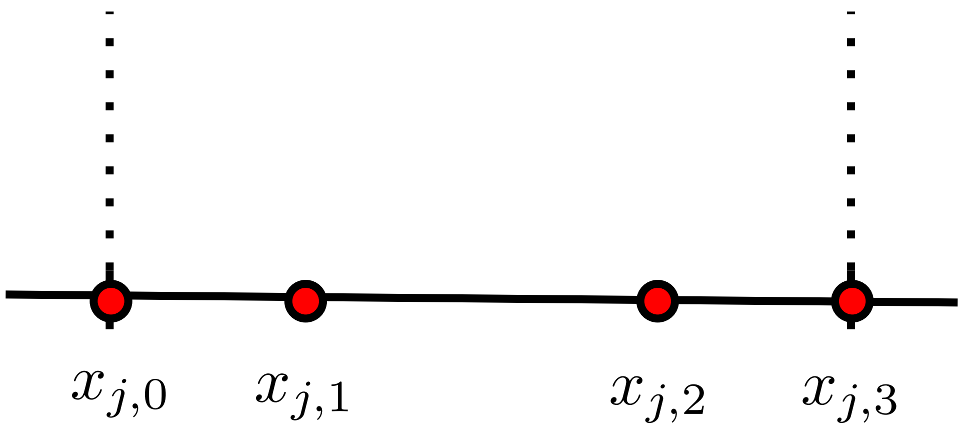
- A nth order finite element method requires n^d basis functions within each element (dimension d).
- To construct basis functions, use GLL nodes within a 2D element.
- Fit polynomials so that each basis function is 1 at one node and 0 at all other nodes.

Image: http://trac.mcs.anl.gov/projects/parvis/wiki/Discretizations

Nodal Finite Element Method

Finite-element methods are a common framework for high-order methods, consisting of multiple number of degrees of freedom per element.

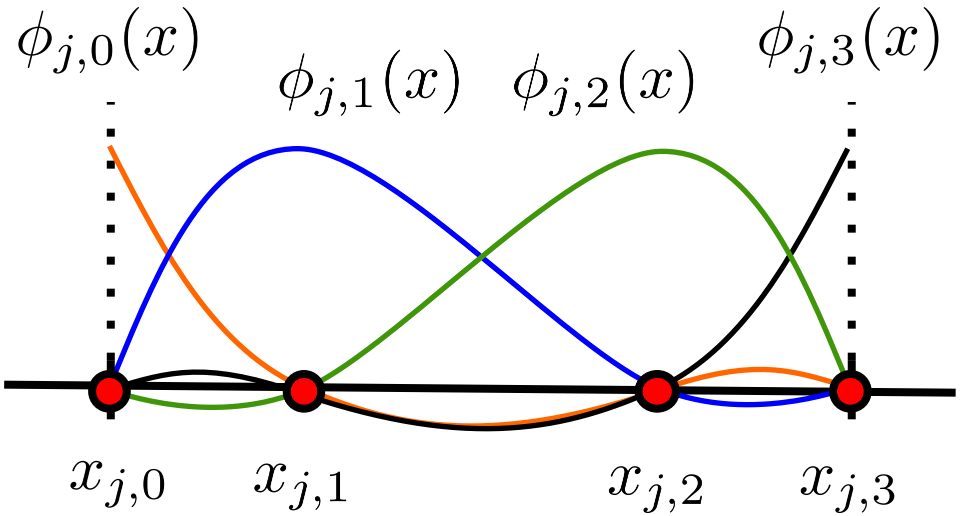
Nodal finite-element methods are one particular flavor, where degrees of freedom are associated with pointwise locations within the element.



Gauss-Lobatto nodes within an element, here for a fourth-order method

Nodal Finite Element Method

Each node is then associated with a unique basis function which is 1 at a particular node and 0 at all other nodes (characteristic functions).



Gauss-Lobatto nodes within an element, here for a fourth-order method

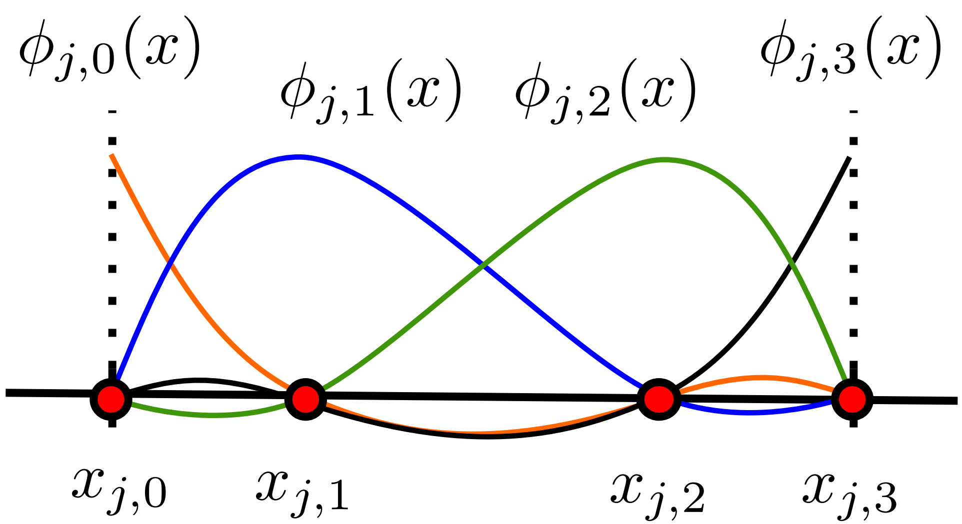
Nodal Finite Element Method

This approach then leads to a discrete / continuous dual for each state variable:

$$q(x,t) = \sum_j \sum_p q_{j,p}(t)\phi_{j,p}(x)$$

Continuous State

Discrete State



Gauss-Lobatto nodes within an element, here for a fourth-order method

Nodal Finite Element Method

General PDE

$$\frac{\partial q}{\partial t} + p(x,q)\frac{\partial}{\partial x} f(x,q) = s(x,q)$$

Question: How can this PDE be solved using nodal FEM?

