**CORRECTION ÉPREUVE CONCOURS D'ENTRÉE EN $1^{ere}$ ANNÉE DE L'INSTITUT UNIVERSITAIRE DE TECHNOLOGIE DE DOUALA, SESSION DE JUILLET 2021**

SUBJECT CORRECTION OF FIRST YEAR COMPETITIVE ENTRANCE EXAMINATION OF TECHNOLOGY UNIVERSITY INSTITUTE OF DOUALA, JYLY 2021 SESSION

**Filière (Speciality) :** *MTI - MTIN*
**Épreuve de (Paper of) :** *Mathématiques (Mathematics)*
**Durée (Duration) :** *02 heures (02 hours)*

## Exercice 1 (Exercise 1) : 04 points (04 marks)

**1.** d

En posant $X = \ln(x)$, on obtient $X^2 - 3X + 2 = 0$.
En résolvant cette equation, on trouve: $X = 1$ ou $X = 2$.
$X = 1 \Longrightarrow \ln(x) = 1 \Longrightarrow x = e$ et $X = 2 \Longrightarrow \ln(x) = 2 \Longrightarrow x = e^2$.
D'où $S_R = \{e, e^2\}$.

**2.** b

En posant $Y = \cos(x)$, on obtient: $3Y^2 - \left(9 + \frac{3}{2}\sqrt{2}\right) Y + \frac{9}{\sqrt{2}} = 0$.

En résolvant cette equation, on trouve: $Y = \frac{\sqrt{2}}{2}$ ou $Y = 3$.
$Y = 3 \Longrightarrow \cos(x) = 3$, absurde car $\forall x \in R, \cos(x) \in [-1, 1]$

$$Y = \frac{\sqrt{2}}{2} \Longrightarrow \cos(x) = \cos(\frac{\pi}{4}) \Longrightarrow \begin{cases} x = \frac{\pi}{4} + 2k\pi \\ x = -\frac{\pi}{4} + 2k\pi \end{cases} \quad (k \in Z)$$

D'où $S_R = \{-\frac{\pi}{4}, \frac{\pi}{4}\}$.

**3.** b

$\arctan(\frac{1}{2}) \approx 0,46 \mid \arctan(\frac{1}{5}) \approx 0,2 \mid \arctan(\frac{1}{8}) \approx 0,12$
On a donc:

$$\begin{aligned} \arctan(\frac{1}{2}) + \arctan(\frac{1}{5}) + \arctan(\frac{1}{8}) &\approx 0,78 \\ &\approx \frac{3,14}{4} \\ &\approx \frac{\pi}{4} \end{aligned}$$

**4.**

**4.1.** a

La moyenne est:

$$\begin{aligned} \bar{x} &= \frac{\sum n_i x_i}{\sum n_i} \\ &= \frac{97 * 15 + 34 * 16 + 43 * 17 + 20 * 18 + 6 * 19}{97 + 34 + 43 + 20 + 6} \\ &= \frac{3204}{200} \\ &= 16,02 \end{aligned}$$

**4.2.** d

La variance est:

$$V = \frac{\sum n_i x_i^2}{\sum n_i} - \bar{x}^2$$

$$= \frac{51602}{200} - 256,6404$$
$$= 1,3696$$

L'écart-type est:

$$\sigma = \sqrt{V}$$
$$= \sqrt{1,3696}$$
$$= 1,17$$

## Exercice 2 (Exercise 2) : 04 points (04 marks)

**1.** b

**2.** e

$$|z-1-i| = |z-1+i|$$
$$\Longrightarrow |z-(1+i)| = |z-(1-i)|$$
$$\Longrightarrow |z-z_A| = |z-z_B|$$
$$\Longrightarrow |z_{\overrightarrow{AM}}| = |z_{\overrightarrow{BM}}|$$
$$\Longrightarrow ||\overrightarrow{AM}|| = ||\overrightarrow{BM}||$$
$$\Longrightarrow AM = BM$$

**3.** b

$$z' = z+1+i$$
$$= z+(1+i)$$
$$= z + z_{\vec{w}} \text{ (avec } z_{\vec{w}} = 1+i)$$

**4.** c

$$Z = \frac{\cos(\frac{\pi}{3})+i\sin(\frac{\pi}{3})}{\cos(\frac{\pi}{5})+i\sin(\frac{\pi}{5})}.$$
$$= \frac{e^{i\frac{\pi}{3}}}{e^{i\frac{\pi}{5}}}$$
$$= e^{i(\frac{\pi}{3}-\frac{\pi}{5})}$$
$$= e^{i\frac{2\pi}{15}}$$

## Exercice 3 (Exercise 3) : 04 points (04 marks)

**1.** b

$$w_{n+1} = v_{n+1} - u_{n+1}$$
$$= \frac{u_{n+1}+v_n-(u_n+v_n)}{2}$$
$$= \frac{u_{n+1}-u_n}{2}$$
$$= \frac{\frac{u_n+v_n}{2}-u_n}{2}$$
$$= \frac{u_n+v_n-2u_n}{4}$$
$$= \frac{v_n-u_n}{4}$$
$$\frac{w_{n+1}}{w_n} = \frac{v_n-u_n}{4} \times \frac{1}{v_n-u_n}$$
$$= \frac{1}{4}$$

