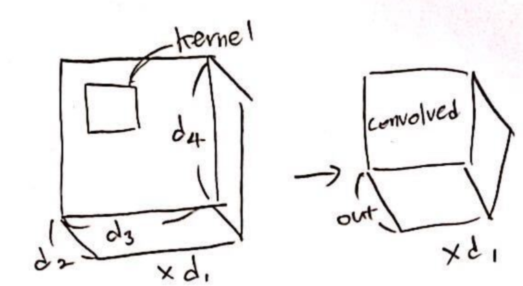
Convolutional Layers

torch.nn.Conv2d

<table>
<tr><td colspan="2"><code>torch.nn.Conv2d(in_channels, out_channels, kernel_size, stride=1, padding=0, dilation=1, groups=1, other_params..)(x)</code></td></tr>
<tr><td rowspan="6">
</td><td>Require</td></tr>
<tr><td>

- $|x| = (d_1, d_2, d_3, d_4) \quad (rank = 4)$
- $d_2 = in\_channels$
- $d_3 + 2 \times padding[0] - dilation[0] \times (kernel\_size[0] - 1) - 1 \geq 0$
- $d_4 + 2 \times padding[1] - dilation[1] \times (kernel\_size[1] - 1) - 1 \geq 0$
- $groups | in\_channels$ and $groups | out\_channels$

</td></tr>
<tr><td>Guarantees</td></tr>
<tr><td>

- $|y| = (d_1, out\_channels, h, w)$ where.. refers to the proof tree.

</td></tr>
<tr><td>Comment</td></tr>
<tr><td>

- Convolution layer입니다. 선배님의 자료를 `pytorch`의 사용에 맞게 풀어 쓴 것입니다.
- $kernel\_size$, $stride$와 같은 옵션은 튜플로 구성될 수도 있습니다. (가로 세로에 대한 필터크기가 서로 다르도록) 이 경우를 위하여 proof 트리에서 $kernel_size[0], [1]$과 같은 표기를 사용하였습니다. 튜플이 아니라 스칼라 입력인 경우, $kernel_size[0], [1]$은 모두 $kernel_size$와 같습니다.
- 뒤의 $other\_params..$ 부분은 텐서 shape에 전혀 영향을 주지 않는 인자입니다.

</td></tr>
</table>

$$\frac{\begin{array}{l}
\sigma \vdash E \Rightarrow e, c \\
h = \left\lfloor \frac{e[3] + 2 \times padding[0] - dilation[0] \times (kernel_size[0] - 1) - 1}{stride[0]} \right\rfloor + 1 \\
w = \left\lfloor \frac{e[4] + 2 \times padding[1] - dilation[1] \times (kernel_size[1] - 1) - 1}{stride[1]} \right\rfloor + 1 \\
e' = (e[1], out, h, w) \\
c_{dim} = \{(\texttt{rank}(e) = 4) \land (e[2] = in)\} \\
c_h = \{(e[3] + 2 \times padding[0] - dilation[0] \times (kernel_size[0] - 1) - 1 \geq 0)\} \\
c_w = \{(e[4] + 2 \times padding[1] - dilation[1] \times (kernel_size[1] - 1) - 1 \geq 0)\} \\
c_{group} = \{(in \% groups = 0) \land (out \% groups = 0)\}
\end{array}}{\sigma \vdash \texttt{Conv2d}(in, out, kernel_size, stride = 1, padding = 0, dilation = 1, groups = 1)(E) \Rightarrow e', c \cup c_{dim} \cup c_h \cup c_w \cup c_{group}}$$

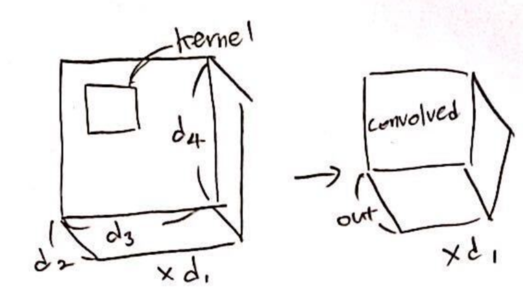
$kernel_size, stride, padding, dilation$는 가로-세로별 2-tuple로도 들어갈 수 있음

이 경우를 위해 $stride[0], stride[1]$으로 표기함

만일 $stride$가 튜플이 아닌 스칼라라면 $stride[0]$ 또는 $[1]$은 $stride$ 값 자체를 의미

(Builtins) `torch.conv2d`, `torch.nn.functional.conv2d`

```
torch.conv2d(input, weight, bias=None, stride=1, padding=0, dilation=1, groups=1)
```



Require

- $|input| = (batch, in, h_{in}, w_{in}) \quad (rank = 4)$
- $|weight| = (out, in\_group, h_{weight}, w_{weight}) \quad (rank = 4)$
- $bias$ is $None$ or $|bias| = (out)$
- $h_{weight} + 2 \times padding[0] - dilation[0] \times (h_{weight} - 1) - 1 \geq 0$
- $w_{weight} + 2 \times padding[1] - dilation[1] \times (w_{weight} - 1) - 1 \geq 0$
- $groups|in$, $groups|out$ and $in\_group \times group = in$

Guarantees

- $|y| = (batch, out, h, w)$ where.. refers to the proof tree.

Comment

- 컨볼루션 계산을 위해 사용하는 빌트인 함수입니다.
- `torch.conv2d`, `torch.nn.functional.conv2d` 모두 같은 방식으로 작동됩니다.