Differential Approach: (analogous to finite difference methods)

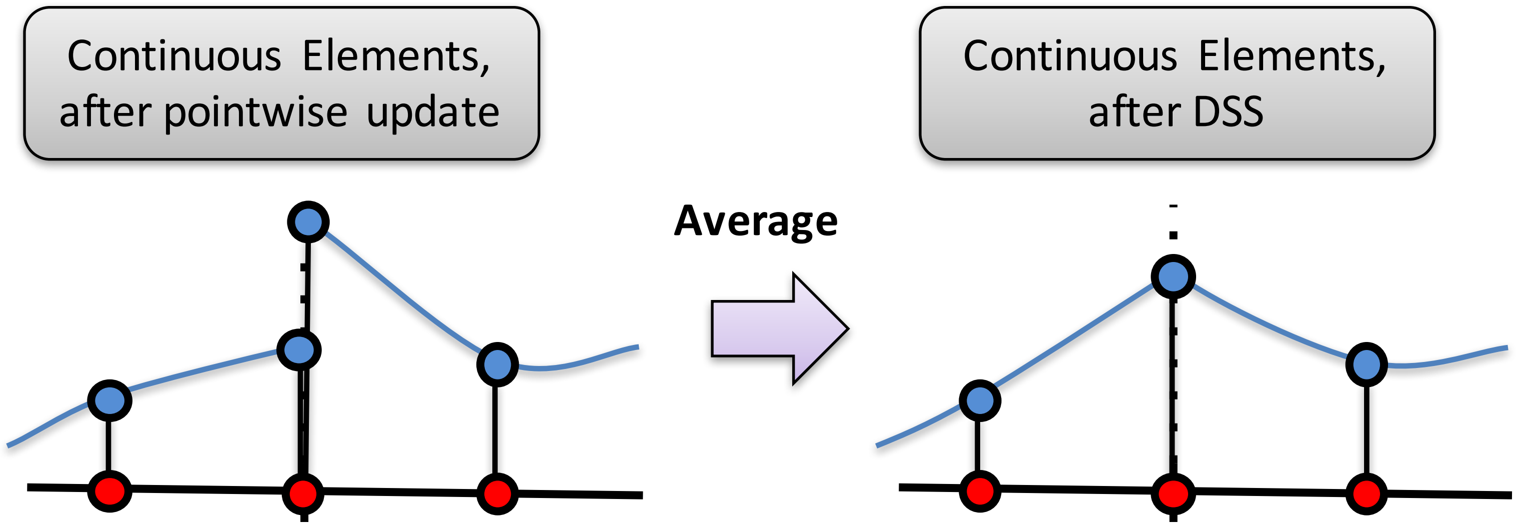
Step 1: Calculate $f_{j,p} = f(x_{j,p}, q_{j,p})$ at each node $x_{j,p}$.

Step 2: Use continuous dual to compute derivatives of f at each node.

Step 3: Apply the update pointwise with an appropriate time integrator.

