BKM - zadaci za vježbu

* sinh/cosh → $\frac{e^x \pm e^{-x}}{2}$

? 4.

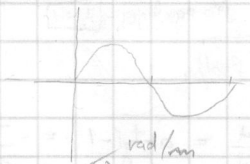
6 - Kad se koristi ta formula za $U(t)$?
- $\beta = 0$??

(1.) $u(t) = \sin(\pi \cdot 10^6 t)$ [V] $\Rightarrow \omega = 10^6 \pi$

$v_p = 2 \cdot 10^8$ m/s

$\beta \cdot x = \frac{\pi}{2}$?

$x = ?$



$\omega = \frac{2\pi}{T}$ $\quad T = \frac{2\pi}{\omega}$

$$u(x) = U_0 \sin\left(\omega t - \frac{x}{v_p}\right) = U_0 \sin(\omega t - \beta x)$$

$\beta \cdot x = \frac{\pi}{2}$

$x = \frac{\pi}{2\beta}$

$\beta = \frac{\omega}{v_p}$ $\quad \left[\frac{\text{rad/s}}{\text{m/s}} = \frac{\text{rad}}{\text{m}}\right]$

$\beta = \frac{10^6 \cdot \pi}{2 \cdot 10^8}$

$\beta \cdot x = \frac{\pi}{2} \Rightarrow x = \frac{\pi}{2\beta} = \frac{\pi}{2 \cdot 10^6 \cdot \pi} \cdot 2 \cdot 10^8 = 10^2 = 100$ m

(2.) $f = 10^6$ Hz $\Rightarrow \omega = 2\pi f$

$v_p = 200000$ km/s $= 2 \cdot 10^8$ m/s

$\alpha = 2$ dB/km

$l = \lambda$ $\quad (\beta \cdot \lambda = 2\pi) \Rightarrow \lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega} \cdot v_p = \frac{2\pi}{2\pi f} \cdot v_p = \frac{v_p}{f} = \frac{2 \cdot 10^8}{10^6} = 200$ m

$P_2 = ?$ / kW = ?

$\lambda = 0.2$ km

$\beta = \frac{\omega}{v_p}$

$P_2 = P_1 - A = P_1 - \alpha \cdot l$

$A = \alpha \cdot P_2 = 2 \cdot 0.2 = 0.4$ dB

?

(3.) f_0

$Z_S = 100 + j100$ [Ω] $\Rightarrow Z_S = Z_0^* = 100 - j100$

$R_L = 150$ [Ω] $\quad Z_L = R_L + jX_L$

uvjet za max snagu $Z_S = Z_L$?

$$\rho = \frac{Z_L - Z_S}{Z_L + Z_S} = \frac{150 - 100 - 100}{150 + 100 + j100} = \frac{50 + j100}{250 - j100} \cdot \frac{250 + j100}{250 + j100} = \frac{50 \cdot 250 + 300 \cdot 100 - 100^2}{250^2 + 100^2}$$

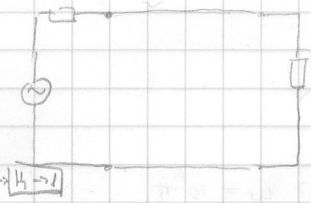
$$= \frac{2500 + 30000j}{72500}$$

$$= \frac{25}{725} + j\frac{300}{725}$$

$$|\rho| = \sqrt{\left(\frac{25}{725}\right)^2 + \left(\frac{300}{725}\right)^2} = 0.415229$$

4. $Z_0 = 120 - j120$

$Z_{ul} = ?$



$Z_L = 0 \rightarrow Z_{ul} = Z_0$

? jako duga linija $\rightarrow |\Gamma_L| = 1$

$Z_{ul} = Z_0 \dfrac{1+\Gamma_L}{1-\Gamma_L}(\beta d)$

$x + iy \quad \operatorname{arctg}\frac{y}{x} =$

$\varphi(Z_{ul}) = \operatorname{arctg}\dfrac{\operatorname{Im} Z_{ul}}{\operatorname{Re} Z_{ul}} = \operatorname{arctg}(-1) = -\dfrac{\pi}{2}$

