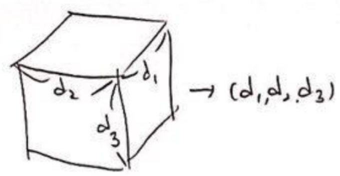
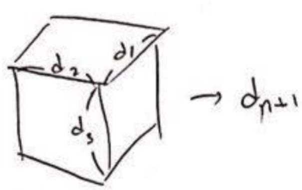
Shape

`torch.Tensor.size`, `torch.Tensor.stride`

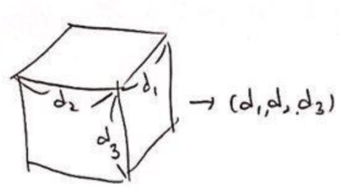
`a.size() or a.stride()`	
	Require
	• $\|\mathsf{a}\| = (d_1, d_2, \ldots, d_k)$
	Guarantees
	• $(d_1, d_2, \ldots, d_k)$를 튜플로 반환

$$\forall \texttt{ft} \in \{\texttt{size}, \texttt{stride}, \}, \qquad \frac{\sigma \vdash E \Rightarrow e, c}{\sigma \vdash E.\texttt{ft}() \Rightarrow shapeToTuple(e), c}$$

`a.size(n)`	
	Require
	• $\|\mathsf{a}\| = (d_1, d_2, \ldots, d_k)$ • $0 \leq n < k$
	Guarantees
	• d_{n+1}을 숫자(int)로 반환 • n에 -1이 들어갈 수도 있음

$$\forall \texttt{ft} \in \{\texttt{size}, \texttt{stride}, \}, \qquad \frac{\begin{array}{l}\sigma \vdash E \Rightarrow e, c \\ k = \texttt{rank}(e) \\ c' = \{(k \geq 1) \land (0 \leq n < k)\}\end{array}}{\sigma \vdash E.\texttt{ft}(n) \Rightarrow e[n+1], c \cup c'}$$

`torch.Tensor.shape`

`a.shape`	
	Require
	• $\|\mathsf{a}\| = (d_1, d_2, \ldots, d_k)$
	Guarantees
	• $(d_1, d_2, \ldots, d_k)$를 튜플로 반환

$$\frac{\sigma \vdash E \Rightarrow e, c}{\sigma \vdash E.\texttt{shape} \Rightarrow shapeToTuple(e), c}$$

Tensor Declarations

`torch.tensor`

`torch.tensor(x)`	
	Comment
	`numpy`나 파이썬 리스트로 선언된 객체를 `torch`에서 호환가능한 형태로 바꾸는 함수

$$\frac{\sigma \vdash E \Rightarrow e, c}{\sigma \vdash \texttt{tensor}(E) \Rightarrow e, c}$$

`torch.range`, `torch.arange`

<table>
<tr><td colspan="2"><code>torch.range(start=0, end, step=1, out=None, ...), torch.arange(...)</code></td></tr>
<tr><td rowspan="6">$(s, s+d, s+2d, \cdots, s+\lfloor\frac{e-s}{d}\rfloor d)$</td><td>Require</td></tr>
<tr><td>• $step \neq 0$
• $(end - start)/step > 0$</td></tr>
<tr><td>Guarantees</td></tr>
<tr><td>• $|y| = (1 + \lfloor(end - start)/step\rfloor)$</td></tr>
<tr><td>Comment</td></tr>
<tr><td>• $(start, start + step, start + 2 \cdot step, \ldots)$를 반환
• out-텐서 인자가 있는 함수
• <code>torch.arange</code>도 똑같은 방식으로 작동함</td></tr>
</table>

$$\forall \texttt{ft} \in \{\texttt{range}, \texttt{arange}\}, \qquad \frac{c = \{(d \neq 0) \wedge ((e - s)/d > 0)\}}{\sigma \vdash \texttt{ft}(s, e, d, out = None, ...) \Rightarrow (1 + \lfloor(e - s)/d\rfloor), c} \qquad \text{Default: } s = 0, d = 1$$

