Practicum Control Theory 1 – Q1 Week 3-4

Requirements: Matlab R2018a or higher with the Symbolic Math Toolbox.

Last weeks, you got acquainted with the symbolic Math Toolbox to define functions, to draw functions, to compute Laplace transforms and perform partial fraction decompositions. The next two weeks' assignments give you the means to check your skills in computing the inverse Laplace transforms and solving initial value problems. We use the Symbolic Math Toolbox to solve initial value problems directly in the time domain (only by Matlab) and we use what we learned in the past assignments to solve the initial value problem in the Laplace domain first and transform it back to the time domain by the inverse Laplace transform. We will do this by hand first and verify it by Matlab.

Inverse Laplace transform

The Symbolic Math Toolbox has the functionality to perform the inverse Laplace transform of a given function. For example

```
syms s
H(s) = 1/(s-1);
ilaplace(H(s))
```

ans = e^t

provides the inverse Laplace transform of the function $H(s) = \frac{1}{s-1}$. Please study the manual on `ilaplace` to see more examples.

Assignment Inverse Laplace Transform

Compute the inverse Laplace transform by hand or by looking up their corresponding transformation in the table of the following functions

1. $F(s) = \frac{2s^2 - 3s + 1}{(s-3)(s^2+1)}$
2. $G(s) = \frac{-(s+6)}{2(s^2+6s+8)}$
3. $H(s) = \frac{-8s-2}{s^3 - s^2 - 2s}$
4. $I(s) = \frac{s^2+3s+2}{s^3+2s^2+2s}$
5. $J(s) = \frac{3s+5}{s^2+2s+5}$
6. $K(s) = \frac{s^2-5s+1}{s^2-4}$

and verify your answers using the Symbolic Math Toolbox.

Solving initial value problems

The Symbolic Math Toolbox has the functionality to solve an initial value problem directly. As an example, we try to solve the first order system

$$y'(t) - y(t) = 0, y(0) = 2$$

with the following commands:

```
syms y(t)
y(t) = dsolve(diff(y,t) - y == 0, y(0) == 2)
```

y(t) = $2\,e^t$

which gives the solution $y(t) = e^{2t}$.

In a similar way, we can solve the second order system:

$$y''(t) + y(t) = \cos(2t), y(0) = 1, y'(0) = 0$$

with the following commands:

```
syms y(t)
Dy = diff(y);
y(t) = dsolve(diff(y, t, 2) + y == cos(2*t), y(0) == 1, Dy(0) == 0);
y(t) = simplify(y)
```

y(t) =

$$1 - \frac{8\sin\left(\frac{t}{2}\right)^4}{3}$$

which gives the solution $y(t) = 1 - \frac{8}{3}\left(\sin\left(\frac{t}{2}\right)\right)^4$.

Alternatively, with the Symbolic Math Toolbox we can also monitor the steps of solving the initial value problem using Laplace transforms.

```
syms s t Y y(t)
de = diff(y(t),t,2)+y(t)-cos(2*t);
DE = laplace(de)
```

DE =

$$s^2\,\mathrm{laplace}(y(t), t, s) - \frac{s}{s^2+4} - s\,y(0) - \left(\left(\frac{\partial}{\partial t}\,y(t)\right)\Big|_{t=0}\right) + \mathrm{laplace}(y(t), t, s)$$

```
DE = subs(DE,{y(0),subs(diff(y(t), t), t, 0)},{1,0})
```

DE =

$$s^2\,\mathrm{laplace}(y(t), t, s) - \frac{s}{s^2+4} - s + \mathrm{laplace}(y(t), t, s)$$

```
DE = subs(DE,laplace(y(t),t,s),Y)
```

```
DE =
```

$$Y - s - \frac{s}{s^2+4} + Y\,s^2$$

```
Y = solve(DE,Y);
y(t) = ilaplace(Y,s,t);
simplify(y(t))
```

```
ans =
```

$$-\frac{2\cos(t)^2}{3} + \frac{4\cos(t)}{3} + \frac{1}{3}$$

This script will return each individual step of the process of solving the initial value problem. Hence, you can verify each step separately. The solution is $y(t) = \frac{4}{3}\cos(t) - \frac{2}{3}\cos^2(t) + \frac{1}{3}$. Although the answers of solving the initial value problem in the time domain and Laplace domain do not seem to correspond, they should and they also do correspond (how?).

Please study the help documentation on the commands `diff`, `dsolve`, `subs` and `solve` to find out about the details of each one of them.

Assignment Solving Initial Value Problems

Solve by hand using Laplace transforms the following initial value problems:

- $x'(t) - 2x(t) = 1 - 2t^2, x(0) = 0$
- $x'(t) + x(t) = 1 + t + 2\cos(t), x(0) = 1$
- $4x''(t) + x(t) = 3, x(0) = 3, x'(0) = -0.5$
- $x''(t) + x'(t) - 2x(t) = \cos(t) - 3\sin(t), x(0) = 2, x'(0) = -3$
- $x''(t) - 2x'(t) = 2e^{2t}, x(0) = 0, x'(0) = 1$
- $x''(t) + 2x'(t) + 5x(t) = 0, x(0) = 1, x'(0) = -1$

and verify your answers using the Symbolic Math Toolbox in two ways:

1. *By performing the Laplace transform of the differential equation, fill in the initial values, solve the algebraic equation in the Laplace domain and finally perform the inverse Laplace transform to get the solution of the initial value problem in the time domain.*
2. *By solving the initial value problem directly in the time domain.*