5. $u(t) = \sin(\pi 10^5 t) \rightarrow \omega = 10^5\pi \ \text{rad/s}$

$v_p = 2\cdot 10^8 \ \text{m/s}$

$\beta = \dfrac{\omega}{v_p} = \dfrac{10^5\pi}{2\cdot 10^8} = \dfrac{1}{2\cdot 10^3}\pi \ \text{rad/m} = \dfrac{1}{2}\pi \ \text{rad/km}$

6. $f = 2\ \text{km}$

$f > 10\ \text{kHz} \rightarrow Z_s = Z_L = Z_0 \rightarrow$ Pasica idealno zaključena $Z_L = 0 \rightarrow$ nema refleksije $\rightarrow \boxed{V(x) = V_0^+ e^{-\beta x} + \underbrace{V_0^- e^{\beta x}}_{0}}$

$f = 100\ \text{kHz}$

$A = 20\ \text{dB} \rightarrow \alpha = 10\ \text{dB/km} \approx 1.15\ \text{Np/km}$ $\quad \boxed{[\text{dB}] = \dfrac{k_n b}{z_0}[\text{Np}] = \dfrac{1}{20\log_{10} e} \approx 0.115 z_0 [\text{Np}]}$

$\omega = 2\pi f = 200\cdot 10^3\pi$

$\beta = \dfrac{\omega}{k}$ / 4 Np! $\quad \beta x = 2\pi \quad \beta = 0$?

$\gamma = \alpha + i\beta$

$\Rightarrow \beta = \alpha = \underline{1.15\ \text{Np/km}}$

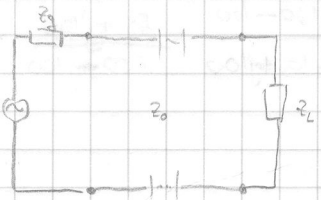
$$\boxed{\text{prijenosna fja } H(f) = \frac{V(d)}{V(0)} = \frac{1}{\cosh(\beta d) + \frac{Z_0}{Z_g}\sinh(\beta d)}} = \frac{1}{\cosh(1.15\cdot 2) + \frac{Z_0}{Z_0}\sinh(1.15\cdot 2)} = 0.1$$

$\text{kada } Z_g = Z_0$

amplituda kvadratična $H(f) \Rightarrow |H(f)| = \sqrt{\operatorname{Re}\{H(f)\}^2 + \operatorname{Im}\{H(f)\}^2} = 0.1$

7. $f_0\ (> 10\ \text{kHz})$

pretpostavi $Z_g = R + jX\ [\Omega]$



$\boxed{Z_L = Z_g = Z_0^* \quad \text{uvjet za max snagu}}$

$R = 10X \Rightarrow X = \dfrac{R}{10}$

$f_L = ?$

$$\boxed{f_L = \frac{Z_L - Z_0}{Z_L + Z_0}} = \frac{10x + jx - 10x + jx}{10x + jx + 10x - jx} = \frac{2jx}{20x} = \frac{j}{10}\,10 = \frac{10j}{100} = 0.1j$$

8. $IL = 20\ \text{dB}$

$IL = 10\log\dfrac{P_1}{P_2}$

$H(f) = \dfrac{U_2}{U_1} = \sqrt{\dfrac{P_2}{P_1}} = 0.1$

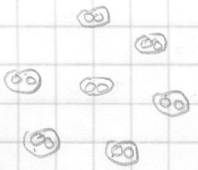
$|H(f)| = \sqrt{0.1^2} = 0.1$

$P = \frac{1}{2}\left|\frac{U}{Z_u}\right|^2 \cdot R_L = \frac{1}{2}\left|U\right|$

