Volatility and Multivariate Analysis

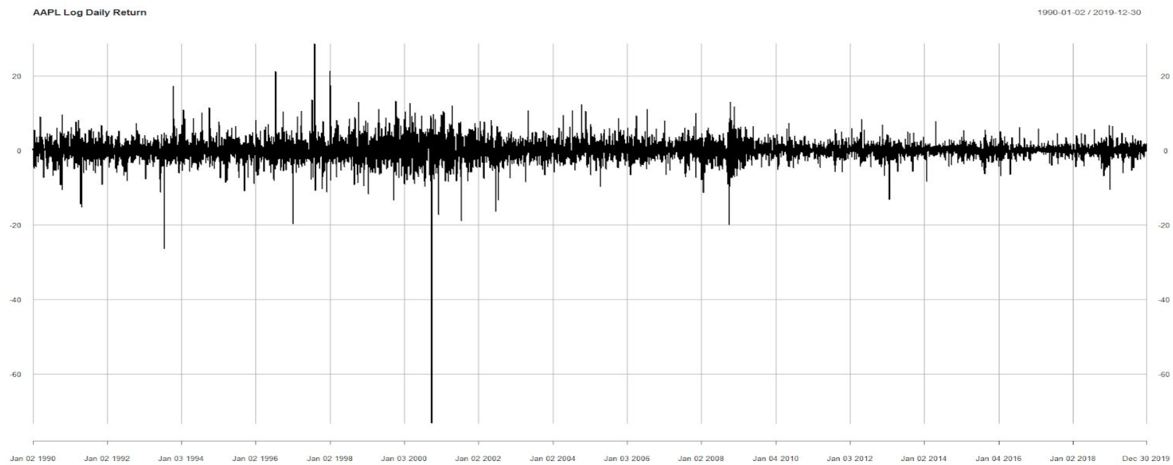
1. Volatility Analysis of Apple stock return

This section was contributed by Nguyen Long Son

```
# We use quantmod package to download Apple stock data
fromDate <- "1990-01-01"; toDate <- "2019-12-31"

getSymbols("AAPL",  from = fromDate, to = toDate)

# Get the log returns of Apple stock
ret.aapl <- 100*dailyReturn(Cl(AAPL), type = "log")

# PLotting the process
plot(ret.aapl, main = "AAPL Daily Return")
```

We can see from the plot that

- The process is clearly zero-mean reverting, and is not a unit root process: no tendency to drift away from its means or has any upward or downward trend.
- There are periods of high volatility, followed by periods of relatively tranquility, hence the ARCH/GARCH effect.
- There was an "outlier" - excessive volatility (60% downside) during 2000 (the Dot-com Bubble) followed with less, but also significant, around 20% downside, within a short period of time (a few months)

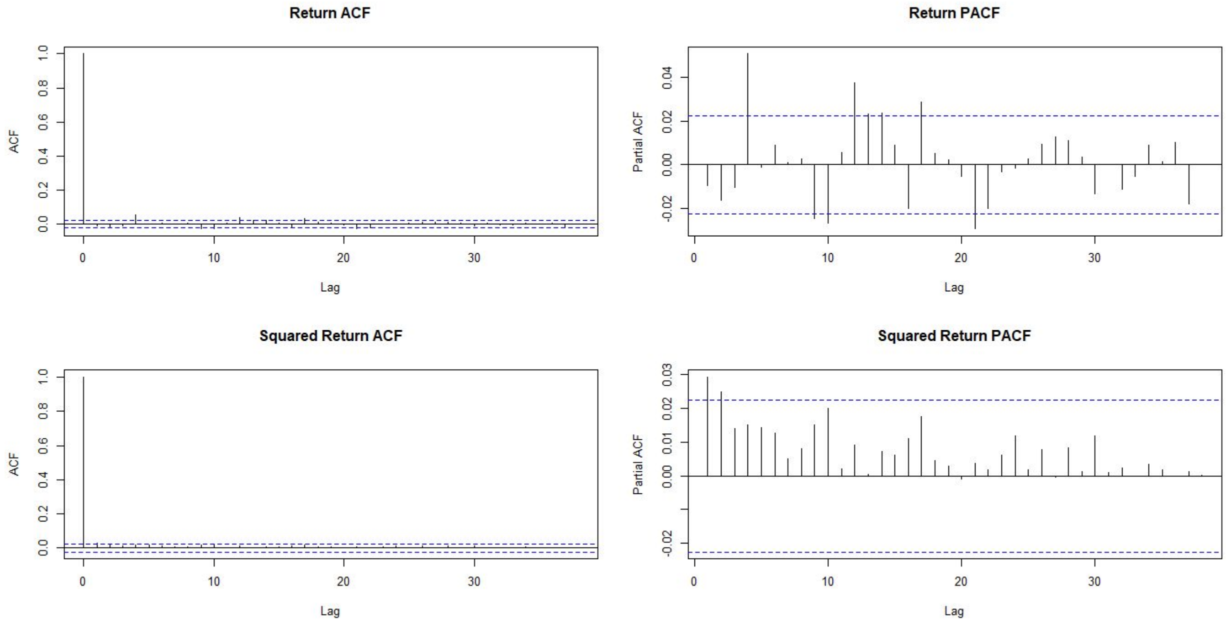


The log-return from 1990 - 2019

Next we will explore the ACF and PACF

```
# Explore the ACF and PACF
par(mfrow = c(2,2))
acf(ret, main = "Return ACF")
pacf(ret, main = "Return PACF")
acf(ret^2, main = "Squared Return ACF")
pacf(ret^2, main = "Squared Return PACF")
par(mfrow = c(1,1))
```

Choosing different time frame, we can see that if the time frame covers the period of the excessive Dot-com bubble 60% downside, then both the return and squared return demonstrate no auto-correlation, i.e., we cannot capture the variance for the whole period just by using ARCH/GARCH model.