`torch.linspace`

<table>
<tr><td colspan="2"><code>torch.range(start, end, steps=100, out=None, ...), torch.arange(...)</code></td></tr>
<tr><td rowspan="6">$(s, s+d, s+2d, \cdots, s+\lfloor\frac{e-s}{d}\rfloor d)$</td><td>Require</td></tr>
<tr><td>• $steps \geq 0$</td></tr>
<tr><td>Guarantees</td></tr>
<tr><td>• $|y| = (steps)$</td></tr>
<tr><td>Comment</td></tr>
<tr><td>• $(start, start + d, start + 2d, \ldots, end)$인데 원소가 $steps$개인 텐서 반환
• out-텐서 인자가 있는 함수</td></tr>
</table>

$$\frac{c = \{(steps \geq 0)\}}{\sigma \vdash \texttt{ft}(s, e, steps = 100, out = None, ...) \Rightarrow (steps), c}$$

`torch.zeros`, `torch.empty`, `torch.rand`, `torch.randn`

<table>
<tr><td colspan="2"><code>torch.zeros(t1, t2, ..., tl, out=None, ...) or .empty, .rand, .randn</code></td></tr>
<tr><td rowspan="6"></td><td>Require</td></tr>
<tr><td></td></tr>
<tr><td>Guarantees</td></tr>
<tr><td>$|y| = (t_1, t_2, \ldots, t_l)$</td></tr>
<tr><td>Comment</td></tr>
<tr><td>입력받은 형태대로 0, uninitialized, uniformly random, gaussian random 텐서를 반환
$(t_1, t_2, \ldots, t_l)$ 입력이 하나의 튜플로 들어오는 경우도 있음
out-텐서 인자가 있는 함수</td></tr>
</table>

$$\forall \texttt{ft} \in \{\texttt{zeros}, \texttt{empty}, \texttt{rand}, \texttt{randn}\}, \quad \frac{}{\sigma \vdash \texttt{ft}(t_1, t_2, \ldots, t_l, out = None) \Rightarrow (t_1, t_2, \ldots, t_l), \emptyset}$$

$$\forall \texttt{ft} \in \{\texttt{zeros}, \texttt{empty}, \texttt{rand}, \texttt{randn}\}, \quad \frac{}{\sigma \vdash \texttt{ft}((t_1, t_2, \ldots, t_l), out = None) \Rightarrow (t_1, t_2, \ldots, t_l), \emptyset}$$

`torch.randint`

<table>
<tr><td colspan="2"><code>torch.randint(low=0, high, shape, ..., out=None, ...)</code></td></tr>
<tr><td rowspan="6"></td><td>Require</td></tr>
<tr><td>$low < high$
$shape$가 well-defined인 텐서 shape. (스칼라 타입은 X)</td></tr>
<tr><td>Guarantees</td></tr>
<tr><td>$|y| = shape$</td></tr>
<tr><td>Comment</td></tr>
<tr><td>out-텐서가 있는 함수</td></tr>
</table>

$$\frac{}{\sigma \vdash \texttt{randint}(low = 0, high, (t_1, t_2, \ldots, t_l), ..., out = None, ...) \Rightarrow (t_1, t_2, \ldots, t_l), \{(low < high)\}}$$

`torch.randperm`

<table>
<tr><td colspan="2"><code>torch.randperm(n, out=None, ...)</code></td></tr>
<tr><td rowspan="6"></td><td>Require</td></tr>
<tr><td>$n \geq 0$</td></tr>
<tr><td>Guarantees</td></tr>
<tr><td>$|y| = (n)$</td></tr>
<tr><td>Comment</td></tr>
<tr><td>out-텐서가 있는 함수</td></tr>
</table>

$$\frac{}{\sigma \vdash \texttt{randint}(n, out = None, ...) \Rightarrow (n), \{(n \geq 0)\}}$$