$P = \dfrac{U^2}{R} \Rightarrow U = \sqrt{PR}$

$\dfrac{U_1}{U_2} = \sqrt{\dfrac{P_1}{P_2}} = 10 = \sqrt{\dfrac{P_2}{P_1}} = 0.1$

8.



f_0

$A_b = 10 \log \frac{P_{2P}}{P_{1P}}$

$|A_b| = 70\,dB$

gubitak $\rightarrow < 0 \rightarrow A_b = -70\,dB$

$10^{A_b/10} = \left(\frac{P_{2P}}{P_{1P}}\right)^{10} \rightarrow 10^{-A_b/10} = \frac{P_{2P}}{P_{1P}} = \underline{10^{-7}}$

$NEXT = 10 \log (6 \cdot 10^{-7}) = \underline{-62.218\,dB}$

9. $f = 10^6\,Hz \qquad d \rightarrow \infty$

$\lambda = 200\,m$

$v_p = ?$

$u(t) = \sin\left(\omega t - \frac{x}{v_p}\right) = \sin(\omega t - \beta x)$

$v_p = \frac{\omega}{\beta} = \frac{2\pi f \cdot 200}{2\pi}$

$x \cdot \beta = 2\pi$

$\beta = \frac{2\pi}{x} = \frac{2\pi}{200} = \frac{\pi}{100}$

$v_f = \frac{\lambda \cdot \omega}{2\pi} = \frac{200 \cdot 2\pi \cdot 10^6}{2\pi} = \underline{2 \cdot 10^8\,m/s}$

10. $v_s(t) = E \sin(\omega t) \qquad d \rightarrow \infty$

$f_0 = 1\,kHz \rightarrow V_s(f_0, t) = 1\,V$; $Z_s(f_0) = 100 + j100\,[\Omega]$; max snaga: $Z_s = Z_L = \overline{Z_0}$

$= U_{max} = \sqrt{2} \cdot U_{efektivno}$

$Z_0 = 100 - j100$

$Z_u = Z_0$

$P_s = ?$

$P(t) = \frac{1}{2} Re\{U \cdot I\} = \frac{1}{2} |I|^2 \cdot R_L = \frac{1}{2} \left|\frac{U}{Z_u}\right|^2 \cdot R_L = \frac{1}{2} \left|\frac{1/\sqrt{2}}{100 - j100}\right|^2 \cdot 100 = \frac{1}{2}\left(\frac{1/\sqrt{2}}{\sqrt{100^2 + 100^2}}\right)^2 \cdot 100$

$= 0.00125$

11. Maks. snaga $\rightarrow Z_L = Z_S = \overline{Z_0}$

$Z_L = 100 + j100\,[\Omega]$

$\rho = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{100 + j100 - 100 + j100}{100 + j100 + 100 - j100} = \frac{j200}{200} = j$

$RL = 10 \log_{10}\left(\frac{1}{|\rho|^2}\right) = 10 \log_{10} \frac{1}{1} = \underline{0\,dB}$

12. $l = 100\,m = 0.1\,km$

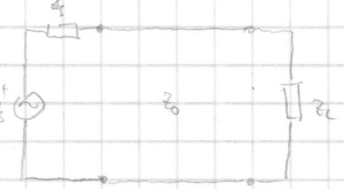
$V_s(t) = 0.1 \sin(2\pi 10^7 t)\ [V]$

$f = 10^7\,Hz$

$A_{NEXT} = -45\,dB$; $ACR = 40\,dB$

$\alpha = ?$

$Z_s = Z_L = Z_0$



$A = \alpha \cdot l \Rightarrow A = 10 \log_{10} P_A = 10 \log 10^{-5} = \underline{-5\,dB}$

$\alpha = \frac{|A|}{l} = \frac{5}{0.1} = \underline{50\,dB/km}$

$A_{NEXT} = 10 \log_{10}\left(\frac{P_1}{P_2}\right) \Rightarrow \frac{P_1}{P_2} = 10^{A_{NEXT}/10}$