Nodal Finite Element Method

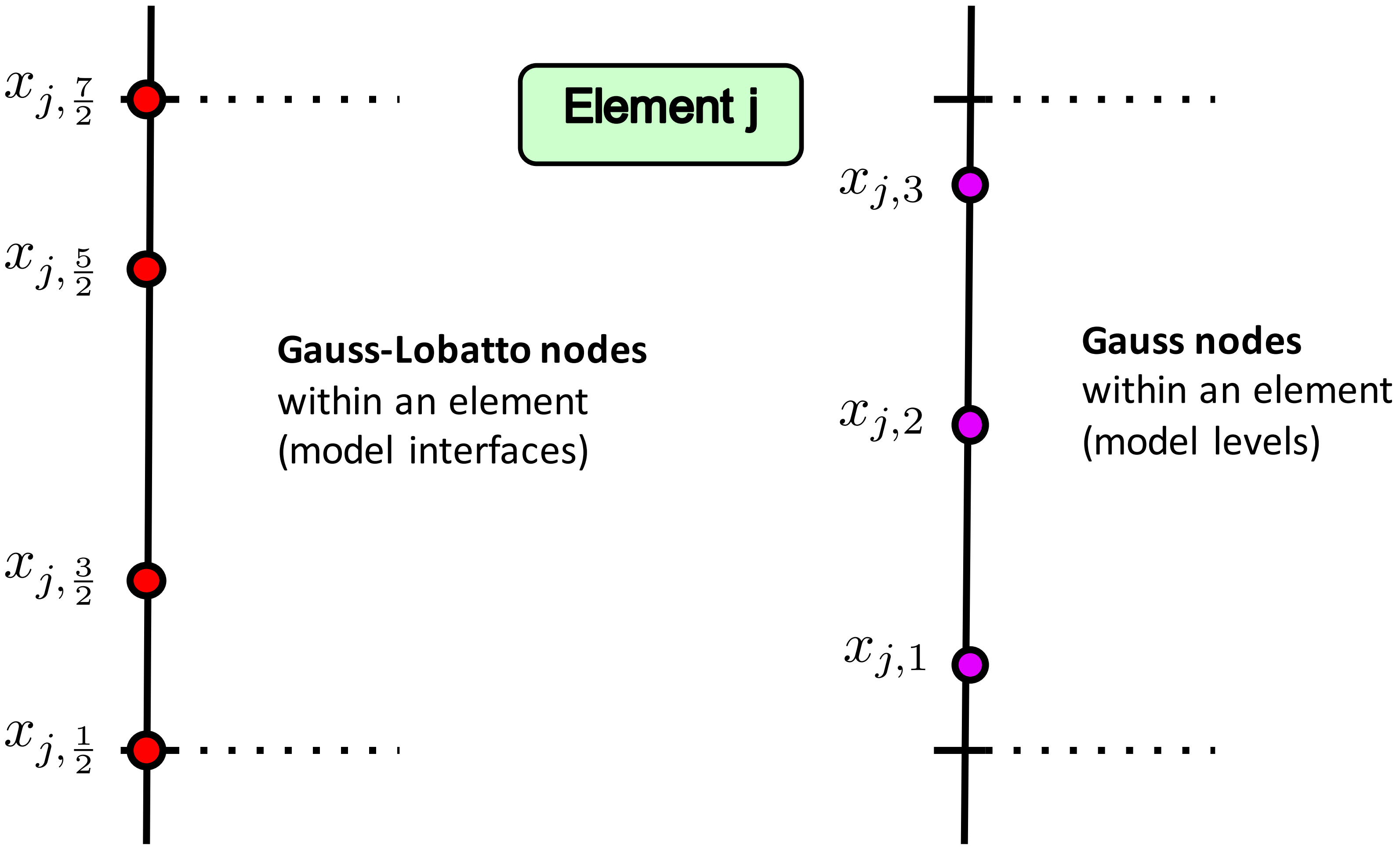
For continuous elements, a projection operator (**Direct Stiffness Summation**) is used to project back into the continuous space:



Leads to **Spectral Element Method (SEM)**.

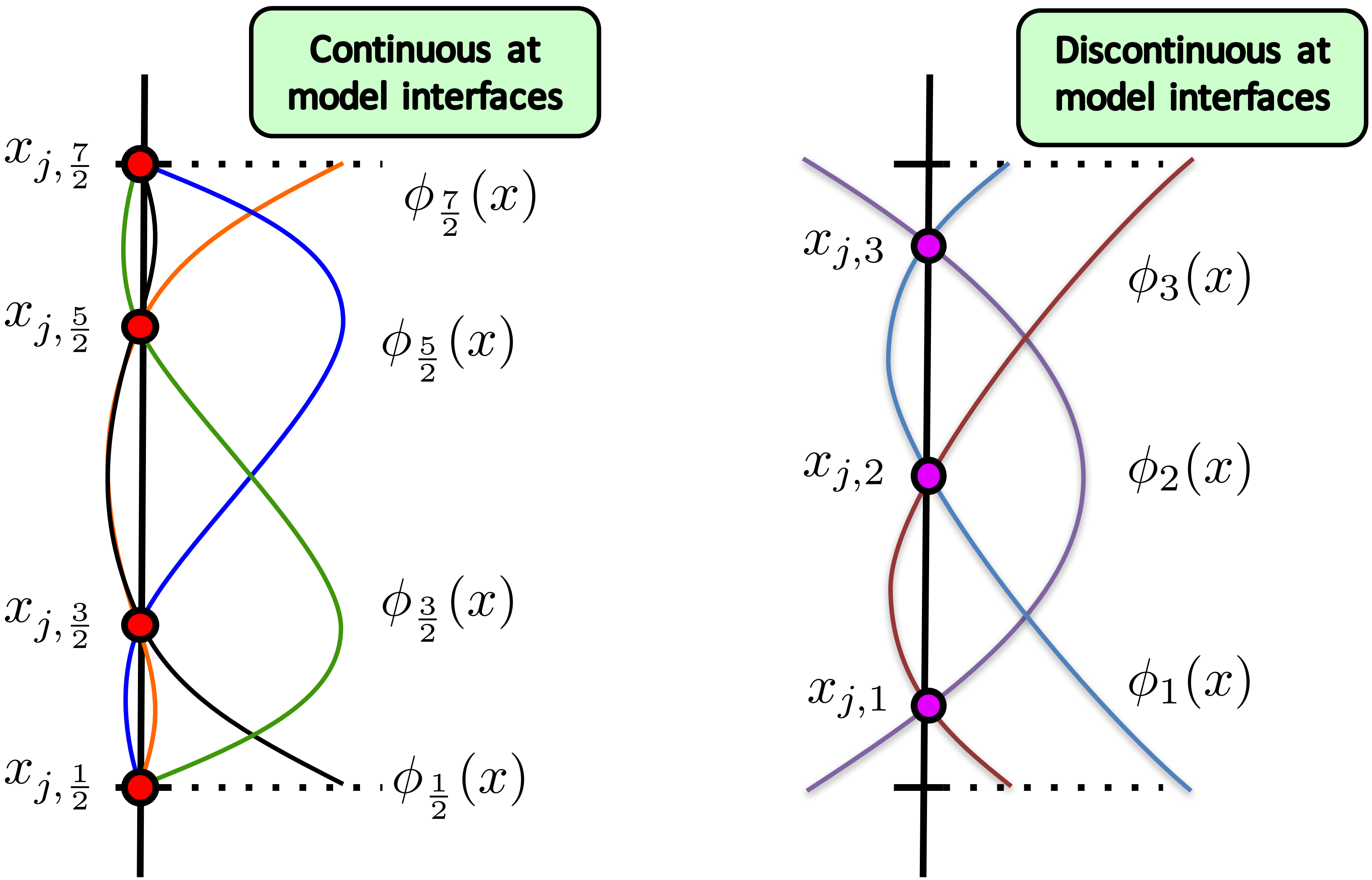
Nodal Mixed Finite Element Methods

Consider an element which contains state variables on model levels and model interfaces.