`torch.full`

`torch.full((t1, t2, ..., tl), fill_value, out=None, ...)`	
	Require
	Guarantees
	• $\lvert y \rvert = (t_1, t_2, \ldots, t_l)$
	Comment
	• 모든 원소가 *fill_value*로 채워진 텐서 반환 • `empty`랑 비슷해보이는데, size인자를 항상 튜플로만 받음 • *out*-텐서가 있는 함수

$$\frac{}{\sigma \vdash \texttt{full}((t_1, t_2, \ldots, t_l), \mathit{fill\_value}, \mathit{out} = \mathit{None}) \Rightarrow (t_1, t_2, \ldots, t_l), \emptyset}$$

`torch.normal`

`torch.normal(mean, std, (t1, t2, ..., tl), out=None`	
	Require
	• $\mathit{std} > 0$
	Guarantees
	• $\lvert y \rvert = (t_1, t_2, \ldots, t_l)$
	Comment
	• `empty`랑 비슷해보이는데, size인자를 항상 튜플로만 받음 • *out*-텐서가 있는 함수

$$\frac{}{\sigma \vdash \texttt{normal}(\mathit{mean}, \mathit{std}, (t_1, t_2, \ldots, t_l), \mathit{out} = \mathit{None}) \Rightarrow (t_1, t_2, \ldots, t_l), \{(\mathit{std} > 0)\}}$$

`torch.Tensor.new_empty`

`x.new_empty((t1, t2, ..., tl), ...)`	
	Require
	Guarantees
	• $\lvert y \rvert = (t_1, t_2, \ldots, t_l)$
	Comment
	• `empty`랑 비슷해보이는데, `Tensor` 클래스에서만 사용 가능하고, *out* 인자도 없으며, size인자를 항상 튜플로만 받음 • 텐서 `x`의 shape도 똑같이 변함

$$\frac{}{\sigma \vdash x.\texttt{new\_empty}((t_1, t_2, \ldots, t_l), \ldots) \Rightarrow (t_1, t_2, \ldots, t_l), \emptyset}$$

x의 shape도 똑같이 $(t_1, t_2, \ldots, t_l)$로 변함

torch.Tensor.new_full

`x.new_full((t1, t2, ..., tl), fill_value, ...)`	
	Require
	Guarantees
	• $\lvert y\rvert = (t_1, t_2, \ldots, t_l)$
	Comment
	• `new_empty`랑 비슷해보이는데, `fill_value` 인자가 추가적으로 있음 • 텐서 `x`의 shape도 똑같이 변함

$$\frac{}{\sigma \vdash x.\texttt{new\_full}((t_1, t_2, \ldots, t_l), \mathit{fill\_value}...) \Rightarrow (t_1, t_2, \ldots, t_l), \emptyset}$$

x의 shape도 똑같이 $(t_1, t_2, \ldots, t_l)$로 변함

torch.Tensor.clone

`x.clone(...)`	
	Require
	Guarantees
	• $\lvert y\rvert = \lvert x\rvert$

$$\frac{\sigma \vdash E \Rightarrow e, c}{\sigma \vdash E.\texttt{clone}(...) \Rightarrow e, c}$$

torch.zeros_like, torch.empty_like, torch.rand_like, torch.randn_like

`torch.zeros_like(input, ...) or .empty_like, .rand_like, .randn_like`	
	Require
	Guarantees
	• $\lvert y\rvert = \lvert input\rvert$
	Comment
	• 입력받은 텐서와 shape이 같은 0, uninitialized, uniformly random, gaussian random 텐서를 반환 • *out*-텐서 인자가 없음!

$$\forall \texttt{ft} \in \{\texttt{zeros\_like}, \texttt{empty\_like}, \texttt{rand\_like}, \texttt{randn\_like}\}, \quad \frac{\sigma \vdash E \Rightarrow e, c}{\sigma \vdash \texttt{ft}(E, ...) \Rightarrow e, c}$$

