HOFEM-AIRBUS Transmission/Reflection

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Outline

1. On the FEM Implementation of TX/RX Conditions in HOFEM
   - Existing FEM Formulation in HOFEM
   - Two-Port Network Parameters

2. $[Z]/[Y]$ Approach
   - FEM Formulation
   - Testing
   - HOFEM Implementation

Outline

We start considering different alternatives to implement the TX/RX conditions in the context of the present FEM formulation coded in HOFEM

Outline

FEM Formulation

- Formulation based on double curl vector wave equation (use of **E** or **H**).

$$\nabla \times \left(f_r^{-1} \nabla \times \mathbf{V}\right) - k_0^2 g_r \mathbf{V} = -jk_0 H_0 \mathbf{P} + \nabla \times f_r^{-1} \mathbf{L}$$

Table: Formulation magnitudes and parameters

	$\mathbf{V}$	$\mathbf{V}^d$	$\bar{\bar{f}}_r$	$\bar{\bar{g}}_r$	h	$\mathbf{P}$	$\mathbf{L}$
Form. **E**	$\mathbf{E}$	$\mathbf{H}$	$\bar{\bar{\mu}}_r$	$\bar{\bar{\epsilon}}_r$	η	$\mathbf{J}$	$\mathbf{M}$
Form. **H**	$\mathbf{H}$	$\mathbf{E}$	$\bar{\bar{\epsilon}}_r$	$\bar{\bar{\mu}}_r$	$-\frac{1}{\eta}$	$\mathbf{M}$	$-\mathbf{J}$

FEM Formulation (cont.)

- Use of $\mathbf{H}(\mathrm{curl})$ spaces:

$$\mathbf{H}(\mathrm{curl})_0 = \{\mathbf{W} \in \mathbf{H}(\mathrm{curl}),\ \hat{\mathbf{n}} \times \mathbf{W} = 0 \ \text{ on } \ \Gamma_\mathrm{D}\} \tag{1}$$

$$\mathbf{H}(\mathrm{curl}) = \{\mathbf{W} \in L^2,\ \boldsymbol{\nabla} \times \mathbf{W} \in L^2\} \tag{2}$$

- and Galerkin method leads to (with respect to the double-curl term only) to:

$$\int_\Omega (\boldsymbol{\nabla} \times \mathbf{F}) \cdot \left(\overline{\overline{f}}_r^{\,-1} \boldsymbol{\nabla} \times \mathbf{V}\right) d\Omega + \underbrace{\int_\Gamma \mathbf{F} \cdot (\hat{\mathbf{n}} \times \underbrace{(\overline{\overline{f}}_r^{\,-1} \boldsymbol{\nabla} \times \mathbf{V})}_{-jk_0 h_0 \mathbf{V}^d}) d\Gamma}_{\text{“natural” b.c.}}$$

- More precisely
  - Form. $\mathbf{E}$: $\overline{\overline{f}}_r^{\,-1} \boldsymbol{\nabla} \times \mathbf{V} = -jk_0 \eta_0 \, \mathbf{H}$
  - Form. $\mathbf{H}$: $\overline{\overline{f}}_r^{\,-1} \boldsymbol{\nabla} \times \mathbf{V} = +j \dfrac{k_0}{\eta_0} \, \mathbf{E}$

FEM Formulation (cont.)

The integral boundary term $\int_\Gamma \mathbf{F} \cdot (\hat{\mathbf{n}} \times (\bar{\bar{f}}_r^{-1} \boldsymbol{\nabla} \times \mathbf{V})) d\Gamma$

- Can be used to weakly impose some boundary conditions of the problem
- For instance, Neumann boundary condition

$$\hat{\mathbf{n}} \times \left(\bar{\bar{f}}_r^{-1} \boldsymbol{\nabla} \times \mathbf{V}\right) = \boldsymbol{\Psi}_\mathrm{N}$$

$$\longrightarrow \quad \int_\Gamma \mathbf{F} \cdot (\hat{\mathbf{n}} \times (\bar{\bar{f}}_r^{-1} \boldsymbol{\nabla} \times \mathbf{V})) d\Gamma = \int_\Gamma \boldsymbol{\Psi}_\mathrm{N} \, d\Gamma$$

