1 Basic equations

Basic equations to start from: MHD with non-adiabatic effects added:

$$\begin{aligned}
\frac{\partial \rho}{\partial t} &= -\nabla \cdot (\rho \boldsymbol{v}) \\
\rho \frac{\partial \boldsymbol{v}}{\partial t} &= -\nabla p - \rho \boldsymbol{v} \cdot \nabla \boldsymbol{v} + (\nabla \times \boldsymbol{B}) \times \boldsymbol{B} + \rho \boldsymbol{g} \\
\rho \frac{\partial T}{\partial t} &= -\rho \boldsymbol{v} \cdot \nabla T - (\gamma - 1) p \nabla \cdot \boldsymbol{v} - (\gamma - 1) \rho \mathscr{L} + (\gamma - 1) \nabla \cdot (\boldsymbol{\kappa} \cdot \nabla T) \\
\frac{\partial B}{\partial t} &= \nabla \times (\boldsymbol{v} \times \boldsymbol{B})
\end{aligned} \tag{1}$$

Linearize them with flow, so $\boldsymbol{v}_0 \neq 0$. Also use a vector potential $\boldsymbol{B}_1 = \nabla \times \boldsymbol{A}_1$ for the perturbed magnetic field and replace p_1 by $T_0 \rho_1 + \rho_0 T_1$ (from the linearized ideal gas law):

$$\begin{aligned}
\frac{\partial \rho_1}{\partial t} &= -\nabla \cdot (\rho_0 \boldsymbol{v}_1) - \nabla \cdot (\rho_1 \boldsymbol{v}_0) \\
\rho_0 \frac{\partial \boldsymbol{v}_1}{\partial t} &= -\rho_0 \boldsymbol{v}_0 \cdot \nabla \boldsymbol{v}_1 - \rho_0 \boldsymbol{v}_1 \cdot \nabla \boldsymbol{v}_0 - \rho_1 \boldsymbol{v}_0 \cdot \nabla \boldsymbol{v}_0 - \nabla (T_0 \rho_1 + \rho_0 T_1) + (\nabla \times \boldsymbol{B}_0) \times (\nabla \times \boldsymbol{A}_1) \\
&\quad + [\nabla \times (\nabla \times \boldsymbol{A}_1)] \times \boldsymbol{B}_0 + \rho_1 \boldsymbol{g} \\
\rho_0 \frac{\partial T_1}{\partial t} &= -\rho_0 \boldsymbol{v}_1 \cdot \nabla T_0 - \rho_0 \boldsymbol{v}_0 \cdot \nabla T_1 - (\gamma - 1) \rho_0 T_0 \nabla \cdot \boldsymbol{v}_1 - (\gamma - 1) \rho_1 \mathscr{L}_0 - (\gamma - 1) \rho_0 (\mathscr{L}_T T_1 + \mathscr{L}_\rho \rho_1) \\
&\quad + (\gamma - 1) \nabla \cdot (\boldsymbol{\kappa}_0 \cdot \nabla T_1) + (\gamma - 1) \nabla \cdot (\boldsymbol{\kappa}_1 \cdot \nabla T_0) \\
\frac{\partial \boldsymbol{A}_1}{\partial t} &= \boldsymbol{v}_1 \times \boldsymbol{B}_0 + \boldsymbol{v}_0 \times (\nabla \times \boldsymbol{A}_1) \\
\frac{p_1}{p_0} &= \frac{T_1}{T_0} + \frac{\rho_1}{\rho_0}
\end{aligned} \tag{2}$$

Denote coordinate system by (u_1, u_2, u_3), which is (x, y, z) in Cartesian and (r, θ, z) in cylindrical coordinates. Equilibrium profiles are only dependent on u_1, taken to be

$$\begin{aligned}
\rho_0 &= \rho_0(u_1) \\
p_0 &= p_0(u_1) \\
T_0 &= T_0(u_1) \\
\boldsymbol{v}_0 &= v_{02}(u_1) \hat{\boldsymbol{e}}_2 + v_{03}(u_1) \hat{\boldsymbol{e}}_3 \\
\boldsymbol{B}_0 &= B_{02}(u_1) \hat{\boldsymbol{e}}_2 + B_{03}(u_1) \hat{\boldsymbol{e}}_3
\end{aligned} \tag{3}$$