`torch.full_like`

`torch.full_like(input, fill_value, out=None, ...)`	
	Require
	Guarantees
	• $\|y\| = \|input\|$
	Comment
	• 입력받은 텐서와 shape이 같은 $fill_value$로 가득찬 텐서 반환 • 희한하게 이건 out-텐서 인자가 있음..

$$\frac{\sigma \vdash E \Rightarrow e, c}{\sigma \vdash \texttt{full\_like}(E, fill\_value, out = None...) \Rightarrow e, c}$$

`torch.scalar_tensor`

`torch.scalar_tensor(scalar, ...)`	
	Require
	Guarantees
	• $\|y\| = ()$
	Comment
	• $scalar$ 값 하나만 가지는 `rank-0` 텐서 반환

$$\frac{}{\sigma \vdash \texttt{scalar\_tensor}(scalar, ...) \Rightarrow (), \emptyset}$$

`torch.eye`

`torch.eye(n, m=None, out=None, ...)`	
	Require
	• $n \geq 0$ • $m = None$ or $m \geq 0$
	Guarantees
	• $\|y\| = \begin{cases} (n, n) & \text{if } m \text{ is } None \\ (n, m) & \text{otherwise} \end{cases}$
	Comment
	• 곱셈의 항등원 I_n를 리턴 • out 인자가 있는 함수

$$\frac{\begin{array}{l} e = \texttt{if } m = None \texttt{ then } (n, n) \texttt{ else } (n, m) \\ c = \{(n \geq 0) \wedge (m = None \vee m \geq 0)\} \end{array}}{\sigma \vdash \texttt{eye}(n, m, out = None, ...) \Rightarrow e, c}$$

Same Shape, Elementwise Operators

All these builtin functions `torch.*` are used with `torch.ft(input, out=None)` that output the same shapes of the `inputs`.

- `round`, `floor`, `ceil`
- `exp`, `log`, `log10`, `log2`, `log1p`, `sigmoid`
- `sqrt`, `rsqrt`
- `cos`, `sin`, `tan`, `angle`
- `sign`, `neg`, `frac`
- `torch.Tensor.contiguous`
  - 텐서 객체에서 사용 가능한 함수.
  - 똑같은 내용물이지만, 메모리 상에서 원소들이 연속하도록 배치해주므로 같은 shape을 반환
  - 즉, 얘는 `a.contiguous()` 이런 식으로 많이 쓰임

input 인자는 무조건 텐서 type이어야 합니다. (스칼라 X)

$$\frac{\sigma \vdash E \Rightarrow e, c}{\sigma \vdash \texttt{ft}(E, out = None) \Rightarrow e, c}$$

`torch.clamp(input, min, max, out=None)` 함수는 *input* 텐서의 모든 원소가 $min \leq \cdot \leq max$가 성립하도록 만들어주는 것으로, 역시 텐서 shape이 보존됩니다.

$$\frac{\sigma \vdash E \Rightarrow e, c}{\sigma \vdash \texttt{clamp}(E, min, max, out = None) \Rightarrow e, c}$$

Broadcasted Shape, Binary Operators

All these builtin functions `torch.*` are used with `torch.ft(input, other, out=None)` that output the same shapes of the `inputs`.

- `eq`, `le`, `lt`, `ge`, `gt`
- `mul`, `div`, `fmod`

한 가지 독특한 성질은 *input*, *other* 인자가 스칼라로 들어오면 [ ]-shape 텐서로 변환된 후 계산됩니다.