- For instance, ABC boundary condition

$$\hat{\mathbf{n}} \times \left(\bar{\bar{f}}_r^{-1} \boldsymbol{\nabla} \times \mathbf{V}\right) + \gamma \, \hat{\mathbf{n}} \times \hat{\mathbf{n}} \times \mathbf{V} = \boldsymbol{\Psi}_\mathrm{C}$$

$$\longrightarrow \quad \int_\Gamma \mathbf{F} \cdot (\hat{\mathbf{n}} \times (\bar{\bar{f}}_r^{-1} \boldsymbol{\nabla} \times \mathbf{V})) \, d\Gamma = \int_\Gamma \boldsymbol{\Psi}_\mathrm{C} d\Gamma + \gamma \int_\Gamma (\hat{\mathbf{n}} \times \mathbf{F}) \cdot (\hat{\mathbf{n}} \times \mathbf{V}) \, d\Gamma_\mathrm{C}$$

FEM Formulation (cont.)

- Note that the implementation of above boundary conditions is straightforward because
  - Either, the integrand of the boundary term was a known function
  - or, it can be expressed in terms of the primal unknown $\mathbf{V}$
- At the end of the day, the contribution of the boundary term is translated into algebra as
  - extra contributions for the values of the matrix coefficients (related to the existing g_i associated to the corresponding boundary).
  - I.e., no extra degrees of freedom g_i are needed

What does it happen when that is not the case?

FEM Continuity Conditions

- Assuming $\mathbf{V} = \sum_i g_i \mathbf{N}_i \leftrightarrow \{g\}$

Between Neighbour Elements 1 and 2

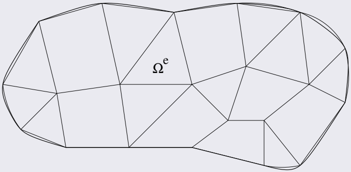
- Two continuity conditions:
  - Strong continuity of $\hat{\mathbf{n}} \times \mathbf{V}$

$$\{g\}_1 = \{g\}_2$$

  - Weak continuity of $\hat{\mathbf{n}} \times \mathbf{V}^d$

$$\int_\Gamma \mathbf{F} \cdot (\hat{\mathbf{n}}_1 \times (\bar{\bar{f}}_r^{-1} \nabla \times \mathbf{V}_1)) d\Gamma$$

$$+ \int_\Gamma \mathbf{F} \cdot (\hat{\mathbf{n}}_2 \times (\bar{\bar{f}}_r^{-1} \nabla \times \mathbf{V}_2)) d\Gamma = 0$$



We simply omit the term

Note that $\hat{\mathbf{n}} \times \mathbf{F}_1 = \hat{\mathbf{n}} \times \mathbf{F}_2$ and that

$$\mathbf{F} \cdot (\hat{\mathbf{n}} \times (\bar{\bar{f}}_r^{-1} \nabla \times \mathbf{V}) = (\hat{\mathbf{n}} \times \mathbf{F}) \cdot (\hat{\mathbf{n}} \times (\bar{\bar{f}}_r^{-1} \nabla \times \mathbf{V})$$

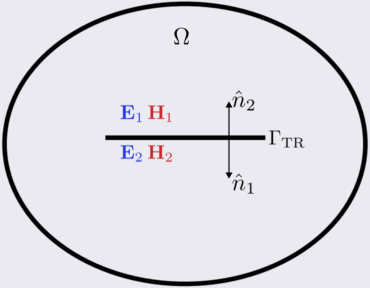
FEM Continuity Conditions (cont.)