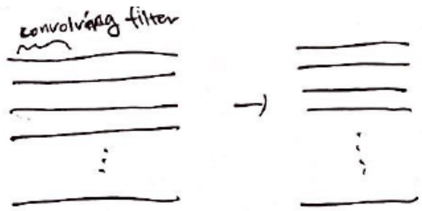
$$
\frac{
\begin{array}{l}
\sigma \vdash E \Rightarrow e, c \\
\sigma \vdash F \Rightarrow f, c \\
\sigma \vdash B \Rightarrow b, c \quad \text{if } B \text{ is not } None \\
(batch, in, h_{in}, w_{in}) = e \\
(out, in\_group, h_{filter}, w_{filter}) = f \\
h = \left\lfloor \frac{h_{in} + 2 \times padding[0] - dilation[0] \times (h_{filter} - 1) - 1}{stride[0]} \right\rfloor + 1 \\
w = \left\lfloor \frac{w_{in} + 2 \times padding[1] - dilation[1] \times (w_{filter} - 1) - 1}{stride[1]} \right\rfloor + 1 \\
e' = (batch, out, h, w) \\
c_{dim} = \{(\texttt{rank}(e) = 4) \wedge (\texttt{rank}(f) = 4)\} \\
c_{bias} = \{((B = None) \vee (\texttt{rank}(b) = 1 \wedge b[1] = out))\} \\
c_h = \{(h_{in} + 2 \times padding[0] - dilation[0] \times (h_{filter} - 1) - 1 \geq 0)\} \\
c_w = \{(w_{in} + 2 \times padding[1] - dilation[1] \times (w_{filter} - 1) - 1 \geq 0)\} \\
c_{group} = \{(in\% groups = 0) \wedge (out\% groups = 0) \wedge (in\_group \times groups = in)\}
\end{array}
}{
\sigma \vdash \texttt{conv2d}(E, F, B = None, stride = 1, padding = 0, dilation = 1, groups = 1) \Rightarrow e', c \cup c_{dim} \cup c_{bias} \cup c_h \cup c_w \cup c_{group}
}
$$

$kernel_size, stride, padding, dilation$는 가로-세로별 2-tuple로도 들어갈 수 있음

이 경우를 위해 $stride[0], stride[1]$으로 표기함

만일 $stride$가 튜플이 아닌 스칼라라면 $stride[0]$ 또는 $[1]$은 $stride$ 값 자체를 의미

`torch.nn.Conv1d`

<table>
<tr><td colspan="2"><code>torch.nn.Conv1d(in_channels, out_channels, kernel_size, stride=1, padding=0, dilation=1, groups=1, other_params..)(x)</code></td></tr>
<tr><td rowspan="6">
</td><td>Require</td></tr>
<tr><td>

- $|x| = (d_1, d_2, d_3) \quad (rank = 3)$
- $d_2 = in\_channels$
- $d_3 + 2 \times padding - dilation \times (kernel\_size - 1) - 1 \geq 0$
- $groups | in\_channels$ and $groups | out\_channels$

</td></tr>
<tr><td>Guarantees</td></tr>
<tr><td>

- $|y| = (d_1, out\_channels, w)$ where.. refers to the proof tree.

</td></tr>
<tr><td>Comment</td></tr>
<tr><td>

- Convolution 1차원 레이어입니다.
- $kernel\_size$, $stride$ 등은 1차원 튜플로 구성될 수도 있습니다.

</td></tr>
</table>

$$
\frac{\begin{array}{l}
\sigma \vdash E \Rightarrow e, c \\
w = \left\lfloor \frac{e[3] + 2 \times padding - dilation \times (kernel\_size - 1) - 1}{stride} \right\rfloor + 1 \\
e' = (e[1], out, w) \\
c_{dim} = \{(\mathtt{rank}(e) = 3) \land (e[2] = in)\} \\
c_w = \{(e[3] + 2 \times padding - dilation \times (kernel\_size - 1) - 1 \geq 0)\} \\
c_{group} = \{(in \% groups = 0) \land (out \% groups = 0)\}
\end{array}}{\sigma \vdash \mathtt{Conv1d}(in, out, kernel\_size, stride = 1, padding = 0, dilation = 1, groups = 1)(E) \Rightarrow e', c \cup c_{dim} \cup c_w \cup c_{group}}
$$

$kernel_size, stride, padding, dilation$는 1-length-tuple로 들어올 수 있음

(Builtins) `torch.conv1d`, `torch.nn.functional.conv1d`

$$
\frac{\begin{array}{l}
\sigma \vdash E \Rightarrow e, c \\
\sigma \vdash F \Rightarrow f, c \\
\sigma \vdash B \Rightarrow b, c \quad \text{if } B \text{ is not } None \\
(batch, in, w_{in}) = e \\
(out, in\_group, w_{filter}) = f \\
w = \left\lfloor \frac{w_{in} + 2 \times padding - dilation \times (w_{filter} - 1) - 1}{stride} \right\rfloor + 1 \\
e' = (batch, out, w) \\
c_{dim} = \{(\mathtt{rank}(e) = 3) \land (\mathtt{rank}(f) = 3)\} \\
c_{bias} = \{((B = None) \lor (\mathtt{rank}(b) = 1 \land b[1] = out))\} \\
c_w = \{(w_{in} + 2 \times padding - dilation \times (w_{filter} - 1) - 1 \geq 0)\} \\
c_{group} = \{(in \% groups = 0) \land (out \% groups = 0) \land (in\_group \times groups = in)\}
\end{array}}{\sigma \vdash \mathtt{conv1d}(E, F, B = None, stride = 1, padding = 0, dilation = 1, groups = 1) \Rightarrow e', c \cup c_{dim} \cup c_{bias} \cup c_w \cup c_{group}}
$$