- 즉, $\texttt{torch.mul}(1, 2)\texttt{.shape}$은 [ ] 입니다.

$$\frac{\sigma \vdash E_1 \Rightarrow e_1, c_1 \quad \sigma \vdash E_2 \Rightarrow e_2, c_2}{\sigma \vdash \texttt{ft}(E_1, E_2, out = None) \Rightarrow broadcast(e_1, e_2), c_1 \cup c_2 \cup broadcastable(e_1, e_2)}$$

The following builtin functions `torch.*` are slightly different. `torch.ft(input, other, out=None, alpha=1)` that output the same shapes of the `inputs`. (Additional *alpha* option!)

- `add`, `sub`
  - Calculates broadcasted $input \pm \alpha \cdot output$

마찬가지로 *input*, *other* 인자로 스칼라가 들어오면 [ ]-shape 텐서로 변환된 후 계산됩니다.

$$\frac{\sigma \vdash E_1 \Rightarrow e_1, c_1 \quad \sigma \vdash E_2 \Rightarrow e_2, c_2}{\sigma \vdash \texttt{ft}(E_1, E_2, out = None, alpha = 1) \Rightarrow broadcast(e_1, e_2), c_1 \cup c_2 \cup broadcastable(e_1, e_2)}$$

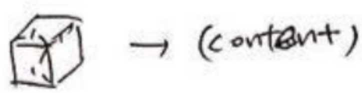
torch.pow

`torch.pow(input, exponent, out=None)`	
	Require
	• [*input* or *exponent* are scalar] or $broadcastable(\|input\|, \|exponent\|)$
	Guarantees
	• $broadcast(\|input\|, \|exponent\|)$
	Comment
	• elementwise binary operator라는 점에서 `mul`과 비슷하나, 인자 이름(exponent)이 달라서 따로 뺐습니다. (`pow(a, exponent=b)`같은 문제..) • 마찬가지로 인자에 스칼라가 들어오면 [ ]-shape 텐서로 변환된 후 계산됩니다. • *out* 인자가 있는 함수

$$\frac{\begin{array}{c}\sigma \vdash E_1 \Rightarrow e_1, c_1 \\ \sigma \vdash E_2 \Rightarrow e_2, c_2\end{array}}{\sigma \vdash \texttt{pow}(E_1, E_2, out = None) \Rightarrow broadcast(e_1, e_2), c_1 \cup c_2 \cup broadcastable(e_1, e_2)}$$

Indexing

torch.Tensor.item

`a.item()`	
 	Require
	• $\|a\| = ()$ or $\|a\| = (1, 1, 1, \ldots, 1)$
	Guarantees
	• $\|y\| = e_n$
	Comment
	• Singular element tensor의 원소(스칼라 타입으로 반환)

$$\frac{\begin{array}{l}\sigma \vdash E \Rightarrow e, c \\ k = \texttt{rank}(e) \\ c' = \{(\forall i = 1, 2, \ldots, k,\ e[i] = 1)\}\end{array}}{\sigma \vdash E.\texttt{item}() \Rightarrow e_n, c \cup c'}$$

Shape Polymorphism

`torch.Tensor.view`

`a.view(n1, n2, ..., nl)`	
	Require
	• $\lvert \mathsf{a} \rvert = (d_1, d_2, \dots, d_k)$ • $\mathit{reshapable}((d_1, d_2, \dots, d_k), (n_1, n_2, \dots, n_l))$
	Guarantees
	• $(n_1, n_2, \dots, n_l)$ as tensor
	Comment
	• 단, $n_1, n_2, \dots, n_l$이 하나의 튜플로 입력이 들어올 수도 있음

$$\frac{\sigma \vdash \texttt{reshape}(E, n_1, n_2, \dots, n_l) \Rightarrow e, c}{\sigma \vdash E.\texttt{view}(n_1, n_2, \dots, n_l) \Rightarrow e, c}$$