Between Elements Simulating a Thin Sheet

- In general, we need to break continuity on
  - $\hat{\mathbf{n}} \times \mathbf{V} \Rightarrow$ Duplication of degrees of freedom: $\{g\}_1, \{g\}_2$
  - $\hat{\mathbf{n}} \times \mathbf{V}^d \Rightarrow$ The integral boundary term can not be omitted
- Two linearly independent equations involving four magnitudes $\mathbf{E}_1, \mathbf{E}_2, \mathbf{H}_1, \mathbf{H}_2$:

$$[\ ]_{2x2}\,\{X\}_{2x1} = \{B\}_{2x1}$$

- Different possibilities depending on the magnitudes chosen as sources (right hand side B) and responses X
- Clear analogy with two-port network parameters of linear circuit analysis: $\mathbf{E} \leftrightarrow V \qquad \mathbf{H} \leftrightarrow I$



The Immittance Approach

The Simplest (Least Invasive) Approach

- We express $\hat{\mathbf{n}} \times \mathbf{V}^d$ for region 1 and region 2 in terms of $\hat{\mathbf{n}} \times \mathbf{V}$ of region 1 and region 2, i.e.,

$$\begin{Bmatrix} \hat{\mathbf{n}} \times \mathbf{V}_1^d \\ \hat{\mathbf{n}} \times \mathbf{V}_2^d \end{Bmatrix} = \begin{bmatrix} YZ_{11} & YZ_{12} \\ YZ_{21} & YZ_{22} \end{bmatrix} \begin{Bmatrix} \hat{\mathbf{n}} \times \mathbf{V}_1 \\ \hat{\mathbf{n}} \times \mathbf{V}_2 \end{Bmatrix}$$

  - This is equivalent to an immitance characterization of the equivalent two-port network connecting upper and lower regions separated by the sheet.
    - ⋆ Form. $\mathbf{E}$: $\mathbf{V} = \mathbf{E}$, $\mathbf{V}^d = \mathbf{H}$, immittance $\equiv$ admitance
    - ⋆ Form. $\mathbf{E}$: $\mathbf{V} = \mathbf{E}$, $\mathbf{V}^d = \mathbf{H}$, immittance $\equiv$ impedance
- We substitute $\hat{\mathbf{n}} \times \mathbf{V}^d$ on the integral boundary terms corresponding to region 1 and region 2 by its linear combination of $\hat{\mathbf{n}} \times \mathbf{V}_1$ and $\hat{\mathbf{n}} \times \mathbf{V}_2$.

- For instance, with **E**-formulation we have

$$\int_\Gamma \mathbf{F}_1 \cdot (\hat{\mathbf{n}} \times (\bar{\bar{\mu}}_r^{-1} \boldsymbol{\nabla} \times \mathbf{E}_1)) d\Gamma = y_{11} \int_\Gamma \mathbf{F}_1 \cdot (\hat{\mathbf{n}} \times \mathbf{E}_1)) d\Gamma + y_{12} \int_\Gamma \mathbf{F}_1 \cdot (\hat{\mathbf{n}} \times \mathbf{E}_2)) d\Gamma$$

$$\int_\Gamma \mathbf{F}_2 \cdot (\hat{\mathbf{n}} \times (\bar{\bar{\mu}}_r^{-1} \boldsymbol{\nabla} \times \mathbf{E}_2)) d\Gamma = y_{21} \int_\Gamma \mathbf{F}_2 \cdot (\hat{\mathbf{n}} \times \mathbf{E}_1)) d\Gamma + y_{22} \int_\Gamma \mathbf{F}_2 \cdot (\hat{\mathbf{n}} \times \mathbf{E}_2)) d\Gamma$$

where $\hat{\mathbf{n}} \times \mathbf{F}_1 = \hat{\mathbf{n}} \times \mathbf{F}_2$ (conformal mesh above-below the sheet).

The Immittance Approach (cont.)

The Simplest (Least Invasive) Approach

- Thus, the TX/RX continuity conditions imposed by the sheet are translated into algebra simply as extra contributions for the values of the matrix coefficients (related to the duplicated degrees of freedom $\{g\}_1$, $\{g\}_2$ associated to the sheet).

Advantages

- Simple (non code invasing)
  - No new variational unknowns
  - No new equations but simply additional terms to the existing ones
  - Only requires replication of uknonws on sheet: $\{g\}_1 \neq \{g\}_2$
- Reciprocity of the material ($y_{12} = y_{21}$) $\Rightarrow$ symmetry of FEM matrix
- Short circuit (PEC) and open circuit (PMC) conditions naturally reproducible

The Immittance Approach (cont.)