$kernel_size, stride, padding, dilation$는 1-length-tuple로 들어올 수 있음

`torch.nn.Conv3d`

$$
\frac{
\begin{array}{l}
\sigma \vdash E \Rightarrow e, c \\
z = \left\lfloor \frac{e[3] + 2 \times padding[0] - dilation[0] \times (kernel\_size[0] - 1) - 1}{stride[0]} \right\rfloor + 1 \\
h = \left\lfloor \frac{e[4] + 2 \times padding[1] - dilation[1] \times (kernel\_size[1] - 1) - 1}{stride[1]} \right\rfloor + 1 \\
w = \left\lfloor \frac{e[5] + 2 \times padding[2] - dilation[2] \times (kernel\_size[2] - 1) - 1}{stride[2]} \right\rfloor + 1 \\
e' = (e[1], out, z, h, w) \\
c_{dim} = \{(\texttt{rank}(e) = 5) \wedge (e[2] = in)\} \\
c_z = \{(e[3] + 2 \times padding[0] - dilation[0] \times (kernel\_size[0] - 1) - 1 \geq 0)\} \\
c_h = \{(e[4] + 2 \times padding[1] - dilation[1] \times (kernel\_size[1] - 1) - 1 \geq 0)\} \\
c_w = \{(e[5] + 2 \times padding[2] - dilation[2] \times (kernel\_size[2] - 1) - 1 \geq 0)\} \\
c_{group} = \{(in \% groups = 0) \wedge (out \% groups = 0)\}
\end{array}
}{
\sigma \vdash \texttt{Conv3d}(in, out, kernel\_size, stride = 1, padding = 0, dilation = 1, groups = 1)(E) \Rightarrow e', c \cup c_{dim} \cup c_z \cup c_h \cup c_w \cup c_{group}
}
$$

$kernel_size, stride, padding, dilation$는 깊이-가로-세로별 3-tuple로도 들어갈 수 있음

이 경우를 위해 $stride[0], [1], [2]$으로 표기함

만일 $stride$가 튜플이 아닌 스칼라라면 $stride[0], [1]$ 또는 $[2]$는 $stride$ 값 자체를 의미

(Builtins) `torch.conv3d`, `torch.nn.functional.conv3d`

$$
\frac{
\begin{array}{l}
\sigma \vdash E \Rightarrow e, c \\
\sigma \vdash F \Rightarrow f, c \\
\sigma \vdash B \Rightarrow b, c \quad \text{if } B \text{ is not } None \\
(batch, in, z_{in}, h_{in}, w_{in}) = e \\
(out, in\_group, z_{filter}, h_{filter}, w_{filter}) = f \\
z = \left\lfloor \frac{z_{in} + 2 \times padding[0] - dilation[0] \times (z_{filter} - 1) - 1}{stride[0]} \right\rfloor + 1 \\
h = \left\lfloor \frac{h_{in} + 2 \times padding[1] - dilation[1] \times (h_{filter} - 1) - 1}{stride[1]} \right\rfloor + 1 \\
w = \left\lfloor \frac{w_{in} + 2 \times padding[2] - dilation[2] \times (w_{filter} - 1) - 1}{stride[2]} \right\rfloor + 1 \\
e' = (batch, out, z, h, w) \\
c_{dim} = \{(\texttt{rank}(e) = 5) \wedge (\texttt{rank}(f) = 5)\} \\
c_{bias} = \{((B = None) \vee (\texttt{rank}(b) = 1 \wedge b[1] = out))\} \\
c_z = \{(z_{in} + 2 \times padding[0] - dilation[0] \times (z_{filter} - 1) - 1 \geq 0)\} \\
c_h = \{(h_{in} + 2 \times padding[1] - dilation[1] \times (h_{filter} - 1) - 1 \geq 0)\} \\
c_w = \{(w_{in} + 2 \times padding[2] - dilation[2] \times (w_{filter} - 1) - 1 \geq 0)\} \\
c_{group} = \{(in \% groups = 0) \wedge (out \% groups = 0) \wedge (in\_group \times groups = in)\}
\end{array}
}{
\sigma \vdash \texttt{conv2d}(E, F, B = None, stride = 1, padding = 0, dilation = 1, groups = 1) \Rightarrow e', c \cup c_{dim} \cup c_{bias} \cup c_z \cup c_h \cup c_w \cup c_{group}
}
$$

$kernel_size, stride, padding, dilation$는 가로-세로별 3-tuple로도 들어갈 수 있음

이 경우를 위해 $stride[0], [1], [2]$으로 표기함

만일 $stride$가 튜플이 아닌 스칼라라면 $stride[0], [1]$ 또는 $[2]$는 $stride$ 값 자체를 의미

`torch.nn.ConvTranspose1d`

<table>
<tr><td colspan="2"><code>torch.nn.ConvTranspose1d(in_channels, out_channels, kernel_size, stride=1, padding=0, output_padding=0, groups=1, bias=True, dilation=1, padding_mode='zeros')(x)</code></td></tr>
<tr><td rowspan="6">target → bigger convolved
filter
(backpropagation에서 target은 out. filter는 flipped filter, output은 input gradient에 해당)</td><td>Require</td></tr>
<tr><td>• $|x| = (d_1, d_2, d_3) \quad (rank = 3)$
• $d_2 = in\_channels$
• $kernel\_size \leq d_3 + 2 \times padding$
• $groups|in\_channels$ and $groups|out\_channels$</td></tr>
<tr><td>Guarantees</td></tr>
<tr><td>• $|y| = (d_1, out\_channels, w)$ where.. refers to the proof tree.</td></tr>
<tr><td>Comment</td></tr>
<tr><td>• Convolution에서 gradient를 구하기위한 레이어로 보시면 됩니다.
• $kernel\_size$, $stride$ 등은 1차원 튜플로 구성될 수도 있습니다.
• <code>bias</code>, <code>pad_mode</code> 옵션은 shape에 영향을 주지 않습니다.</td></tr>
</table>

$$\frac{\begin{array}{l} \sigma \vdash E \Rightarrow e, c \\ w = (e[3] - 1) \times stride - 2 \times pad + dilation \times (kernel - 1) + out\_pad + 1 \\ e' = (e[1], out, w) \\ c_{dim} = \{(\texttt{rank}(e) = 3) \wedge (e[2] = in) \wedge (w > 0)\} \\ c_{group} = \{(in\%groups = 0) \wedge (out\%groups = 0)\} \end{array}}{\begin{array}{l} \sigma \vdash \texttt{ConvTranspose1d}(in, out, kernel, stride = 1, pad = 0, out\_pad = 0, groups = 1, bias = True, dilation = 1, pad\_mode)(E) \\ \quad \Rightarrow e', c \cup c_{dim} \cup c_{group} \end{array}}$$

