My mathematical and psycholgical fight

Olympiad Journey

By Aleph

Part 1

Warning!

The entire contents of this book may not correspond to what happened, especially the small details. All names here are nicknames and do not correspond to real names. Furthermore, this book may contain some controversial content and inappropriate language. Finally, it should be noted that this book contains writing errors because it was written in a hurry.

Chapter 1: Who I am.

Hello!

Welcome to this short book I am writing which is about my journey in mathematical Olympiads. Why I am writing this? Well... I really like writing and I just don't want forget the beautiful trip I did. But anyway, I hope you enjoy, but first let me introduce myself and my journey before the world of olympiads. I'm from Portugal, a country which, compared to compared to some countries in Europe and the East, it doesn't have tradition in mathematics and the away math is thought at schools is bad in my opinion. I mean, math is mostly taught as a bunch of formulas and concepts you have to learn to solve exercises mechanically and I suffered (and I believe that most young people are suffering now) from this education system that hid the beauty behind mathematics from me. By the way, I don't blame the teachers, I had very nice math teachers along my journey at school, but now I looked at the past and it makes me unhappy for not gaining a passion for math so early... maybe its my blame? Never mind, even though I was slow to realize what mathematics was, I would inevitably fall in love with it, and this happened in summer holidays during the Covid-19 pandemic. But before I tell this love story, I still want to emphasize my math skills in my along my school journey.

I wasn't good at math, I remember that I didn't care about math in primary school and in 5th grade I participated in my first math contest: Math Kangaroo Without Borders. Math Kangaroo is a math contest that schools do, it isn't a national level contest. By the way, I got the last place in that competition. At the time, I went because my teacher said it would be fun, but in reality I didn't care much about it, I just did it for the sake of it. Anyway, my interest in mathematics was only awakened in the 7th year. That year, I had a different math teacher than in past years and she had a very special way of teaching that made me like the subject. I remember the conversations I had with the teacher at the end of the classes and the classes themselves, that's when I started to improve in school math, but I still didn't see math as something so special. I also wasn't very social since I joint

school and around the 7th grade I started to suffering bullying from others students, mainly because of my way of being, and in my way of thinking and seeing things. This left me quite psychologically shaken, but I knew a lot of cool people who supported me, especially a girl who we were very good friends with and also my parents who gave me all support I needed (thanks mom). I also want to emphasize that I didn't participated in more math contests, that's because at least my school never warned me about the existence of these contests or the school has not signed up for these contests.

In the 9th grade, I changed school to another city in the north of Portugal. It was a very sad moment in my life, because I was too shy and unable to make friends, but this didn't affect my studies and I continued to do better in general subjects. It was in that year that I participated for the first time in the Physics Olympiad, a national competition. I remember that at the time I didn't want to participate, but my teacher insisted that I go and, in fact, I went to take the test. As usual, the problems are totally different from those at school, but those problems piqued my curiosity. I remember recreating the problem scenarios in my head and finding it amusing. So I solved the problems with what I had recreated in my head and it wasn't long before I received the test results and knew I had been the best in my school. Funny, because my teacher corrected my test and said the following words: "I would like to, but I can't, but if I could, I would put your test in a frame and hang it on the wall." I miss that teacher, I really liked the way she taught. I didn't passed the 2nd round of this Olympiad, but it was really fun and I didn't care to much.

Also, it was that year that I got interested in programming and I learned Python and JavaScript online. I remember that I did a website for a portuguese work about Luis de Camões and coded my first game, which was a game of reflexes. I think I still have a copy of that game haha! By the way, much of my experience with programming came from that time and today I continue to program when I have time. It was also at that time that the quarantine took place in Portugal because of the covid-19 pandemic and I remember that it shook my psyche a lot. With no friends and not much to do during the 9th grade

vacation, I decided to take advantage of this time to study programming and realize several ideas I had in mind. I remember at the time, I wanted to build a simple artificial intelligence to be able to play my game, but I didn't have the slightest idea how to do that and so I decided to look more about it on the internet and that's when I realized that it used mathematics quite different from what I knew. I was faced with terms such as derivatives, linear regression, limits, gradient descent and among others that piqued my interest. Instead of getting scared and giving up, I decided to study these terms and that's when my love for math was awakened. Probably, that was one of the most productive vacations of my life, every day I immersed myself in new things and became more and more fascinated by mathematics. One of the things that impressed me was how a logical system was able to create a totally perfect world, where everything works in such an orderly way by its theorems, lemmas and axioms. The results that are extracted from simple propositions made me shiver. You must probably be finding this strange, but that was how I saw mathematics, a world where everything has a pre-established, perfect order and beautiful and fancy results.

By the way, despite studying quite new advanced concepts for me at that time, I never actually used them to solve problems, I just used these concepts as great tools that allowed me to model scenarios that I was curious to explore through programming, the real mathematics I would only discover few years later. It was a really fun vacation and at the end I manage to make a neural network to play my game! I was very proud of myself at the time and continued to study advanced math concepts.

In my 10th grade, I had a math teacher who taught in a very dynamic way and I really liked her. I remember in the first class showing her what I had studied on vacation and she was quite impressed with me. Anyway, I really liked that teacher and I spent a 10 year with very good grades. I mean, I didn't care much about grades, I was always more worried about learning than memorizing for the test. And that helped me a lot for the national physical-chemistry exam, I studied very little for the exam and I got 18/20, which is more than enough grade to get into the course I want for university. It was also in that

year that I first heard about the Portuguese Mathematical Olympiads. My school didn't sign up for the olympids I guess (at least no one ever told me they did) and I never got to take a deeper look at it, causing me to lose interest in it pretty quickly and I deeply regret never paying attention to it so soon...

But anyway, life moves on. I continued to study advanced topics in mathematics and programming during the summer holidays, moved to another city (to my hometown) and started my 11th year of schooling.

This year was kinda fun and I participated in two contests: Physics National Olympiad and Math Kangaroo Without Borders. It went quite well this time: I got 4th position in Physics Olympiad and $8^{\rm th}$ position in Math Kangaroo Without Borders. They are good results, since it participated more than 120 students in both of them and I was very proud by the results. But, I still didn't have that desire/interest to participate in scientific competitions. I also participated in several chess tournaments between schools and managed to obtain good results, given that that year I dedicated myself a lot to studying chess. By the way, I only participated in scientific olympiads to pass time and have a little fun in this boring life without friends. It was at that time that I started to get a little depressed about life, because I looked at the world around me and only saw chaos and disorder, I saw the absurdity of the world. It was then that I accidentally discovered Anton Lavey's Satanism. Basically, Lavey's Satanism is a religion that does not believe in deities or a devil, that does not believe in the supernatural and that preaches rationalism and living in the best way possible, characterized by bringing Epicurean philosophy. I was curious and bought the Satanic Bible, I read it with an open mind, always researching the meaning of some phrases and discovered that I agreed with it by 90%. Contrary to what many people think or judge without knowing it, Lavey Satanism preaches valuing the self, being strong to face life's difficulties and that's what made me feel a little better about my mental health. The philosophy that this religion provided, influenced by Friedrich Nietzsche, Ayn Rand and Arthur Desmond, made me not feel alone, because Satanism saw reality in the same way I saw it. Then I bought "The Satanic Scriptures" and studied a lot

about this philosophy. Some believers in God would be a little disgusted reading these words, but as a human being, I respect others regardless of belief, as long as they know how to live in society, of course. People deserve to be respected, ideas no longer deserve it.

By the way, finally the summer vacation had started earlier because of the exams and I took advantage of this time to reflect on life and of course to study. A few days after the school exams, news appeared on my cell phone that would change the course of things. It was news about the results of the Portuguese team in the International Mathematical Olympiad, also known as IMO and that's where my journey into the world of the olympiads really begins!

Chapter 2: My first battle

I was very surprised when I saw this news: how young the time were and I was amazed that young people of that age compete in international olympids and the most impressive were the results. I wonder, if they can, why can't I? I mean, this may sound stupid and in fact at the time I had no ambition to participate in an olympiad of that level, but that news, in a way, was the motto for me to start to be interested in the mathematical olympiads. After this news, I started to research more deeply and I was reminded again of the Portuguese Mathematical Olympiads. From what I researched at the time, students who won medals in that olympid were invited to the Delfos project, an olympid mathematics school / math camp that does monthly internships to train students for the international olympids.

And that's where my ambition to win a medal at the olympids began, I wanted to be in the national final and be among people who share the same taste for mathematics and in a way that impacted me a lot, because I wanted to be part of a group and that was the my chance. But maybe I was asking too much? How could a boy without experience in olympids like me be able to reach a national final? Well, maybe I was planting a poison there, which is focusing on the result instead of the path I should have taken. I realized that right away and I started training. My training routine was very simple, as I had a lot of time because of the summer vacation, I started by doing problems from previous tests of this olympiad and I had even bought a book with a collection of problems from the older tests. All day, without exception, I solved problems for past tests, sometimes I went to a park near to my house and sat there, sitting in a chair, developing my reasoning while enjoying the warm summer air. It was there that my poison that I planted initially was being healed quickly, because during my training, I had fun solving these problems. I found the strategies used to destroy obstacles that eventually appear beautiful and funny!

Perhaps it was at that moment that I began to train not only to achieve a goal, but also to have fun. If I didn't find that amusing, I would have quickly given up and followed my solitary life, but... the problems were... beautiful and beautiful that brought me moments of great joy... and also of sadness. Of course, I faced several obstacles and there were times when I cried for not being able to solve a single problem, but it took time to say that evolution is something gradual and does not happen overnight. You don't become an olympiad champion in a few days or weeks, that's a matter of months or years. What were my inspirations? Well, the existence of those olympids was an inspiration for me, but students from Delfos project because one thing I realized is that most of the people on this project were normal people, just ordinary people who have fun doing math and who attend internships to learn more and better. In a way, this was an incentive, because if normal young people can be part of Delfos, why can't I? I knew a lot about mathematics and I wanted to be among people who shared the same taste, something I never had in my life and I may never have... But anyway, this was important to continue with my goal of acquiring a medal while having fun along this journey.

I continued to spend my summer vacation time solving problems from national olympiads tests, until shortly before school started, I found a place that would probably become my home to this day: MODS - Mathematical Olympiad Discord Server. I accidentally found this server in discord math server and joining this server was one of the best things I could have done. On this server, I met an important person who would accompany me on this journey, his nickname is "darth_sundays" and he was the only active portuguese person I found on the server. We exchanged a few messages and noticed that both of us wanted the same thing: to reach the final and get the medal. It was so cool! I was so happy to have found someone from the same country as me and who is in this world of the olympiads, such a rarity!

Also, on this server I came across problems much more difficult than those of my national olympiad, for example problems of the British Math Olympiad, United States of America Math Olympiad and many others. The people on the server were mostly young people from around this world who had goals in this world of the olympiads and many were sharks! I felt like an ant compared to these people, some of them have already been selected to do IMO and others who were part of olympiad training camps. But I wasn't

scared at first, on the contrary, I was even more curious! I wanted to learn all those techniques, to be able to solve the problems that were posted there (both on potd and on the math channels), I wanted to meet all those people! I felt happy to be part of a group.

Every day I tried to solve the potds that were posted there, when it got to the end of the week it was almost impossible to solve them, I remember trying to solve a Sunday potd without knowing that it was a high difficulty potd haha! By the way, that server was a home to me and I won't forget the good times I spent there.

The school had started and since I was in the 12th grade, I didn't have an excess of disciplines as I had last year and therefore I had more free time to train for the olympids. I'm not a person who has a lot of friends and I'm not very social, so I spent most of my breaks headsolving some problems that came to mind and sometimes I went to the school library to read books. I was probably seen as a weirdo by others, but... why should I care? I mean, I'm not going to stop doing what I like because others think it's stupid.

Unfortunately, not all young people today have a clear view of what mathematics is... also, classes started to be boring for me too, not because the subjects were boring, but because I had already learned what was taught some time ago, that's what being self-taught is all about. I simply started doing solving oly problems during portugueses classes and the best thing is that my teacher didn't care haha! Finally, after a few days, the teachers started to enroll students who wanted to participate in the National Mathematical Olympiad. Here anyone can participate without any requirements, unlike some countries where competitors have to have good results in small competitions to participate in the country's national math olympiad. In all, I think 120 students from my school participated, a very large number of students that initially scared me a little. Oh! I forgot to say, this participation was to make the first round of the olympiad, where the 50 best from each region of the country (north, center or south) are selected, plus the best from each school that was not on that list. Until the day of the first round, I was training with some previous tests from the second round and I was solving problems that were posted in the MODS. I had improved a lot in geometry and number theory, but I had some difficulty in

combinatorics and algebra, fields that I urgently needed to improve. But it wasn't just studying either. I remember the poker nights I played at MODS. Me, Darth, and other people playing and I had a lot of fun. Darth and I also liked to troll on #get-to-know-you-well asking the stupidest questions and laughing at the funniest answers. It turns out that one day we went a little too far and we got warned and managed to get the chat blocked for everyone xD. I laughed so much about it and it was fun, it was a good time.

Anyway, time passed and the day of the 1st round arrived. I remember that I had slept a little badly last night, too excited for the test. But I knew it would go well, because the problems of the 1st round are not so difficult, so I didn't let myself be shaken by fear. I remember going to the bathroom about 4 times before taking the test, maybe I exaggerated the amount of water I drank haha. Anyway, there was little time left to start the test, I left the chess club that I used to have that afternoon and I went to the place of the test. For some curious reason, I was the only boy in the room and there were like 14-15 girls around me, the teachers had to be making fun of me xD. Finally, the vigilant teachers handed the tests to the students and the 1st round had begun!

The test is composed by 4 problems, the first one is divided into 4 multiple choice problems, the second one was an easy combinatorics problem, the third problem is a geometry problem that were very similar to one I did from 2014 1st round test and the last problem was a number theory problem. The first problem I attacked was the geometry one and I could solve that one in 10 minutes, using symmetry and pythagorean theorem. After that, I attacked the combinatorics problem, which was practically bash and I solved it in a short time. Happy with my performance I continued solving and I started attacking the number theory problem. This problem could be transcribed into the language of a Diophantine equation that could be solved with brutal force, but I thought there was a more elegant solution and therefore I took a break from this problem and went to solve the multiple choice ones. The multiple choices were quite easy which made me afraid, as I was afraid of making a mistake due to distraction and this could be fatal because for each wrong multiple choice it is minus 1 point, but I took the time I needed to solve them.

The test had a multiple choice that was relatively difficult compared to the others and I spent some time on it, but I solved it using algebra. Finally, some time left and I started solving again the number theory problem. For some reason, I didn't brutal force the problem and spent the remaining time looking for a better solution. With 2 minutes left to finish the test, I had to leave the problem, I wouldn't be able to solve it in that time. I turned in the test and walked home, happy and sad that I hadn't completely solved the test. Then I started to doubt whether I was going to move on to the second round, because the problems were very easy compared to previous years and the cutoff could be very high. With that in mind, I started to get a little sad. Not passing the first round would be a cold shower for me, since I was capable of solving more difficult problems than those on the test. If I didn't pass it would be the greatest shame in my life.

