

Model of computation assignment 2
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1.

a. $S \rightarrow a \mid b \mid aAa \mid bBb$

$$F \rightarrow a \mid b$$

$$A \rightarrow FAF \mid a$$

$$B \rightarrow FBF \mid b$$

b. $S \rightarrow aSa \mid Ab \mid bA$

$$A \rightarrow aA \mid a$$

2. b) part 1. prove $L(G) \subseteq C$, $C: \{a, b\}^*$ a string that $\text{no. } a \geq \text{no. } b$
(no negative no. a or no. b)

Induction hypothesis: $P(S) ::= (S \in C)$

base case 1: $C(S = \epsilon)$, 0 no. a and 0 no. b

base case 2: $C(S = a)$, 1 no. a and 0 no. b

$\therefore P(\epsilon)$ and $P(a)$ are true.

Inductive case 1: $f = (aSb)$, by induction hypothesis, $S \in C$.

no. a in $f = \text{no. a in } S + 1$

no. b in $f = \text{no. b in } S + 1$

Since $\text{no. a in } S \geq \text{no. b in } S$ ($S \in C$)

$\text{no. a in } S + 1 \geq \text{no. b in } S + 1$

$\therefore \text{no. a in } f \geq \text{no. b in } f$

$P(f)$ is true

Inductive case 2: $f = (bSa)$, same prove as above, it only switch a and b position, so no. a and no. b is the same.

$\therefore P(f)$ is true.

Inductive case 3: $f = (SS)$, by induction hypothesis, $S \in C$

no. a in $f = (\text{no. a in } S) \times 2$

no. b in $f = (\text{no. b in } S) \times 2$

Since $\text{no. a in } S \geq \text{no. b in } S$ ($S \in C$)

$(\text{no. a in } S) \times 2 \geq (\text{no. b in } S) \times 2$

$\therefore \text{no. a in } f \geq \text{no. b in } f$

$P(f)$ is true

Part 1 proved: $L(G)$ is in C ($L(G) \subseteq C$)

2. b) part 2. prove $C \subseteq L(G)$, $C: \{a, b\}^*$
 a string that no. a \geq no. b
 (no. a and no. b can't be negative)

Let n be the length of string in C

$S(n)$ be the string set that with length n , and in C .

Induction hypothesis: all strings in $S(n)$ also in $L(G)$

base step: when $n=0$, $S(n) = \{\epsilon\}$, (ϵ is in $L(G)$)

when $n=1$, $S(n) = \{a\}$, (a is in $L(G)$)

$\therefore S(n) \subseteq L(G)$ while $n \leq 1$

When $n \geq 1$:

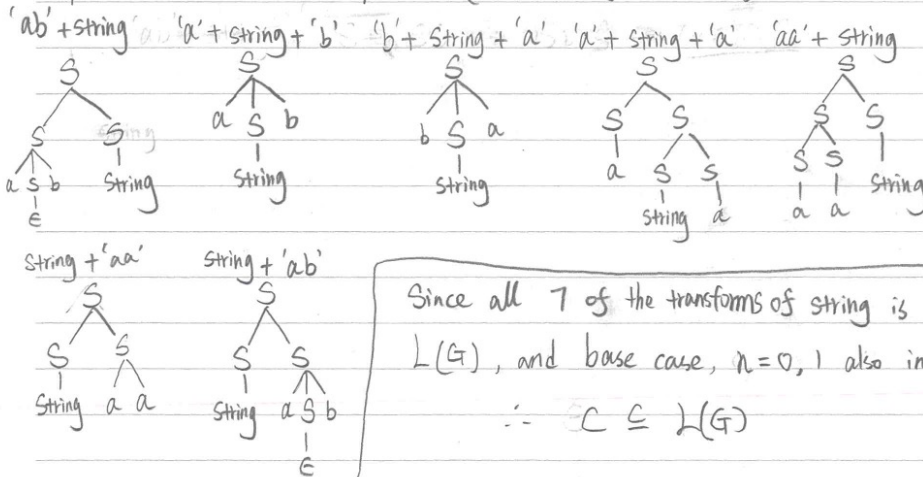
$S(n+1) : \{$ for each string in $S(n-1)$:

for string s in $S(n-1)$:

- 'ab' + string,
- 'a' + string + 'b',
- 'b' + string + 'a',
- 'a' + string + 'a',
- 'aa' + string,
- string + 'aa',
- string + 'ab'.

}

express each statement in the loop as $L(G)$: (string is in S)



Since all 7 of the transforms of string is in the $L(G)$, and base case, $n=0, 1$ also in the $L(G)$.

$\therefore C \subseteq L(G)$

Since, we prove $C \subseteq L(G)$ and $L(G) \subseteq C$,
 now we know that the $L(G)$ generates C .

Challenge 3.

a) Yes, regular language closed in concatenation.

therefore $R \circ R$ is regular language if R is,

and $R \circ R \circ R = (R \circ R) \circ R$ is regular language.

Since $(R \circ R)$ is Regular language.

and R is also Regular language.

b) ^{No} ~~Yes~~, function triple(L) concatenate the string in L two times.
if R is a regular language, triple(R)

Let R be $\{a^m b^l \mid m, l \geq 0\}$,

triple(R) become $\{a^m b^l a^m b^l a^m b^l \mid m, l \geq 0\}$

assume triple(R) is a regular language.

and p is the pumping length.

- Consider $a^p b^1 a^p b^1 a^p b^1 \in \text{triple}(R)$ with length greater than p .

By the pumping lemma, $a^p b^1 a^p b^1 a^p b^1 = xyz$, with xy^iz in triple(R) for all $i \geq 0$, $y \neq \epsilon$, and $|xy| \leq p$.

Since $|xy| \leq p$, y consists entirely of a 's.

But then $xyyz \notin \text{triple}(R)$, a contradiction.

\therefore even R is a regular language, Triple(R) not necessarily regular.

Challenge 3.

c) Let $R = \{101\}^*$ which is a regular language with DFA



In $\text{snip}(R)$, since R 's length always $3K$, $K \in \mathbb{N}$, (possibly 0) and base case of R is a palindrome (101) with length 3.

therefore $\text{snip}(R) = \{w \mid w \in (101)^m (11 \cup 1001 \cup \epsilon) (101)^m, m \geq 0, m \in \mathbb{N}\}$

Assume $\text{snip}(R)$ is regular, and P is the pumping length

consider $(101)^P 1001 (101)^P \in \text{snip}(R)$ with length greater than P

By pumping lemma, $(101)^P 1001 (101)^P = xyz$, with $xy^iz \in \text{snip}(R)$ for all $i \geq 0$, $y \neq \epsilon$, and $|xy| \leq P$

Since $|xy| \leq P$, and $y \neq \epsilon$.

No matter how you pump, the '1001' part of the string will be moved and no longer in the middle of the string.

But, the language $\text{snip}(R)$ only allow one '1001' in the string and only allow it in the middle.

$xyy^iz \notin \text{snip}(R)$, contradiction.

\therefore when R is regular, $\text{snip}(R)$ not necessary regular.