$kernel_size, stride, padding, dilation$는 1-length-tuple로 들어올 수 있음

(Builtins) `torch.conv_transpose1d`, `torch.nn.functional.conv_transpose1d`

$$\frac{\begin{array}{l} \sigma \vdash E \Rightarrow e, c_e \\ \sigma \vdash F \Rightarrow f, c_f \\ \sigma \vdash B \Rightarrow b, c_b \quad \text{if } B \text{ is not } None \\ w = (e[3] - 1) \times stride - 2 \times pad + dilation \times (f[3] - 1) + out\_pad + 1 \\ e' = (e[1], f[2] \times groups, w) \\ c_{dim} = \{(\texttt{rank}(e) = 3) \wedge (\texttt{rank}(f) = 3) \wedge (f[1] = e[2]) \wedge (w > 0)\} \\ c_{bias} = \{(B = None \vee b = (f[2] \times groups))\} \\ c_{group} = \{(in\%groups = 0)\} \end{array}}{\begin{array}{l} \sigma \vdash \texttt{conv\_transpose1d}(E, F, B = None, stride = 1, pad = 0, out\_pad = 0, groups = 1, dilation = 1) \\ \quad \Rightarrow e', c \cup c_{dim} \cup c_{bias} \cup c_{group} \end{array}}$$

$kernel_size, stride, padding, dilation$는 1-length-tuple로 들어올 수 있음

Folding

torch.nn.functional.unfold

`torch.nn.functional.unfold(input, kernel_size, dilation=1, padding=0, stride=1)`	
	Require
	• $\lvert input \rvert = (N, C, d_0, d_1) \quad (rank = 4)$ • $d_0 + 2 \times padding[0] - dilation[0] \times (kernel\_size[0] - 1) - 1 \geq 0$ • $d_1 + 2 \times padding[1] - dilation[1] \times (kernel\_size[1] - 1) - 1 \geq 0$
	Guarantees
	• $\lvert y \rvert = (N, C \times kernel\_size[0] \times kernel\_size[1], h \times w)$ where.. refers to the proof tree.
	Comment
	• $kernel\_size$, $stride$와 같은 옵션은 튜플로 구성될 수도 있습니다. (가로 세로에 대한 필터크기가 서로 다르도록) 이 경우를 위하여 proof 트리에서 $kernel_size[0], [1]$과 같은 표기를 사용하였습니다. 튜플이 아니라 스칼라 입력인 경우, $kernel_size[0], [1]$은 모두 $kernel_size$와 같습니다. • `torch.unfold`는 없고 `torch.nn.functional.unfold`로 쓸 수 있습니다.

$$
\frac{
\begin{array}{l}
\sigma \vdash E \Rightarrow e, c \\
h = \left\lfloor \frac{e[3] + 2 \times padding[0] - dilation[0] \times (kernel\_size[0] - 1) - 1}{stride[0]} \right\rfloor + 1 \\
w = \left\lfloor \frac{e[4] + 2 \times padding[1] - dilation[1] \times (kernel\_size[1] - 1) - 1}{stride[1]} \right\rfloor + 1 \\
e' = (e[1], e[2] \times kernel\_size[0] \times kernel\_size[1], h \cdot w) \\
c_{dim} = \{(\texttt{rank}(e) = 4)\} \\
c_h = \{(e[3] + 2 \times padding[0] - dilation[0] \times (kernel\_size[0] - 1) - 1 \geq 0)\} \\
c_w = \{(e[4] + 2 \times padding[1] - dilation[1] \times (kernel\_size[1] - 1) - 1 \geq 0)\}
\end{array}
}{
\sigma \vdash \texttt{unfold}(E, kernel\_size, dilation = 1, padding = 0, stride = 1) \Rightarrow e', c \cup c_{dim} \cup c_h \cup c_w
}
$$

$kernel_size, stride, padding, dilation$는 가로-세로별 2-tuple로도 들어갈 수 있음

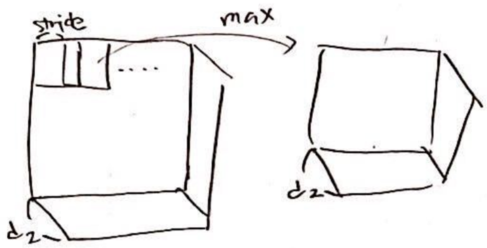
이 경우를 위해 $stride[0], stride[1]$으로 표기함

만일 $stride$가 튜플이 아닌 스칼라라면 $stride[0]$ 또는 $[1]$은 $stride$ 값 자체를 의미

Activations

`torch.nn.MaxPool2d`

`torch.nn.MaxPool2d(kernel_size, stride=kernel_size, padding=0, dilation=1)(x)`



Require

- $|x| = (d_1, d_2, d_3, d_4)$ or (d_2, d_3, d_4)
- $d_3 + 2 \times padding[0] - dilation[0] \times (kernel\_size[0] - 1) - 1 \geq 0$
- $d_4 + 2 \times padding[1] - dilation[1] \times (kernel\_size[1] - 1) - 1 \geq 0$

Guarantees

- (d_1, d_2, h, w) or (d_2, h, w) where.. proof tree.