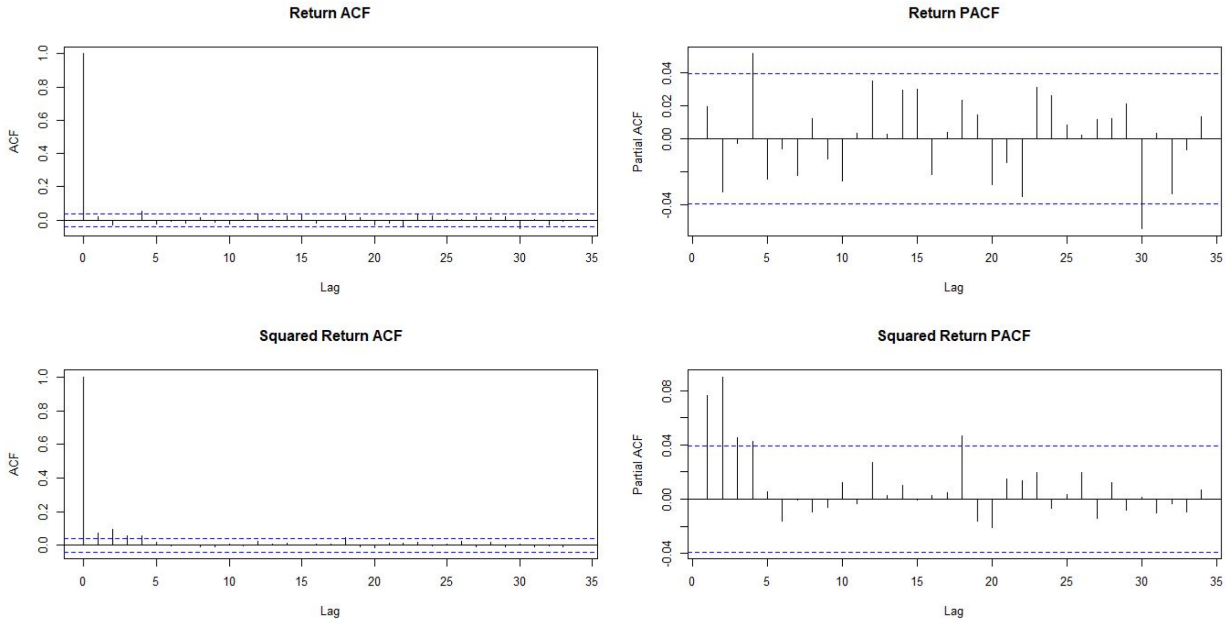


ACF/PACF for the period 1990 - 2019

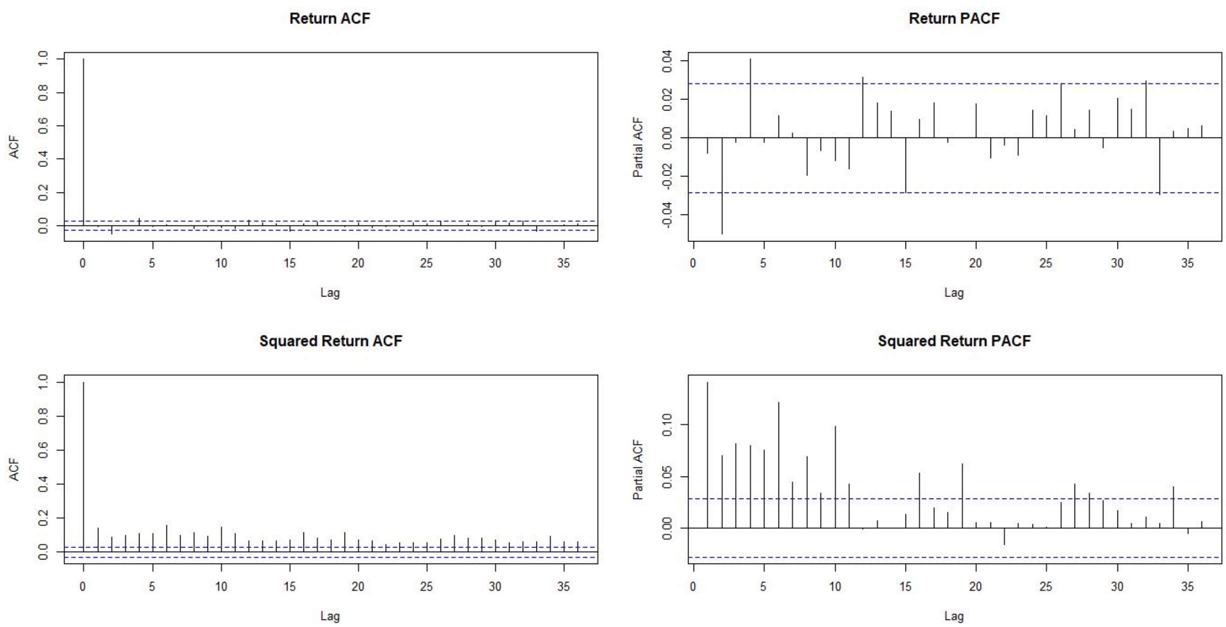
Therefore, it would be appropriate if we divide the process into 2 different periods, before 2000 and after 2000, so that the Dot-com bubble volatility was not inside.

Below are ACF/PACF for the period of 1990 - 1999 and 2001 - 2019, taken separately. The plot shows what we expect: the return itself is not auto-correlated, but there are several auto-correlations in the squared return. Thus, the return process is not auto-correlated, but not independent. Hence, we can use ARCH/GARCH to capture the volatility part, and ARMA(0,0) for the mean part.

We also see significant PACF for several lags in the squared return. Therefore, we should expect that GARCH family is more suitable than simple ARCH model.