The Simplest (Least Invasive) Approach

Disadvantages

- Total transmission (sheet transparency) not reproducible
  - Singular system of equations
  - This situation must be either avoided or taken into account explicitly

Short Circuit (PEC) Sheet

- We have $y_{12} = y_{21} = 0$ and $y_{11} = y_{22} = \infty$
- Thus, the equation associated to $\{g\}_1$ has a dominant term

$$\ldots + y_{11} \int_\Gamma \mathbf{F}_1 \cdot (\hat{\mathbf{n}} \times \mathbf{E}_1)) d\Gamma + y_{12} \int_\Gamma \mathbf{F}_1 \cdot (\hat{\mathbf{n}} \times \mathbf{E}_2)) d\Gamma \approx \infty \int_\Gamma \mathbf{F}_1 \cdot (\hat{\mathbf{n}} \times \mathbf{E}_1)) d\Gamma$$

that corresponds to impose $\hat{\mathbf{n}} \times \mathbf{E}_1 = 0$, i.e., algebraically $\{g\}_1 = 0$

- Analogously, with equation associated to $\{g\}_2$

$$\ldots + y_{21} \int_\Gamma \mathbf{F}_2 \cdot (\hat{\mathbf{n}} \times \mathbf{E}_1)) d\Gamma + y_{22} \int_\Gamma \mathbf{F}_2 \cdot (\hat{\mathbf{n}} \times \mathbf{E}_2)) d\Gamma \approx \infty \int_\Gamma \mathbf{F}_2 \cdot (\hat{\mathbf{n}} \times \mathbf{E}_2)) d\Gamma$$

that corresponds to impose $\hat{\mathbf{n}} \times \mathbf{E}_2 = 0$, i.e., algebraically $\{g\}_2 = 0$

The Immittance Approach (cont.)

Special Cases

Open Circuit (PMC) Sheet

- We have $y_{12} = y_{21} = 0$ and $y_{11} = y_{22} = 0$
- Thus, the equation associated to $\{g\}_1$ is equivalent to have

$$\int_\Gamma \mathbf{F}_1 \cdot (\hat{\mathbf{n}} \times (\bar{\bar{\mu}}_r^{-1} \nabla \times \mathbf{E}_1)) d\Gamma = 0$$

that corresponds to weakly impose $\hat{\mathbf{n}} \times \bar{\bar{\mu}}_r^{-1} \nabla \times \mathbf{E}_1 = 0$ (Neumann b.c.) i.e., $\hat{\mathbf{n}} \times \mathbf{H}_1 = 0$.

- Analogously, with equation associated to $\{g\}_2$

$$\int_\Gamma \mathbf{F}_2 \cdot (\hat{\mathbf{n}} \times (\bar{\bar{\mu}}_r^{-1} \nabla \times \mathbf{E}_2)) d\Gamma = 0$$

that corresponds to weakly impose $\hat{\mathbf{n}} \times \bar{\bar{\mu}}_r^{-1} \nabla \times \mathbf{E}_2 = 0$ (Neumann b.c.) i.e., $\hat{\mathbf{n}} \times \mathbf{H}_2 = 0$.

The Immittance Approach (cont.)

Special Cases

Total Transmission (sheet transparency)

- It is clear that from

$$\begin{Bmatrix} \hat{\mathbf{n}} \times \mathbf{V}_1^d \\ \hat{\mathbf{n}} \times \mathbf{V}_2^d \end{Bmatrix} = \begin{bmatrix} YZ_{11} & YZ_{12} \\ YZ_{21} & YZ_{22} \end{bmatrix} \begin{Bmatrix} \hat{\mathbf{n}} \times \mathbf{V}_1 \\ \hat{\mathbf{n}} \times \mathbf{V}_2 \end{Bmatrix}$$

  is not possible to impose full continuity of $\hat{\mathbf{n}} \times \mathbf{V}$, $\hat{\mathbf{n}} \times \mathbf{V}^d$, i.e.,

$$\begin{Bmatrix} \hat{\mathbf{n}} \times \mathbf{V}_1 \\ \hat{\mathbf{n}} \times \mathbf{V}_1^d \end{Bmatrix} = \begin{Bmatrix} \hat{\mathbf{n}} \times \mathbf{V}_2 \\ \hat{\mathbf{n}} \times \mathbf{V}_2^d \end{Bmatrix}$$

