Matmul Layers

`torch.nn.Linear`

`torch.nn.Linear(in_features, out_features, bias=True)(x)`	
	Require
	• $\|x\| = (d_1, d_2, \ldots, d_k)$ • $\texttt{rank}(\|x\|) \geq 1$ • $d_k = in\_features$
	Guarantees
	• $\|y\| = (d_1, d_2, \ldots, d_{k-1}, out\_features)$
	Comment
	• $y = xA^T + b$를 계산하는 dense 레이어 • 1차원인 경우에도 잘 작동합니다. • $bias$ 옵션은 출력 shape에 영향을 주지 않습니다.

$$\frac{\begin{array}{l}\sigma \vdash E \Rightarrow e, c \\ k = \texttt{rank}(e) \\ e' = e[1:k-1]@(out) \\ c' = \{(k \geq 1) \wedge (d_k = in)\}\end{array}}{\sigma \vdash \texttt{Linear}(in, out, bias = True)(E) \Rightarrow e', c \cup c'}$$

Activations

`torch.nn.ReLU`, `torch.nn.ReLU6`, `torch.relu`, `torch.nn.functional.relu`

`torch.nn.ReLU(inplace=True)(x)`	
	Require
	Guarantees
	• $\|y\| = \|x\|$ (same shape)
	Comment
	• $inplace$ 옵션은 shape에 영향을 주지 않습니다. • `ReLU6`도 `ReLU`와 똑같은 방식으로 shape 계산 • Bulitins인 `torch.relu`와 `torch.nn.functional.relu`는 같은거

$$\forall \texttt{ft} \in \{\texttt{ReLU}, \texttt{ReLU6}\}, \quad \frac{\sigma \vdash E \Rightarrow e, c}{\sigma \vdash \texttt{ft}(inplace = True)(E) \Rightarrow e, c}$$

$$\frac{\sigma \vdash E \Rightarrow e, c}{\sigma \vdash \texttt{relu}(E, inplace = True) \Rightarrow e, c}$$

End Points

`torch.nn.CrossEntropyLoss`

`torch.nn.CrossEntropyLoss(weight=None, size_average=None, ignore_index=-100, reduction='mean')(input, target)`	
	Require

	• $\|input\| = (n, c, d_1, d_2, \ldots, d_k) \quad (\texttt{rank}(\|input\|) \geq 2)$
	• $\|target\| = (n, d_1, d_2, \ldots, d_k)$
	  – $\texttt{rank}(\|target\|) + 1 = \texttt{rank}(\|input\|)$
	  – $\|target\|[1] = \|input\|[1]$, $\|target\|[2] = \|input\|[3]$, ...
	• if $weight \neq None$, then $\|weight\| = (c)$

	Guarantees
	• $\|y\| = \texttt{if}\ reduction == \text{`}none'\ \texttt{then}\ (n, d_1, d_2, \ldots, d_k)\ \texttt{else}\ ()$
	Comment
	• *size_average*, *reduce* 인자는 deprecated로 도큐먼트되어 있습니다. • *ignore_index*는 *reduction*이나 *size_average* 옵션에서 평균을 계산하면서 생략하는 인덱스 번호를 지정할 때 쓰는 것으로, shape에 영향을 주지 않습니다.

$$\frac{\begin{array}{l}\sigma \vdash E \Rightarrow e, c_e \\ \sigma \vdash T \Rightarrow t, c_t \\ \sigma \vdash weight \Rightarrow w, c_w \quad \text{if } weight \neq None, \text{otherwise } c_w = \emptyset \\ c_{dim} = \{(\texttt{rank}(e) \geq 2) \wedge (\texttt{rank}(e) = \texttt{rank}(t) + 1)\} \\ c_{elt} = \{(e[1] = t[1]) \wedge (e[3] = t[2]) \wedge (e[4] = t[3]) \wedge \cdots\} \\ c_{weight} = \{(weight = None \vee w = (e[2]))\} \\ e' = \texttt{if}\ reduction = \text{`}none'\ \texttt{then}\ t\ \texttt{else}\ ()\end{array}}{\sigma \vdash \texttt{CrossEntropyLoss}(weight = None, ..., reduction = \text{`}mean')(E, T) \Rightarrow e', c_e \cup c_t \cup c_w \cup c_{dim} \cup c_{elt} \cup c_{weight}}$$

`torch.nn.TripletMarginLoss`

NOT DONE YET!!