ACF/PACF for the period 1990 - 1999



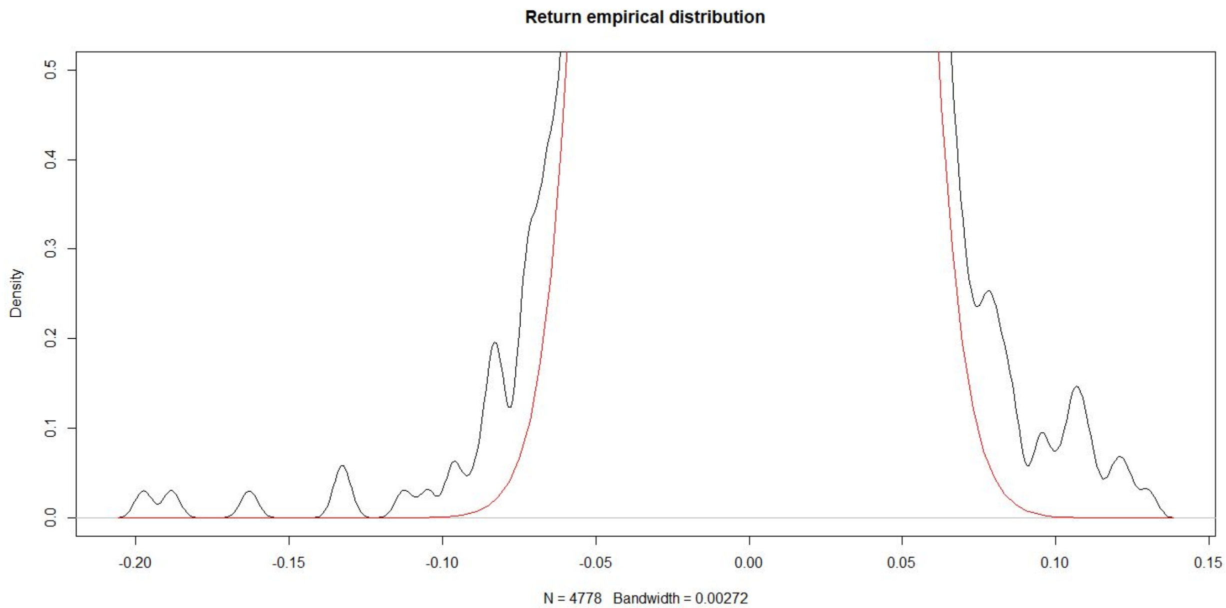
ACF/PACF for the period 2001 - 2019

From now on we will look at the log-return from 2001 - 2009 and trying to model the volatility using GARCH and its extensions.

Look at some statistical characteristics of the log-returns

```
# Some statistical measures
```

```r
mean(ret); sd(ret); kurtosis(ret); skewness(ret)
# Plotting the density vs. normal distribution of the normal with the same mean and standard deviation. Zoom in to see the fatness of the tail

plot(density(ret), main = "Return empirical distribution", ylim = c(0, 0.5))
curve(dnorm(x, mean = mean(ret), sd = sd(ret)), add = TRUE, col = "red")
```




```
> kurtosis(ret)
daily.returns
     8.797226
> skewness(ret)
daily.returns
   -0.1692948
> mean(ret)
[1] 0.00117507
> sd(ret)
[1] 0.02279116
```

We can observe for the log-returns

- zero - mean reverting
- Much heavier tails than a normal distribution (Kurtosis > 3)
- Skewness into the negative.

Therefore we should expect the extensions of GARCH (E-GARCH/T-GARCH) will be more efficient than a standard GARCH in modelling the volatility. Also, in fitting the GARCH family model we will use skew-student distribution model for the innovations.

The fitting using standard GARCH, E-GARCH and T-GARCH and the results are shown below

```
# Fitting standard GARCH model for ARMA(0,0)-GARCH(1,1)
spec <- ugarchspec(variance.model = list(model = "sGARCH", garchOrder =
c(1,1)),
                   mean.model     = list(armaOrder = c(0,0)),
                   distribution.model = "sstd")
model <- ugarchfit(spec, data = ret, out.sample = 20)
show(model)
```

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model : sGARCH(1,1)
Mean Model  : ARFIMA(0,0,0)
Distribution      : sstd

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001548    0.000253   6.1098 0.000000
omega   0.000002    0.000001   1.9971 0.045815
alpha1  0.052524    0.006325   8.3039 0.000000
beta1   0.946476    0.006458 146.5698 0.000000
skew    1.028095    0.020093  51.1681 0.000000
shape   4.722147    0.309589  15.2530 0.000000

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001548    0.000272   5.6946 0.000000
omega   0.000002    0.000003   0.7934 0.427544
alpha1  0.052524    0.021064   2.4935 0.012649
beta1   0.946476    0.020227  46.7926 0.000000
skew    1.028095    0.020798  49.4313 0.000000
shape   4.722147    0.412013  11.4612 0.000000
```

```
LogLikelihood : 11995.1

Information Criteria
------------------------------------

Akaike       -5.0396
Bayes        -5.0314
Shibata      -5.0396
Hannan-Quinn -5.0367

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                     0.9432  0.3314
Lag[2*(p+q)+(p+q)-1][2]    2.1187  0.2438
Lag[4*(p+q)+(p+q)-1][5]    5.9563  0.0921
d.o.f=0
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                      1.355  0.2443
Lag[2*(p+q)+(p+q)-1][5]     1.686  0.6942
Lag[4*(p+q)+(p+q)-1][9]     2.737  0.8015
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]    0.1702 0.500 2.000  0.6800
ARCH Lag[5]    0.6000 1.440 1.667  0.8537
ARCH Lag[7]    1.1518 2.315 1.543  0.8877

Nyblom stability test
------------------------------------
Joint Statistic:  36.1612
Individual Statistics:
mu     0.1239
omega  4.2141
alpha1 1.4589
beta1  1.7338
skew   0.4173
```

```
shape  1.9299

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:         1.49 1.68 2.12
Individual Statistic:    0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value    prob sig
Sign Bias           1.4133 0.15763    
Negative Sign Bias  1.1492 0.25054    
Positive Sign Bias  0.2358 0.81358    
Joint Effect        7.9924 0.04617  **


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     25.69       0.1390
2    30     34.06       0.2372
3    40     48.68       0.1378
4    50     53.83       0.2946


Elapsed time : 0.7569761
```

```
# Fitting  E-GARCH model for ARMA(0,0)-GARCH(1,1)
spec <- ugarchspec(variance.model = list(model = "eGARCH", garchOrder = 
c(1,1)),
                   mean.model     = list(armaOrder = c(0,0)),
                   distribution.model = "sstd")
model <- ugarchfit(spec, data = ret, out.sample = 20)
show(model)
```

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model : eGARCH(1,1)
Mean Model  : ARFIMA(0,0,0)
```

```
Distribution      : sstd

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001406    0.000302   4.6575 0.000003
omega  -0.097540    0.057820  -1.6870 0.091612
alpha1 -0.052188    0.023838  -2.1893 0.028577
beta1   0.987593    0.007629 129.4601 0.000000
gamma1  0.159428    0.100379   1.5883 0.112229
skew    1.029688    0.021018  48.9898 0.000000
shape   4.919479    0.485502  10.1328 0.000000

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001406    0.001230  1.14318 0.252963
omega  -0.097540    0.419629 -0.23244 0.816194
alpha1 -0.052188    0.162773 -0.32062 0.748500
beta1   0.987593    0.055444 17.81258 0.000000
gamma1  0.159428    0.736234  0.21654 0.828563
skew    1.029688    0.043739 23.54147 0.000000
shape   4.919479    2.550253  1.92902 0.053729

LogLikelihood : 12030.31

Information Criteria
------------------------------------

Akaike       -5.0539
Bayes        -5.0444
Shibata      -5.0539
Hannan-Quinn -5.0506

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                      2.065  0.1507
Lag[2*(p+q)+(p+q)-1][2]     2.973  0.1427
Lag[4*(p+q)+(p+q)-1][5]     6.239  0.0790
d.o.f=0
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
```

```
------------------------------------
                        statistic p-value
Lag[1]                     0.4051  0.5245
Lag[2*(p+q)+(p+q)-1][5]    1.1935  0.8146
Lag[4*(p+q)+(p+q)-1][9]    1.6580  0.9417
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]     1.038 0.500 2.000  0.3082
ARCH Lag[5]     1.052 1.440 1.667  0.7175
ARCH Lag[7]     1.192 2.315 1.543  0.8805

Nyblom stability test
------------------------------------
Joint Statistic:  4.884
Individual Statistics:
mu     0.51709
omega  2.54350
alpha1 0.85338
beta1  2.46236
gamma1 0.07127
skew   0.35428
shape  1.04580

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:         1.69 1.9 2.35
Individual Statistic:    0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value    prob sig
Sign Bias           1.7216 0.08522   *
Negative Sign Bias  0.6533 0.51360
Positive Sign Bias  0.4261 0.67007
Joint Effect        3.3819 0.33641


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     20.04       0.3921
```

```
2    30     31.03        0.3641
3    40     41.97        0.3435
4    50     44.46        0.6575


Elapsed time : 1.095111
```

