

# HOFEM-AIRBUS

## Transmission/Reflection

Group of Radiofrequency, Electromagnetism, Microwaves  
and Antennas (GREMA)

<http://grema.webs.tsc.uc3m.es/>

Departamento de Teoría de la Señal y Comunicaciones (TSC)  
Universidad Carlos III de Madrid, Spain

Contact: Luis Emilio García-Castillo [legcasti@ing.uc3m.es](mailto:legcasti@ing.uc3m.es),  
Adrián Amor [aamor@ing.uc3m.es](mailto:aamor@ing.uc3m.es), Sergio Llorente [sllorent@ing.uc3m.es](mailto:sllorent@ing.uc3m.es)

**uc3m** | Universidad **Carlos III** de Madrid

# Outline

- 1 On the FEM Implementation of TX/RX Conditions in HOFEM
  - Existing FEM Formulation in HOFEM
  - Two-Port Network Parameters
  
- 2  $[Z]/[Y]$  Approach
  - FEM Formulation
  - Testing
  - HOFEM Implementation

# Outline

- 1 On the FEM Implementation of TX/RX Conditions in HOFEM
  - Existing FEM Formulation in HOFEM
  - Two-Port Network Parameters
  
- 2  $[Z]/[Y]$  Approach
  - FEM Formulation
  - Testing
  - HOFEM Implementation

We start considering different alternatives to implement the TX/RX conditions in the context of the present FEM formulation coded in HOFEM

# Outline

- 1 On the FEM Implementation of TX/RX Conditions in HOFEM
  - Existing FEM Formulation in HOFEM
  - Two-Port Network Parameters
- 2  $[Z]/[Y]$  Approach
  - FEM Formulation
  - Testing
  - HOFEM Implementation

# TITULO

Hola

# Outline

## 1 On the FEM Implementation of TX/RX Conditions in HOFEM

- Existing FEM Formulation in HOFEM
- Two-Port Network Parameters

## 2 $[Z]/[Y]$ Approach

- FEM Formulation
- Testing
- HOFEM Implementation

# TITULO

Hola



# Outline

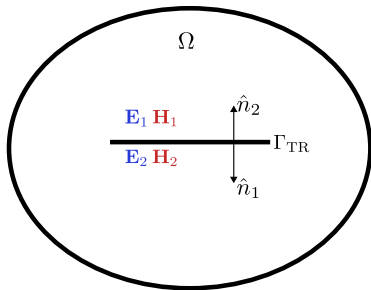
- 1 On the FEM Implementation of TX/RX Conditions in HOFEM
  - Existing FEM Formulation in HOFEM
  - Two-Port Network Parameters
  
- 2  $[Z]/[Y]$  Approach
  - FEM Formulation
  - Testing
  - HOFEM Implementation

We describe the implementation of the TX/RX conditions using the characterization of the material sheet in terms of its immittance (impedance/admittance) matrix

# Outline

- 1 On the FEM Implementation of TX/RX Conditions in HOFEM
  - Existing FEM Formulation in HOFEM
  - Two-Port Network Parameters
- 2 **[Z]/[Y] Approach**
  - **FEM Formulation**
  - Testing
  - HOFEM Implementation

# Formulation



$$\hat{n}_1 \times (\mu_r^{-1} \nabla \times \mathbf{E}_1) - \frac{jk_0}{\eta} y_{11} \hat{n}_1 \times (\hat{n}_1 \times \mathbf{E}_1) - \\ - \frac{jk_0}{\eta} y_{12} \hat{n}_2 \times (\hat{n}_2 \times \mathbf{E}_2) = 0,$$

$$\hat{n}_2 \times (\mu_r^{-1} \nabla \times \mathbf{E}_2) - \frac{jk_0}{\eta} y_{21} \hat{n}_1 \times (\hat{n}_1 \times \mathbf{E}_1) - \\ - \frac{jk_0}{\eta} y_{22} \hat{n}_2 \times (\hat{n}_2 \times \mathbf{E}_2) = 0,$$

Note that  $y_{xx}$  are relative to the vacuum admittance.

## Formulation (cont.)

Find  $\mathbf{E} \in \mathbf{H}_0(\text{curl}, \Omega)$  such that

$$\begin{aligned} & \left( \nabla \times \mathbf{w}, \mu_r^{-1} \nabla \times \mathbf{E} \right)_{\Omega} - k_0^2 \left( \mathbf{w}, \varepsilon_r \mathbf{E} \right)_{\Omega} + jk_0 \left\langle \hat{n} \times \mathbf{w}, \hat{n} \times \mathbf{w} \right\rangle_{\Gamma_c} = \\ & \left( \mathbf{w}, \mathbf{F} \right)_{\Omega} - \left\langle \hat{n} \times (\mathbf{w} \times \hat{n}), \boldsymbol{\Psi}_N \right\rangle_{\Gamma_N} - \left\langle \hat{n} \times (\mathbf{w} \times \hat{n}), \boldsymbol{\Psi}_C \right\rangle_{\Gamma_c} \quad \forall \mathbf{w} \in \mathbf{H}_0(\text{curl}, \Omega). \end{aligned}$$

with

$$\begin{aligned} \left( \mathbf{w}, \mathbf{v} \right)_{\Omega} &= \int_{\Omega} \mathbf{w}^* \cdot \mathbf{v} d\Omega, \\ \left\langle \mathbf{w}, \mathbf{v} \right\rangle_{\Gamma} &= \int_{\Gamma} \mathbf{w}^* \cdot \mathbf{v} d\Gamma. \end{aligned}$$