- Technically, we have $y_{12} = y_{21} = \infty$ and $y_{11} = y_{22} = \infty$

The Immittance Approach (cont.)
Special Cases

Total Transmission (sheet transparency)

- Thus, the equation associated to $\{g\}_1$ has two dominant terms

$$\ldots + \infty \int_\Gamma \mathbf{F}_1 \cdot (\hat{\mathbf{n}} \times \mathbf{E}_1)) d\Gamma + \infty \int_\Gamma \mathbf{F}_1 \cdot (\hat{\mathbf{n}} \times \mathbf{E}_2)) d\Gamma$$

- Analogously, with equation associated to $\{g\}_2$

$$\ldots + \infty \int_\Gamma \mathbf{F}_2 \cdot (\hat{\mathbf{n}} \times \mathbf{E}_1)) d\Gamma + \infty \int_\Gamma \mathbf{F}_2 \cdot (\hat{\mathbf{n}} \times \mathbf{E}_2)) d\Gamma$$

Total Transmission (sheet transparency)

- Both equations tends to be the same equation

  $\Rightarrow$ Singular FEM matrix

  (I mush still check signs for y_{ij}) Nevertheless, the above resulting equations can never impose continuity of $\hat{\mathbf{n}} \times \mathbf{V}$

The Immittance Approach (cont.)

Special Cases

Workaround

- We must identify the case, i.e., when y_{ij} is "large enough"
- Then, we have two main possibilities:
  - Ignore TR/RX boundary conditions for that sheet, i.e., do nothing
    - Do not replicate degres of freedom $\{g\}$
    - Do not alter FEM equations
  - Recover full continuity, i.e., continuity of $\hat{\mathbf{n}} \times \mathbf{V}$ and $\hat{\mathbf{n}} \times \mathbf{V}^d$ once replication of $\{g\}$ has been perfomed
    - Several possibilities

How to Recover Full Continuity?

- We keep the integral boundary term $\int_\Gamma \mathbf{F} \cdot (\hat{\mathbf{n}} \times (\bar{\bar{f}}_r^{-1} \nabla \times \mathbf{V})) d\Gamma$ for region 1 and region 2
- We add two equations to recover continuity of $\hat{\mathbf{n}} \times \mathbf{V}$ and $\hat{\mathbf{n}} \times \mathbf{V}^d$

Continuity of $\hat{\mathbf{n}} \times \mathbf{V}$

- We add one strong condition equation

$$\{g\}_1 = \{g\}_2$$

- Perfomed at the algebraic level, i.e., operating in the matrix by fixing all coefficients on the corresponding row to zero except the two being related (paired)
- The above operation is simple, trivial I would say.
- However, if I am not wrong, it introduces asymmetry in the FEM matrix

The Immittance Approach (cont.)

Special Cases

Continuity of $\hat{\mathbf{n}} \times \mathbf{V}^d$, i.e., of $\hat{\mathbf{n}} \times (\bar{\bar{f}}_r^{\,-1} \nabla \times \mathbf{V})$

- We add a equation to weakly force continuity of the dual variable

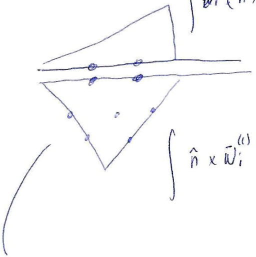
$$\int_\Gamma \mathbf{F}_1 \cdot (\hat{\mathbf{n}} \times (\bar{\bar{\mu}}_r^{\,-1} \nabla \times \mathbf{E}_1)) d\Gamma = \int_\Gamma \mathbf{F}_2 \cdot (\hat{\mathbf{n}} \times (\bar{\bar{\mu}}_r^{\,-1} \nabla \times \mathbf{E}_2)) d\Gamma \qquad (3)$$

- Note that the above equation after discretization introduces asymmetry in the matrix
  - Although, $\hat{\mathbf{n}} \times \mathbf{F}_1 = \hat{\mathbf{n}} \times \mathbf{F}_2$ due to conformity of the mesh
  - the discretization of $\hat{\mathbf{n}} \times \nabla \times \mathbf{E}$ involves all basis functions of the finite element, i.e., not only basis functions on the common boundary (sheet)

The Immittance Approach (cont.)