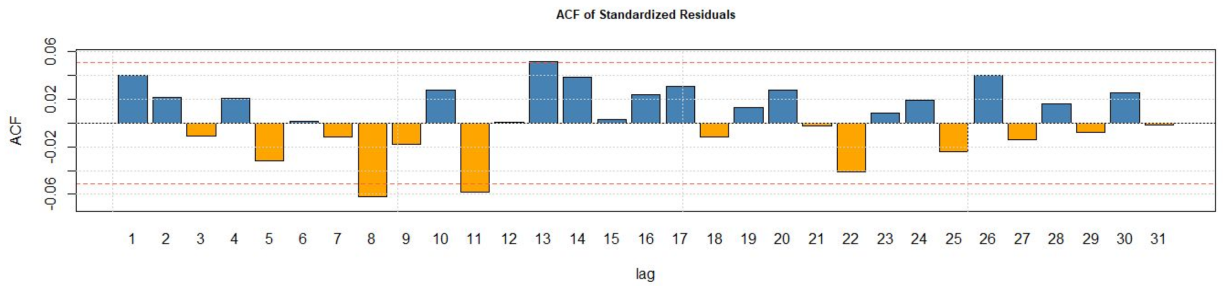
```
# Fitting  T-GARCH model for ARMA(0,0)-GARCH(1,1)
spec <- ugarchspec(variance.model = list(model = "fGARCH", submodel =
"TGARCH", garchOrder = c(1,1)),
                   mean.model     = list(armaOrder = c(0,0)),
                   distribution.model = "sstd")
model <- ugarchfit(spec, data = ret, out.sample = 20)
show(model)
```

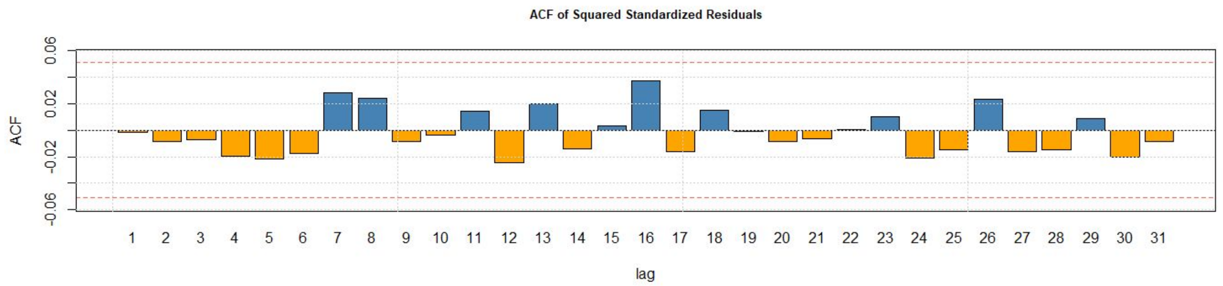
```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model : fGARCH(1,1)
fGARCH Sub-Model  : TGARCH
Mean Model  : ARFIMA(0,0,0)
Distribution      : sstd

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001407    0.000255   5.5234 0.000000
omega   0.000277    0.000101   2.7487 0.005984
alpha1  0.092558    0.015256   6.0670 0.000000
beta1   0.919373    0.014842  61.9444 0.000000
eta11   0.336204    0.059825   5.6197 0.000000
skew    1.030225    0.020390  50.5258 0.000000
shape   4.891184    0.343492  14.2396 0.000000

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001407    0.000273   5.1569 0.000000
omega   0.000277    0.000190   1.4581 0.144824
alpha1  0.092558    0.030096   3.0754 0.002102
```

```
beta1   0.919373    0.030120  30.5238 0.000000
eta11   0.336204    0.062608   5.3700 0.000000
skew    1.030225    0.021039  48.9666 0.000000
shape   4.891184    0.354728  13.7885 0.000000

LogLikelihood : 12028.85

Information Criteria
------------------------------------

Akaike       -5.0533
Bayes        -5.0438
Shibata      -5.0533
Hannan-Quinn -5.0500

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                      2.441 0.11818
Lag[2*(p+q)+(p+q)-1][2]     3.367 0.11166
Lag[4*(p+q)+(p+q)-1][5]     6.675 0.06213
d.o.f=0
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                     0.5953  0.4404
Lag[2*(p+q)+(p+q)-1][5]    1.7413  0.6807
Lag[4*(p+q)+(p+q)-1][9]    2.2643  0.8712
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]     1.117 0.500 2.000  0.2906
ARCH Lag[5]     1.125 1.440 1.667  0.6962
ARCH Lag[7]     1.319 2.315 1.543  0.8568

Nyblom stability test
------------------------------------
Joint Statistic:  5.1975
Individual Statistics:
```