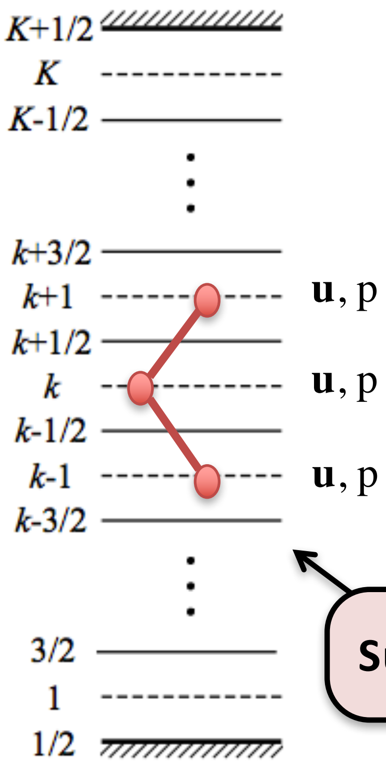
Nodal Mixed Finite Element Methods

Again a set of basis functions are constructed using the characteristic functions.



The Need for Staggering

Unstaggered grid



Supports 2Δx mode!

Linearized vertical velocity equation:

$$\frac{\partial w}{\partial t} + \frac{1}{\overline{\rho}} \frac{\partial p'}{\partial z} = 0$$

Pressure perturbation

Discretization on the unstaggered grid:

$$\frac{\partial w}{\partial t} + \frac{1}{\overline{\rho}} \left(\frac{p_{k+1} - p_{k-1}}{z_{k+1} - z_{k-1}} \right) = 0$$

Result holds for ALL unstaggered numerical methods, including arbitrary order FEM! Even worse at high horizontal / vertical aspect ratios

The Need for Staggering

Linearized vertical velocity equation:

$$\frac{\partial w}{\partial t} + \frac{1}{\overline{\rho}} \frac{\partial p'}{\partial z} = 0$$

Discretization on staggered grid:

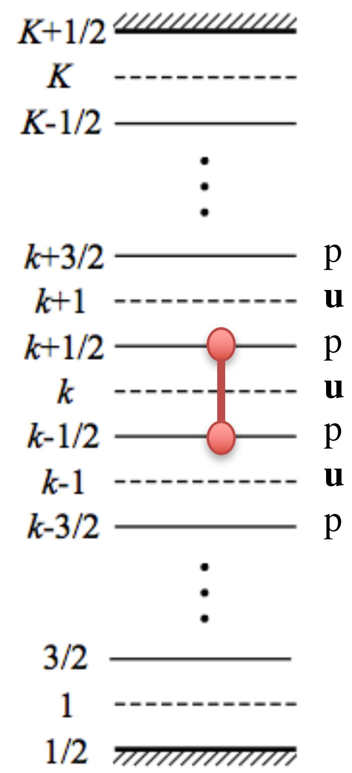
$$\frac{\partial w_k}{\partial t} + \frac{1}{\overline{\rho}} \left(\frac{p_{k+1/2} - p_{k-1/2}}{z_{k+1/2} - z_{k-1/2}} \right) = 0$$

No separation of odd/even modes in w/p. Still can get computational mode in other variables depending on staggering.

Also see:

- Tokioka (1978)
- Arakawa and Moorthi (1988)
- Arakawa and Konor (1996)