Comment

- Convolution 다음 activation으로 자주 쓰이는 MaxPool 레이어 입니다.

$$\frac{\begin{array}{l} \sigma \vdash E \Rightarrow e, c \\ k = \texttt{rank}(e) \\ h_{orig} = e[k-1] \\ w_{orig} = e[k] \\ h = \left\lfloor \frac{h_{orig} + 2 \times padding[0] - dilation[0] \times (kernel\_size[0]-1) - 1}{stride[0]} \right\rfloor + 1 \\ w = \left\lfloor \frac{w_{orig} + 2 \times padding[1] - dilation[1] \times (kernel\_size[1]-1) - 1}{stride[1]} \right\rfloor + 1 \\ e' = e[1{:}k-2]@(h, w) \\ c_{dim} = \{(k = 3 \vee k = 4)\} \\ c_h = \{(h_{orig} + 2 \times padding[0] - dilation[0] \times (kernel\_size[0] - 1) - 1 \geq 0)\} \\ c_w = \{(w_{orig} + 2 \times padding[1] - dilation[1] \times (kernel\_size[1] - 1) - 1 \geq 0)\} \end{array}}{\sigma \vdash \texttt{MaxPool2d}(kernel\_size, stride = kernel\_size, padding = 0, dilation = 1)(E) \Rightarrow e', c \cup c_{dim} \cup c_w \cup c_h}$$

$kernel_size, stride, padding, dilation$는 가로-세로별 2-tuple로도 들어갈 수 있음

이 경우를 위해 $stride[0], stride[1]$으로 표기함

만일 $stride$가 튜플이 아닌 스칼라라면 $stride[0]$ 또는 $[1]$은 $stride$ 값 자체를 의미

`torch.nn.MaxPool2d(kernel_size, stride=..., dilation=1, return_indices=False, ceil_mode=False)(x)`

return_indices가 True이면,
어떤 인덱스로부터 왔는지
튜플로 반환 (같은 shape 두 개)

Require

- $|x| = (d_1, d_2, d_3, d_4)$ or (d_2, d_3, d_4)
- $d_3 + 2 \times padding[0] - dilation[0] \times (kernel\_size[0] - 1) - 1 \geq 0$
- $d_4 + 2 \times padding[1] - dilation[1] \times (kernel\_size[1] - 1) - 1 \geq 0$

Guarantees

- (d_1, d_2, h, w) or (d_2, h, w) where.. proof tree.
- $return_indices$가 $True$이면 인덱스 번호까지 튜플로 반환
- $ceil_mode$가 $True$이면 $floor$대신 $ceil$로 shape 계산

$$
\frac{
\begin{array}{l}
\sigma \vdash E \Rightarrow e, c \\
k = \texttt{rank}(e) \\
h_{orig} = e[k-1] \\
w_{orig} = e[k] \\
h = \left\lfloor \frac{h_{orig}+2\times padding[0]-dilation[0]\times(kernel\_size[0]-1)-1}{stride[0]} \right\rfloor + 1 \\
w = \left\lfloor \frac{w_{orig}+2\times padding[1]-dilation[1]\times(kernel\_size[1]-1)-1}{stride[1]} \right\rfloor + 1 \\
h_{ceil} = \left\lceil \frac{h_{orig}+2\times padding[0]-dilation[0]\times(kernel\_size[0]-1)-1}{stride[0]} \right\rceil + 1 \\
w_{ceil} = \left\lceil \frac{w_{orig}+2\times padding[1]-dilation[1]\times(kernel\_size[1]-1)-1}{stride[1]} \right\rceil + 1 \\
e' = \texttt{if } ceil\_mode \texttt{ then } e[1{:}k-2]@(h_{ceil}, w_{ceil}) \texttt{ else } e[1{:}k-2]@(h, w) \\
e_{out} = \texttt{if } return\_indices \texttt{ then } (e', e') \texttt{ else } e' \\
c_{dim} = \{(k = 3 \vee k = 4)\} \\
c_h = \{(h_{orig} + 2 \times padding[0] - dilation[0] \times (kernel\_size[0] - 1) - 1 \geq 0)\} \\
c_w = \{(w_{orig} + 2 \times padding[1] - dilation[1] \times (kernel\_size[1] - 1) - 1 \geq 0)\}
\end{array}
}{
\begin{array}{ll}
\sigma \vdash & \texttt{MaxPool2d}(kernel\_size, stride, padding, dilation, return\_indices, ceil\_mode)(E) \\
& \Rightarrow e', c \cup c_{dim} \cup c_w \cup c_h
\end{array}
}
$$

$return_indices$가 $True$이면 (결과, 인덱스) 튜플 형태로 반환

$ceil_mode$가 $True$이면 $floor$대신 $ceil$함수로 계산

$$
\frac{\sigma \vdash \texttt{torch.nn.MaxPool2d}(E, other\_params...) \Rightarrow e, c}{\sigma \vdash \texttt{max\_pool2d}(E, other\_params...) \Rightarrow e, c}
$$

(Builtins) `torch.max_pool2d`나 `torch.nn.functional.max_pool2d`에 대한 적용

(Builtins) `torch.max_pool2d`, `torch.nn.functional.max_pool2d`

<table>
<tr><td colspan="2"><code>torch.max_pool2d(input, kernel_size, stride=kernel_size, padding=0, dilation=1, ceil_mode=False)</code></td></tr>
<tr><td rowspan="6"></td><td>Require</td></tr>
<tr><td>

- $|input| = (d_1, d_2, d_3, d_4)$ or (d_2, d_3, d_4)
- $d_3 + 2 \times padding[0] - dilation[0] \times (kernel\_size[0] - 1) - 1 \geq 0$
- $d_4 + 2 \times padding[1] - dilation[1] \times (kernel\_size[1] - 1) - 1 \geq 0$

</td></tr>
<tr><td>Guarantees</td></tr>
<tr><td>

- (d_1, d_2, h, w) or (d_2, h, w) where.. proof tree.

</td></tr>
<tr><td>Comment</td></tr>
<tr><td>

- `torch.max_pool2d` ≡ `torch.nn.functional.max_pool2d`
- Builtin 함수인데, 특이한 점은 `return_indices` parameter가 없다는 것입니다. (`max_pool2d_with_indices`라는 다른 함수로 분리되어있습니다.)