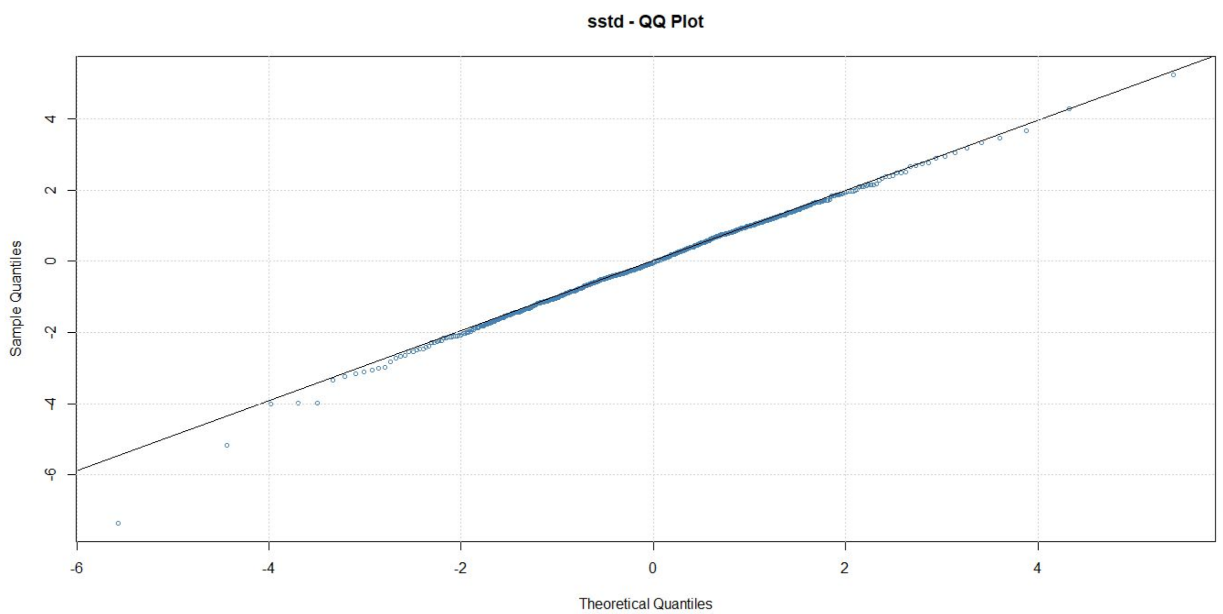
```
mu     0.5713
omega  2.4327
alpha1 2.3724
beta1  2.6724
eta11  1.4026
skew   0.3696
shape  2.0039

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:          1.69 1.9 2.35
Individual Statistic:     0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value    prob sig
Sign Bias           1.7229 0.08497   *
Negative Sign Bias  1.0120 0.31158
Positive Sign Bias  0.2713 0.78618
Joint Effect        3.4656 0.32526


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     14.71       0.7408
2    30     25.10       0.6729
3    40     28.78       0.8848
4    50     42.90       0.7174


Elapsed time : 1.804209
```

Based on the Goodness-of-Fit test and the Ljung-Box test on residuals, squared residuals, we can conclude that ARMA(0,0) + E-GARCH(1,1) with t-distribution is the most suitable among others. We can confirm that visually by looking at the ACF of the residuals and squared residuals below. As expected there is no auto-correlation in both.



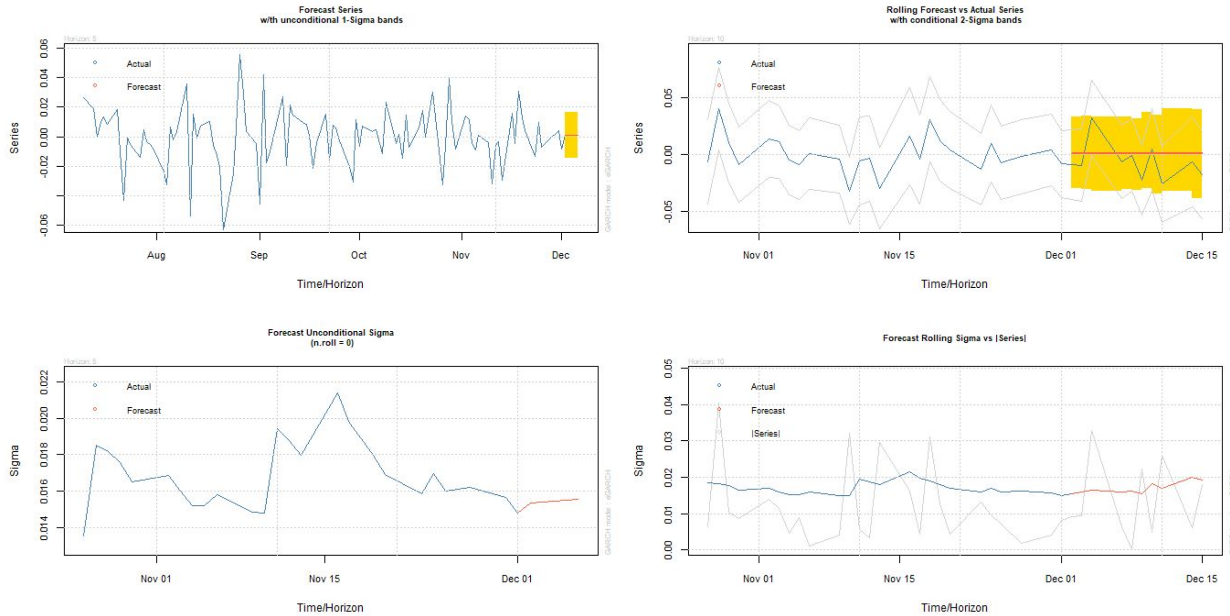


Also, the residuals closely follow t-distributions



Next, we use this model for forecasting

```
# Forecasting using the chosen model
forecast <- ugarchforecast(model, n.ahead = 5, n.roll = 10)
plot(forecast, which = "all")
show(forecast)
```




```
*------------------------------------*
*       GARCH Model Forecast         *
*------------------------------------*
Model: eGARCH
Horizon: 5
Roll Steps: 10
Out of Sample: 5

0-roll forecast [T0=2015-12-01]:
      Series    Sigma
T+1 0.001085 0.01537
T+2 0.001085 0.01542
T+3 0.001085 0.01548
T+4 0.001085 0.01552
T+5 0.001085 0.01556
```

We can see that the conditional mean (column Series) remains constant (zero) with the conditional volatility varying.

2. Multivariate Analysis

2.1. Indicating economic theories and models for calculating equilibrium FX

This section was contributed by Mateusz Dubaniowski.