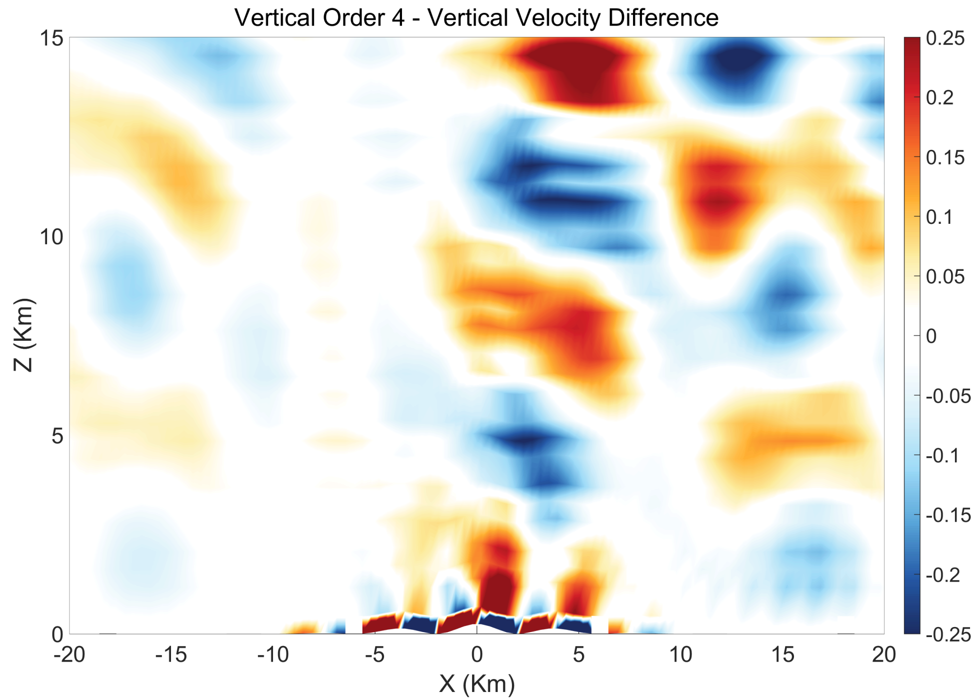
Staggered grid



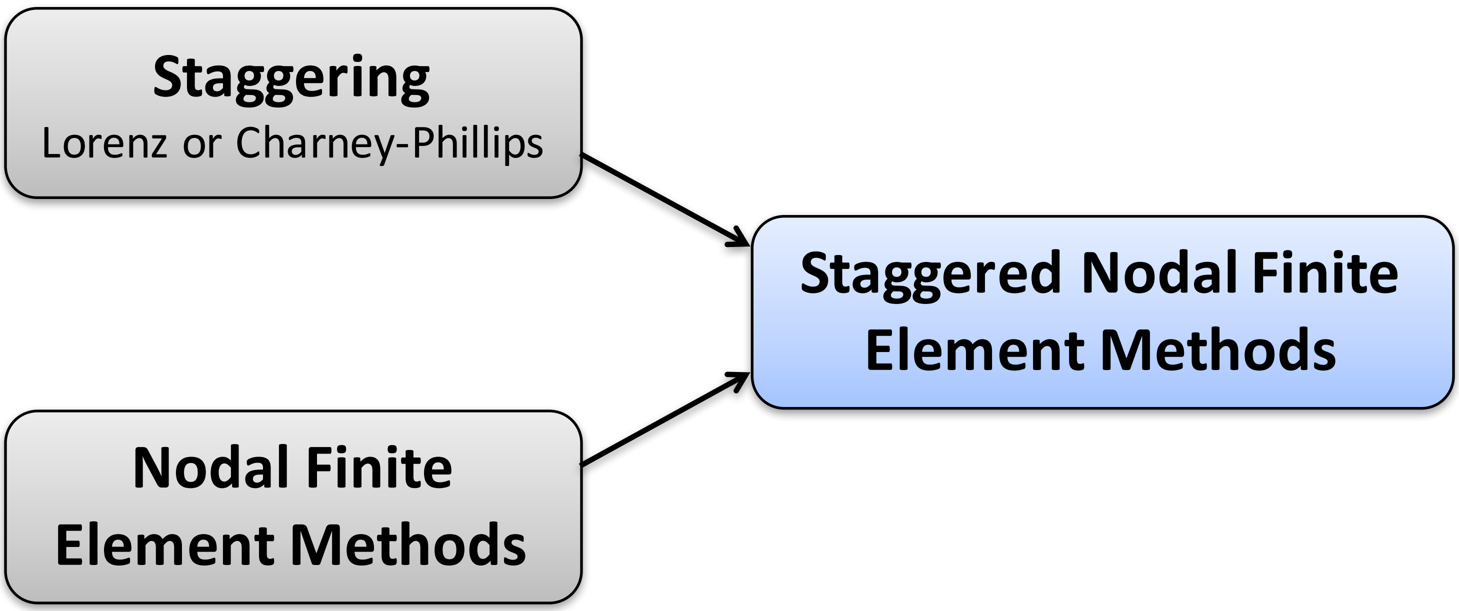
The Need for Staggering

The unstaggered formulation retains a stationary 2dx mode even at high-order accuracy. When the vertical coordinate is not staggered, imprinting from this mode needs to be filtered.

Right: Vertical velocity difference (unstaggered minus reference) for the Schär mountain test without vertical damping.