</td></tr>
</table>

$$\dfrac{\begin{array}{ll} \sigma \vdash E \Rightarrow e, c & \\ k = \texttt{rank}(e) & \\ h_{orig} = e[k-1] & \\ w_{orig} = e[k] & \\ h = \left\lfloor \frac{h_{orig} + 2 \times padding[0] - dilation[0] \times (kernel\_size[0] - 1) - 1}{stride[0]} \right\rfloor + 1 & \text{(if } ceil\_mode \text{ is } True\text{, then use } \lceil \cdot \rceil) \\ w = \left\lfloor \frac{w_{orig} + 2 \times padding[1] - dilation[1] \times (kernel\_size[1] - 1) - 1}{stride[1]} \right\rfloor + 1 & \text{(if } ceil\_mode \text{ is } True\text{, then use } \lceil \cdot \rceil) \\ e' = e[1{:}k-2]@(h, w) & \\ c_{dim} = \{(k = 3 \vee k = 4)\} & \\ c_h = \{(h_{orig} + 2 \times padding[0] - dilation[0] \times (kernel\_size[0] - 1) - 1 \geq 0)\} & \\ c_w = \{(w_{orig} + 2 \times padding[1] - dilation[1] \times (kernel\_size[1] - 1) - 1 \geq 0)\} & \end{array}}{\sigma \vdash \texttt{max\_pool2d}(E, kernel\_size, stride = kernel\_size, padding = 0, dilation = 1) \Rightarrow e', c \cup c_{dim} \cup c_w \cup c_h}$$

$kernel_size, stride, padding, dilation$는 가로-세로별 2-tuple로도 들어갈 수 있음

이 경우를 위해 $stride[0], stride[1]$으로 표기함

만일 $stride$가 튜플이 아닌 스칼라라면 $stride[0]$ 또는 $[1]$은 $stride$ 값 자체를 의미

`torch.nn.functional.max_pool2d_with_indices(input, kernel_size, stride=..., dilation=1, ceil_mode=False)`	
	Require
	• $\lvert x \rvert = (d_1, d_2, d_3, d_4)$ or (d_2, d_3, d_4) • $kernel\_size[0] \leq d_3 + 2 \times padding[0]$ • $kernel\_size[1] \leq d_4 + 2 \times padding[1]$
	Guarantees
	• 2-tuple of (d_1, d_2, h, w) or (d_2, h, w) where.. proof tree.
	Comment
	• `torch.`에는 없고, `torch.nn.functional`에만 있습니다.

$$\dfrac{\begin{array}{ll} \sigma \vdash E \Rightarrow e, c & \\ k = \texttt{rank}(e) & \\ h_{orig} = e[k-1] & \\ w_{orig} = e[k] & \\ h = \left\lfloor \frac{h_{orig} + 2 \times padding[0] - dilation[0] \times (kernel\_size[0] - 1) - 1}{stride[0]} \right\rfloor + 1 & \text{(if } ceil\_mode \text{ is } True\text{, then use } \lceil \cdot \rceil) \\ w = \left\lfloor \frac{w_{orig} + 2 \times padding[1] - dilation[1] \times (kernel\_size[1] - 1) - 1}{stride[1]} \right\rfloor + 1 & \text{(if } ceil\_mode \text{ is } True\text{, then use } \lceil \cdot \rceil) \\ e' = e[1{:}k-2]@(h, w) & \\ c_{dim} = \{(k = 3 \vee k = 4)\} & \\ c_h = \{(h_{orig} + 2 \times padding[0] - dilation[0] \times (kernel\_size[0] - 1) - 1 \geq 0)\} & \\ c_w = \{(w_{orig} + 2 \times padding[1] - dilation[1] \times (kernel\_size[1] - 1) - 1 \geq 0)\} & \end{array}}{\begin{array}{ll} \sigma \vdash & \texttt{max\_pool2d\_with\_indices}(E, kernel\_size, stride, padding, dilation, ceil\_mode) \\ & \Rightarrow (e', e'), c \cup c_{dim} \cup c_w \cup c_h \end{array}}$$

`torch.nn.AvgPool2d`

<table>
<tr><td colspan="2"><code>torch.nn.AvgPool2d(kernel_size, stride=kernel_size, padding=0, other_params, ...)(x)</code></td></tr>
<tr><td rowspan="6"></td><td>Require</td></tr>
<tr><td>

- $|x| = (d_1, d_2, d_3, d_4)$ or (d_2, d_3, d_4)
- $kernel\_size[0] \leq d_3 + 2 \times padding[0]$
- $kernel\_size[1] \leq d_4 + 2 \times padding[1]$

</td></tr>
<tr><td>Guarantees</td></tr>
<tr><td>

- (d_1, d_2, h, w) or (d_2, h, w) where.. proof tree.

</td></tr>
<tr><td>Comment</td></tr>
<tr><td>

- 셀들의 최대값이 아닌 평균으로 정규화하는 레이어
- `MaxPool2d`와 비슷하나 $dilation$과 $return\_indices$ 옵션이 없음

</td></tr>
</table>

$$
\frac{\begin{array}{ll}
\sigma \vdash E \Rightarrow e, c & \\
k = \texttt{rank}(e) & \\
h_{orig} = e[k-1] & \\
w_{orig} = e[k] & \\
h = \left\lfloor \frac{h_{orig} + 2 \times padding[0] - kernel\_size[0]}{stride[0]} \right\rfloor + 1 & \text{(if } ceil\_mode \text{ is } True\text{, then use } \lceil \cdot \rceil\text{)} \\
w = \left\lfloor \frac{w_{orig} + 2 \times padding[1] - kernel\_size[1]}{stride[1]} \right\rfloor + 1 & \text{(if } ceil\_mode \text{ is } True\text{, then use } \lceil \cdot \rceil\text{)} \\
e' = e[1{:}k-2]@(h, w) & \\
c_{dim} = \{(k = 3 \vee k = 4)\} & \\
c_w = \{(kernel\_size[0] \leq h_{orig} + 2 \times padding[0])\} & \\
c_h = \{(kernel\_size[1] \leq w_{orig} + 2 \times padding[1])\} &
\end{array}}{\sigma \vdash \texttt{AvgPool2d}(kernel\_size, stride = kernel\_size, padding = 0, other\_params, ...)(E) \Rightarrow e', c \cup c_{dim} \cup c_w \cup c_h}
$$