Anyway, I started training for the 2nd round that would be in two months and I needed to improve on some of my weak points. The 2nd round was the most important test, it was the only one that gave access to the national final of the olympiads and only 10 people from each region passed this test. Therefore, until the 2nd round, I had to work hard to reach the final and I had to pass, my level was quite high to be able to solve the problems of the 2nd round.

Oh! I forgot to tell you, as I was training for the Olympiads, I came across problems that I thought were quite beautiful and I wanted to share these problems. So, I started posting videos with proposed solutions to some olympiads problems on my YouTube channel. I tried to make the videos as clear as possible for people see how art math problems can be and I had a lot of fun making these videos. By the way, a few days after the 1st round, I met a guy who made something possible that I never thought would be possible at the time: my first contact with the Delfos project.

Chapter 3: The day is close

I was on YouTube searching more information about the 2^{nd} round of my national Olympiad when suddenly I found a video of student reacting the medal he won in one of the editions of the olympiads and since he passed the 2^{nd} round, he could say what could be the cutoff to get to the finals (The organizers of the olympiad did not reveal this data and therefore I did not know how many points + or - were needed to pass the 2nd round). His name is EricXYX

I commented on his video and luckily for me he replied and passed his name on discord so we could talk. He was a silver medalist and was part of the Delfos project. I took advantage of this moment to clarify some doubts about the olympiads and the cutoff of the 2nd round. He was quite surprised at me because of the videos I made solving problems of my national olympiad and because I was also an "outsider" boy who already knew something about the olympiads. He even challenged me with some problems to find out my level and the problems were IMO and Balkan, which despite having already solved some of the shortlists, I didn't see myself (for the time being) capable of solving them.

Anyway, from time to time I tried to solve the problems and also took the opportunity to learn a little more theory during my training. One day at night, I received a message on discord from someone I didn't know. He was asking if I was Portuguese, but as I saw that he was in MODS, I was immediately happy to think that I had found another person who was also doing the olympiads, but the best was yet to come....

His name was Doctor, I didn't recognized his name at first seen and I thought it would be some teen like me that is also interested in olympiads. As long we talk I noticed I was in front of someone with a lot experience in math that was connected with Delfos project and I got totally surprised by that. At first I found it a little strange and I suspect a person with such knowledge could come completely out of nowhere, but then I remembered that as EricXYX was from the Delfos, he might have told Doctor about me. With curiosity, Doctor asked me some questions to know me well and I told him the journey I was doing and after some messages, he asked me if I was available for a video

call. Again, I got quite afraid. because it was already 11:00 pm I think, I was alone at home and a stranger that I barely met a few minutes ago wanted a video call. Thinking for a little, I accepted, what can go wrong?

The video call went well and I got quite happy after it. We both talked a bit about the situation of Delfos and I got very surprised by the words of Doctor and this motivated me, because I never thought I would meet someone from Delfos or something like that, so I was very amazed by that. I also enjoyed that moment to talk about Darth and the path we were doing in this olympiad world, probably because Darth deserved more than me such gift of meeting Doctor. Sad thing that at the time Darth was quite offline on discord because school stuff and other things, but during holidays he often is online on discord and on MODS. Also, after that video call I remember the last time I heard the name of Doctor: it was from a video made by TV at the time Portugal got gold medal at IMO, if I am not mistaken, the video was from 2012 so its a bit old. Not only that, but he appears on a math page made by Portuguese Mathematical Society, where he is interviewed and answers some questions.

Fascinated with that, my desire to be part of Delfos project increased and the only way at that moment to be there is to get a medal in the Portuguese MO and the next hurdle I had to face was the 2nd round of the olympiad, so I started training a lot.

My training at that moment was doing some past problems from 2nd round and finals, solving potds and problems from MODS and learning some theory that could be useful, not only to solve problems from Portuguese MO, but also to solve hard problems from other countries. I remember at that time I got fascinated by the theory in geometry that I learnt from EGMO (Euclidean Geometry in Mathematical Olympiads by Evan Chen), and that's when I started to have contact with geometry a little more advanced and the more I learned, the more I became fascinated with it and the more my desire to learn increased. Not only in geometry, I also was getting fascinated with number theory, learning very nice cool theorems and solving funny problems from the book MONT (Modern Olympiad Number Theory by Aditya Khurmi). The book is very good and free for

everyone one who wants to improve at number theory, I really recommend. In MODS, I started finally understanding some things that the most experienced used to talk and usually enjoyed some activities there like ACPS sessions made by Tony Wang (the owner of the server) where we solve problems from some chapter of the book *Art and Craft of Problem Solving* by Paul Zeitz, a very good book for people who are new at math olympiads, I really enjoyed some problems from there.

By the way, the only goal at my life now was to get a medal in this olympiad and this made me lost some interest in learning the subjects I have at school. Of course, I always did my best in group works at school and I could increased my grade with that (especially in portuguese subject, because there we do more oral presentations and group works than tests). I really don't like do oral presentations of subjects I don't like (like portuguese), but I did very well in my past presentations. The thing is I wasn't caring about my grades and stuff like that but I could also manage to increase them, maybe because in the 12th grade is more relaxing year than 10th or 11th. At least at my school, I had a free day (friday!!!) and this is so amazing because I have more time to do more math. At school, during breaks I usually I stay alone at the library thinking about problems and trying to solve some problems, but why you ask. You don't have friends or you don't like to be with others? Well, I just feel in peace when I am alone, I actually like my class, they are really amazing but... you know, being alone is where I feel comfortable, where I can forget the worries of the world and travel in my most philosophical or mathematical thoughts. Oh, I don't know if I have already said that, but I really like philosophy. Probably my love for mathematics increased even more because of philosophy. Most teens when they join at 10th grade, they tend to think philosophy is boring and just another stupid obligatory subject they have to study to get good grades, but I don't think so. I fell in love with philosophy the very first class I took in 10th grade. My teacher did a great job showing me how magnificent and important philosophy is. The art of solving philosophical problems through critical thinking fascinated me. I don't know if I'm the only one who thinks this, but I always like to look for different answers to the same

philosophical problem, not because I wanted to know all the theories, but because I liked the thought process of different philosophers in formulating their theories and if you have noticed, in mathematics it's almost the same.

The world of ideas of Plato also influenced me a lot at the beginning and still makes me thinking sometimes. Thinking about that mathematics is just a universe ruled by its axioms that we humans simply assume they are true. This universe is perhaps kinda bizarre, and our human brains can't explore it completely by the restriction of our logic and this was proved by Kurt Godel (mathematics is more than logic). Isn't that strange to you, thinking that assuming that some prepositions are true, we can create a universe that escapes our reasoning tools? Mathematics has something special, something that we can't even know exactly and we only can explore some of this universe through our human brains, that's how at least I see math: a universe ruled by its axioms that can be explored thorough reasoning machines. Note that you can only explore this universe if you accept the axioms, otherwise no game for you. Some people think that axioms reflect reality, but that's not correct. Axioms are just prepositions we assume they are true, but we don't prove. If you want, you can assume God exists as a preposition and use it to extract theorems, many philosophers have done this, like Baruch Spinoza or Rene Descartes. By the way, it was beautiful to see amazing results that sometimes escape our intuition are single ruled by simple claims. This made me think about life. Life is such an absurdity that makes you to search some sense, unlike chess or math were everything is ruled. So, if your universe is ruled, where comes the free will?

Ok, December has arrived and Doctor said there were going to start an online competition at Delfos Forum and me and Darth signed up. The competition works like this: we have 73 hours to do three problems at first round and second round, its kinda similar to the IMO style, each problem gives at maximum 7 points. This is kinda strange because nobody of Delfos know us (except EricXYX) and we are participating even not being delfics, I am very grateful by Doctor for this opportunity and of course, this is an

opportunity to show that people who don't attempt training camps can be good at math. I don't know about you, but this was probably the best christams gift I receive that christmas. Of cource, at christams I received a lot of things: cloths, the presence of my family and other nice things, but since I was in olympiad mode, the participation of this competition was a very good gift, and the best I am participating with Darth! Oh man, that was amazing.

The day of the first round has arrived, I checked the forum of the math camp and I download the pdf with the problems. The first problem was an algebra one, the second problem was a geometry one, and the third problem was number theory. Since we had 73 hours, pressure by time was not a problem, so it was a relax competition.

The first problem I attacked was the geometry one, with the following statement:

Problem 2: Let O be the circumcenter of the triangle ABC. The line through O intersects the sides CA and CB at points D and E, respectively, and intersects the circle of ABO again at the point $P \neq O$ inside the triangle. Let Q be the point on side AB such that $\frac{AQ}{OB} = \frac{DP}{PE}$. Prove that $\angle APQ = 2\angle CAP$

It's a interesting problem, because I have never saw I problem with such ratio of lengths. So probably trig as required to solve it. After 30 minutes thinking about it and without no progress, I decided to do problem 1, as follows.

Problem 1: Let $a_1, a_2, ..., a_n$ with $n \ge 2$ real numbers with the same sign greater than -1. Show that $(1 + a_1)(1 + a_2)...(1 + a_n) \ge 1 + a_1 + a_2 + ... + a_n$.

I started reduced the problem to small cases like n=2 and I noticed this could be solve with the principle of induction. The base case was easy and it was pretty chill to prove the induction step with case work. After some minutes writing the solution, the first

problem was solved, but I wanted a more hard thing so solve, so I returned to the problem 2.

I spent tons of time on that problem and I couldn't find something that allowed me to make progress. I simply didn't know how to use the fact that $\frac{AQ}{QB} = \frac{DP}{PE}$, maybe Thales theorem or trigonometry, but I have tried both and I couldn't do nothing useful. Darth also didn't get nothing on problem 2, so he tried to solve problem 3, since he like more algebra problems than geometry ones. After sometime, I still was doing problem 2 and nothing so far and Darth found out the formula... oh wait, I forgot to share the problem, my bad xD.

Problem 3: Define the sequence $a_1 = 1 < a_2 < ...$ of positive integers as follows. As soon as $a_1, a_2, ..., a_n$ were chosen, a_{n+1} is the smallest positive integer not of the form a_i or $a_i + i$ for $1 \le i \le n$. Find a simple expression for the sequence. (Possible hint: use φ , where $\varphi = \frac{1 + \sqrt{5}}{2}$)

This problem is interesting because it gives a hint to you about the expression of the sequence. I didn't think the problem was very good and so I continued to hammer my head on problem 2, where I hadn't made any progress.

With some hours left to submit the answers, I tried problem 3. I couldn't find the expression, but I found out some interesting facts about the sequence that could help me later. So at the end, I just wrote my ideas for problem 2 and 3 to gain some partials. I was quite sad for not being able to solve problem 2 and after me and Darth discussing the problems, he said he could quite prove the expression he found, so I was behind and I was afraid of being in last place in this competition, which would be a tremendous disaster, not gonna lie. But after that first round, I reflected and concluded that I shouldn't spend too much time on a problem because it is the genre I like, this was a very stupid decision at that contest. I promised that I would never repeat it again. But by the way, me and darth talked for a while before the second round and I enjoy this time to share the problems to

the people at MODS. They solved problem 1 as the same way like us, but they got stuck at problem 2, even Matteddy (almost perfect score at IMO 2022) was a little stuck on problem 2, but he solved with sine rule and phantom points. I didn't understand it at first seen, but others agreed Matteddy was right, so I just accepted xD.

Oh yeah, how I could forget. At school some days ago before this contest, I received a letter from my math club teacher, warning me I passed the first round! I was quite happy of course, but passing the first round isn't that important, I needed to pass the 2nd round if I wanted to be at the finals and have the opportunity to win a medal.

Realizing that the results of the first round were released, I decided to check the official website of the contest to see how many people from my school passed and I just realized only 4 passed from B category! I was like... really? How is it possible? Compared to the last year, more than 10 students B category from my school passed the first round. This was very strange... I thought a lot of students of my school would pass the first round, since it seemed to me a lot easier than last year first round. Maybe because it was so easy that the competitive factor increased a lot and you would need a very high score to pass?

At the time, I talked with a guy at chess club and he said he solved problem 2 and problem 4 and got one multiple choice wrong, so the cutoff was like 27/40? I don't know and it doesn't matter, what matters is that I passed and I needed to focus on 2nd round now. But before I continue with the contest that me and darth were participating, I have to share the most unexpected thing.

There was me, checking the people who passed the first round of my school and I had the brilliant (ok maybe stupid) idea of searching their names on google, to see if I could find something special or something and... it worked! There was a guy called Zex who won a silver medal at last year portuguese MO, in A category. I got totally surprised, there is a silver medalist in my school, which means, a delfic in my school! I got totally possessed by happy, I couldn't believe there was another math oly guy I could meet. I remember at time it was night when I did this discovery and I immediately opened discord to see if I could find him on delfos discord server and for my lucky... he was there! I sent a message to

him and after some minutes he answered! We talked for a while, I though about me and my desire to be at national finals and he shared a bit about delfos and his new experience at my school, because he was new there. Also we marked an hour to meet at school tomorrow, I wanted to talked personally. Next day, we both talked and I discovered that he was also medalist at sixth grade! He was doing math oly long time ago and I got amazed by that. After that meeting, we never talked again until the day of 2nd round, but before we get there I still need to share the 2nd round of the constest of Delfos.

After a good day of Christmas with the family I loved so much, after eating a lot and after enjoying the first round of the contest, the 2^{nd} round has arrived and the problem 4 was a number theory one, problem 5 was a combinatorics one and problem 6 was a geometry one.

The first problem I attacked was the geometry one, with the following statement:

Problem 6: Let ABC be an acute triangle, with AB < AC < BC, inscribed in the circumference Ω of center O. The A-circle has center I_A and is tangent to the lines BC, AC and AB of the triangle ABC at the points D, E and F, respectively. The line AI_A intersects Ω at the point $M(M \neq A)$ and the circumcircle ω of the triangle AEF intersects Ω at $K(K \neq A)$. The circumcircle of the triangle OKM intersects ω at the point $N(N \neq K)$. Prove that the intersection of the lines AN and KIA meet at Ω .

The statement was quite long and it took me a while to draw the diagram correctly on paper. After drawing it, I start get proprieties of the diagram and after some minutes without any progress, I took a look at problem 4:

Problem 4: Determine the largest positive integer n that divides abcd whenever a, b, c and d are integers that satisfy the $a^2 + b^2 + c^2 = d^2$

It seemed interesting problem, so I decided to give it a try. First I tried with small cases and I got $2^2 + 2^2 + 1^2 = 3^2$ and so 12 is the largest number that divides $2 \cdot 2 \cdot 1 \cdot 3$. After that, I also found that $2^2 + 3^2 + 6^2 = 7^2$ and 12 also divides $2 \cdot 3 \cdot 6 \cdot 7$. So I conjectured that 12 is the largest n we are searching. Using modular arithmetic and working with divisibility, I just did casework and solved the problem (at least I thought it was solved).

After that, I take a look at problem 5, which a funny statement:

Problem 5: Let $N \geq 3$ be a positive integer. In a congress held by N ex-delfics, they noticed that each of them maintained contact with at least one of the other ex-delfics present. Determine all values of N for which, given any integer n with 1 < n < N, it is necessarily possible to select one group of n delfics in which each one maintains contact with at least one other of the selected ex-delfics.