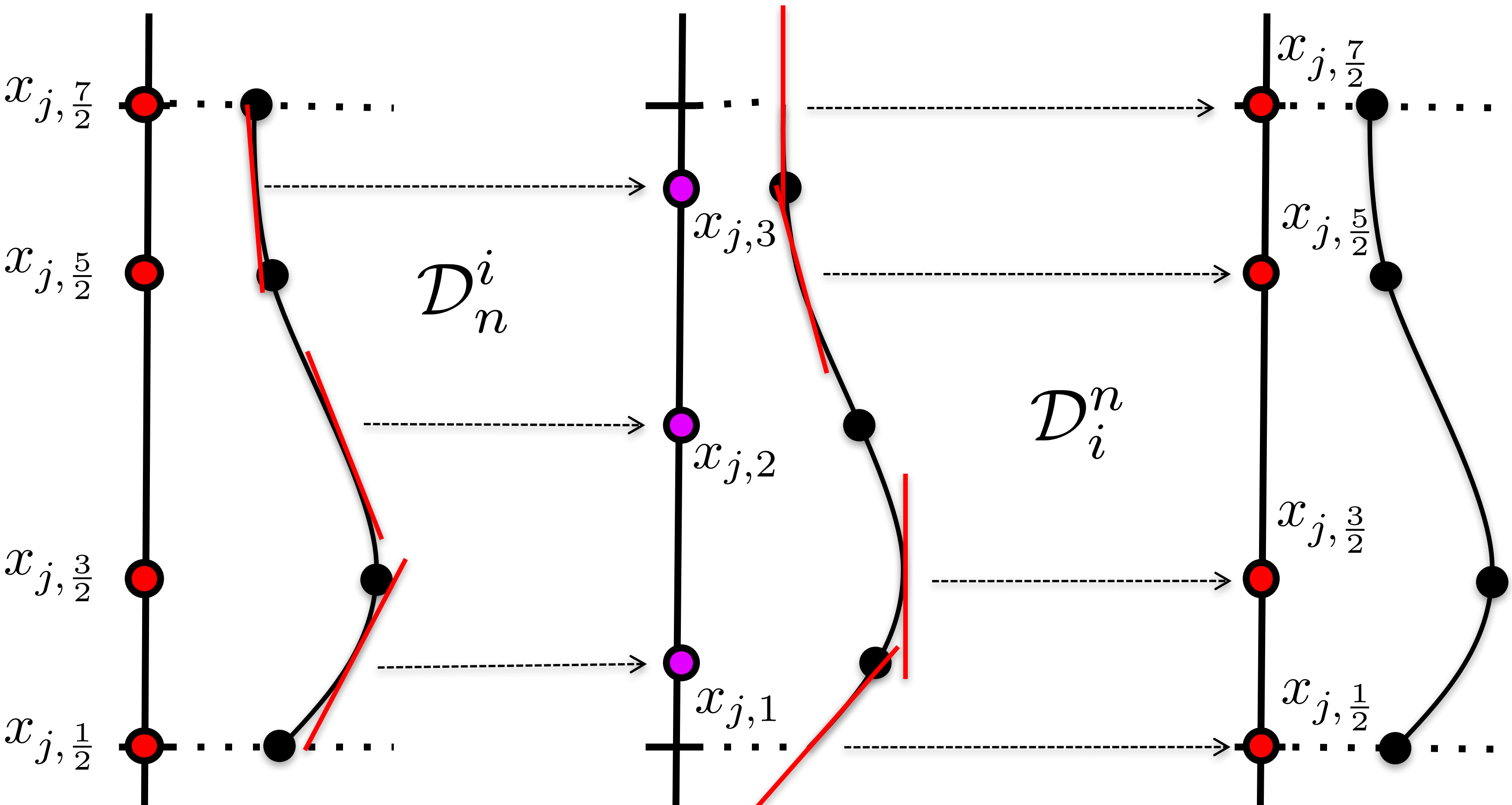


Staggered Nodal Finite Element Methods



Staggered Nodal Finite Element Methods

Simple linear differentiation operators can be defined in order to map derivatives between grid point locations.



Non-Hydrostatic Dynamics

Lorenz Staggering

Continuity

$$\frac{\partial \rho}{\partial t} = -\frac{1}{J}\frac{\partial}{\partial \alpha}(J\rho u^\alpha) - \frac{1}{J}\frac{\partial}{\partial \beta}(J\rho u^\beta) \boxed{- \frac{1}{J}\mathcal{D}_n^i(J(\mathcal{I}_i^n \rho)u^\xi)}$$

Horizontal Velocity

$$\frac{\partial u_\alpha}{\partial t} = -\frac{\partial}{\partial \alpha}(K + \Phi) - \theta\frac{\partial \Pi}{\partial \alpha} + (\boldsymbol{\eta} \times \mathbf{u})_\alpha \,,$$

$$\frac{\partial u_\beta}{\partial t} = -\frac{\partial}{\partial \beta}(K + \Phi) - \theta\frac{\partial \Pi}{\partial \beta} + (\boldsymbol{\eta} \times \mathbf{u})_\beta \,,$$

Vertical Velocity

$$\boxed{\left(\frac{\partial r}{\partial \xi}\right)\frac{\partial w}{\partial t} = -\mathcal{D}_i^n(K + \Phi) - (\mathcal{I}_i^n \theta)(\mathcal{D}_i^n \Pi) + (\boldsymbol{\eta} \times \mathbf{u})_\xi}$$

Thermodynamics

$$\frac{\partial \theta}{\partial t} = -u^\alpha\frac{\partial \theta}{\partial \alpha} - u^\beta\frac{\partial \theta}{\partial \beta} \boxed{- u^\xi(\mathcal{D}_n^n \theta)}$$

The vertical linear operators can be directly utilized to support staggering.

Staggered Nodal Finite Element Methods

The staggered nodal finite element methods provide a natural extension of traditional vertical discretizations to finite elements.