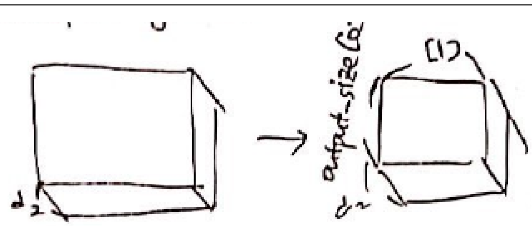
$kernel_size, stride, padding$는 가로-세로별 2-tuple로도 들어갈 수 있음

이 경우를 위해 $stride[0], stride[1]$으로 표기함

만일 $stride$가 튜플이 아닌 스칼라라면 $stride[0]$ 또는 $[1]$은 $stride$ 값 자체를 의미

`torch.nn.AdaptiveAvgPool2d`

<table>
<tr><td colspan="2"><code>torch.nn.AdaptiveAvgPool2d(output_size)(x)</code></td></tr>
<tr><td rowspan="6">



</td><td>Require</td></tr>
<tr><td>

- $|x| = (d_1, d_2, d_3, d_4)$ or (d_2, d_3, d_4)
  - $\texttt{rank}(|x|) = 3$ or 4

</td></tr>
<tr><td>Guarantees</td></tr>
<tr><td>

- $(d_1, d_2, output\_size[0], output\_size[1])$ or $(d_2, output\_size[0], output\_size[1])$

</td></tr>
<tr><td>Comment</td></tr>
<tr><td>

- 출력 shape를 강제하는 평균 pool 입니다.
- $output_size$는 2-tuple이 될 수도 있습니다.
- $output\_size > d_3$ or d_4인 상황에도 오류없이 작동합니다.

</td></tr>
</table>

$$\frac{\begin{array}{l}\sigma \vdash E \Rightarrow e, c \\ k = \mathtt{rank}(e) \\ e' = e[1{:}k-2]@(output\_size[0], output\_size[1]) \\ c_{dim} = \{(k = 3 \vee k = 4)\}\end{array}}{\sigma \vdash \mathtt{AdaptiveAvgPool2d}(output\_size)(E) \Rightarrow e', c \cup c_{dim} \cup c_w \cup c_h}$$

$output_size$는 가로-세로별 2-tuple로도 들어갈 수 있음

이 경우를 위해 $output_size[0], output_size[1]$으로 표기함

만일 $output_size$가 튜플이 아닌 스칼라라면 $output\_size[0]$ 또는 $[1]$은 $output\_size$ 값 자체를 의미

torch.nn.AdaptiveAvgPool3d

<table>
<tr><td colspan="2"><code>torch.nn.AdaptiveAvgPool3d(output_size)(x)</code></td></tr>
<tr><td rowspan="6"></td><td>Require</td></tr>
<tr><td>$|x| = (d_1, d_2, d_3, d_4, d_5)$ or (d_2, d_3, d_4, d_5) $\mathtt{rank}(|x|) = 4$ or 5</td></tr>
<tr><td>Guarantees</td></tr>
<tr><td>$(d_1, d_2, d_3, output\_size[0], output\_size[1])$
or $(d_2, d_3, output\_size[0], output\_size[1])$</td></tr>
<tr><td>Comment</td></tr>
<tr><td>출력 shape를 강제하는 평균 pool 입니다.
$output_size$는 3-tuple이 될 수도 있습니다.
$output\_size > d_3, d_4$ or d_5인 상황에도 오류없이 작동합니다.</td></tr>
</table>

$$\frac{\begin{array}{l}\sigma \vdash E \Rightarrow e, c \\ k = \mathtt{rank}(e) \\ e' = e[1{:}k-2]@(output\_size[0], output\_size[1]) \\ c_{dim} = \{(k = 4 \vee k = 5)\}\end{array}}{\sigma \vdash \mathtt{AdaptiveAvgPool3d}(output\_size)(E) \Rightarrow e', c \cup c_{dim} \cup c_w \cup c_h}$$

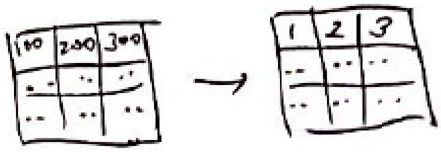
$output_size$는 깊이-가로-세로별 3-tuple로도 들어갈 수 있음

이 경우를 위해 $output_size[0], [1][2]$으로 표기함

만일 $output_size$가 튜플이 아닌 스칼라라면 $output\_size[0], [1]$ 또는 $[2]$은 $output\_size$ 값 자체를 의미

Normalizations

`torch.nn.BatchNorm2d`

<table>
<tr><td colspan="2"><code>torch.nn.BatchNorm2d(num_features, other_params...)(x)</code></td></tr>
<tr><td rowspan="4">
</td><td>Require</td></tr>
<tr><td>

- $|x| = (d_1, d_2, d_3, d_4) \quad (\texttt{rank} = 4)$
- $d_2 = num\_features$

</td></tr>
<tr><td>Guarantees</td></tr>
<tr><td>

- $|y| = (d_1, d_2, d_3, d_4) \quad \text{(same shape to } x\text{)}$

</td></tr>
</table>

$$\frac{\begin{array}{l}\sigma \vdash E \Rightarrow e, c \\ c' = \{(\texttt{rank}(e) = 4) \land (e[2] = num\_features)\}\end{array}}{\sigma \vdash \texttt{BatchNorm2d}(num\_features, other\_params)(E) \Rightarrow e, c \cup c'}$$