Playing with problem tell us that when N is odd, we can necessarily select a group that satisfies the problem. But I noticed this would require me some knowledge of graph theory to prove it, so I wrote some notes for later I write the full solve. At the time, I was kinda lazy to write the solve, so I take a break on the problem and I returned to the problem 6. After playing around with it, I could prove that A, O and N were colinear and after that I create an argument using incenter-excenter lemma and phantom points to prove the result, but I was a little afraid of my argument, although at the time I didn't find something that makes it weak (this smells fakesolve, doesn't it?). After sometime, I wrote my full solve for problem 6 and then I returned again to the problem 5.

Again with my laziness, I didn't write a decent solve for this problem and if you have already done combinatorics, you know sometimes you have to write walls of text to solve such combinatorics problem. But anyway, I wrote a solve using box principle with some casework. I could have write more, but since I was lazy to write a wall of text to this problem, I just wrote the more important things.

The time of 2nd round has ended, me and Darth have both submitted our answers and we start discussing. He said could solve problem 4 and 5 and he wrote 4 pages solution for problem 5! What dedication! But since he wasn't interested with geometry, he did not spend his energies on the last problem and he decided to solve problem 5 in a more decent way.

By the way, we would have to wait some days until the results of the contest be released, which is kinda boring waiting. But now I had to prepare for the 2nd round of my national olympiad and I started doing all past 2nd rounds papers that existed, from 1982 to 2022. I also have done some papers from finals because older 2nd rounds tend to be more easy, because... well, they are old. So basically every day, I solved problems, always trying to see my weak points and trying to improve them. One thing I noticed during my training is that I sometimes couldn't solve a problem because of a simple arithmetic mistake or because I couldn't concentrate on the problems, getting distracted very easily. Maybe this is because last days I wasn't sleep well: I couldn't sleep until 1 AM and I woke up quite late. Also, for those who are preparing for some contest or something, I really recommend get good habits, because believe or not, you habits have a big impact on your future performance, you will see this in the next pages.

Anyway, I continued focusing on my training, feeling very sad every time I failed to solve a problem or I couldn't improve my weak points, but I couldn't stop, I couldn't give up because I was addicted to problem solving and I couldn't stop solving problems even after fail to solve 10 problems in a row. Maybe if I didn't like problem solving so much, I would have give up very ealier, because motivation should come to the desire of do problem-solving and not getting a medal or an award. Of course, I am not saying for you to forget your goals, everybody can't scape the desire of getting nice results, but its important to keep in mind that this isn't your main reason to train for math olympiads. If you train to math olympiads to get a medal or something and you don't like to do problem solving or you don't like math, then the probability of you give up very ealier is very high.

It was 01/01/2023, 3 AM approximately and I had just come back from a restaurant where my family went to celebrate New Year. I confess that I preferred to spend the New Year at home calmly, but it wasn't bad at all and what matters is that my family had fun. Anyway, I was in my room, looking at the stars through the window, spending my energy thinking a little about life and the world. It is in those moments when the philosopher that lives in me awakens and begins to wander in the world of ideas, without a direction, without a need, just taking comfort in ideas, imagining hypothetical scenarios and thinking of solutions to simple questions. Sometimes I question my role here, my existence and why I stay here. All lives, all people are here for a reason, is it because of a God? Perhaps by simple evolution of nature? Well, the reality is that no one knows and we never will, even if you are a religious fanatic or atheist. Maybe that sounds a little suicidal? Oh, it reminds me of "The Myth of Sisyphus" by Albert Camus. A very small book but very profound and difficult to understand, but which is well worth reading, at least for people who like to satisfy their philosophical thirst. I was thinking about my beliefs and my points of view, it had been a while since I discovered Anton Lavey's Satanism and I still continued to felt very comfortable and pleased with the philosophy it offers. It helped me a lot to face the challenges of life. Again, this religion does not believe in the devil or a deity, on the contrary, it is an atheist religion that promotes living a good life in an epicurean and healthy way. It sounds stupid and maybe it's stupid to believe, but I just don't want to be misunderstood because of this religion and suffer stupid prejudices, but since this is like a kind of a diary, I'm happy to talk a little about myself, to talk about things I've never dare to speak in public.

By the way, I kept thinking about that night, remembering that in a few days I would be fighting with more than 120 students from my region to occupy one of the 10 places in my region for the national final. Maybe I was doing too much drama around this 2nd round, but maybe that's my taste for math olympiads that made me quite anxious and happy for this moment to be approaching. After a while, sleep wanted to take control of me. I closed the blinds and my philosopher went back to sleep again, dreaming of the

reflections he had that night. I turned off the lamp, lay down on the bed, looked at the darkness of the room for a few seconds and slept, because my mind needed to clean up the mess that philosopher caused.

Chapter 4: The torment

It was 11 January of 2023, the day of the 2nd round. I have slept well last night, more well than the night before the 1st round. I took the breakfast, looking at the window the sun that was arising. I was shaking, not too much, but sufficient to mess me a bit. I took a deep breath, started to pack my things for school and left for my teaching center at 8:00 am while I listened heavy metal. Oh! I forgot to mention, I'm a big heavy metal fan, but I don't only listen heavy metal, I also listen lofi, very relaxing and calm. Why not listen to lofi that morning? Well, I was more excited and motivated, I waited a long time for that moment and decided that I was going to make the most of that day, starting with my favorite songs.

It was Wednesday and the first class was math, the teacher was teaching complex numbers if I remember well. By the way, it was boring, because I already knew complex numbers and the teacher was teaching basic operations and giving us trivial exercises, but since I had nothing interesting to do at the moment, I just spent the class doing exercises. As that involved more calculation than problem solving, I took the opportunity to train my mental calculation speed, as it would be useful for the 2nd round that would take place in the afternoon at 3:30 PM. This class made me relax a little, lowered the frequency of my tremors a little and I felt a little more relieved.

After math class, at school break I went to the room where I would take chemistry class, looking around the students with their funny lifes, I mean, I don't know. Is it just for me, or does your school have students who are very social? Like, it's quite rare to find a group of young people on their cell phones alone, most of them tend to have fun during breaks, some tend to be flirtatious and the lonely ones (where I include myself), tend to either look at nothing or see things on their cell phones. The chemistry teacher arrived and the classes started again.

I'm not a chemistry lover, but I like the discipline and as the class was a laboratory class, I could entertain myself and help my work group. My body was shaking slightly and I drank a lot of water frequently, it seemed that thirst took over me that morning. Finally

after a great chemistry class, the break finally started and I only had a Portuguese class before I could have lunch and relax a little in the chess club before going to do the 2nd round.

At break, I saw Zex talking with other guy who is at chess club very often, it seemed they were talking about math, probably discussing some problem I don't know. I just stayed in a corner pretending I was using my cell phone, when I was actually trying to listen in on their conversation, not wanting to interrupt. I know I should have been less shy and joined in the conversation, but I don't. My mind was dominated by the idea that Zex was a Delfic and that somehow I was not worthy of being in front of him and talking. I know, it's stupid. I usually talk every day with IMOers on discord and I can't get close to a camper personally. Damn, I'm really weird, but whatever.

After a few seconds, they walked somewhere and since I also had other things to do with my life, I went to the Portuguese class. To be quite honest, I tend to completely skip Portuguese classes, solving problems in class. Like, I don't recommend anyone to do that, but since my teacher didn't care and I didn't disturb the class, I ended up spending Portuguese classes doing math. But as that day was special, I just watched that Portuguese class, which was about the analysis of poems by Fernando Pessoa's heteronyms. As you can see, it was very boring, but at same time good because it allowed my body to relax a bit somehow. After a boring 45 minutes of Portuguese, the class ended and the first thing I did was go to the bathroom to urinate before luch. Oh man! I forgot to say, before the Portuguese class, I also went to bathroom. I'm not going to lie that it's very, very rare to go to the bathroom at school. Not because the hygienic conditions are terrible, on the contrary, it's just that I don't really go to the bathroom at school, but since I was more nervous that day, it made sense.

I finally went to lunch and my classes for the day were already over. I don't remember what lunch was, maybe fish and chips? I don't usually complain about school food, sometimes I like it and I'm grateful for that. At lunch, I was shaking a little unusually for that morning, but I enjoyed my lunch.

After lunch, I went to the library. As I walked there, I noticed that they were setting up something outside. Well, of course, it could only be a student union thing or something like that. Like, around this time of the year, there are usually lists made up of students who aim to bring improvements to the school and very often they do different types of campaigns, some a little boring like loud music during breaks, banquets, shirts, etc... you know, I always thought it was an unnecessary waste of time and I didn't care much about it, I just thought it was stupid.

By the way, in the library, I took my computer and watched some helluva boss art on twitter while I waited for the chess club to start, which was 1:20 PM. Damm, I forgot to tell, I'm also a big fan of helluva boss, a very funny webseries for +18 that has an animation style and a universe that I really like. Well, not only that, but I also see some philosophical issues in the series and I wish to write a philosophical essay about it some day. This is me seeing philosophy where there is probably no philosophy, but who cares xD.

After some time, it was time for the chess club, I could already see the teacher picking up the boards in the library room. There were 3 people at the club and its common not too many people come to it, but it's still cool have someone to play chess on the board. By the way, we did some puzzles, analyzed a game and we played some matches. My interest in chess was very big last year, I got federated and stated playing federated competitions, which is a nice way to improve at chess because actually long games (90+30) are very good to improve at chess. The thing is after I started get interest at math olympiads, I have abandoned chess quite a bit, I start playing less federated competitions and getting more rusty at calculating variations, but I still came to the chess club every week just for the fun, since there are funny people who come to the club and I enjoy a lot the time there.

After some games, there were 30 minutes left before the beginning of the 2nd round. I said goodbye to the teacher and the others and went to the outside of school to get some fresh air and walking slowly around the school. The sun seemed almost to set, the exterior

of school had few people, there were just people from student association that were playing some music outside, but I just ignored because the most important moment was way to come.

I walked slow around the soccer field in the exterior of school, thinking again, but this time, thinking what I would do when I sit in front of the exam, I shouldn't be afraid or nervous, I have trained a lot for this moment, I can't fail, that reality is impossible, I thought to myself. I looked at the plants, the cars moving, the other students have fun with the student association thing, the birds, the sky. There were two things that could happen after the exam: do very good at it and come to my home very happy or do bad at it and come to home very sad and unhappy about my performance.

After a short walk, I sat down on a bench outside the school. I took a deep breath, my hands were shaking, my heart was beating fast, but I managed to control myself enough to feel comfortable. Again, I went to the bathroom to urinate, this was the fourth time I had gone to the bathroom, as I had also gone there while in the chess club. There were a few minutes left, I went to the student's room to pack my material that I would use in the test: a compass, a ruler, an eraser, a pencil, a pen and a sharpener. As I was packing my things, I saw my math teacher who walked towards me. He asked what I was doing there and I said I was going to do the 2nd round of Portuguese mo. After all he forgot that today was the 2nd round and he wished me good luck. I thanked him and went straight to the room where the test would take place.

You know, this math teacher of mine wasn't very interested in math olympiads, but I won't deny that he is a good teacher and he made me laugh a lot throughout that school year. Oh! I have to tell this! There was a support math class in which I proposed to solve a geometry problem from the 2nd round of some year to the teacher, the problem was as follows:

Problem: Let ABC be a triangle such that $\angle ABC = \angle BCA = 40$ and D be the point of intersection of the bisector of $\angle ABC$ with AC. Show that BC = BD + AD.

I remember I solved that problem last night before this support class in 20 minutes and I wanted to share to the class. The result was that the teacher couldn't solve, nor the class. Of course, they didn't solve that because they don't have the mindset for doing math oly problems, but it made me laugh a little to see them trying to solve it xD.

Ok, I was outside the room waiting for the vigilant teachers to give a signal to enter. As usual, students from other schools tend to be there to take the exam and I remember there were about 10 or 11 people I think. Zex hadn't shown up yet, but he would. I remember that I was there waiting in front of the classroom door when a girl came up to me, probably from the 9th grade. She asked if this was my first time doing the 2nd round and I said yes. She also replied yes and we wished each other luck. There wasn't much to talk about in the nervous climate we were going to be subject to.

Finally, the teachers gave the signal to enter. I sat down in a comfortable place for me and put all the work material on the table and waited for them to deliver the test. Before that, they gave us a sheet to write our data and scratch sheets. Zex appeared and we greeted each other. I asked if I could use my smartwatch to see the hours and they said I couldn't. I was like.... ARE YOU FUCKING JOKING, RIGHT? The damn room didn't have a clock and I was left without a clock, or rather, I had no control of the time. I just hope they tell the time at regular time intervals, or I'll get lost in time during the exam. I didn't let it get to me, after all, I just had to solve problems quickly and I was prepared for that.

After some minutes, they finally gave the test papers to the students. The war had officially begun!

The 2nd round is composed by 4 problems, each 10 points and the duration is about 2 hours. The first problem had a quite long statement and it seemed to be an invariant

problem, the second problem was an inequality, something that never appeared before at Portuguese MO 2^{nd} rounds, the third problem was a geometry one and the fourth problem was a combinatorics one.

The first problem I attacked was the geometry one, with the following statement:

Problem 3: In the triangle ABC, M is the midpoint of AC and P is the intersection of the angle bisector of $\angle BAC$ with the side BC. Knowing that AP and BM are perpendicular, prove that CP = 2PM.

After drawing the diagram, the first thing I did was to connect the point P with the point M. Since AP and BM are perpendicular and $\angle BAP = \angle PAM$, the line AP is the perpendicular bisector of BM and so we get BP = PM. At this moment I got stuck for some minutes, after realizing I could use angle bisector theorem, which give us $\frac{AB}{BP} = \frac{AC}{PC}$. Ok, since BP = PM and $AB = AM = MC = \frac{AC}{2}$, I got $\frac{MC}{PM} = \frac{2MC}{PC}$, which gave me immediately the result I wanted. I got quite surprise of solving this problem so fast, so much so that I reread my solution several times to see if it had any flaws, but it was impossible. Yey! I solved my first problem from 2^{nd} round!

By the way, next problem I attacked was the inequality one, which the follow statement:

Problem 2: How many pairs of positive integers (x, y) verify the inequality:

$$2^{2x^2} + 4^{4y^4} < 8^{43}$$

This inequality was quite strange, so the first thing I tried to do was to simplify as much as I could, so I did the following thing:

$$2^{2x^2} + 4^{4y^4} < 8^{43} \Rightarrow 2^{2x^2} + 2^{8y^4} < 2^{129} \Rightarrow 2^{x^2} + 2^{4y^2} < 2^{128}$$

And from that I noticed that $x \le 11$ and $y \le 5$ and I got stuck for a while. After think about the problem, I just skipped and tried to solve the first one:

Problem 1: Zex, Bruno and Joao are players from the same team and play with jerseys 2, 8 and 17. They noticed that if they added the numbers of their jerseys two by two, you would obtain the numbers of Bernardo's jerseys, from Nuno and Otavio. As they found this procedure amusing, they added up the numbers again which got two by two. If you do this procedure 2023 times, what is the difference between the two largest numbers obtained?