**2.**

**2.1.** b

$$w_n = w_0 q^n$$

$$= 1(\frac{1}{4})^n$$
$$= \frac{1}{4^n} \text{ (avec } \begin{cases} w_0 = v_0 - u_0 = 1 \\ q = \frac{1}{4} \end{cases} \text{)}$$

**2.2.** b

**3.** c

**4.** a

Le premier terme de la suite est:

$$T_1 = 7 + \frac{1}{2}(1) = \frac{15}{2}$$

La somme des n premiers termes est:

$$S = n\left(\frac{T_1 + T_n}{2}\right)$$
$$= n\left(\frac{\frac{15+n}{2} + 7}{2}\right)$$
$$= \frac{n}{4}(29+n)$$

**Exercice 4 (Exercise 4) : 04 points (04 marks)**

**1.** b

$$\int_1^e \frac{\ln(x)}{x}\, d_x = \int_1^e \frac{1}{x}\ln(x)\, d_x$$
$$= \int_1^e (\ln(x))' \ln(x)\, d_x$$
$$= \left[\frac{1}{2}(\ln(x))^2\right]_1^e$$
$$= \frac{1}{2}$$

**2.** ?

$$x^2 = (x-1)^2 + 2x - 1$$
$$= (x-1)^2 + (2x-2) + 1$$

$$\int_{-3}^0 \frac{x^2}{(x-1)^2}\, d_x = \int_{-3}^0 \frac{(x-1)^2 + (2x-2) + 1}{(x-1)^2}\, d_x$$
$$= \int_{-3}^0 \left(\frac{(x-1)^2}{(x-1)^2} + \frac{2x-2}{(x-1)^2} + \frac{1}{(x-1)^2}\right) d_x$$
$$= \int_{-3}^0 \left(1 + \frac{((x-1)^2)'}{(x-1)^2} + \frac{(x-1)'}{(x-1)^2}\right) d_x$$
$$= \left[x + \ln(x-1)^2 + \left(-\frac{1}{(2-1)(x-1)^{2-1}}\right)\right]_{-3}^0$$
$$= \left[x + \ln(x-1)^2 - \frac{1}{x-1}\right]_{-3}^0$$
$$= (0+0+1) - (-3 + \ln(16) + \frac{1}{4})$$
$$= 4 - 4\ln(2) - \frac{1}{4}$$
$$= \ldots$$

**3.** ?

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{4}} \frac{1}{\cos^2(x)}\, d_x = \int_{\frac{\pi}{3}}^{\frac{\pi}{4}} (\tan(x))'\, d_x$$
$$= [\tan(x)]_{\frac{\pi}{3}}^{\frac{\pi}{4}}$$
$$= \tan(\frac{\pi}{4}) - \tan(\frac{\pi}{3})$$
$$= 1 - \sqrt{3}$$

**4.** c

$$\begin{aligned}\int_0^1 (\ln(1+x)+3)\,d_x + \int_0^1 \ln(\frac{1}{1+x})\,d_x &= \int_0^1 \left(\ln(1+x)+3+\ln(\frac{1}{1+x})\right)\,d_x\\ &= \int_0^1 \left((\ln(1+x)+\ln(\frac{1}{1+x})+3\right)\,d_x\\ &= \int_0^1 \left(\ln\left((1+x)(\frac{1}{1+x})\right)+3\right)\,d_x\\ &= \int_0^1 \left(\ln(\frac{1+x}{1+x})+3\right)\,d_x\\ &= \int_0^1 (\ln(1)+3)\,d_x\\ &= [3x]_0^1\\ &= 3-0=3\end{aligned}$$

## Exercice 5 (Exercise 5) : 04 points (04 marks)

**1.** c

On a : $G(\bar{x},\bar{y})$

$\bar{x} = \dfrac{\sum n_i x_i}{\sum n_i} = \dfrac{1*1+1*2+1*5+1*7+1*11+1*13}{1+1+1+1+1+1} = \dfrac{1+2+5+7+11+13}{6} = 6,5$

$\bar{y} = \dfrac{\sum n_j y_j}{\sum n_j} = 31,575$

**2.** b

**3.** c

On a:

$M_1(x_1=1, y_1=24)$ et $M_6(x_6=13, y_6=39)$

L'équation de la droite $(M_1M_6)$ est de la forme: $y = ax+b$

$M_1, M_6 \in (M_1M_6) \Longrightarrow \begin{cases} 24 = a+b \\ 39 = 13a+b \end{cases}.$

En résolvant ce système, on trouve: $a = 1,25$

**4.** a

x passe de 1 à 13, alors x augmente de: $(\dfrac{13-1}{1})\times 100 = 1200\%$