$ACR = 10 \log \frac{P_A}{P_1/P_2} \Rightarrow P_A = \frac{P_1}{P_2} \cdot 10^{ACR/10}$

$P_A = 10^{A_{NEXT}/10} \cdot 10^{ACR/10}$

$P_A = 10^{-4.5} \cdot 10^{4} = 10^{-0.5}$

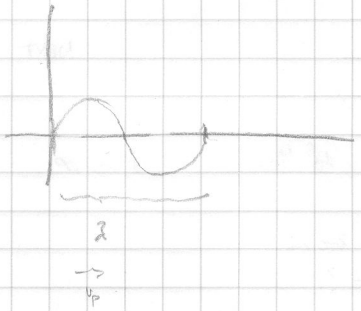
(13.) $10\,Np = ?\ dB$

$$1\,dB = \frac{\ln 10}{20}\,Np \quad \Rightarrow \quad 1\,Np = \frac{20}{\ln 10}\,dB \quad /\cdot 10$$

$$10\,Np = \underline{86.86\ dB}$$

(14.) $v_p = 2\cdot 10^5\ km/s = 2\cdot 10^8\ m/s$

$u = 5\sin(10^6 t)$

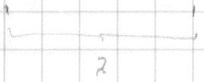


$\Delta f = ?$

$s = 200\ m$

$\omega = 2\pi f \Rightarrow \quad \frac{1}{f} = \frac{2\pi}{\omega} = \frac{2\pi}{10^6}\ s = T$

$$\lambda = \frac{2\pi}{\omega} \cdot v_p$$



$v_p = \frac{\omega}{\beta}$

$v = \frac{s}{t}$

$t_{200} = \frac{s}{v} = \frac{200}{2\cdot 10^8} = \underline{10^{-6}\ s}$

$$\beta x = \frac{\omega}{v_p} \cdot x = \frac{10^6}{2\cdot 10^8} \cdot 200 = \frac{10^6}{2\cdot 10^8} \cdot 2\cdot 10^2 = \underline{1\ rad}$$

$\Delta u = ?$ $\quad \Delta u = u(t_{200}) - u(0) = 5\left(\sin(10^6 \cdot 10^{-6}) - \sin(0)\right) = 5\cdot\left(\sin(\underset{t_1}{1}) - \sin(\underset{t_2}{0})\right)$

$\rightarrow \frac{\sin 1}{\sin 0} =$

(15.) $Z_0(f)$

f_1; $\quad Z_0(f_1) = 10x + jX$

$Z_s(f_1) = Z_0(f_1)$

$Z_L(f_1) = \overline{Z_0(f_1)}$

$RL = ?$

$RL = 10\log\left(\frac{1}{|p|^2}\right)$

$$r_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{10x - jX - 10x - jX}{10x - jX + 10x + jX} = \frac{-2jX}{20x} = -\frac{1}{10}j$$

$$RL = 10\log\left(\frac{1}{\left|\frac{1}{10}\right|^2}\right) = 10\log 100 = \underline{20\ dB}$$

(16.) $Z_0(f_1) =$

$Z_s(f_1) = Z_0(f_1)$

$Z_L(f_1) = Z_0(f_1)$

$\alpha_{f_1} = 5\ dB/km$

$A = 50\ dB$

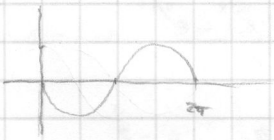
$AL = \alpha \cdot l \Rightarrow \quad l = \frac{A}{\alpha} = \underline{10\ km}$

(17.) $d \to \infty$

$f = 1000\ Hz$

$t = 0 \to \pi = \varphi = \beta \cdot x$ $\quad u(t) = A \sin(\omega t + \varphi)$