The statement is fun and quite long and strange, what the fuck is Bernardo, Nuno and Otavio that appeared from nothing xD. Ok, the first thing I did was to make the procedure with the numbers 2, 8 and 17 and see what I could got. After adding numbers, I couldn't see pattern yet. I asked the vigilant teacher the time and they said it had already passed 1 hour... WHAT? (I was like). One hour passed and I only solved one problem. At this moment, I start getting more and more nervous, but I took a deep breath, and I went back to the problem again. After carefully looking at the numbers, I noticed that there was an alternation in the difference of numbers that I obtained and it seemed always repeated, that is... I found the invariant of the problem! After that, it was just basic steps to get the difference between the two numbers after 2023 operations.

Ok ok, I was more calm now, since I solved two problems, it just remains to solve one more and my pass to the final stage is guaranteed. More calming now, I returned to the problem 2.

Taking a look at what I solved, I noticed that I just have the answer in my hands, because since I have $1 \le x \le 11$ and $1 \le y \le 5$, then I have $11 \cdot 5 = 55$ pairs which satisfied the condition I got. So amazing! I had solved 3 problems already! My ticket to the finals is guaranteed!

I remembered I was so calm at that moment that could just relax and don't continue anymore. I asked how many time I had left and the vigilant teacher said 30 minutes. Since I had some time remaining, I decided to solve problem 4, which the following statement.

Problem 4: In a chess tournament, 10 players participated and each pair of players played exactly once each round. After each game, each player received 1 point if they won, ½ in case of a draw and 0 if they lost. At the end of the tournament, how many players, at the most, could have been left with exactly 2 points?

Since the problem has chess on it, I already liked it. My first approach was to construct some chess tournaments, but I noticed this would take me a while and just gave up this approach. I didn't think they want the contestants see the constructions and probably the key idea was to look at the numbers, numbers of games, points, and so on.

After playing around with the numbers I concluded that, if N is the number of players that end up the tournament with exact 2 points, then $N \leq 5$. Great! I did some progress, a bit more happy now. At this moment, my conjecture was that N=5 was the answer and so I returned to play with the numbers and I got that N=5 doesn't work, but N=4 worked and so I wrote the solution for the problem on time.

The exam time was up at the same moment and I didn't have time to check my solutions for all the problems, but if did everything right, it was guaranteed my presence in the finals. At that moment when I handed in the exam, the room was almost empty, there was only the teacher watching, a 9th grade girl, me and Zex. When I went to hand in the test, the teacher saw that I had answered everything and kindly asked if I made it to the final. I just said it was very likely and left the room quite happy.

When I left the room, I had the feeling that I had done something wrong in problem 2, but I didn't pay much attention to it and before I left the school towards home, I went

to the bathroom because I was very upset, since I didn't go to the bathroom during the exam, because teachers didn't allowed it. The night was dark, the clouds partially covered the sky and I walked home, calm, as if I had done my duty that afternoon.

When I got home, I packed my things and saw on my phone that I had received some discord messages. When I went to see it, it was from Delfos server and it was EricXYX with the answers for the problems, at that moment I got a shock: my answers to problem 2 and 4 were wrong.

I started to have panic, I couldn't believe I gave wrong answers for two problems and my opportunity to be in final stage could be completely destroyed by that. In my head, I couldn't accept I was wrong, I immediately open MODS and asked for the answer of problem 4, but it was useless since the original answer was different from my answer. I remember being very very sad, I couldn't accept the end of my journey, I just wanted to do something decent in my life, something I could be proud of and being at the finals was my dream since I saw that news about portuguese IMO team. By the way, I simply that eat my dinner and before I go to sleep, I opened discord to see what was happening in the servers, but I realized Doctor sent a message to me asking if it went good the exam. I explained to him what I did and he decided to make a video call for we talk together. I thought that was probably the last video call ever and I couldn't believe that I had (maybe) given so many expectations that they could have been thrown in the trash after this exam.

After some time talking, we end up the meeting and I went to sleep, but before I sleep, I thought about everything that happened that day, I still couldn't deal with the part of failing this exam and not being in the finals. I looked at the darkness of my room and small tears started to run down my face, when finally sleep came to put an end to that day.

But anyway, time passed after that disastrous 2nd round, I had to deal with school stuff and that was boring since school subjects didn't interest me much and also because without the math oly my life seemed to lose meaning, it lost its grace. After all, the spirit of math oly were what filled my emotional days, that desire to take the test, that desire to know more and more about it and the best of all was talking to other people online about it, that was which made me happy over the days, since I didn't really have any friends to go out with, I spent most of the time alone at home because my parents had to go to work and my life ended up getting a little gray that way. But then, it destroyed me quite a bit psychologically and I continued to live my boring life I had at the time, always looking for something interesting to do in MODS, like solving problems or talking about math oly. If I didn't pass the 2nd round, I wouldn't practically give up on math oly, since I had created "very strong bonds" with that community in MODS and talking to those people made me a little happier, but as it was obvious the fact that I wasn't being able to participate in math oly anymore made me sad but that wasn't why I was going to give it up completely. After all, that for me was fun that made me happy.

After some time, the official solutions of the 2nd round were published and as it was obvious EricXYX was right, I had in fact wrong both problems. But what did that matter, right? I couldn't do anything else now but wait for the finalists to be published. It reminds me how every day I blamed myself for getting the problems answer wrong and always speculating whether I would make it to the final or not. This is what happens when you have an empty mind that seek to satisfy its thirst for knowledge at all costs.

By the way, carnival was approaching and I could finally have some free days where I could entertain my mind with things. At that time I remember that I was watching a series called NUMB3RS, which is really cool and I really like it. Basically it's like a cross between CSI and math. Like, it's focused on two brothers, one who is very math genius and one who works for the FBI and they together use their knowledge to solve crimes, with real applications of math to that. At least I thought the series was pretty cool and I spent my Carnival marathoning it.

Things were normal when on a certain day, I was getting ready to go out with my mother to take care of some personal documents, when suddenly I received an email that

almost made me have a heart attack for the first time I saw it: it was an email from the Portuguese MO organization advising that I had been selected for the national final of the olympiads! I remember that I was so excited that my mother almost thought I was having a heart attack from all the screaming I went through the house. I was crying with happiness because being in the national final was my dream since the beginning and I really wanted my olympiad journey to continue, that it would never end. Undoubtedly, I must have been very lucky to have been selected and that maybe I wouldn't have passed if I had been in the country's capital region, but what matters is that it wasn't the end!

Even that day, we sent everything necessary for the organization of the Olympiad to confirm my presence in the national final, which would be in Faro in the south of the country! It was going to be a long trip and I was going to prepare myself as much as possible so that I could get a medal and enter the math training camp in my country: Delfos! It was then that my life started to gain more color and meaning and I started to train the best I could, solving problems from past exams and solving some problems posted in MODS, with people helping me on the server whenever they wanted! I had to somewhat maintain my school test performance while I trained for the olympiad, even so, I managed to do well in school, although I sometimes skipped classes to solve problems (maybe I can't count the number of times I didn't pay attention in Portuguese classes to do geometry xD), but it was pretty fun. My classmates didn't bother me and I didn't bother them, so I was fine. I remember that was when I started playing with some advanced geometry techniques like inversion, projective and a bit of MMP. It was all new and fascinating to me and I loved it. Not only that, but I also liked p-adic evaluation and quadratic residuals which really piqued my curiosity for number theory, although I actually practiced more geometry than NT xD. But anyway, I was enjoying a lot learning new things and trying to solve problems with new techniques and of course, exploring and having fun that the various problems that come in front of me. Of course, there were many downfalls, I wasn't always able to solve part of the problems I attempted, but when that happened and I read the solution, I tried to understand what motivated the solution, the reasoning path,

because that's what really matters when practicing problem-solving: knowing whether or not we are walking in the right place and solving a number of problems helps a lot to recognize this instinct to identify the best path. Problem solving is like a sense, like touch and taste, seeing things that other people don't see and that's only possible if you have some previous ideas of past problems. Once the owner of the MODS server said that a person is a pro because he can have several ideas and choose the ones that allow him to demolish the problem and I quite agree with that.

So I worried more about solving problems to improve my problem-solving sense and of course I enjoyed a lot.

Chapter 5: More contests?

Before the national finals, I still had two math competitions to participate, but they are kinda different: one was the 2023 math kanguru and the other was the national math games competition. Math kanguru is a type of competition I don't really like, because it is short answer (multiple choice style) and you need to be very very fast to do well and this is terrible because you feel very pressured during the contest, but that's my opinion and I understand why some people like it.

After some days since I received that mail about national finals, the Math Kangaroo competition day arrived and it was at afternoon. I remembered that it was raining that day and I was quite nervous because I wanted to do well in this contest to win an award that the organization gives for top 25 students around the country. When it was time to enter the room and take the test, I was quite nervous, but like, a nervousness that wasn't necessary for a silly test. I remember that three more people from my class decided to participate and that I was also with middle school kids, and the most curious thing I noticed was that some of them were interested in the kangaroo and that made me very happy, but also sad, because I wish I had had this curiosity for mathematical competitions earlier...

After a few minutes, the test had begun. My plan was to answer all the questions as quickly as possible, without thinking too much, in order to be able to answer as many questions as possible. As I was solving, I looked at the clock and saw time pass at the speed of light and that bothered me a lot during the test. After some time, I had already solved all the 3-point questions, I had left one or two 4-point questions behind and started focusing on the 5-point questions, where I spent a lot of time on those just to be able to answer 3 questions. There was little time left to finish the test and I barely had time to review the work I had done, I just took a quick look and handed the test to the vigilant teacher. In the end, it was just me and another classmate. I remember that I was very sad and furious with my performance, because taking that test in a hurry could have been a mistake and I could not get the necessary score for the award. But I just returned home

normally and completely forgot about the kangaroo, after all, PorMO was the most important thing I had to worry about at that time. Maybe if I had adopted a better strategy to do the kangaroo, I would have performed better, maybe getting all the 3 and 4 points questions right was enough for get the award, but on second thought, I shouldn't have been focused on the award but more on the competition and that maybe because I was so pressured during the race...

Oh! Now it's time to talk about the competition that was quite fun: the math games!

The national math games competition is a very fun contest where several students from various schools in the country compete in mathematical games, such as domineering, atari go, trails, product and other fun games. I didn't take that competition so serious and I wanted to enjoy a nice day in Aveiro, where it was held in the year 2023. Now, participation in this competition is not free: in order for me to be selected at my school, I had to come first place in my school's math games competition and I choosed atari go game to compete. The thing was that I didn't know nothing about atari go and the contest was in a few days, so I just speedrunned some videos about atari go on YT and I asked EricXYX some tips, since EricXYX got first place in his school too and he had some experience in the game. Unforently, EricXYX decided to not go to Aveiro compete because of school works, math camp and PorMO finals, so he was kinda busy and I couldn't meet him personally yet.

The day of the contest in my school arrived and I was quite nervous, but it was fine, since I could play and do well in a few matches and get the first place in atari go I need to participate in the national contest. Maybe I would be very upset if I didn't get the first place because at the same time that the competition took place at my school, I had a course on chess referees that needed to be completed and I had to risk this dilemma. I really don't like being stuck in this kind of dilemma because there are always risks that we are subject to, but in the end there isn't much to do to avoid that, since life is full of these risky decisions.

But anyway, the day of the national competition of mathematical games had arrived and I was very happy to enjoy a day in Aveiro. But I wasn't alone, Zex had also taken first place in the domineering, a boy who is part of the chess club in my city also managed to take first place in the product game and another boy with two other girls (one of them who was part of the club my school's chess team) from middle school also achieved the same result. That is, I was surrounded by very cool people with me for this day. The trip to Aveiro went quite well, we went to the university of Aveiro where the competition took place and each one went to compete in their respective game with other students. The competition worked by knockouts, where groups of some people were made and only one of them was qualified for the next round. Unfortunately, all of our school's team ended up tied with other students and none of us had a chance to move on to the next round due to tiebreakers and I found that frustrating. But that didn't matter to me, because we had the whole afternoon to walk around Aveiro. After the games, we went for lunch at the university canteen where I really enjoyed the food, then from what I remember we went for a walk around the city center and it was a lot of fun as it had been a long time since I had had fun with other people on walks. We also visited some churches just to get to know the monuments a little bit, ate ovos moles (and maybe this was my first time eating ovos moles) and visited some shops there, it was really fun.

Finally, we visited the city park which I thought was quite beautiful and finally we went back to the university where we received our diplomas and traveled to home. It's been a long time since I've had this much fun, as maybe I've already said, I don't have friends I can go out with or have fun with. When I got home, I realized that I was a little sick and I had already noticed that on the drive to home, so my day came to this unpleasant end.

Days passed, my mother and I were getting ready for the event that I had been waiting for since the day I saw that IMO 2022 news. My mother, always worried about me, bought some clothes and a big bag in which I could store my clothes for the days I was

going to stay in Faro. Finally, after months of waiting for this moment, after a lot of suffering and a lot of struggle to get there, after a lot of training I had with the people at MODS, after everything and more, I was finally going to fulfill my dream of participating in the PorMO national final! There was only one way to make this olympiad journey go on and last a little longer, and that was to earn the medal to be invited to training camp. I felt that this was my obligation, but I didn't let myself be shaken by it, I knew that this was all for fun, otherwise I would have given up a long time ago...

Before the long-awaited day, I had received a message from Doctor on discord last night, asking if I was prepared for the final. I remember that I had received several messages of support on MODS, wishing me good luck and fun in this great event. I replied to Doctor that I was very happy and very anxious to meet the other delfics and he replied that he was also going to be at the station in Porto, that is, I was going to meet Doctor in person for the first time. I was very happy to have received the message and got ready to sleep, I had to wake up in the best way for tomorrow.

This was my only chance to do something useful in my life, it was my last chance to go further and I was going to take every opportunity that came my way to go further.

Wait for me, because the beast is prepared for the fight!

I'll see you on the battlefield!

Chapter 6: PorMO National finals!

It was day 30 of March of 2023, and I just woke up at 6:00 AM. The mail I received from the organization said all contestants from the north region should be at Porto-Campanhã train station at 10 AM. Given that the city where I live is a little far from Porto and that the possibility of getting into traffic that day was high (since it was a Thursday), my parents and I decided to wake up a little earlier so that there is no possibility of me not being able to catch the train. When I woke up, I looked out the window and it was dawn outside, I took the opportunity to have my breakfast with my parents. My parents had no knowledge of how the math olympiads worked and they didn't even know the benefits it could bring, but they wished me good luck in the competition.

After breakfast, I took the opportunity to pack my things and get dressed, and I also took the opportunity to flex in MODS my participation in the PorMO final xD. After a few minutes to prepare everything, we leave for Porto-Campanhã station at 7 AM. I was quite happy that my parents were able to take me to the station and I remember that during the trip I was a little nervous and anxious for what could happen when I arrived in Porto, given that I had mixed expectations. After about two hours of travel to Porto, we had arrived there around 9 AM, quite early for what we had expected. I had never been to the train station in Porto and to be quite honest I had never taken a train or been to Faro, so all this for me was new experiences and that made me anxious and also a little afraid, since for the first time I would be a few days away from my parents and therefore I was a little afraid of all this, but everything in life has a first time and maybe it was time to explore new experiences.