There are several theories for calculating equilibrium FX rates . Here are presented some of these:

- **PPP theory** – in this theory it is considered that price levels determine the equilibrium exchange rates. However, it is a limited model that fails to capture many shocks to exchange rate prices. Cassel (1916 and 1918).
- **Fundamental Equilibrium Exchange Rate (FEER)** – Approach developed Williamson (1994). In this approach the equilibrium exchange rate is the real effective exchange rate when the economy is operating at full capacity i.e. under full employment, low inflation and a sustainable current account balance. This approach is fundamental in the way that it considers medium and long-term factors, while ignoring the short-term factors. The evaluation of exchange rate is completed by comparing the actual current level with the calculated FEER level. This approach is reasonable and has strong support in fundamental economic analysis theories and presents good results over the long term, however, its weakness is the lack of short term factors inclusion on the calculation of exchange rate.
- **Behavioural Equilibrium Exchange Rate (BEER)** – This model was described by Clark and MacDonald (1999). This model extends FEER by including also transitory factors that affect exchange rates also in the short-term. Hence, the model attempts to capture that short term effects of exchange rates and find equilibrium in response to short term factors. This model therefore reminds of Vector Error Correction Model in general econometric theory.
- **Permanent Equilibrium Exchange Rate (PEER)** – this model attempts to build upon BEER by decomposing individual factors further down into temporary and permanent component in order to improve upon the model. For example
- **Natural Rate of Exchange Rate (NATREX)** – a model described by Stein (1995). In this model the trajectory and equilibrium of exchange rates is decomposed into three different stages short-term/spot rate, medium-term, and long-term. Long-run is consistent with PPP theory, while the medium term is similar to FEER. Short-term depends on speculative capital flows and stocks of capital and debt.

There exist several more models of equilibrium exchange rates, which are listed for example in Siregar (2011). The above are, however, the most prevalent and most widely accepted and described in the literature.

2.2. Indicating macroeconomic variables used for calculating equilibrium FX

This section was contributed by Mateusz Dubaniowski.

Macroeconomic variables that can be used for calculating equilibrium foreign exchange rates:

- **Consumer price index (CPI)**, and **producer price index (PPI)** – to compare price levels under all models and to estimate inflation.
- **Unit labour cost** – to estimate productivity of economy similarly under FEER and PPP theory models
- **Composition of the economy** – percentages of GDP derived from various sectors. Similarly under PPP model.
- **Interest rates across different maturity terms** – to calculate risk premium across different economies. This is used in BEER and PEER models.
- **Share of public consumption and investment in GDP** – to estimate the stocks of capital in the economy.
- **Total trade over GDP** – to estimate openness of the economy and competitiveness.
- **Real wage** – to estimate productivity.
- **M2/GDP** - to estimate medium and short term stocks of financial capital in the economy.
- **Public debt as a percentage of GDP** – to estimate stocks of debt.

Above are listed some of the variables that can be used in the models mentioned in the previous section. There exist many other variables that can be used to calculate equilibrium exchange rates, however, these are the most common in literature.

2.3. Explaining the connection between the linear regression and Vector Error Correction (VEC)

This section is contributed by Zhang Shiqi

Major similarities

- Both models are designed to find to model the relationship between a dependent variable and one or more explanatory variables (or independent variables).
- Both models are linear
- Both can be estimated by OLS (ordinary least square regression)
    - Linear regression models are often fitted using the least squares approach

    - Being a restricted VAR designed for use with nonstationary series that are known to be cointegrated, VEC model is still linear in parameters, and hence can be simply estimated by OLS
- Relationships among variables are important for both models in determining which variables should be used in model estimation and interpretation
    - Linear regression: Principal component regression is used when strong correlations exist among the predictor variables. It first reduces the predictor variables using principal component analysis then uses the reduced variables in an OLS regression fit.
    - VEC: Johansen test is required to get the best estimate of the number of cointegration relationships in the system, which is then used to re-estimate the system imposing exactly the number of cointegration relationships as identified by the Johansen test and interpreting the results.

Major differences:

- Multivariate vs. Univariate:
    - Linear regression focuses on the conditional probability distribution of all of the response given the values of the predictors, rather than on the joint probability distribution of all of these variables, which is the domain of multivariate analysis
    - Vector Error Correction Modelling is one of the modelling in the multivariate time series.
- Non-stationary vs. Stationary
    - VEC is a restricted VAR designed for nonstationary series that are known to be cointegrated. Whereas Linear Regression does not have this restriction for its variables.
- Correlation in error terms
    - Linear regression: one of the standard assumptions in the regression model is that the error terms are uncorrelated. Correlation in the error terms suggests that there is additional information in the data that has not been exploited in the current model.
    - VEC: it allows a general correlation structure between the variables by allowing a general variance-covariance matrix in the errors/residuals. A valid variance-covariance matrix will always be positively definite.