`torch.Tensor.expand`

`a.expand(n1, n2, ..., nl)`	
	Require
	• $\lvert \mathsf{a} \rvert = (d_1, d_2, \dots, d_k)$ • $k \leq l$ • $\forall i = 1, 2, \dots, l-k,\ n_i > 0$ • $\forall i = l-k+1, l-k+2, \dots, l,\ [(n_i = -1) \text{ or } ((n_i > 0) \text{ and } ((d_{i-(l-k)} = 1) \text{ or } (d_{i-(l-k)} = n_i)))]$
	Guarantees

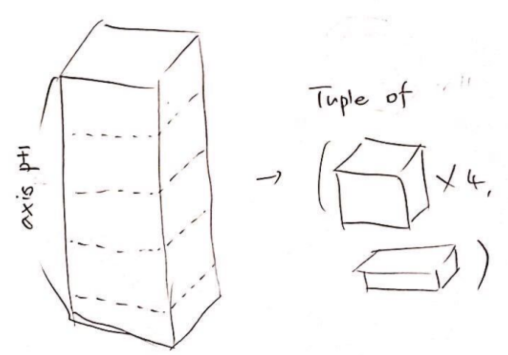
	• $(m_1, m_2, \dots, m_l)$ as tensor where
	  – $m_1 = n_1, m_2 = n_2, \cdots, m_{l-k} = n_{l-k}$
	  – $m_{l-k+i} = \texttt{if } (n_{l-k+i} = -1) \texttt{ then } d_i \texttt{ else } n_{l-k+i}$ for the rests

	Comment
	• 일방향 broadcast 함수; $(n_1, n_2, \dots, n_l)$ shape이 목표 • $n_i = -1$인 경우, 본래 크기만큼 그대로 유지 • 단, $n_1, n_2, \dots, n_l$이 하나의 튜플로 입력이 들어올 수도 있음

$$\frac{\begin{array}{l} \sigma \vdash E \Rightarrow e, c \\ k = \texttt{rank}(e) \\ m_1 = n_1, m_2 = n_2, \cdots, m_{l-k} = n_{l-k} \\ \forall i \in \{1, 2, \dots, k\},\ m_{l-k+i} = \texttt{if } (n_{l-k+i} = -1) \texttt{ then } d_i \texttt{ else } n_{l-k+i} \\ c_{base} = \{(k \leq l) \wedge (n_1 > 0) \wedge (n_2 > 0) \wedge \cdots \wedge (n_{l-k} > 0)\} \\ c_{expandable} = \{(\forall i \in \{1, 2, \dots, k\},\ (n_{l-k+i} = -1) \vee [(n_{l-k+i} > 0) \wedge ((d_i = 1) \vee (d_i = n_{l-k+i}))])\} \end{array}}{\sigma \vdash E.\texttt{expand}(n_1, n_2, \dots, n_l) \Rightarrow (m_1, m_2, \dots, m_l), c \cup c_{base} \cup c_{expandable}}$$

torch.split

`torch.split(tensor, split_size_or_section, dim=0)`



Require

- $|tensor| = (d_1, d_2, \dots, d_k)$
- $k \geq 1$
- $0 \leq dim < k$

Guarantees

- 아래 proof tree와 같이 $dim + 1$번째 axis가 최대 $split_size_or_section$개의 원소를 가지도록 $\lceil d_{dim+1}/\cdot \rceil$개 튜플 형태로 반환

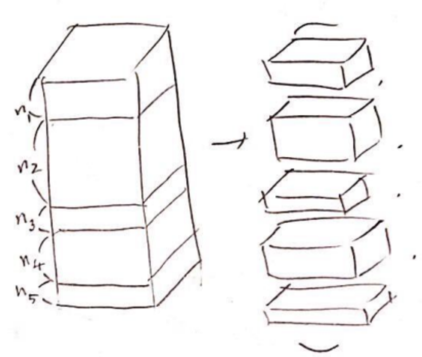
Comment

- Concat의 반대 역할 함수
- Divisible 여부 assert하지 않음
- 학습 및 테스트 데이터를 배치단위로 쪼개는 용도로 쓰일 것으로 추측