We arrived at the station and sat down on some benches that were there, I remember that neither I nor my parents knew exactly where it was to wait for the guide, so much so that my mother was a little worried that she tried to call one of the guides, but he did not answer (after all, it was too early for the guide to be there and I tried to calm my mother as best I could). It was almost 10 AM, when I decided to go to the place where people buy the tickets to see if there was someone from the organization. When I walked

there, I saw that there was a man very similar to Doctor's face, but I wasn't sure it was him (after all, I only knew his face because of a picture of him I found on the internet). I approached him slowly, while each one looked at the other and when I finally got close to him I found out in fact that he was Doctor. We talked a little, I was very happy to have met him and we were there waiting for the other contestants. I took the opportunity to call my parents who were there waiting on the bench and it was at that moment that they left me with Doctor and they said goodbye to me, I would miss them a lot these days. We stayed there for a while and some contestants started to appear, among them some delfics whose names I recognized from that competition I participated in December.

Finally after a few minutes we were all ready to catch the trains and I remember I was really, really shy at that point. I really wanted to talk to the delfics, but my shyness just wouldn't let me and I stayed quiet until the moment we got on the train. There were quite a few people, as some of the different categories participated: those from 5th grade, those from 6th to 9th and those from 10th to 12th, where I was.

I remember that I got on the train and I could barely say goodbye to Doctor, Zex was there too and he was the closest person I could talk to, but I didn't talk to him much again because of my shyness. Finally inside the train, I was sitting next to Zex and in front of us were two more dolfics and they were sitting in front of us because of the chairs.

I remember that the train carriage was full of contestants and I was very happy to see all these young people there, some I could even see sleep on their faces xD.

Next to the carriage was a former Olympian who was also an IMOer for 3 years! And you know, I remember that at the time my desire was to talk to him, since I knew he was going to be there, but my shyness didn't allow it again. I don't know, I wasn't sure what I was going to talk to him about, what it's like to be an IMOer, or math subjects that you like the most xD? But at the moment I felt a mixture of thoughts, fear, happiness, apprehension, nervousness and others that which can't be said by words. After a few minutes of the train leaving, the IMOer asked everyone in the carriage to follow the following tradition: in a certain order, each person says hello and everyone else says hello

and then that person introduces himself with his name, age, favorite number and what they are for breakfast. I thought that was a little strange but funny at the same time and I remember being nervous when it was my turn. After this kind of "ritual", one of the Delfics in front of me suggested the other people to play UNO. I've probably played this once in my life, but I couldn't remember the rules for sure. So they invited me to play and as I had no other choices I accepted. For some reason that morning I didn't feel like playing cards and after a few rounds I started to get sad, because maybe I wasn't enjoying it and I was finding it boring and every round they created different game versions that left me totally confused and I didn't have the courage to ask to leave the game.

After a few minutes, we stopped playing because of the train's turbulence and because there wasn't even space to play. At one point, most of the people around me were talking about everyday matters and I kept to myself, silent, afraid to say something. As I didn't have anything to say, I decided to take my cell phone and see messages on MODS.

I took advantage of the fact that one of the chats was quite active and started exchanging messages with other people on the server, telling them how my experience was going.

And it was during that exchange of messages that I started to get happy and almost laugh uncontrollably on the train, but I had to make an effort not to make a fool of myself xD.





If there's a moment on that train ride that I really enjoyed, it was during that message exchange, MODS people never fail when it comes to cracking jokes and making others laugh, that server was a real circus xD. After several exchanges of messages, the signal was given to change trains, as I was taking the intercity train. When we left, we waited outside for a few minutes until the other train appeared. This time, I sat in the carriage as far away as possible from the other delfics and I was next to a young man who I didn't know if he was a delfic or not, but I didn't care, so I felt more relaxed. At that moment, the train had just left towards Faro, I thought how my parents would be at that moment, and so I took the opportunity to make a call. Also, the organization took care of us with snacks and entertainment, as the guides liked to play some games. While I ate my snack, I looked out the window and I was amazed by the places I passed. After all, I had never traveled to other areas of Portugal other than the northern region and that fascinated me. I chatted a little in MODS, as my cell phone battery was running out and I had to preserve it to make calls to my parents. I remember that train trip to Faro took a long time and I was quite bored. I couldn't sleep and felt homesick. After a few hours, we finally arrived to Faro. When I got off the train, I saw the other contestants from the southern region at the station, I could see EricXYX in the distance, but I wasn't sure if it

was him. I stayed quiet in mine, without motivation or desire to talk to those people, which was strange to me, after all, I've waited so long for this moment to... I don't know... not live up to my expectations? I didn't know, it was a mixture of feelings that attacked me at the time, from time to time I felt some nihilistic feelings. I looked at that, I knew that everything was going to end soon and that my life would have no meaning without the olympiads, but I wasn't going to let that happen, I was going to get that medal, it was my obligation, I wasn't going to let this journey end! After a few minutes, we started walking. I didn't know where we were going, but it was probably to the school or the hotel to check in. During this walk, I concentrated on all the things I saw around me, the conversations, the contestants, everything.

The city was really beautiful, I fell in love with that place: the houses, the smell of the beach, the sea, etc. I had never been to the south of Portugal and people usually go there every summer to take a vacation, now I understood why. After a few steps, we arrived at what looked like a hotel where we were going to stay. I remember when we were near the hotel, I saw an Asian guy closed to me and I was thinking he was EricXYX and so I said hello to him and he just replied with a hello, then I thought, is this EricXYX or did I say hello for a random guy xD? But anyway, we move on.

The hotel was quite stylish, but I had never been to a hotel in my life either so I didn't know if that hotel was decent or not. We all checked in and Zex and I stayed in the same room, which was a relief because I didn't want to be with someone I didn't know. There was a great coexistence between the delfics, they talked a lot and were quite friendly, but I rarely saw them talking about mathematics, and this completely escaped my expectations. But everyone seemed quite excited and happy, so was I, but along with other feelings. Afterwards, Zex and I went to our hotel room, which happened to be quite comfortable. We left some of our things in the room and took the essentials because we had to go down to the school where the tests would be done. Zex and I talked a little, he was also a little shy, but he got along well with the other delfics, so I wasn't the only shy one among those people

After we getting ready, we went down to the lobby where some competitors were waiting. When we got there, I saw the same Asian guy, who got up from the chair he was sitting on and came over to greet me. At that moment I realized he was EricXYX xD. I also greeted Hydrogen, the guy who got the perfect score at that math oly in December. I didn't talk much with them, I just talked about my participation in the opp and nothing else. We waited a while in the lobby for the other contestants and when we were all together, we walked to the Tomás Cabreira school, where we were going to take the tests tomorrow. The walk was pretty easy, the school wasn't too far away, Zex was talking to Hydrogen and I was quiet as usual. The architecture of that place really fascinated me, when we arrived at the school, I was totally surprised by the size and the fact that the school has balconies! I had never seen a school with balconies before, but it also made sense, as the climate in the city is somewhat tropical. We stayed a little while waiting in front of the school door for the other junior contestants, this time was perfect to contemplate that beautiful place. They were there too, some teachers were there in a good mood and many of the contestants were very excited, talking to each other. After some time, a sign was given for us to enter the school, which by the way was very beautiful inside.

The opening ceremony was about to start, and we all entered a very spacious room with several posters related to mathematics and tables covered with delicious snacks. I took a look at the posters, all very well done, probably the students at the school had made them. At the tables with the snacks, there were several students very well dressed, that's what a Gala event is! All my effort to reach that national final was being rewarded in the best possible way, for me that was extraordinary and I was loving it, despite being a little lonely by my side, but that wasn't a bad thing, after all, I wanted to share my glory with myself and with the people of MODS, these were my true companions on this olympiad journey that I would do my best to continue. Afterwards, bags containing pens, a small notebook and a key ring were given to the competitors, I love free stuff!

After everyone had explored the environment, all the conversations that took place in that room were interrupted by the teachers, who were going to speak about the beginning of this olympiad! After a somewhat inspiring speech, we could enjoy the snacks that were available to us. I felt like a king being treated in that environment, something I never had the pleasure of enjoying, after all, the event was so well organized that I couldn't be happier than I already was.

The day was already starting to get dark, the sunset in that place was wonderful.

After a while, we were all called to dinner in the school canteen.

While I was waiting in line for me bring my food, they were distributing ID cards to the contestants, I heard EricXYX's voice calling me, but I was about to get my food, so I couldn't make it to his meeting. Anyway, a few seconds later, he came over to hand me the card. I stayed a few seconds looking at it, I couldn't believe I made it this far.

After having taken my food, I pondered where I was going to sit down to eat, I remember some delfics were sitting at a very spacious table, only as expected, I sat alone. After all, maybe my presence among them would only create an uncomfortable environment and make me feel insecure.

I was alone, sitting on those chairs in a school canteen, enjoying the meal there: it was potatoes with peas and a soup that I don't remember the taste of, but it was decent for my taste. After a few seconds, EricXYX came over to the table where I was sitting to leave his board and then I left to get something maybe. Suddenly two young men asked to sit next to me and I kindly let them. I didn't know them and they didn't seem delfics to me, which relieved me a little. One of the guys asked how the 2nd round had gone for the three of us. Each one talked about the test and everyone was amazed at the fact that I solved the geometry one in a short time. Thus, a conversation was generated between the 4 of us, these two boys were from the same school and in the same class! Then I gave them some hints on how to tackle the geometry problems, and it seem to have been of some use. After everyone had finished their meal, we stayed outside the school waiting for the bell to go to the hotel. While we waited, the sky was already dark and that darkness comforted

me. I don't know if it's just me who has this strange feeling of being comfortable in the dark, the absence of light put me in a calm state of mind.

I don't remember exactly what happened, but EricXYX and I ended up starting a conversation at some point, if I'm not mistaken the topic was about the USAMO 2023 geometry problem. Then, when we were called to go to the hotel, we continued to talk. chatting halfway through about olympiad problems and about the techniques I was learning that were inversion and projective.

Walking in the dark in the middle of the city gives a feeling of more comfort, it seems counter intuitive, since it is at these times that crimes tend to happen, but to be honest I was never afraid, on the contrary, the night left me at peace in spirit. Despite not going out at night because my parents wouldn't let me, sometimes I had that experience, one of them was the walks I took to my chess club when there were tournaments at night.

When we arrived at the hotel, those responsible made a call to confirm everyone's presence and then everyone went to their respective rooms. I remember the delfics trying to get into the hotel elevator all together, but the elevator was too small and the attendant caught their attention. Most of them were full of that spirit of fun, leaving me a little to be desired, but I could never imagine having fun like that.

Inside the hotel room, Zex took the opportunity to show his notebook that he used to take notes on Delfos, along with some problem sheets. From what I could see from the topics in the notebook, Delfos wasn't that advanced in terms of olympic content, at least in terms of what I've already started to discover in MODS. I remember that we were discussing geometry and at a certain point I started giving a mini class on inversion, at least the basics of the technique and what it was used for. I probably didn't explain it very well, but I believe I managed to arouse some curiosity in Zex, after all, that was more or less how I discovered the inversion.

We talked for a few minutes, I took a look at MODS and took the opportunity to share what was happening that big event and after a few minutes we went to sleep. I remember I wasn't very nervous or anxious when I went to sleep compared to the night before the 2nd round. It was probably the walk around town that sweet night that made me calm and relaxed. It wasn't long before both of us fell asleep. Tomorrow morning I was going to take the first test, I knew that was my only opportunity to show myself that I'm good at something.

It was early, the noise of the alarm clock of our cell phones woke us up from our dreams. It looked like both of us had slept well. We opened the window, the view from our hotel window was not that great, but it gave access to the total view of a street, the architecture of the place was very different and that made me very curious. I brushed my teeth, dressed accordingly, in order to do the test in the best possible way, called my parents to let them know that everything was fine with me. After a few minutes, we went down to the breakfast area of the hotel.

The breakfast area was very nice and tidy, I don't know about the others, but that hotel was doing a great job in welcoming us. There were several types of food that we could choose for breakfast. I got a good coffee with a rice cake and some bread and butter, after all, I needed to be well fed since that test would probably consume a lot of ATP from my body. Zex and I had breakfast together and we also saw some contestants doing the same. The music playing in the breakfast area comforted my nerves, when I woke up, my heart was already beating a little above normal that morning and my nerves were already quite agitated, but I was calmer compared to the day I went to do the 2nd round.

After having finished breakfast, we went back to our room to get ready a little better so that we could go to school to take the test. We were both ready, so we took this time to explore our hotel room a bit. I turned on the television and noticed that many of the channels were foreign, which left me a little surprised.

After exploring all the TV channels, it was time to go down to the lobby so we could go to school. Those responsible called to confirm the presence of all contestants and we all walked to the school. The air was cold, the sun was beginning to light up the streets, the movement of people at that hour was decent for that Friday morning. I took my kit

with me which contained all the material I needed to carry out the test: compass, ruler, pencil, pen and eraser, pencil sharpener.

When we arrived at the school, we immediately entered its interior, passing through its corridors until we reached the room where I was going to take the test. My heart was beating fast, but I managed to control the nervousness with my breathing. Arriving at the room, there were 3 rows of tables, I sat a little behind in the last row, by the window, I could observe a little of the outside. EricXYX was in the same row as me, sitting a little further in front of me. There were a few minutes left until the exam started, draft sheets were distributed and the answer sheets were already on each table. I was shaking a little, I knew that this was his only chance to enter Delfos and I had an obligation to be among the top 12 of the 29 people I was competing with. Only oneshot! It's all or nothing.

Finally, the vigilant teacher (who I only found out later was an IMOer), began to distribute the sheet with the statements for each one. The national final is composed of 3 problems that have to be solved in 3 hours. I read problem 1 and found that one boring, so I skipped to problem 2, which was worded as follows.

Problem 2: Let [AB] be a diameter of a circle with center O and radius 1. Let P be a point on the circle different from A and B and let Q be the midpoint of the arc AP. The line parallel to PQ passing through O intersects the line PB at the point S. Determine PS.

After drawing the diagram, I quickly realized that OQPS formed a parallelogram. To prove it, I just had to use the fact that OS//PQ and by the inscribed angle theorem that $\angle AOQ = \angle ABP$ which quickly concluded that OQ//PS. So PS = 1. I think I managed to solve the problem in 10 minutes and I was already quite satisfied with it, now there are only two left.

The first problem had a rather tedious aspect, with the following wording:

Problem 1: Ana, Bruno and Carolina played table tennis with each other. In each game, only two of the friends played, leaving the third to rest. Every time one of the friends won a game, rest during the next match. Ana played 12 games, Bruno played 21 games and Carolina rested for 8 games. Who rested in the last match?

The way I did this problem was working with cases, after all, it was more like a logic problem than a combinatorial one, after some time, I wrote the solution on my answer sheet, it took some time because I had been wrong a few times when doing the build for the problem, but I managed to write that as best I could. There were approximately 1 hour and half left to finish the test and I already had two problems done! At first I thought the test was going to be difficult but I quickly changed my mind.