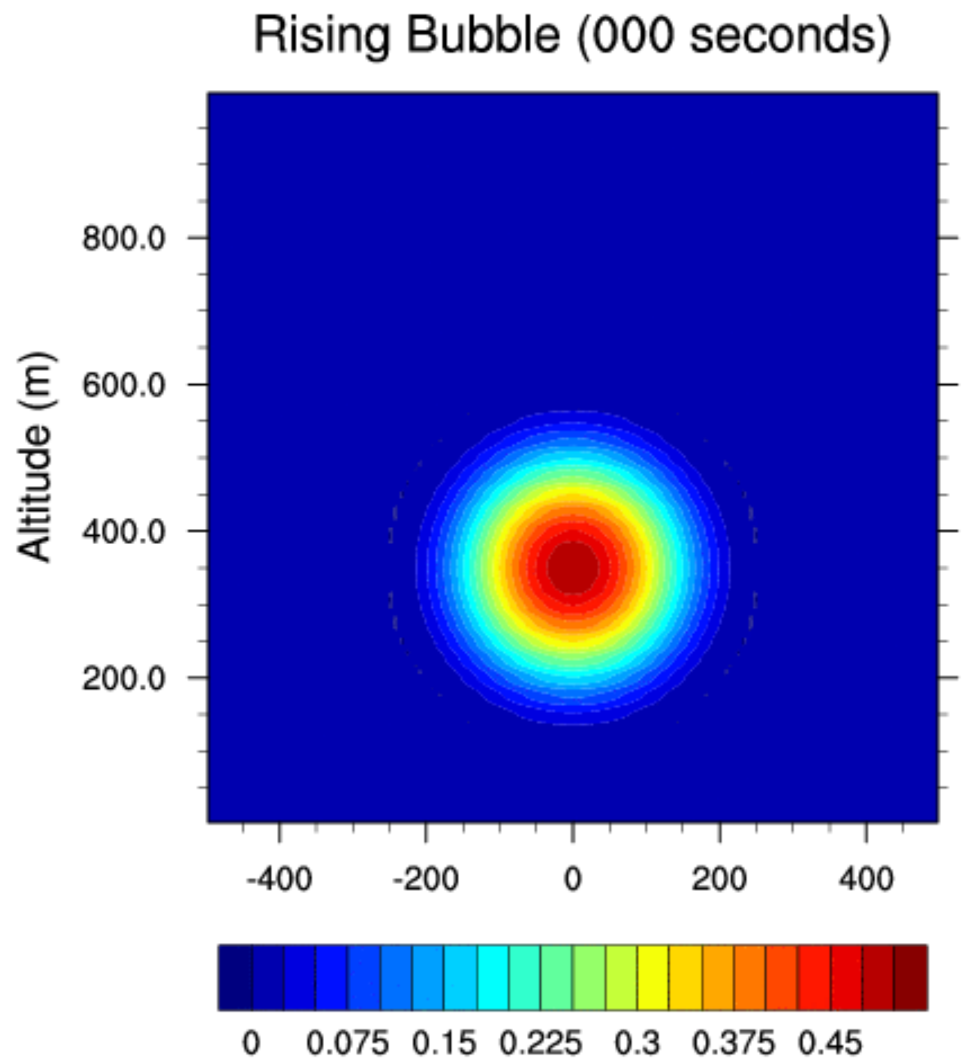
Advantage: These methods support **arbitrary order of accuracy** and allow for practically any choice of vertical variables. In conjunction with improved horizontal-vertical coupling, this approach can improve pressure gradient errors.

Advantage: **Hydrostatic balance** can be captured more effectively by a high-order reconstruction (and so does not require the use of a background reference profile in non-hydrostatic models).

Advantage: With a centered interface flux this scheme can be **mimetic** (energy conserving) at all orders of accuracy.

Rising Thermal Bubble

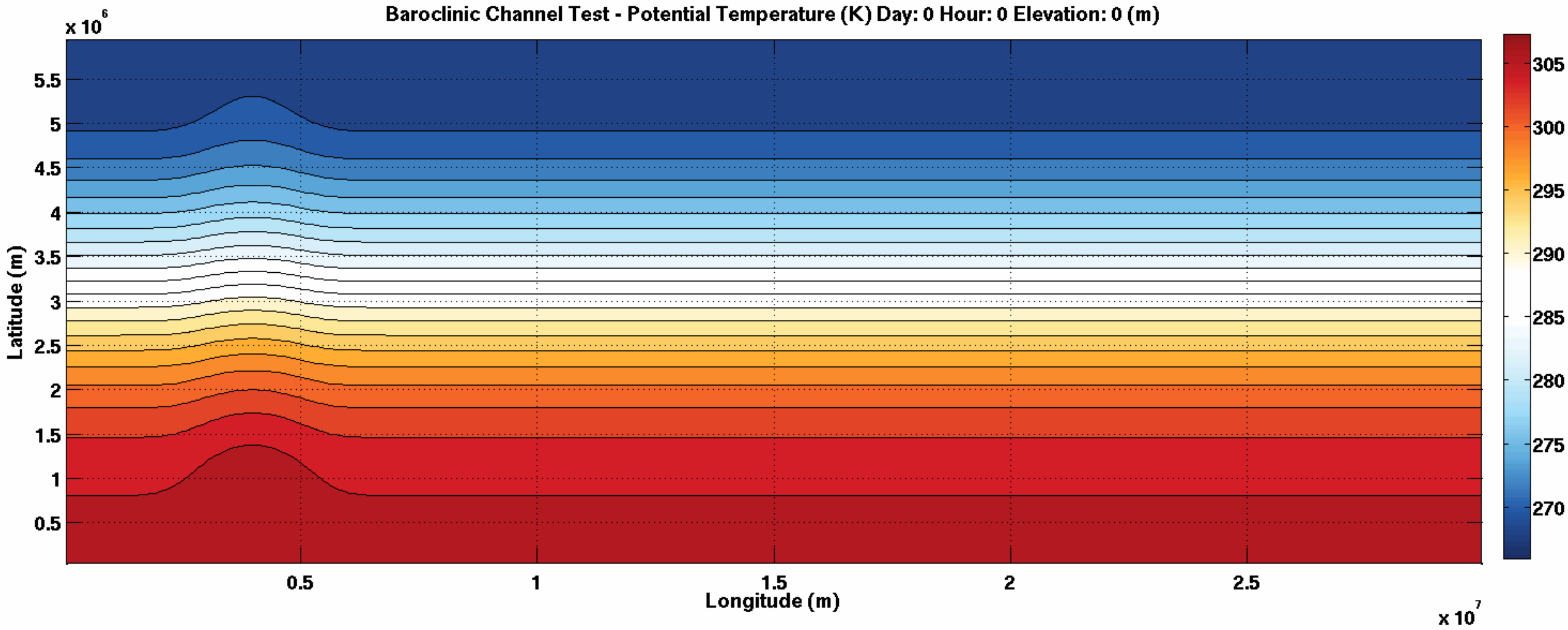
Fourth-order staggered FEM vertical discretization, fully explicit, no reference profile