Reading Papers...

`torch.nn.TripletMarginLoss(margin=1.0, p=2.0, eps=1e-6, swap=False, size_average=False, reduce=None, reduction='mean')(anchor, positive, negative)`	
	Require

	• $broadcastable(\|anchor\|, \|positive\|, \|negative\|)$
	• $\texttt{rank}(broadcast(\|anchor\|, \|positive\|, \|negative\|)) \geq 2$
	• if $swap$ is $True$ then,
	  – $\texttt{rank}(\|anchor\|), \|positive\|, \|negative\|)$
	  – $\texttt{rank}(broadcast(\|anchor\|, \|positive\|, \|negative\|)) \geq 2$
	• $\|target\| = (n, d_1, d_2, \ldots, d_k)$
	  – $\texttt{rank}(\|target\|) + 1 = \texttt{rank}(\|input\|)$
	  – $\|target\|[1] = \|input\|[1]$, $\|target\|[2] = \|input\|[3]$, ...
	• if $weight \neq None$, then $\|weight\| = (c)$

	Guarantees
	• $\|y\| = \texttt{if}\ reduction == \text{`}none'\ \texttt{then}\ (n, d_1, d_2, \ldots, d_k)\ \texttt{else}\ ()$
	Comment
	• $size\_average$, $reduce$ 인자는 deprecated로 도큐먼트되어 있습니다. • $ignore_index$는 $reduction$이나 $size\_average$ 옵션에서 평균을 계산하면서 생략하는 인덱스 번호를 지정할 때 쓰는 것으로, shape에 영향을 주지 않습니다. • 토치 구현의 버그인지 모르겠는데, $swap$이 $True$인 경우가 너무 복잡합니다.. 도큐먼트에도 논문 하나만 첨부되어 있습니다.

$$\frac{\begin{array}{l}\sigma \vdash E \Rightarrow e, c_e \\ \sigma \vdash T \Rightarrow t, c_t \\ \sigma \vdash weight \Rightarrow w, c_w \quad \text{if } weight \neq None, \text{ otherwise } c_w = \emptyset \\ c_{dim} = \{(\texttt{rank}(e) \geq 2) \wedge (\texttt{rank}(e) = \texttt{rank}(t) + 1)\} \\ c_{elt} = \{(e[1] = t[1]) \wedge (e[3] = t[2]) \wedge (e[4] = t[3]) \wedge \cdots\} \\ c_{weight} = \{(weight = None \vee w = (e[2]))\} \\ e' = \texttt{if}\ reduction = \text{`}none'\ \texttt{then}\ t\ \texttt{else}\ ()\end{array}}{\sigma \vdash \texttt{CrossEntropyLoss}(weight = None, ..., reduction = \text{`}mean')(E, T) \Rightarrow e', c_e \cup c_t \cup c_w \cup c_{dim} \cup c_{elt} \cup c_{weight}}$$

Technique

`torch.nn.Dropout`, `torch.dropout`, `torch.nn.functional.dropout`

`torch.nn.Dropout(...)(x)`	
	Require
	Guarantees
	• $\|y\| = \|x\|$ (same shape)
	Comment
	• 모든 옵션은 shape에 영향을 주지 않습니다. • Bulitins인 `torch.dropout`와 `torch.nn.functional.dropout`는 서로 역할이 같습니다.

$$\frac{\sigma \vdash E \Rightarrow e, c}{\sigma \vdash \texttt{Dropout}(...)(E) \Rightarrow e, c}$$

$$\frac{\sigma \vdash E \Rightarrow e, c}{\sigma \vdash \texttt{dropout}(E, ...) \Rightarrow e, c}$$

Wrapper

`torch.nn.Sequential`

`torch.nn.Sequential(l1, l2, l3, ..., ln)(x)`	
	Require
	• 순차적으로 shape이 맞아떨어져야함
	Guarantees
	• $\lvert y \rvert = \lvert l_n \circ l_{n-1} \circ \cdots \circ l_1(x) \rvert$

$$\frac{\sigma \vdash l_n \circ l_{n-1} \circ \cdots l_1(E) \Rightarrow e, c}{\sigma \vdash \texttt{Sequential}(l_1, l_2, \ldots, l_n)(E) \Rightarrow e, c}$$