Now only the last problem remained, which was supposed to be the hardest of the three. It was a combination problem and had the following wording:

Problem 3: A box of base 4×2 and height 2 is open at the top. Tomas wants to completely fill the crate with some of your cubes. He has 16 equal cubes of volume 1 and two equal cubes of volume 8. One cube of volume 1 can only be placed on the top layer if the cube on the bottom layer has already been placed. In how many ways can Tomas fill the box with cubes, placing them one by one?

At the start, I didn't understand the problem, after all, the statement didn't say whether we had to disregard rotations, but the vigilant teacher clarified that the box was fixed, which made me quite relieved. I made some scribbles on the scratch paper to see how I could count that, until I realized that this problem had something very similar to a problem that I had done weeks ago in MODS, this problem was the problem of spiders with socks and shoes, has the following statement.

A spider has one sock and one shoe for each of its eight legs. In how many different orders can the spider put on its socks and shoes, assuming that, on each leg, the sock must be put on before the shoe?

The fact that cube of volume 1 can only be placed on the top layer if the cube on the bottom layer has already been placed and that the sock must be put on before the shoe established a bijection! The argument used for the spider problem also served in this problem 3. It was just working with the case and applying the spider problem idea, which led me to the answer very quickly. There was still an hour left to finish the test and I took advantage of that time to reread all my solutions.

After confirming everything, I handed in my answer sheets and left the room, EricXYX had already left the room, but before meeting him I went to the bathroom, it was perhaps the third or fourth time I had gone to bathroom that day.

When I left, I went down to the outside of the school, I was a little confused because there were several corridors, but I quickly found an exit. The day already had a new face, the sun was shining brightly, several contestants from the junior category were already downstairs outside and EricXYX was near the canteen talking to another delfic. I took advantage of that moment to call my parents to let them know that everything was fine with me and that the test had gone well.

A few minutes later, Zex appeared and came up to me to discuss the test. He said that the last problem was similar to the spider problem and I quickly agreed, he said that he had done the problem in Delfos and that he didn't remember the solution to the problem and that he only did the first two.

After discussing the problems a bit, we went to the canteen for lunch. I walked past EricXYX and the first thing he did was ask if he had wiped the evidence. I said yes and then he said the test was trivial. Even though I solved everything, I don't know if the word "trivial" was the best to describe the difficulty of a problem. A problem can be trivial or not depending on the person who solves it, that is, it ends up being a little subjective.

Easier would be to compare the difficulty of a problem relative to other problems. If I were to measure the difficulty of the first day in this way, I would say that this test was easier compared to previous years, which I found a little strange, maybe the team that makes up the problems has changed?

After getting my food from the canteen, I sat down with Zex and another boy from the 10th grade and at the table next to us sat EricXYX and the other delfic. We had the problem solving discussed and I found that the last problem resolution was 11 or 12 digits long if I remember correctly. I'm glad the vigilant teacher let me write the answer in simplified form otherwise I would spend a long time calculating it...

I remember that after dinner, EricXYX introduced me to another delfic that I didn't know, let's call him Mars, Mars came to greet me, it seemed from the look of him that he already knew me or something like that, but I didn't give much thought. importance. Then there was nothing else, I don't know if it was EricXYX who forced him to greet me or something like that, maybe some delfics already knew me because of EricXYX and I didn't even know it, but the fact was that I was just a new contestant just like some who were there, I didn't want to have people with expectations of me, it was like it was more pressure that I had to endure, what would happen if expectations weren't fulfilled? There was still one more day to go, the medal was not yet guaranteed. Having the expectations of a result is almost like celebrating before the result and that is a disgrace.

By the way, after having eaten the delicious food in the canteen, some people started to walk towards the school zone and I had no option but to follow along. When we got to that place, there was a football field there, one of those very common ones that probably all schools have a similar one. I didn't have much interest in playing soccer, but some contestants were playing volleyball in a circle, where each one passes the ball to someone else, with the aim of not making the ball fall. It was quite fun and I was enjoying it, at least I was doing a little physical exercise, something I didn't usually do very often except at school. As we were playing, some people started to leave the game, either

because they were finding it boring, or because they were tired. It didn't take long for me to get tired and stop playing, even the few who were playing football had already left.

I left the field and went to the large patio in front of the canteen door, where the other contestants were. I don't remember exactly what they were doing, but a lot of people were on small stairs in the courtyard and some were standing in front of them, they were playing a game and I quickly deduced that it was the "chairs psychologist". I remember just watching them play, but it wasn't for long, as it was time to start the day's activities. The plan for the afternoon of that day was to take a walk around the city, see the city museum, have the "musical moment" that I certainly didn't know what it was. We all had to carry our train ticket given to us the day before.

We left school, the brightness and heat of the sun bathed that city in a wonderful way, as if the city was made to be in those conditions, which was true due to the tropical climate in that region. We were walking around town, probably walking to the museum or something like that. We arrived there once to take a short break, we could see the sea and the boats on the horizon, I had never seen anything more beautiful than that. We had stopped in front of a very large building, what would that be? My doubts evaporated when the signal was given for us to walk again. That rural environment reminded me of the time I was in Guimarães, a very beautiful city and I enjoyed being there for two years, but those houses, those streets where they walked were much prettier, most of the houses painted yellow and white and it fascinated me, it was quite beautiful.

We stopped at the entrance to the museum, I was a little tired, but in shape to walk a little further. The junior and senior contestants had different day plans, but the activities were the same. After a while a sign was given for us to enter, the interior of the museum was very beautiful, at least I liked that architecture. During this journey through the museum, our guide showed us a lot, told us a little about the place, the vast amount of works of art that the museum contained and much more, the route somehow made my thoughts go to another place, starting to reflect a little on history and life. I don't know why, my love of philosophy was a little awakened by that place. I have the gift of

sometimes seeing philosophy in what doesn't seem to have any philosophy, maybe because my mind likes to create relationships between things, but I always admired the fact that philosophical theories gave a kind of comfort in what I observed, what was mysterious. Although I had a very skeptical mind, I loved the way various philosophers approached problems in different ways.

The tour of the museum was really nice, but my thoughts had to be stopped as we were about to leave the museum. Outside, I took the opportunity to eat a little snack that the organizers gave us, always very kind. While we ate, it was the turn of the junior contestants to explore the museum. While some had a snack, others brought several puzzles: a magic cube, a puzzle with several clocks, chess, among other funny puzzles. EricXYX apparently was a big Rubik's Cube fan, there was that mirrored cube that was pretty funny, even funnier was seeing others break their heads to solve it. Meanwhile, others played brain games. The environment that formed there was quite dynamic and fun, but despite that, I didn't see much interest in those games, maybe because I'm too old for that? I just didn't feel like or motivated to play. Furthermore, my shyness prevented me from creating any friendship bond, I didn't even have the desire to create a friendship bond, maybe out of fear? Yes, quite capable.

At one point, some contestants were distributed in a circle, playing a reasoning game and invited me to play. Since I had nothing else to do, I accepted. They explained the rules of the game but I was always confused to understand this type of games and I only understood the rules shortly afterwards. It was a game in which you had to discover a pattern in a kind of base 3 string. Each had three options for speaking: up up; down down; up down. If you said the wrong option, those who knew the rule said it was wrong and the turn was passed. I wasn't finding it fun and I was very confused, not even concentrating on the game.

I was already a little fed up with that until finally the junior contestants left the museum and we could continue with the day's activities.

We walked a little to a place where we could hear some background music, would that be the musical moment?

A large metal door gave access to the place, there were several tables spread out in the place with the large yellow walls surrounding the space, giving a view of that beautiful afternoon sky. Me, Zex, EricXYX, a delfic boy and a 10th grade boy, were sitting at a table. No one seemed to have anything interesting to talk about so everyone stayed comfortable: EricXYX and Zex were watching something on their cell phones, the delfic boy was playing chess, the 10th grade boy was talking to someone on his cell phone about the test, more specifically the fact that the answer to the last problem has 11 digits and I was looking at the MODS messages, telling others a little about my experience. After some time, the delfic and EricXYX left the table and went somewhere, until that moment, what was he doing there, was that supposed to be the musical moment? There was only a little music in the background, but nothing too special; however, the atmosphere created by the contestants was wonderful, I loved seeing the little crowd having fun, but on the other hand it made me sad, as I was never introduced to the existence of this national olympiad for several years... but anyway, it wasn't too late, despite this being my only chance to do something good in my life, something that would show anyone that I can be better at something, that I am different from everyone else.

After some time, I was already getting bored with it, but luckily the organizers asked us to leave, we had to walk again.

The afternoon of that day was really fun, at least I was enjoying the walk a lot without having to be forced to socialize with someone. We walked through the old town "inside the village" if I'm not mistaken, our guide was happy to explain why that place had that architecture. To be honest, it wasn't something that piqued my interest, again stuck with the philosophies that played in my mind.

After a good walk, I still had plenty of energy, maybe my physical fitness favored that factor, but the day's activities weren't over yet. At one point, we stopped at a place where it had already passed the day before. This place overlooked the sea and the boats on the horizon and the view was very beautiful and sweet, it reminded me of the beaches in Brazil when I was there for two years.

Not far away was one of those tourist trains that tour the city, that would be our next activity. The train was quite long with small carriages. I didn't know where I was going to stay, each carriage had a limit of people, I wanted to get into the carriage that EricXYX was in but it filled up quickly, so I went to the first carriage which was more spacious than the others. Some contestants were already there, the delfic boy was sitting a little further in front and the two boys from the same class with whom I spoke yesterday at dinner were on my left and on my right a boy I didn't know.

My carriage filled up quickly and it didn't take long for all the contestants who were present to get on the train, we gave our tickets and shortly afterwards the train started to move, starting this little tour.

It didn't take long for a conversation to start inside my carriage, I remember one of the boys starting to discuss the test on the 1st day. I saw that this was something that interested me and I decided to join the conversation when they asked me how the test went and how I had solved the problems. The guy on my right had cleaned up the test and was already suspecting he was delfic, when one of the guys asked if he was delfic, the guy next to me said he was there for a while during the pandemic. Let's call this guy Mars. The train measure and make its way, the conversation was getting more and more interesting, so interesting that even my vigilant teacher that day decided to participate and talk a little about the olympiad. As I already said, he is an IMOer, he has participated in IMO twice and was a deputy leader later for a few years. He made a friendly flex when he showed the deck of cards from the IMO 2011, the year he was a deputy and witnessed Portugal take its first gold medal at the IMO.

Mars was also discussing the junior contestants test, a bit suppressed that our day 1 test didn't contain problem involving the number 2023.

That conversation began to extend in a pleasant way, I had never talked so much with someone since I arrived at that place. Mars kept discussing math with us, it was clear that he knew a lot and I was happy to finally have someone I could talk to without feeling insecure. As we talked, I admired the city through the train window, while a little music from the carriage's loudspeaker provided a slightly more lively atmosphere.

At one point, the train suddenly stopped right next to the metal door of the "musical moment" place, where some junior contestants were, it seemed that the driver had run over a kid, so much so that another boy who was in the carriage had the same thought as me, but that was not the case. At one point, music played from the loudspeaker that made me go into an existential crisis for a few seconds, but it was quickly cut short by the hustle and bustle of the conversation.

Mars continued to spout some math facts, whenever a given conversation ended he would start another one with whatever math stuff was stored in his head and my desire to talk more was strictly increasing. After a while, the tourist train stopped. Time seemed to fly by with that conversation, but whether it was short or not, I loved it a lot.

We stopped at the same place where we had taken the train. The view to the sea at that time was more beautiful, because of the approach of sunset. Being on the train or not, Mars would not stop talking and none of us who were following the conversation managed to get bored. The conversations were of the most diverse subjects of mathematics, of which a large part I knew and could discuss. Some of the boys who were with us on the train, who were not delfic, were amazed by each subject, captivated by the wonder that mathematics has.

We started walking to school, apparently there weren't any activities for that day other than having dinner at school and then going to sleep at the hotel. I felt like I had more energy after that brainstorming with Mars, I remember at one point as we were walking, we were talking about numerical bases and really amazing things like base matrix or base quartenion, he had this peppery taste. I remember that on the tourist train, he told me that, as he had finished the test earlier, he decided to solve that simple problem 2

with barycentric coordinates, which is almost equivalent to killing ants with nuclear bombs.

Finally, after a few minutes of walking and talking, we arrived at the school. The organizers asked us to stay on the patio until the canteen opened for dinner. Meanwhile, the conversation with Mars continued and apparently we were the only ones talking in the middle of those contestants, attracting a few looks, but nothing that bothered me. Some contestants who were with us on the train continued to observe the conversation and I liked that, seeing people interested in the countless wonders that mathematics has. At some point, we formed a small group, and we went to the school's soccer field, staying behind it, if I remember correctly our group was the only one there, which gave me more freedom to talk.

And we stayed there, as if we were old men in a cafe discussing politics. Mars picked the most diverse subjects out of his hat: quarters; differential geometry, relativity theory, number theory, gaussian integral, multivariable calculus, and the list goes on.

We spent a good amount of time in that corner of the school discussing the most diverse topics in mathematics, as if we had nothing else to do with our lives but to learn more and more about the queen of science. I didn't feel so uncomfortable talking anymore, it seemed that talking about something I like and I'm open to talking made me feel a little more at ease, something I hadn't felt for some time, after all, I hadn't lots of people to discuss math with other than MODS and the math servers. The conversation seemed like a semi-direct, which had a beginning and no end, at a given moment a contestant appeared in our corner to tell us to go to the canteen for dinner, and that was it. We walked to the canteen and stood in line, still talking. It seemed that the heads of the boys who were with us were bursting with so much knowledge. I remember that the subject that seemed to surprise them the most was Godel's incompleteness theorem, the way Mars explained it to the others was sensational and I even concluded that "this is just a result that mathematics goes far beyond our heads".

After each one had taken their food, which by the way looked appetizing, we sat down closest to Mars, that conversation didn't seem to have an end. This time he the topic he brought to the table was googology, a branch of mathematics that I didn't know and which by the way was the branch that Mars liked the most. Basically, googology is dedicated to the study of large numbers and I didn't know that such a branch could have very interesting things, after all, who would have thought that a large number could have something interesting? He introduced me to Graham's number and its history, simply the largest number ever to appear in a mathematical proof, for a problem involving painting the edges and diagonals of a cube. While I ate and Mars told more googology facts, the delfic group were sitting at a table behind us, I don't remember exactly, but they were talking about life issues, I think it was about abortion but I didn't care much. One thing I noticed is that the delfics spoke little about mathematics, they seemed more social when discussing everyday things with each other and maybe that was one of the reasons why I didn't approach them. But the fact that Mars started a conversation with me made me feel more comfortable talking, that is, more at ease.

After dinner, we went to the patio, the sky was already dark and the harmony of the night made me more comfortable. Before we went to our hotel, some took advantage of the time to chat, others admired the night, while me, Mars and my delfic friend were talking about fractals and a Gabriel's trumpet paradox. So far everything was going well, I had cleared the first day's test, I had already had a lot of fun talking about math topics and I really enjoyed the pass I had made during the afternoon. While Mars and Zex talked, I thought about tomorrow. Tomorrow I had to do everything masterfully during the race or the dream of going further would be destroyed, but I believed in myself and I wasn't going to let any opportunity slip away.