`torch.nn.BatchNorm3d`

<table>
<tr><td colspan="2"><code>torch.nn.BatchNorm3d(num_features, other_params...)(x)</code></td></tr>
<tr><td rowspan="4"></td><td>Require</td></tr>
<tr><td>

- $|x| = (d_1, d_2, d_3, d_4, d_5) \quad (\texttt{rank} = 5)$
- $d_2 = num\_features$

</td></tr>
<tr><td>Guarantees</td></tr>
<tr><td>

- $|y| = (d_1, d_2, d_3, d_4, d_5) \quad \text{(same shape to } x\text{)}$

</td></tr>
</table>

$$\frac{\begin{array}{l}\sigma \vdash E \Rightarrow e, c \\ c' = \{(\texttt{rank}(e) = 5) \land (e[2] = num\_features)\}\end{array}}{\sigma \vdash \texttt{BatchNorm3d}(num\_features, other\_params)(E) \Rightarrow e, c \cup c'}$$

`torch.nn.BatchNorm1d`

<table>
<tr><td colspan="2"><code>torch.nn.BatchNorm1d(num_features, other_params...)(x)</code></td></tr>
<tr><td rowspan="4"></td><td>Require</td></tr>
<tr><td>

- $|x| = (d_1, d_2, d_3) \quad (\texttt{rank} = 3)$
- $d_2 = num\_features$

</td></tr>
<tr><td>Guarantees</td></tr>
<tr><td>

- $|y| = (d_1, d_2, d_3) \quad \text{(same shape to } x\text{)}$

</td></tr>
</table>

$$\frac{\begin{array}{l}\sigma \vdash E \Rightarrow e, c \\ c' = \{(\texttt{rank}(e) = 3) \land (e[2] = num\_features)\}\end{array}}{\sigma \vdash \texttt{BatchNorm1d}(num\_features, other\_params)(E) \Rightarrow e, c \cup c'}$$

(Builtins) `torch.batch_norm`, `torch.nn.functional.batch_norm`

<table>
<tr><td colspan="2"><code>torch.batch_norm(input, running_mean, running_var, weight=None, bias=None, training=False, other_params,...)(x)</code></td></tr>
<tr><td rowspan="6"></td><td>Require</td></tr>
<tr><td>

- $|input| = (d_1, d_2, d_3, \ldots, d_k)$
- $k \geq 2$
- If $training$ is $False$ then
  - $|running\_mean| = |running\_var| = (d_2)$

  Otherwise,
  - $running\_mean$ is $None$ or $|running\_mean| = (d_2)$ (also for $running\_var$)
- $weight$ is $None$ or $|weight| = (d_2)$ (also for $bias$)

</td></tr>
<tr><td>Guarantees</td></tr>
<tr><td>

- $|y| = (d_1, d_2, d_3, \ldots, d_k) = |x|$

</td></tr>
<tr><td>Comment</td></tr>
<tr><td>

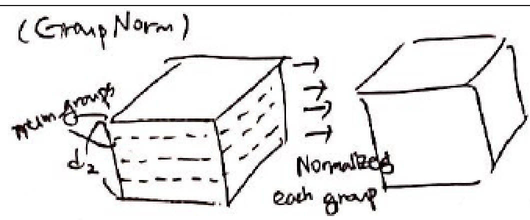
- `BatchNormNd`를 사용하기 위한 일반화된 함수입니다.

</td></tr>
</table>

$$\frac{\begin{array}{rl} \sigma \vdash E \Rightarrow e, c_e & \\ \sigma \vdash M \Rightarrow m, c_m & \text{if } M \text{ is not } None \\ \sigma \vdash V \Rightarrow v, c_v & \text{if } V \text{ is not } None \\ \sigma \vdash W \Rightarrow w, c_w & \text{if } W \text{ is not } None \\ \sigma \vdash B \Rightarrow b, c_b & \text{if } B \text{ is not } None \\ c_{rank} = \{(\texttt{rank}(e) \geq 2)\} & \\ c'_m = \{((training = True \wedge M = None) \vee (m = (d_2))\} & \\ c'_v = \{((training = True \wedge V = None) \vee (v = (d_2))\} & \\ c'_w = \{((W = None) \vee (w = (d_2))\} & \\ c'_b = \{((B = None) \vee (b = (d_2))\} & \end{array}}{\sigma \vdash \texttt{batch\_norm}(E, M, V, W, B, training, other\_params, ...) \Rightarrow e, c_e \cup c_m \cup \cdots \cup c_b \cup c_{rank} \cup c'_m \cup \cdots \cup c'_b}$$

`torch.nn.GroupNorm`

<table>
<tr><td colspan="2"><code>torch.nn.GroupNorm(num_groups, num_channels, other_params, ...)(x)</code></td></tr>
<tr><td rowspan="4">



</td><td>Require</td></tr>
<tr><td>

- $|x| = (d_1, d_2, \ldots, d_k)$
- $\texttt{rank}(|x|) \geq 2$
- $num\_groups | num\_channels$
- $d_2 = num\_channels$

</td></tr>
<tr><td>Guarantees</td></tr>
<tr><td>

- $|y| = |x|$ (same shape)

</td></tr>
</table>

$$\frac{\begin{array}{l} \sigma \vdash E \Rightarrow e, c \\ c' = \{(\texttt{rank}(e) \geq 2) \wedge (e[2] \% num\_groups = 0) \wedge (e[2] = num\_channels)\} \end{array}}{\sigma \vdash \texttt{GroupNorm}(num\_groups, num\_channels, other\_params, ...)(E) \Rightarrow e, c \cup c'}$$