Special Cases

$$\int (\hat{n} \times \bar{W}_i^{(1)}) \cdot \left(\hat{n} \times \mu_r^{-1} (\nabla \times N_j^{(1)})\right) d\Gamma$$



$$\int \hat{n} \times \bar{W}_i^{(1)} \cdot \left(n \times \mu_r^{-1} \nabla \times N_j^{(2)}\right) d\Gamma$$

$\nabla \times E$ depends on all g_i of the element

- The above procedure of recovering full continuity can also be approached by

The $[g]$, $[h]$, $[ABCD]$ Approaches

Outline

TITULO

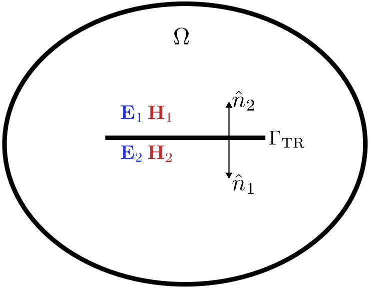
Hola

Outline

We describe the implementation of the TX/RX conditions uisng the characterization of the material sheet in terms of its immittance (impedance/admittance) matrix

Outline

Formulation



$$\hat{n}_1 \times (\mu_r^{-1}\nabla \times \mathbf{E}_1) - \frac{jk_0}{\eta} y_{11}\hat{n}_1 \times (\hat{n}_1 \times \mathbf{E}_1) - \\ - \frac{jk_0}{\eta} y_{12}\hat{n}_2 \times (\hat{n}_2 \times \mathbf{E}_2) = 0,$$

$$\hat{n}_2 \times (\mu_r^{-1}\nabla \times \mathbf{E}_2) - \frac{jk_0}{\eta} y_{21}\hat{n}_1 \times (\hat{n}_1 \times \mathbf{E}_1) - \\ - \frac{jk_0}{\eta} y_{22}\hat{n}_2 \times (\hat{n}_2 \times \mathbf{E}_2) = 0,$$

Note that y_{xx} are relative to the vacuum admittance.

Formulation (cont.)

Find $\mathbf{E} \in \mathbf{H}_0(\mathrm{curl}, \Omega)$ such that

$$\Big(\nabla \times \mathbf{w}, \mu_r^{-1} \nabla \times \mathbf{E}\Big)_\Omega - k_0^2 \Big(\mathbf{w}, \varepsilon_r \mathbf{E}\Big)_\Omega + jk_0 \Big\langle \hat{n} \times \mathbf{w}, \hat{n} \times \mathbf{w} \Big\rangle_{\Gamma_\mathrm{C}} =$$

$$\Big(\mathbf{w}, \mathbf{F}\Big)_\Omega - \Big\langle \hat{n} \times (\mathbf{w} \times \hat{n}), \boldsymbol{\Psi}_\mathrm{N} \Big\rangle_{\Gamma_\mathrm{N}} - \Big\langle \hat{n} \times (\mathbf{w} \times \hat{n}), \boldsymbol{\Psi}_\mathrm{C} \Big\rangle_{\Gamma_\mathrm{C}} \quad \forall\, \mathbf{w} \in \mathbf{H}_0(\mathrm{curl}, \Omega).$$

with

$$\Big(\mathbf{w}, \mathbf{v}\Big)_\Omega = \int_\Omega \mathbf{w}^* \cdot \mathbf{v} d\Omega,$$

$$\Big\langle \mathbf{w}, \mathbf{v} \Big\rangle_\Gamma = \int_\Gamma \mathbf{w}^* \cdot \mathbf{v} d\Gamma.$$

Formulation (cont.)