After a while, the guides called us and we all started walking to the hotel. Again, the dark environment that the night provided left me calm and at peace of mind. When we arrived at the hotel, each couple went to their rooms. Zex and I talked a little about the day and before I got ready for bed, I took the time to look at the messages on discord.

MODS was not very active at that time, so I took the opportunity to share my experience of the day. Doctor was online and responded to my messages, feeling happy that I was having fun at this event. EricXYX was also online and also appeared for the conversation. However, suddenly an individual appeared in the chat with an arrogant and ignorant tone called Zay, exchanging provocative messages with Doctor and I couldn't do anything other than laugh at this poor Zay. He also wished me bad luck for the 2nd day, but I wasn't shaken, although I felt like blocking this son of a bitch who doesn't know how much I suffered psychologically to reach this national final. After the circus that was happening in the chat, I said I was going to sleep and Doctor and others wished me good luck. I took the opportunity to call my friend to say that everything was fine and went to sleep, meanwhile, my delfic friend was watching a video about Hilbert's hotel paradox. Finally, we turned on the lights in the room and went to sleep. Tomorrow was a new day, it was my only chance to do something I was proud of, it was my only chance to go further.

Chapter 7: The Fall?

The alarm clocks went off and then we both woke up. I felt like I woke up a little sick, after all it didn't seem like I had recovered 100% from the flu I caught in the math games championship, but nothing that could affect my performance. It also seemed like Zex had slept well, we opened the window to bathe our room in light and got ready to have breakfast in the hotel lobby. Arriving at the breakfast place, I decided that I would eat a rice cake with some bread and a great coffee to wake up my dormant nerves. Everyone seemed very calm and the music playing in the lobby (it seemed to be LoFi) made the atmosphere even more pleasant. While eating breakfast, I thought about what could be on the test. As yesterday problem 2 was about geometry, today it was quite possible that problem 4 was about geometry and perhaps combinatorics in problem 5 as PorMOs like to put combinatorics problems in the tests.

After Zex and I had breakfast, we went to our room to prepare for the long day we were going to have. We took all the necessary material and then went back to the lobby. The guide made the attendance call and we left the hotel, walking to the school where the battle would take place. Mars was talking about chess with two classmates from the same class and despite wanting to participate in the conversation I didn't, my attention was focused on the test that was about to take place.

Once there, we went to our room where the test would take place, I sat in the middle of the three table rows and waited until the test started. Meanwhile, the two boys in the same class were watching a video about modular arithematic while Mars was talking to them. They seemed to be fascinated with the subject, but this was soon interrupted by the upcoming test. There were just a few minutes left, my heart was beating and I felt the adrenaline taking over me, the adrenaline I loved while taking the tests. Now was the opportunity to achieve glory and go far. Finally, the vigilant teacher began handing out the exams and the battle had begun again!

I picked up the test and started quickly reading the problems, the first was a geometry problem and that was the first one I attacked without mercy or mercy.

Problem 4: Let [ABC] be an equilateral triangle and P be a point on AC such that PC = 7. The straight line that passes through P is perpendicular to AC intersects CB at point M and intersects AB at point Q. The midpoint N of [MQ] is such that BN = 14. Determine the side of the triangle [ABC].

After drawing the diagram, I immediately noticed that it was a semblance of a triangle that I had to work with there and it took me a few minutes to find the answer to the problem. The test had barely started and there were only two problems left to kill. Everything was going well, until the next problem changed the course of things.

Problem 5: In the village of numbers, the houses are numbered from 1 to n. Meanwhile, one of the houses was demolished. Duarte calculated that the average number of houses that still exist is 202/3. How many houses were there in the village and what is the number of the demolished house?

When I read the statement, I quickly remembered that this problem was very similar to one that had appeared in MODS, only it used different numbers instead of 202/3. The problem is that it works with large numbers and that's what started to scare me a little. What I started to do was transcribe the problem into mathematical language and what I had to solve was the following quadratic equation.

$$6x = 3n^2 - 401n + 404$$

Where x is the number of the demolished house and n is the number of houses. The numbers in coefficients were quite big and I started by trial and error some values of n, and after lots of time frantically calculating numbers, I hadn't even found an

approximation to the number I was looking for and I was starting to get worried. He just had to solve this problem to be among the medal candidates and even then it was not guaranteed. My heart started to beat faster, I thought of new ways to reach the amount, all of which required heavy bills and I didn't have the energy for that. There had to be an elegant solution, after all, I had forgotten that this was an olympiad.

A good amount of time had already passed for the test, there were 40 minutes left to hand in the test, I already had several used draft sheets lying around on my work desk and I didn't have any good approximation for problem 5. I didn't even read problem 6 for fear of wasting time on this one and not being able to solve it. I wanted to cry and be angry, I wasn't going to leave this national final with clean hands, in my head I couldn't admit that! I couldn't admit losing after everything I suffered mentally to get here! I took a deep breath, closed my eyes for 10 seconds and opened them again, looking straight at the quadratic equation.

I reviewed all the steps I took until I arrived at the problem equation. I suddenly asked myself, why not just factorize n? I had thought that when I started the problem, but as it never got me anywhere before, I didn't consider doing it. But I had already tried everything, so it didn't cost anything, until I noticed what I had in my hands.

$$6x = n(3n - 401) + 404$$

The path to the answer was always right under my nose! Fucking hell! How had I not noticed that?! This was exactly what I needed! Look how beautiful this is! If 3n < 401, then n=133 is the only value that works otherwise RHS would be negative. If 3n > 401, then n > 134 wouldn't work because x > n. But 134 doesn't work because RHS wouldn't be divisible by 6. Therefore, 133 is the only value!

I quickly, without wasting time, started writing this on my answer sheet with time very close to running out. I started to feel relieved that I had gotten the answer, but I hoped that I hadn't been mistaken as happened in the 2nd round of this Olympiad. This

problem is funny, because it is easily solved with a calculator, but without one it ends up being a little unclear how to find the solution. I should have used my head and not trial and error throughout the entire race.

After doing the math, the test time was up and I was still finishing writing the solution, it might not have been written very well and that bothered me a bit.

I didn't even have the chance to try problem 6, it wasn't worth it anymore because Mars, who was behind me, immediately started discussing this problem with the others. Zex, who was in the row next to me, asked how the test went and I said that I had done the first two problems, he also did it and said that the answer to problem 5 was 133, just as I had found, leaving me with a little more relieved.

But I was quite sad about my performance. Why the hell did I think trial and error was a good idea to solve the problem?! This made me a little furious, but now that the test is in, I can only pray that my work is enough for me to be rewarded with a medal. I also thought, who the hell made these problems? After all, they seemed too easy to be in an olympiad and the reward becomes more competitive. Mars had solved the first two and had arrived at the answer to problem 6, but had not found the proof.

Before we left the room, EricXYX appeared and asked how the test had gone and sadly I said that I had only done the first two problems. He said he solved everything, as I expected, after all this test was too easy for what he knew. We left the room and went to the school courtyard, some contestants were outside and the sun was shining with all its intensity, it seemed like a sign that the day was all ours to enjoy.

I took the opportunity and called my mother to let her know that everything was fine with me, but it wasn't true, I felt a huge disappointment inside me about what I had done in the test, keeping that feeling of disappointment to myself that morning.

After all the contestants had left the interior, we went to the canteen for lunch. The food looked quite appetizing and they were even serving ice cream, but my intestines quickly refused to eat anything, but I had to try hard not to stay hungry all afternoon. Mars was next to me, I told him what I had done in the test and he said that I would

probably have silver or bronze, but I immediately rejected it saying that the test was too easy and that many people had probably managed to solve the first two problems.

My desire now was to go home, I didn't want to wait until tomorrow to find out the result and most likely I wouldn't have fun throughout the day. I felt very sorry for not having found that factorization sooner and I was even embarrassed to tell others what I had done during almost the entire test. To better deal with this situation, I was skeptical about the result, after all, the path I took to get here was the thing that makes me the happiest. But even so, a hint of sadness prevailed.

After all the contestants had eaten, the guides separated us into groups, as we were going to take a boat trip to a beach in Faro. The group I stayed in was the last to be called. While our group wasn't called, the guide who was with us and who was an IMOer came to talk to us about the test, he also commented on a solution to problem 6, now in fact it wasn't worth trying to solve the problem later.

He also asked if anyone had a solution to problem 5 and I told him factorization. This was probably the most optimal solution, but I wasn't sure. Then they started talking about subjects that I don't remember and I sat in a place nearby alone, I wasn't really up for conversations, I just wanted to go home and live with my defeat.

After a while, he gave the signal for us to go. So we left the school and walked to the place where we would catch our boat. As we walked, Mars spoke to our guide, he was also accompanied by those two boys from the same class and others he didn't know, but there were less than 10 of us if I'm not mistaken.

As we were walking, I noticed that our guide was starting a game with us, which is called "the dream": basically, our guide had a very fantastic dream last night, but telling it to us in words didn't have that emotion, So the objective was to try to discover the guide's dream, where we ask questions and he answers with yes or no. At first I didn't find it that interesting, so much so that I didn't even ask any questions, but along the way, it was starting to get a little strange.

While they were trying to figure out the guide's dream, we had finally arrived at the place where we were going to catch our boat. The view of the sea was beautiful and that hot sun only contributed to the sweet view of that place. The boat we took wasn't that big, but it was enough to take around 30 people I think, so much so that we weren't the only ones there. After we got on the boat, the others continued to play the dream game. Mars frantically asked questions to try to figure out what the dream was. From what I remember, the dream had a commode dragon on top of a building that was spitting poison and it jumped to another building that had other commode dragons and when it jumped, it broke the dimensions of time and I don't remember anything else, but it was the most random thing in the world. At first I thought it was a game of rules and not about guessing a dream, but I didn't pay much attention to it. I spent the rest of the boat trip looking at the view from there and enjoying the sweet warm air that that day provided. At least I had calmed down a little since the test was over, but I still felt remorseful about what I had done. I wish I was home, I thought.

After a few minutes of boating, we finally arrived at a small island with several houses and a strong wind. We were given a signal to follow the guides and in the meantime we started walking in the middle of the sand. The bit of sand that got into my shoes bothered me, but I managed to deal with it. While walking along that beach, Mars was playing a game of rules with other people. It was a game that consisted of a journey between countries that I don't remember exactly, but it hadn't piqued my interest. I was just walking wondering when we were going to stop to rest, because it felt like the sand was sucking all my energy.

My motivation to enjoy the day was gone, but it wasn't just because of the test, I felt like I was having an existential crisis. After all, I thought what would I do if I didn't get a medal? It was practically the end and I would be left aimless, without a path, without a meaning. I didn't want to go back to the boring life I had before the math olympiad. I've focused so much on this math olympiad thing in the last few months that it made me afraid that this would all end. I felt like Sisyphus who wanted to lift the stone

without ever reaching the top, I didn't want this to end, and if it did I would feel like I would no longer live but just exist, with a totally boring life. It seems stupid to think about it and ridiculous and dramatic, but for a boy who had never gone this far it was an astonishing thought.

After taking a long walk along the beach, we stopped to rest on a small beach with beach houses around it. Some took the opportunity to talk and socialize, but I sat in a corner alone, looking at the sweet view of the sea that satisfied my eyes. The guides gave us snacks and as I looked at that sweet landscape, I ate my snack with my hand shaking a little. I picked up my cell phone and noticed that it was almost out of battery, I needed it to talk to my mother later. As I looked at that view, I felt my body heat up a little more than it should have. Maybe I would get sick again, the worst time for that to happen. I had started to feel some headaches and the heat from the sun only contributed to that.

Finally after some time, the guides asked us all to follow them. I didn't feel like walking at all and I needed some rest, but I couldn't just stand there like a statue. We walked through a kind of small village on that island, with small beach houses, people enjoying the sweet day, well, for these people it was just another normal and peaceful day, at least that's what it seemed like. After a few steps we had stopped at the place where we were going to catch our boat to return, but for some reason it took a while and everyone started to sit down on stone blocks, and I did the same. There was an ice cream stand nearby, but my lack of motivation prevented me from buying one. I could see some junior contestants having fun there, and all I wanted was to be their age and be there. By this time the sun was less intense and the view was wonderful.

Suddenly a guide came to me, informing me about my train journey to the station where I had started. At least they were kind enough to pay for transportation. After sitting for a while, they signaled some contestants to get on the boat and I went without hesitation.

I sat in the most discreet corner on the boat, I really didn't feel like talking to anyone. I just wanted that headache to go away for a bit. However, some ladies sat next to me, I assumed they were tourists or something like that, but I didn't care much.

The boat started to fill up quickly and I think the contestants sat in an area a little away from mine. Bored as I was, I took the notebook I had with me and thought about solving a problem. The only one that came to mind at that moment was an NT from USAMO, so I started to attack the problem. A few minutes had already passed, the boat sailed smoothly through the sea and I was engrossed in the problem, one of the ladies next to me interrupted me and asked if I was solving a problem from the test and I kindly replied no. Time passed and I made some progress, every now and then I looked at the horizon to see if my headache would ease a little. Maybe I shouldn't be solving problems when I have a headache, but I'm a masochist, damn it.

The boat had finally reached shore and hadn't finished solving the problem yet. When I left, I didn't hesitate and started following the guide, I didn't really want to talk to anyone, but I didn't walk very far. EricXYX appeared next to me and asked if he enjoyed the boat trip. Before I could respond, one of the delfics called EricXYX and he went without hesitation. I think he had already realized that I wasn't well, and in fact I wasn't. As we walked to school, I took the opportunity to look at the sweet view at that time of day. That city was really beautiful, maybe I would think about returning to that place one day. But, my head was also full of thoughts, do you know what it's like to have expectations of something and... they are very far from reality? Well then, maybe that's what I felt. I had sacrificed so much to get there, I dreamed of being in the final, and when I arrived...things didn't go as I expected. I don't even know what I expected! Honestly, my only desire was to get my medal and go to the next level, but the fact that the test went a little badly on the 2nd day and that my social anxiety didn't help much made my joy fade into sadness. It felt like I was being dropped from the very stone I was carrying. I felt like a fallen angel: I wanted to be among the elite of young mathematicians and I was going to end up not getting in and going back to my boring, shitty life.

After a long walk we arrived at the school. I went to check my cell phone and noticed that the battery had run out, but as I had a charger with me, I immediately took the opportunity to charge it inside the school canteen, which was open. Most of the contestants were outside and as I had seen some of them before entering the canteen, they were playing the game "psychological chairs", which I didn't have the slightest desire to play.

While my phone was charging, I sat at a table and picked up my writing material to solve a problem. I had remembered that before coming to the final, I was solving a problem from the EGMO inversion book at home, so I tried to solve the problem again. While I was solving it, EricXYX appeared next to me and told me that according to his statistics, I was going to get a medal, but he still wasn't sure and I was completely skeptical about anything related to the final classification. Having expectations would only fuck with my head more and I didn't want that. Then he asked what I was solving and I said it was an inversion geometry problem, as he didn't know how to use it, he left and went to the others in the school yard to play that game.