Baroclinic Instability in a Channel

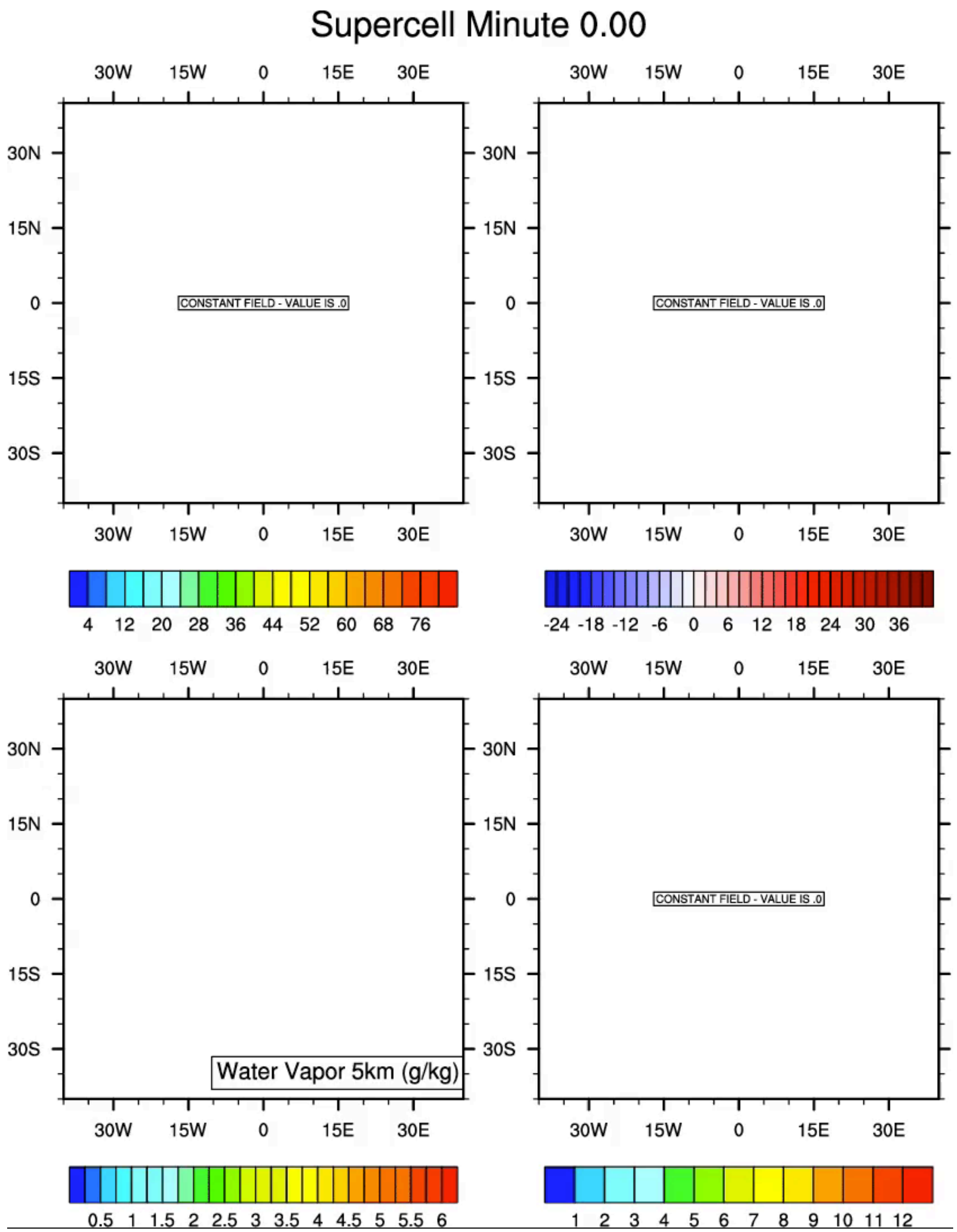
12th-order staggered FEM vertical discretization, Lorenz staggering, Strang splitting, no reference profile

Baroclinic wave is triggered by a topographic ridge.



Supercell

Uniform viscosity against reference profile, Charney-Phillips staggering, ARS232 time integrator



Future Work

- Rigorous comparison of **height-based** and **mass-based** vertical coordinates.
- Rigorous comparison of **Eulerian vertical** and **Lagrangian vertical** coordinates (efficiency, remap frequency and accuracy?)
- Horizontal **mesh refinement** support and studies
- Semi-Lagrangian finite element transport using TempestRemap
- Implement developments in the ***Accelerated Climate Model for Energy spectral element dynamical core.***

ACME
Accelerated
Climate Model for Energy

Thank You!

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