$u(0) = A \sin(\pi)$



$\beta = 0{,}01\pi\ rad/m$

$\beta \cdot x = 2\pi$

$0{,}01\pi \cdot x = 2\pi$

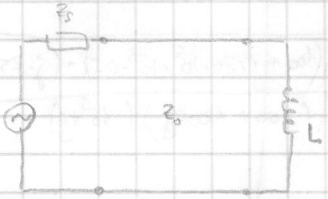
$x = 2 \cdot 100 = 200\ m$

(18.) $f_0 = ?$ $\quad d \to \infty$

$Z_0(f_0) = 100\pi j\ [\Omega]$

$L = 1{,}5 \cdot 10^{-3}\ H$; $Z_L = j\omega L$

$\omega = 2\pi f$



$V_0^+ = 2V_0^-$

$V(x,t) = Z_r + jZ_L$

$$\Gamma_L = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$Z_L + Z_0 = 2(Z_L - Z_0)$

$j\omega L + 100\pi j = 2j\omega L - 200\pi j$

$+300\pi j = j\omega L$

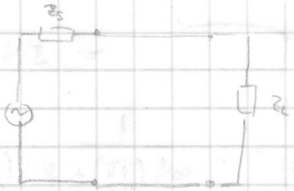
$\omega = \frac{300\pi}{L}$

$f = \frac{300\pi}{2\pi L} = 100000\ Hz$

$= 100\ kHz$

(19.) $E = 10 \sin(2\pi f_0 t)$

$Z_s(f_0) = Z_L(f_0) = Z_0(f) = 100\ \Omega$



$d = 4\ km$

$\alpha = 5\ dB/km$

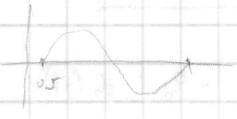
$IL = ?$

$IL = d \cdot \alpha = 20\ dB$

(20.) $u(t) = U \sin(10^6 t + 0{,}5)$ $\quad \beta \cdot x = 2\pi$; $\omega = 10^6$

$x = 1\ km = 1000\ m$ $\quad \beta = \frac{2\pi}{1000}$

$v_p = ?$



$\beta = \frac{\omega}{v_p} \to v_p = \frac{\omega}{\beta} = \frac{\omega}{2\pi} \cdot 1000 = \frac{10^9}{2\pi} = 159.159 \cdot 10^6\ m/s$

(21.) ber qualitativa

$u_s(t) = 10 \sin(2\pi \cdot 10^6 t)$ $\quad [V]$

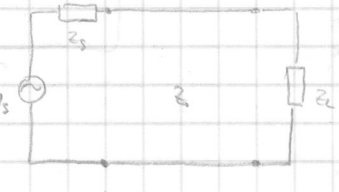
$Z_s = Z_L = Z_0 = 100\ \Omega$

$L = 10^{-3}\ H/km$

$C = 10^{-9}\ F/km$

$u_L(t) = ?$

$l = 0{,}5\ km$



$I_L = \frac{u}{Z_L + Z_0} = \frac{10}{200} = 0{,}05$

$u_L = I_L \cdot Z_L = 0{,}05 \cdot 100 = 5$

$H(f) = \frac{U_L}{U_S}$

$H(f) = \frac{1}{\cosh(\gamma) + 1 \sinh(\gamma)}$

$\gamma = \sqrt{RG} + j\omega\sqrt{LC} \Rightarrow \boxed{\beta = \omega\sqrt{LC}}$

$(\beta \cdot x) = 2\pi \cdot 10^6 \cdot \sqrt{LC} \cdot 0{,}5 = \frac{\pi}{1000}\ rad/km$

$u_L(t) = 5 \sin(2\pi \cdot 10^6 t + \pi) = -5 \sin(2\pi 10^6 t)$

22. $Z_0 = 100j\ (\Omega)$

$Z_L = \overline{Z_0}$

$Z_s = Z_0$

$100\ (\sin + j\cos$

$u_L(t) = 10 \sin(2\pi 10^3 t)$

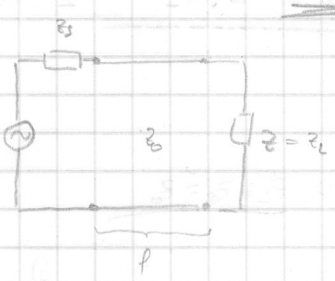
$i_L = ? = \frac{u_L}{Z_L}$

$= \frac{10 \sin(2\pi 10^3 t)}{-j100} = \frac{10 \sin(2\pi 10^3 t + 0)}{100 \angle -90°} = 0.1 \sin(2\pi 10^3 t + 0 - (-\frac{\pi}{2}))$