## Formulation (cont.)

For *upper* elements on  $\Gamma_{\text{TR}}$  (side 1), we have

$$\begin{aligned} &\text{LHS}_1 \\ &+ j \frac{k_0}{\eta} \left\langle \hat{n} \times (\mathbf{w}_1 \times \hat{n}), y_{11} \hat{n} \times (\mathbf{w}_1 \times \hat{n}) \right\rangle_{\Gamma_{\text{TR}}} + j \frac{k_0}{\eta} \left\langle \hat{n} \times (\mathbf{w}_1 \times \hat{n}), y_{12} \hat{n} \times (\mathbf{w}_2 \times \hat{n}) \right\rangle_{\Gamma_{\text{TR}}} = \\ &\text{RHS}_1, \end{aligned}$$

whereas for *lower* elements (side 2), we get

$$\begin{aligned} &\text{LHS}_2 \\ &+ j \frac{k_0}{\eta} \left\langle \hat{n} \times (\mathbf{w}_2 \times \hat{n}), y_{21} \hat{n} \times (\mathbf{w}_1 \times \hat{n}) \right\rangle_{\Gamma_{\text{TR}}} + j \frac{k_0}{\eta} \left\langle \hat{n} \times (\mathbf{w}_2 \times \hat{n}), y_{22} \hat{n} \times (\mathbf{w}_2 \times \hat{n}) \right\rangle_{\Gamma_{\text{TR}}} = \\ &\text{RHS}_2, \end{aligned}$$

# FEM implementation

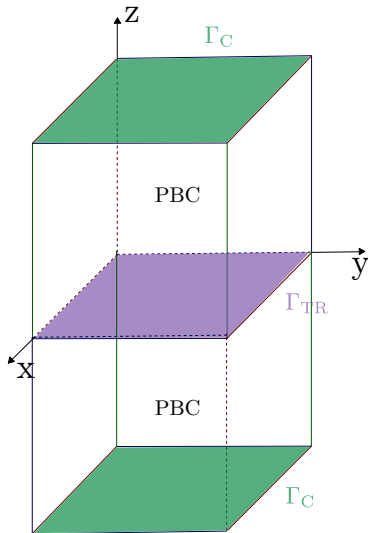
- The DOFs will be doubled for the faces and the interior edges.
- The exterior edges of  $\Gamma_{\text{TR}}$  are not doubled.
  - ▶ Identified by code: the edges associated to two faces are interior.
  - ▶ If the boundaries of the sheet belong to PBC, the edges of  $\Gamma_{\text{TR}}$  are also doubled.

# Outline

- 1 On the FEM Implementation of TX/RX Conditions in HOFEM
  - Existing FEM Formulation in HOFEM
  - Two-Port Network Parameters
- 2  $[Z]/[Y]$  Approach
  - FEM Formulation
  - Testing
  - HOFEM Implementation



# Problem to be solved



Simulation of an infinite medium with transmission/reflection sheet that divides the space into two halves.

- $\Gamma_{TR}$ : Transmission/reflection sheet defined with

$$\mathbf{Y} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}.$$

- $\Gamma_C$ : ABC with excitation with polarization  $E_y$
- The vertical faces are set to PBC

# Testbench

- $\mathbf{Y} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ : sanity check, we should get same result as the two halves with a PMC.
- $\mathbf{Y} = \mathbb{I}$ : sanity check, we should get same result as the two halves with an ABC.
- Change lower  $\Gamma_C$  by PEC and solve analytic problem with four media: final test.
  - ▶ Obtain parameters for  $\mathbf{Y}$  of the equivalent problem.
  - ▶ Get same solutions for the electric field.
  - ▶ Transparent? Puede ser que aproximar con  $1e6$ . Quizás con ABCD.

# Outline

- 1 On the FEM Implementation of TX/RX Conditions in HOFEM
  - Existing FEM Formulation in HOFEM
  - Two-Port Network Parameters
- 2  $[Z]/[Y]$  Approach
  - FEM Formulation
  - Testing
  - HOFEM Implementation

# HOFEM implementation

- New boundary condition: TRBC.
  - ▶ We define a normal,  $\hat{n}_{\text{TRBC}}$  to detect lower and upper side. Upper side is the closer to  $\hat{n}_{\text{TRBC}}$ .
  - ▶ Definition of  $y_{11}$ ,  $y_{12}$ ,  $y_{21}$ , and  $y_{22}$  as relative values with respect to vacuum admittance.
- Two options for implementation
  - ▶ Integers defined in `tetrahedra_element`.
  - ▶ Allocatable array of  $1 \times N_{\text{elem,TR}}$  where the two positions (stored in boundary conditions module, accessible from `mesh_reordering_module` and `elementary_terms_3D`):
    - ①  $10 \times$  Neighbor element identifier (to couple  $\mathbf{w}_2$  and  $\mathbf{w}_1$ ).
    - ② Integer 1,2 (side) (to extract the values of  $y_{11}, y_{12}, y_{21}$ , and  $y_{22}$ ).
- Significant methods involved:
  - ▶ `Postprocessing over reordering_DOF_algorithm_3D`.
  - ▶ `calc_boundary_3D_nxNi_nxNi_term_of_this_element`.
  - ▶ Construction of the MUMPS-related matrix: different number of non-zeros per element, assembly of coupled elements (now single-element assembly).