so $\nabla \cdot \boldsymbol{v}_0 = 0$ and obviously $\nabla \cdot \boldsymbol{B}_0 = 0$ and $\nabla \cdot \boldsymbol{B}_1 = 0$, take gravity as $\boldsymbol{g} = -g \hat{\boldsymbol{e}}_1$. Vectors $\hat{\boldsymbol{e}}_1$, $\hat{\boldsymbol{e}}_2$ and $\hat{\boldsymbol{e}}_3$ are orthonormal basis vectors representing directions along (u_1, u_2, u_3). Apply Fourier transformation using

$$f_1(u_1, u_2, u_3, t) = \widetilde{f}_1(u_1) \mathrm{e}^{-i\omega t + ik_2 u_2 + ik_3 u_3} \tag{4}$$

1.1 Slab geometry

Below all tildes are dropped for convenience.

$$
\begin{aligned}
\omega\rho_1 &= -iv_1\rho_0' - i\rho_0(v_1' + ik_2v_2 + ik_3v_3) + (v_{02}k_2 + v_{03}k_3)\rho_1 \\
\omega\rho_0 v_1 &= \rho_0(k_2v_{02} + k_3v_{03})v_1 - i(\rho_0T_1 + \rho_1T_0)' + iB_{02}'(a_3' - ik_3a_1) - iB_{03}'(a_2' - ik_2a_1) \\
&\quad + iB_{03}\left[k_3^2a_2 - k_2k_3a_3 - (a_2' - ik_2a_1)'\right] - iB_{02}\left[k_2^2a_3 - k_2k_3a_2 + (ik_3a_1 - a_3')'\right] \\
\omega\rho_0 v_2 &= k_2(T_0\rho_1 + \rho_0T_1) + \rho_0(k_2v_{02} + k_3v_{03})v_2 - i\rho_0v_{02}'v_1 - B_{02}'(k_2a_3 - k_3a_2) \\
&\quad - iB_{03}\left[(k_2^2 + k_3^3)a_1 + i(k_2a_2 + k_3a_3)'\right] \\
\omega\rho_0 v_3 &= k_3(T_0\rho_1 + \rho_0T_1) + \rho_0(k_2v_{02} + k_3v_{03})v_3 - i\rho_0v_{03}'v_1 - B_{03}'(k_2a_3 - k_3a_2) \\
&\quad + iB_{02}\left[(k_2^2 + k_3^2)a_1 + i(k_2a_2 + k_3a_3)'\right] \\
\omega\rho_0 T &= -i\rho_0v_1T_0' + \rho_0(k_2v_{02} - k_3v_{03})T_1 - i(\gamma-1)\rho_0T_0(v_1' + ik_2v_2 + ik_3v_3) - i(\gamma-1)\mathscr{L}_0\rho_1 \\
&\quad - i(\gamma-1)\rho_0\mathscr{L}_TT_1 - i(\gamma-1)\rho_0\mathscr{L}_\rho\rho_1 - i(\gamma-1)(\kappa_\parallel - \kappa_\perp)\frac{1}{B^2}(k_2B_{02} + k_3B_{03})^2T_1 \\
&\quad + i(\gamma-1)(\kappa_\perp T_1')' - i\kappa_\perp(\gamma-1)(k_2^2 + k_3^2)T_1 + i(\gamma-1)(\kappa_{1,\perp}T_0')' \\
&\quad + i(\gamma-1)(\kappa_\parallel - \kappa_\perp)\frac{1}{B^2}\left[(k_2k_3B_{02} + k_3^2B_{03})a_2 - (k_2k_3B_{03} + k_2^2B_{02})a_3\right]T_0' \\
\omega a_1 &= iB_{03}v_2 - iB_{02}v_3 + iv_{02}a_2' + iv_{03}a_3' + (k_2v_{02} + k_3v_{03})a_1 \\
\omega a_2 &= -iB_{03}v_1 - v_{03}(k_2a_3 - k_3a_2) \\
\omega a_3 &= iB_{02}v_1 - v_{02}(k_3a_2 - k_2a_3)
\end{aligned}
\tag{5}
$$

where the prime denotes ∂_{u_1}.

1.2 Cylindrical geometry