$= 0.1 \sin(2\pi 10^3 t + \frac{\pi}{2})$

$= 0.1 \cos(2\pi 10^3 t)$

23. $u_s = 10 \sin(2\pi 10 t)$ [V]



$Z_0 = 100\ \Omega$

$R = 100\ \Omega$

$Z_s = 100\ \Omega$

$\alpha = 1\ dB/km$

$l = 2\ km$

$I_L = ?$

$I_L = \alpha \cdot l = 2dB$

24. $P_m(f) = k_6 \cdot f^{1.5}$

$\Delta A = \left| 10 \log P_1 - 10 \log P_2 \right| = \left| 10 \left[\log(k f^{1.5}) - \log(k \cdot (2f)^{1.5}) \right] \right|$

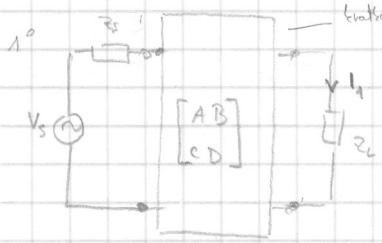
$= \left| 10 \log \frac{k f^{1.5}}{k \cdot 2^{1.5} f^{1.5}} \right| = \left| 10 \log 2^{-1.5} \right| = \left| -15 \log 2 \right| = 15 \log 2 = 4.5154$

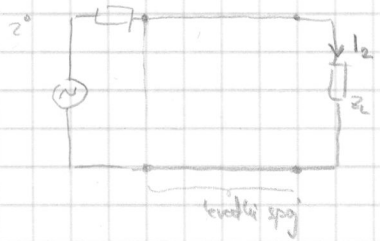
25. $u(t) = 2 \sin(\omega t)$ [V]

$Z_s = Z_L = 1\ \Omega$

$I_1 = I_2$

$U_2 = I_2 \cdot [Z_s + Z_L] = 2V$

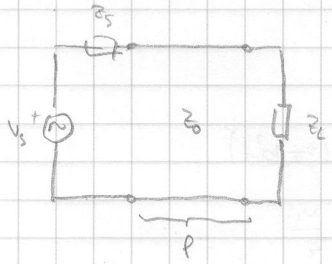




$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \Rightarrow$

jelölésű mátrica

26. <u>linija bez gubitaka</u>



$R' = 0 \ \Omega/km$
$G' = 0$
$\alpha = 0$
$l > 0$

$u(t) = 2 \sin(100t)$
$Z_s =$
Z_L
$Z_0 = \sqrt{\frac{L'}{C'}} =$

$\beta^2(\omega) = \omega^2 L'C' \rightarrow \beta = \omega\sqrt{LC} = 100\sqrt{\ } = 0.01$

$L' = 10^{-3} \ H/km$
$C' = 10^{-5} \ F/km$

$\rho_L = \rho_S = 0 = \frac{Z_L - Z_0}{Z_L + Z_0} \Rightarrow Z_L = Z_S = Z_0$

a) $u_L(t) = ?$

2π

b) $l = ?$

$I_L = \frac{U_s}{Z_L + Z_s} = \frac{2}{2\sqrt{\frac{L}{C}}} = \sqrt{\frac{C}{L}}$

$u_L = I_L \cdot Z_L = \sqrt{\frac{C}{L}} \cdot \sqrt{\frac{L}{C}} = 1 \ V$

$u_L = 1 \sin(100t)$

$\beta x = 24\pi$
$x = \frac{24\pi}{\beta} = 24\pi \cdot 100$