For *upper* elements on Γ_{TR} (side 1), we have

$$\mathrm{LHS}_1 + j\frac{k_0}{\eta}\left\langle \hat{n}\times(\mathbf{w}_1\times\hat{n}), y_{11}\hat{n}\times(\mathbf{w}_1\times\hat{n})\right\rangle_{\Gamma_{\mathrm{TR}}} + j\frac{k_0}{\eta}\left\langle \hat{n}\times(\mathbf{w}_1\times\hat{n}), y_{12}\hat{n}\times(\mathbf{w}_2\times\hat{n})\right\rangle_{\Gamma_{\mathrm{TR}}} = \mathrm{RHS}_1,$$

whereas for *lower* elements (side 2), we get

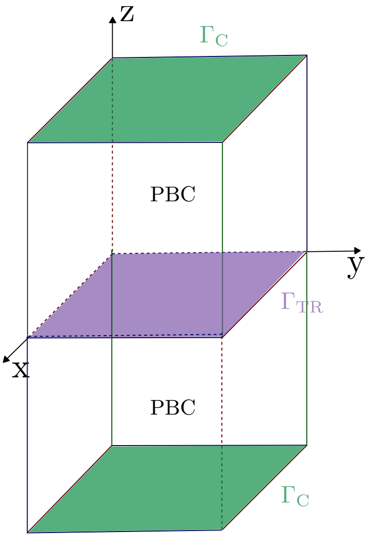
$$\mathrm{LHS}_2 + j\frac{k_0}{\eta}\left\langle \hat{n}\times(\mathbf{w}_2\times\hat{n}), y_{21}\hat{n}\times(\mathbf{w}_1\times\hat{n})\right\rangle_{\Gamma_{\mathrm{TR}}} + j\frac{k_0}{\eta}\left\langle \hat{n}\times(\mathbf{w}_2\times\hat{n}), y_{22}\hat{n}\times(\mathbf{w}_2\times\hat{n})\right\rangle_{\Gamma_{\mathrm{TR}}} = \mathrm{RHS}_2,$$

FEM implementation

- The DOFs will be doubled for the faces and the interior edges.
- The exterior edges of Γ_{TR} are not doubled.
  - Identified by code: the edges associated to two faces are interior.
  - If the boundaries of the sheet belong to PBC, the edges of Γ_{TR} are also doubled.

Outline

Problem to be solved



Simulation of an infinite medium with transmission/reflection sheet that divides the space into two halves.

- Γ_{TR}: Transmission/reflection sheet defined with

$$\mathbf{Y} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}.$$

- Γ_C: ABC with excitation with polarization E_y
- The vertical faces are set to PBC

Testbench

- $\mathbf{Y} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$: sanity check, we should get same result as the two halves with a PMC.
- $\mathbf{Y} = \mathbb{I}$: sanity check, we should get same result as the two halves with an ABC.
- Change lower Γ_C by PEC and solve analytic problem with four media: final test.
  - Obtain parameters for $\mathbf{Y}$ of the equivalent problem.
  - Get same solutions for the electric field.
  - Transparent? Puede ser que aproximar con $1e6$. Quizás con ABCD.

Outline

HOFEM implementation

- New boundary condition: `TRBC`.
  - We define a normal, $\hat{n}_{\mathrm{TRBC}}$ to detect lower and upper side. Upper side is the closer to $\hat{n}_{\mathrm{TRBC}}$.
  - Definition of y_{11}, y_{12}, y_{21}, and y_{22} as relative values with respect to vacuum admittance.
- Two options for implementation
  - Integers defined in `tetrahedra_element`.
  - Allocatable array of $1 \times N_{\mathrm{elem,TR}}$ where the two positions (stored in boundary conditions module, accessible from `mesh_reordering_module` and `elementary_terms_3D`):
    1. $10\times$ Neighbor element identifier (to couple $\mathbf{w}_2$ and $\mathbf{w}_1$).
    2. Integer 1,2 (side) (to extract the values of y_{11},y_{12},y_{21}, and y_{22}).
- Significant methods involved:
  - Postprocessing over `reordering_DOF_algorithm_3D`.
  - `calc_boundary_3D_nxNi_nxNi_term_of_this_element`.
  - Construction of the MUMPS-related matrix: different number of non-zeros per element, assembly of coupled elements (now single-element assembly).