$$\frac{\begin{array}{l}\sigma \vdash E \Rightarrow e, c \\ k = \texttt{rank}(e) \\ e_1 = e[1{:}p]@(n)@e[p+2{:}k] \\ e_2 = e[1{:}p]@(n)@e[p+2{:}k] \\ \quad \dots \\ e_{l-1} = e[1{:}p]@(n)@e[p+2{:}k] \\ e_l = e[1{:}p]@(n')@e[p+2{:}k] \quad \text{where } e[p+1] = n(l-1) + n',\ 0 < n' \leq n \\ c' = \{(k \geq 1) \wedge (0 \leq p < k)\}\end{array}}{\sigma \vdash \texttt{split}(E, n, p = 0) \Rightarrow (e_1, e_2, \dots, e_l), c \cup c'}$$

l-원소 tuple 형태로 반환

`torch.split(input, [n1, n2, ..., nl], dim=0)`



Require

- $|input| = (d_1, d_2, \dots, d_k)$
- $k \geq 1$
- $0 \leq dim < k$
- $d_{dim+1} = n_1 + n_2 + \cdots + n_l$

Guarantees

- 아래 proof tree와 같이 $dim + 1$번째 axis의 크기가 $n_1, n_2, \dots, n_l$인 l개의 텐서 튜플을 반환
- n_i의 합과 d_{dim+1}이 같은지 assert

$$\frac{\begin{array}{l}\sigma \vdash E \Rightarrow e, c \\ k = \texttt{rank}(e) \\ e_1 = e[1{:}p]@(n_1)@e[p+2{:}k] \\ e_2 = e[1{:}p]@(n_2)@e[p+2{:}k] \\ \quad \dots \\ e_l = e[1{:}p]@(n_l)@e[p+2{:}k] \\ c' = \{(k \geq 1) \wedge (0 \leq x < k) \wedge (e[p+1] = n_1 + n_2 + \cdots + n_l)\}\end{array}}{\sigma \vdash \texttt{split}(E, [n_1, n_2, \dots, n_l], p = 0) \Rightarrow (e_1, e_2, \dots, e_l), c \cup c'}$$

l-원소 tuple 형태로 반환

`torch.chunk`

<table>
<tr><td colspan="2"><code>torch.chunk(input, chunks, dim=0)</code></td></tr>
<tr><td rowspan="6"></td><td>Require</td></tr>
<tr><td>$|input| = (d_1, d_2, \ldots, d_k)$ $k \geq 1$ $chunks > 0$ $0 \leq dim < k$</td></tr>
<tr><td>Guarantees</td></tr>
<tr><td>Proof tree와 같이 최소 $chunks$개로 dim 축이 잘라진 텐서 튜플 반환</td></tr>
<tr><td>Comment</td></tr>
<tr><td><code>split</code>과 비슷하나 쪼개는 개수를 명시한다는 점이 다름</td></tr>
</table>

$$\frac{\begin{array}{l}\sigma \vdash E \Rightarrow e, c \\ k = \texttt{rank}(e) \\ d = \min\{e[p+1], chunks\} \\ n = \lceil e[p+1]/d \rceil \\ e_1 = e[1{:}p]@(n)@e[p+2{:}k] \\ e_2 = e[1{:}p]@(n)@e[p+2{:}k] \\ \quad \ldots \\ e_{l-1} = e[1{:}p]@(n)@e[p+2{:}k] \\ e_l = e[1{:}p]@(n')@e[p+2{:}k] \quad \text{where } e[p+1] = n(l-1)+n',\ 0 < n' \leq n \\ c' = \{(k \geq 1) \wedge (chunks > 0) \wedge (0 \leq p < k)\}\end{array}}{\sigma \vdash \texttt{chunk}(E, chunks, p=0) \Rightarrow (e_1, e_2, \ldots, e_l), c \cup c'} \qquad l\text{-원소 tuple 형태로 반환}$$