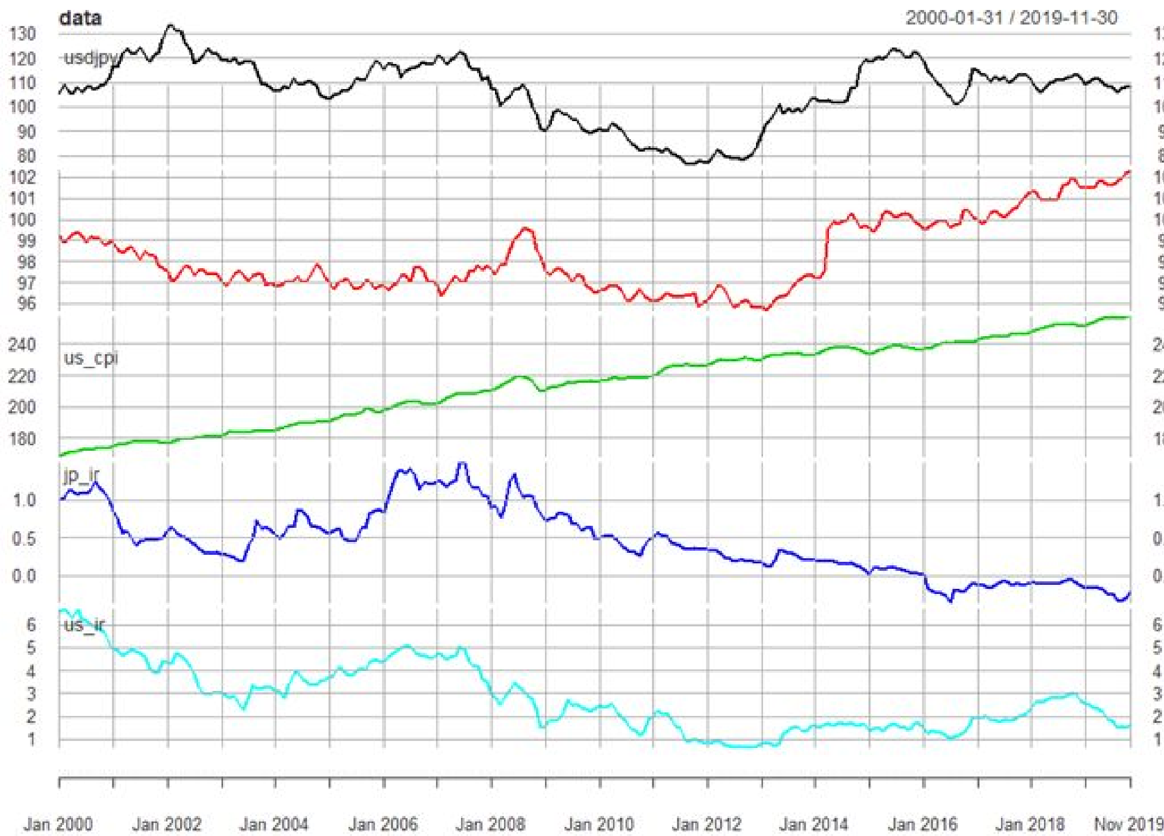
2.4. Calculating Equilibrium FX

This section is contributed by Matthew Ang

For the VECM model, we will be analysing the USD/JPY. The methodology used below closely follows Stephens' (2004) paper, where he incorporates both economic theories of Purchasing Power Parity (PPP) and Uncovered Interest Parity (UIP). He estimates a Behavioural Equilibrium Exchange Rate (BEER) for the NZD/USD currency pair.

First, we retrieve the monthly USD/JPY exchange rate; the Consumer Price Index for both countries; and the 5-year (constant maturity) government bond yield from Quandl:

```
library(Quandl)
library(dplyr)
library(tsDyn)
library(urca)
library(vars)
library(tseries)

# retrieve data from Quandl
quandl_codes = c("FRED/EXJPUS",                          # USD/JPY monthly rate
                 "RATEINF/CPI_JPN", "RATEINF/CPI_USA",      # JP and US CPI
                 "MOFJ/INTEREST_RATE_JAPAN_5Y", "FRED/GS5") # JGB and UST
5Y yield

quandl_data <- Quandl(quandl_codes, start_date="1999-12-31", type="xts")
quandl_data[,2:3] <- lag(quandl_data[,2:3])

# We also format and plot our time series data:
# format data and take first differences

data <- apply.monthly(quandl_data, colMeans, na.rm=TRUE) %>% na.omit()
names(data) <- c("usdjpy", "jp_cpi", "us_cpi", "jp_ir", "us_ir")
sapply(data, adf.test, k=12)   # here, ADF test shows all have unit root
plot(data, multi.panel=TRUE, yaxis.same = FALSE)
```



Here, we see that our time series exhibit idiosyncratic behaviour, although the exchange rate and 5Y US Treasury yield appear positively correlated. We analyse that later with a correlation matrix.

```
sapply(data, adf.test, k=12) # Here, ADF test shows all have unit root
```

```
            usdjpy                       jp_cpi                       us_cpi
statistic   -2.079852                    -2.20459                     -1.950615
parameter   12                           12                           12
alternative "stationary"                 "stationary"                 "stationary"
p.value     0.5422825                    0.4898277                    0.5966295
method      "Augmented Dickey-Fuller Test" "Augmented Dickey-Fuller Test" "Augmented Dickey-Fuller Test"
data.name   "x[[i]]"                     "x[[i]]"                     "x[[i]]"
            jp_ir                        us_ir
statistic   -1.941523                    -2.225999
parameter   12                           12
alternative "stationary"                 "stationary"
p.value     0.6004529                    0.4808248
method      "Augmented Dickey-Fuller Test" "Augmented Dickey-Fuller Test"
data.name   "x[[i]]"                     "x[[i]]"
```

On each individual time series, the Augmented Dickey-Fuller test indicates we cannot reject the existence of a unit root. Hence, they are individually I(1), and may be cointegrated.

```
# take first differences
data.d &lt;- data %&gt;% diff() %&gt;% na.omit()
```

```r
round(cor(data.d), 2)
```

```
       usdjpy jp_cpi us_cpi jp_ir us_ir
usdjpy   1.00  -0.01   0.01  0.22  0.40
jp_cpi  -0.01   1.00   0.32  0.05  0.07
us_cpi   0.01   0.32   1.00  0.19  0.22
jp_ir    0.22   0.05   0.19  1.00  0.49
us_ir    0.40   0.07   0.22  0.49  1.00
```

The correlation matrix confirms our earlier observation that USD/JPY is positively correlated with the 5Y US Treasury yield. Interestingly, price levels and interest rates of both countries also exhibit relatively strong positive correlations.

```r
# check for number of lags using VAR
VARselect(data.d, lag.max=12)
```

```
$selection
AIC(n)  HQ(n)  SC(n) FPE(n)
     2      1      1      2
```

Using VAR(p) and the Akaike Information Criterion (AIC) as our selection criterion, we select number of lags p = 2.

```r
# Johansen Test for # of cointegrating relationships
jotest1=ca.jo(data, type="eigen", K=2, ecdet="const", spec="longrun")
summary(jotest1)
jotest2=ca.jo(data, type="trace", K=2, ecdet="const", spec="longrun")
summary(jotest2)
```

```
> summary(jotest1)

          test 10pct  5pct  1pct
r <= 4 |  2.83  7.52  9.24 12.97
r <= 3 |  8.70 13.75 15.67 20.20
r <= 2 | 10.63 19.77 22.00 26.81
r <= 1 | 16.38 25.56 28.14 33.24
r = 0  | 37.54 31.66 34.40 39.79
```

```
> summary(jotest2)

          test 10pct  5pct  1pct
r <= 4 |  2.83  7.52  9.24 12.97
r <= 3 | 11.53 17.85 19.96 24.60
r <= 2 | 22.16 32.00 34.91 41.07
r <= 1 | 38.54 49.65 53.12 60.16
r = 0  | 76.08 71.86 76.07 84.45
```

The Johansen Test tests for the number of cointegrating relationships. Using the test statistics and critical values above, we infer that we have 1 cointegrating vector.

```
VECM_fit = VECM(data, 2, r=1, include="const", estim="ML", LRinclude
="none")
summary(VECM_fit)
```

```
#############
###Model VECM
#############
Full sample size: 239   End sample size: 236
Number of variables: 5  Number of estimated slope parameters 60
AIC -2464.977   BIC -2243.292   SSR 1340.538
Cointegrating vector (estimated by ML):
   usdjpy    jp_cpi     us_cpi     jp_ir      us_ir
r1      1 -5.693574 0.4172341 2.925376 -5.583159


                ECT                Intercept          usdjpy -1          jp_cpi -1
Equation usdjpy -0.0192(0.0135)    -7.4089(5.0830)    0.2417(0.0720)***  -0.4091(0.5772)
Equation jp_cpi 0.0053(0.0015)***  1.9648(0.5707)***  0.0133(0.0081)     0.1194(0.0648).
Equation us_cpi 0.0016(0.0038)     0.8186(1.4241)     0.0209(0.0202)     0.1431(0.1617)
Equation jp_ir  0.0003(0.0004)     0.1089(0.1597)     0.0050(0.0023)*    0.0171(0.0181)
Equation us_ir  0.0025(0.0012)*    0.9035(0.4509)*    0.0068(0.0064)     -0.0278(0.0512)
                us_cpi -1          jp_ir -1          us_ir -1          usdjpy -2
Equation usdjpy 0.4146(0.2304).    -4.2253(2.3513).  -0.2153(0.8953)   0.1431(0.0713)*
Equation jp_cpi 0.0893(0.0259)***  0.2875(0.2640)    -0.0998(0.1005)   -0.0144(0.0080).
Equation us_cpi 0.5967(0.0646)***  0.8964(0.6587)    0.1719(0.2508)    -0.0270(0.0200)
Equation jp_ir  0.0121(0.0072).    0.1245(0.0739).   0.0174(0.0281)    -0.0055(0.0022)*
Equation us_ir  0.0607(0.0204)**   0.3503(0.2086).   0.1015(0.0794)    0.0007(0.0063)
                jp_cpi -2          us_cpi -2          jp_ir -2          us_ir -2
Equation usdjpy 0.5922(0.5625)     -0.0470(0.2412)    3.3380(2.3761)    -1.2146(0.8806)
Equation jp_cpi -0.2141(0.0632)*** 0.0535(0.0271)*    0.0211(0.2668)    0.0230(0.0989)
Equation us_cpi -0.5787(0.1576)*** -0.2097(0.0676)**  -0.4153(0.6657)   -0.0038(0.2467)
Equation jp_ir  0.0082(0.0177)     -0.0092(0.0076)    -0.0448(0.0747)   0.0468(0.0277).
Equation us_ir  0.0697(0.0499)     -0.0350(0.0214)    0.3015(0.2108)    -0.1119(0.0781)
```

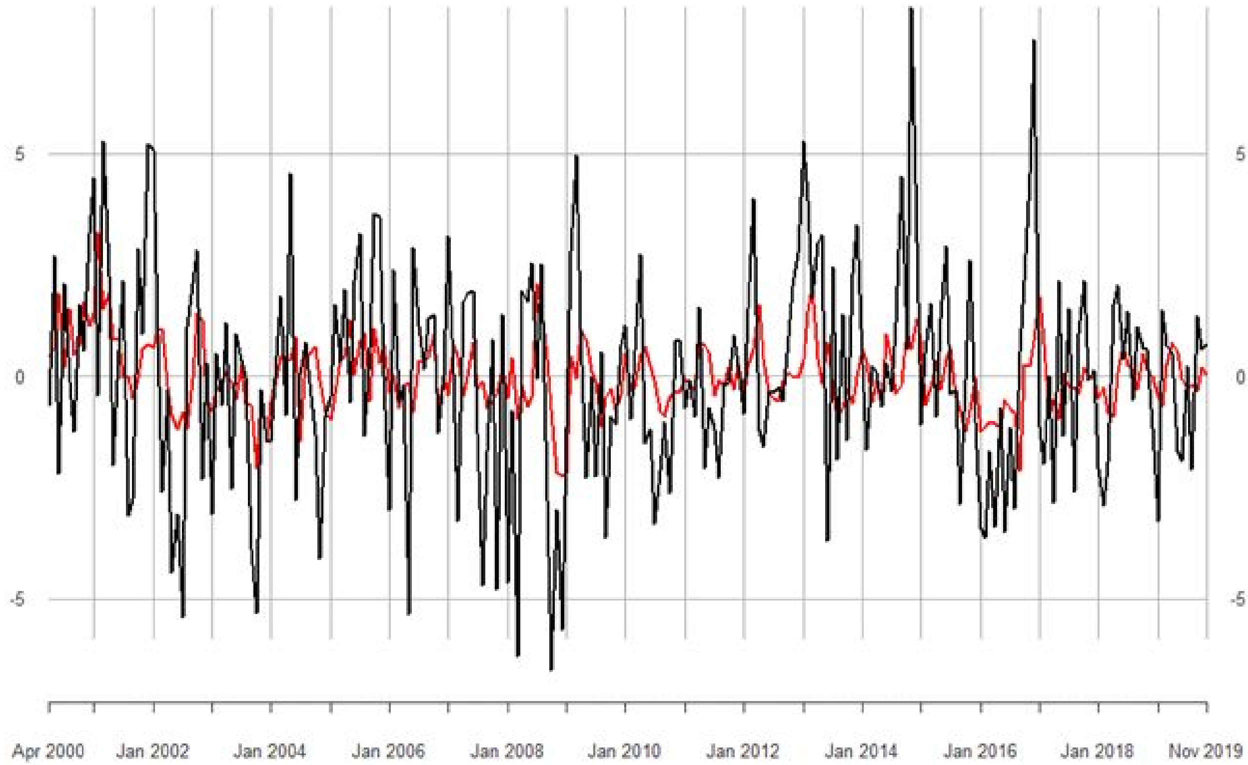
Restricting our model to 2 lags and 1 cointegrating relationship, we derive the VECM model as shown above. The signs of the cointegrating vector are in line with the theory of joint PPP and UIP.

```
# plot fitted vs. actuals
fitted_vals <- fitted(VECM_fit, level="original")$usdjpy
values <- cbind(data$usdjpy, fitted_vals$usdjpy)
plot(values)

fitted_diffs <- fitted(VECM_fit)[,"usdjpy"] %>% as.xts()
index(fitted_diffs) <- index(data.d)[-1:-2]
values <- merge.xts(data.d$usdjpy, fitted_diffs) %>% na.omit()
plot(values)
```

plot(values)

Plotting VECM fitted values against actual monthly exchange rate, we notice that the realised exchange rate tracks the equilibrium exchange rate closely.

```
plot(diff_values)
```



Plotting the differences of fitted against actuals, however, we notice the realised FX rates diverge visibly over several months, but converge to the equilibrium exchange rate thereafter. This is captured by the short-run dynamics of the VECM model (the coefficient terms on explanatory variables), while the error correction term signifies a long-run convergence toward the equilibrium.

3. References

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