After that, there were some junior contestants sitting at a table together and as I couldn't do much progress on the problem, I went to see them and see what they were doing, because I feel more comfortable talking to people younger than me. When I saw it, they were playing mobile games. So I gently asked them how the test had gone and the discussion was a bit funny. As they were young, they didn't care much about their performance in the test and the way they took it as a joke made me happy. One of the boys participated in the olympiad because his school teacher forced him to and he managed to reach the final, I was like: WTF HAHA!

While I was enjoying the temporary happiness of watching them discuss their problems, suddenly EricXYX appeared saying that Doctor had arrived. I was a bit in shock, as he knew from the MODS messages that I was unwell because of social anxiety and even I didn't know exactly what I was dealing with. I kindly told EricXYX that I would stay, I didn't have the courage to go outside and see Doctor, I was so sad. EricXYX

left and I continued to be with the juniors. As I watched one of the juniors doing flex because he "thought" he had solved an old national finals problem and I was laughing a lot, I felt a presence behind me. I thought it was EricXYX but when I went to see it was Doctor. He asked what I was doing and I said I was talking to the juniors and watching them struggle to solve difficult problems.

To talk a little, we sat down at a table and started talking, I told him how it was going (terribly) and also about my disappointment in relation to my expectations.

He felt worried because of my social anxiety, which was normal, after all he saw the walk I took to get here and didn't want him to see me like that. I simply said that I didn't even know, I just felt uncomfortable being with the other delfics and I really imagined myself in a nightmare when talking to them.

I told him about my despair at not getting the medal I needed so much tomorrow and about my Olympic journey ending so soon. While I was talking to Doctor, EricXYX appeared again and came to talk to us, EricXYX was already preparing for the IMO selection tests and I really wanted him to be selected, because from what I had seen EricXYX is a fanatic about this olympiad. After we chatted a little, the canteen was already serving dishes, which meant it was time for dinner. The food looked appetizing again, despite being a little sick. At that time of day I was just existing and not living, with a sadness sweeping my soul. I took my tray and went to sit at a small table there, but Doctor asked me to sit with him at a more spacious table, where the other delfic would probably sit. I couldn't refuse that request, after all, being present next to Doctor already brought me some comfort, I just didn't know if that comfort would later be destroyed next to the other delfics.

But anyway, I sat down next to him and the rest of the delfics started to appear, very excited and social as always, sharing their life stories or whatever it was. I just ate my food quietly, with the topic of discussion entering my ears. They seemed to be talking about some math camp experience or something like that, one of the girls felt embarrassed talking, but they were laughing and having fun, she was aesthetically beautiful by the way.

Even Doctor was having fun in the conversation. As the conversation progressed and I ate hungrily, my mind was free to think. Thinking about how much of a shitty social life I had until then and how it hurt me in so many aspects of my life. It's horrible to live in an environment where no one understands what you feel and like, you feel like you're the only person in the world and that no one else shares the same tastes as you, it was like that at home, at school and everywhere else, social places. It was in these moments that I felt grateful for being born in the 21st century with access to the internet, because it was my greatest refuge and continued to be until then.

Now that I looked at those delfics, they seemed to have a stable social life that allowed them sweet doses of fun, which makes me think again that this world is shit, in which some are born in better conditions than others, people die without knowing why, people there are so many opportunities ahead and others kill themselves to have an opportunity and when you are born you are forced to survive in this injustice that is life, I wish I had a trilobite fossil to remind me that my existence is due to the non-extinction of this animal.

As I finished eating I saw some people behind them setting up musical equipment in the canteen, was it a musical moment? I thought. It looked like he was transforming the school cafeteria into a music show. I had already eaten and the others continued to talk, I quickly felt bored, I wanted to exchange some messages on MODS but as my cell phone was almost running out of battery, I decided not to do it, but it was still no use because the server was not usually active at that time. After everyone had finally finished eating and talking about life, we returned our trays and I could see the lack of satisfaction that Doctor could see on my face.

All the contestants had already eaten the wonderful canteen food and were all sitting around the canteen and people were playing music on the canteen's "stage", filling that place with some animation. I sat a little close to the "stage" where the "show" would take place. I wasn't close to some delfics and Doctor looked like he was talking to last year's IMOers, so I took the opportunity to watch the show, to see if I could relax.

It was fun at first and then funnier when one of our guides decided to go to the "stage" to show the beautiful fun art, but I quickly started to get bored and have a slight existential crisis that made me feel sad again. Now that I thought about it, if I had known that the national final was that psychological hell, I would never have dedicated myself to getting there, but I would also never forgive myself for not having tried to go further ans searched the glory, which made me think that I was in a situation all this time. Trapped in a dichotomy. It reminded me of some words by Albert Camus that I had read in one of his books. I was so grateful for the existence of philosophy, it was my medicine for these absurd thoughts.

To escape these thoughts for a bit, I picked up my notebook again and tried to solve that inversion problem again. Even with the music being a little loud, I was able to just focus on the problem.

After a few minutes, I was already making some progress, I just needed to prove the competition of three straights. Suddenly, Doctor appeared next to me asking what I was doing and I said I was solving an inversion problem in the EGMO book. He pondered a little and quickly noticed what I was trying to prove. After a few seconds, he had already proven the conjecture but didn't say it, leaving me as an exercise haha.

While I was thinking about how to prove that, the musical moment was about to end and it was time to return to our hotel. Before leaving the canteen, one of the delfics came to ask what I was doing and I said I was solving a problem with inversion, which made him react a little scared for some reason, maybe it was because he wanted to go to IMO that year (as he had said that on the train ride). When I left school, the darkness of the night did the favor of easing my head a little. When all the contestants left, our guides took us to our hotels. Walking in the middle of that city at night brought a feeling of calm and relief, but it would be calmer if I were alone.

Finally arriving at the hotel, we all gathered in the lobby. Apparently that night was special, because apparently it is a tradition for every national final to have a game night. This game night would take place at the junior contestants' hotel and our guide

asked who wanted to go. At first I thought about going as it was my day to see if I could have some fun, but as I knew they were going to play those games where you have to be social and taking into account that sleep was already taking over me, I refused to go, maybe I'm too old for that. The vast majority said yes and I think only four people including myself chose to stay. My roommate said he was going, which made me happy because I could finally be alone in my own little corner.

We went up to our room, my delfic friend asked if everything was ok with me, as he had already noticed how sad I had been throughout the day. I just said he was feeling a little sick and decided to stay. I took advantage of the fact that I already had some battery in my cell phone and went into MODS to see what was going on. As always, at that time the chat was not very active and I just sent a message of pure sadness to everyone because this would probably be the end of my Olympiad journey and because I wasn't having fun in this national final, it seemed that from yesterday to today the dream had turned into a nightmare. No one in the chat responded, which I was already expecting. Finally, I said I would try to study a little about MMP and see if I had some fun.

I noticed that my throat was getting a little worse and that I was a little sweaty, I thought it would be better to take a shower first. The hotel bathroom was very comfortable and met all my needs. While I was taking a shower, I heard someone knock on the door of our hotel room and I immediately knew it was Doctor because of the messages I sent on MODS and that he most likely saw. And in fact, it was Doctor who wanted to talk to me, said my roommate. This made me happy and a little sad, because I like Doctor's presence, but I wanted to be alone for a while and try to reorganize my thoughts.

I quickly finished my shower, put on my warm pajamas and when I opened the bathroom door, I could see him on the balcony looking at the beautiful night sky. I felt a little embarrassed about being in my pajamas, but he didn't care.

It came immediately to see if I was okay, because from the messages I had sent it seemed like I was completely depressed. I tried to explain that I wasn't very confident about winning the medal tomorrow, my anger at the problems being too easy, that I didn't

feel comfortable being with the other delfics, in short, my displeasure with this national final. He understood what I was saying and also showed concern for me, because he barely knew who I was, I was just a boy who had come from nothing and was looking for glory in the olympiad. So I took advantage of that moment to tell a little about my past, about the bullying I suffered when I was younger, about all the afternoons I was alone after classes because my mother and stepfather had to work, about my discomfort in being with the other delfics and other reasons why I was like this. I also said that that final was beyond my expectations: I thought the delfics talked more about mathematics but that wasn't what it seemed like. I also said that I didn't want any of the delfics to enter MODS, I didn't want them to enter the only space in which I felt comfortable and happy. Doctor was a little surprised and must have understood my kind of social anxiety, but he tried his best to make me change my mind and go to the junior contestants' hotel for game night. I insisted that I didn't want to and then, as he saw that I was comfortable talking about mathematics, he decided to talk about mathematics with me, in which I had to set the tone for the discussion. I searched through something in my head and noticed a result in geometry that could be proven by MMP, but I didn't know any other way to prove it. So I proposed this to Doctor and he explained to me how to solve it with ceva and trigonometry, taking a sheet of paper in the middle of several sheets with very complex notations. Then, I suggested that we solve an NT from USAMO that I was trying to solve on the boat and couldn't and he kindly showed how to solve it, while I tried to follow his steps. Next we saw an NT from Japan MO and it was a lot of fun as we sorted things out that sweet night.

After we solved problems and talked a little, time passed and it was already around 1 or 2 AM I think. I was already feeling a little better, until it was at that moment that Doctor proposed again that we go to game night. I didn't want it at all and I explained to him that I wouldn't be comfortable or happy among those people. But after those sweet moments solving problems and talking, I couldn't disappoint Doctor, he had no obligation

to be there and he was trying to help me. I pondered for a while while looking at that sheet full of logical reasoning, until I finally accepted Doctor's request.

I got ready quickly, dressed in cozy clothes because of the cold that night, and we left the hotel. The streets were practically deserted of people and the cold of the night spread through the streets. The comfort I felt walking that hour alongside Doctor was enormous, I felt calm and relaxed, but on the other hand, a hint of fear began to grow in me as we approached the junior contestants' hotel. I was impressed by the fact that Doctor knew the way to the hotel at that time of night. Sleep was also beginning to take over me, I wasn't used to staying awake late at night.

After a short walk through the streets of that beautiful city, we finally arrived at the hotel where the junior contestants were staying. It looked like a hotel that was a little old, but comfortable.

When we arrived, Doctor said that the contestants were on a floor below the lobby, where Doctor and I were. But before going down, I told Doctor that I was scared and nervous. It was then that we decided to sit on one of the sofas in the lobby. It was already very late, game night would probably end in a while. I could occasionally hear some voices coming from downstairs. I felt embarrassed to be there, after all, arriving late to a party with so many people was like asking everyone to laugh at me. Before Doctor and I spoke, he went downstairs to see what the others were doing, when he came back, he said that everyone was playing chair psychology, the game I least liked and found boring. I said I was scared and didn't want to go down there, and Doctor insisted that there was nothing to be scared of, he would be by my side. It was then that I lay down on his lap, with tears almost streaming down my face.

Some contestants were slowly leaving downstairs and some noticed our presence in the lobby, but ignored it, leaving me a little more embarrassed. At one point, Doctor returned to one of the things I had said in the hotel room, which was the fact that I didn't want anyone to enter MODS because of something I had said in one of the chats. He asked me what this thing was and I was sure he knew what it was, but I wasn't sure what he was

referring to. I said so much shit on this server that I didn't know exactly what it was. Was it the fact that I was asexual? Lover of the philosophy of Satanism? There was so much that I decided to just answer that I didn't know what it was, but I was hiding a much more important reason for not disclosing the server to others: Preventing others from having access to the server's resources to train for the IMO TSTs. But what was the point, if I didn't receive a medal it would all be in vain. I didn't say this to Doctor so I wouldn't seem too selfish, but this was my last chance I had to go further and I couldn't waste it.

After Doctor insisted a lot that I talk and kept saying that he didn't know what it was specifically, he gave up and asked me again to go with him downstairs to meet the others. Before I could refuse, he asked me what the people at MODS would think I would do, and come to think of it, most of the people I had met at MODS would support me in doing what Doctor was asking. It was then that I gave in.

We left the lobby, went down the stairs and entered a sort of room full of tables and games, sweets scattered around some of them. A little further in the corner, there were the contestants and the guides playing the chairs psychologist. I could see the quick glances in my direction, but it didn't last long and no one spoke of my presence, which made me quite relieved. The contestants stood around in a circle asking questions to a group of 6 or 7 people, who were almost all the IMOers from last year, except one who traveled to study in the USA. It's strange what I thought, because I didn't see them as normal people, the fact that they are IMOers already commands a certain respect and I never imagined myself being in the same room with them, where they are having fun just like the other contestants. I wanted to ask them a lot of questions about math and the olympiad, but social anxiety and shyness just won't let me.

I watched them play and kept quiet, until Doctor suggested we play mime. We then walked to another room on the same floor, which was smaller with tables full of sweets and appetizers. By then 4 more people came in and wanted to play. Basically, one person would volunteer to imitate without speaking while the rest tried to guess what was being imitated. Time passed, I started to get more and more sleepy, it was probably already 4

AM and my body was crying out for sleep. I participated very little, I made a guess but soon regretted doing more, I wasn't in the mood to play that game.

At a certain point, Doctor realized that I was sleepy and came to see me if I wanted him to take me to the hotel where I was staying. I felt so ashamed, as I was 18 and about to turn 19, acting that way as if I were a shy child who needed a guardian to help me. I said yes and we walked back to the hotel.

It seemed like I was almost falling asleep in the dark of the night, only the street lights were there to wake me up. As we walked, Doctor said that he was a little sad that I didn't want to disclose MODS to anyone in Delfos because there was a lot of useful material there and I just replied that he wasn't obliged not to disclose it and do whatever he wanted, but it seemed like he I wouldn't share it for me.

Finally arriving at the hotel, we went to my room and Zex was still awake, probably waiting for me. Doctor took the material he had left in the room and said goodbye to me. I was immensely relieved to be in my room and barely expected to lie down on that bed and fall asleep. I put on my pajamas again and wrapped myself in the covers of the sweet hotel bed. I already felt a little sick and hoped I wouldn't wake up any worse. Tomorrow could be a goodbye to all this, or it could be an invitation to be part of the elite of young mathematicians in the country. I had never gotten this far, I just wanted to prove to myself that I was good at something. That I could somehow be respected with this award.

Well, you've done your job Aleph, tomorrow we'll see if you're worthy of being in the Delfos.

In the middle of the darkness, I closed my eyes, took a deep breath and let sleep take me.

Chapter 8: The moment of truth!

The sound of the alarm clock made me wake up immediately, I felt that I had woken up full of energy and that I was a little sicker: the fever was already starting to show its signs and my throat was starting to get irritated. Maybe the cold last night made my health a little worse, but I was decently well facing a long day, the last day of the national final.

My roommate and I got ready and went to the lobby to have breakfast. Everyone seemed a little sleepy, what time were they awake? I asked myself. I had a bit of a headache, my body probably wanted to punish me for being up so late. After having breakfast with coffee and some bread, we went back to our room to pack our bags and leave them in the lobby and then check out. I felt happy and a little sad that this national final was about to come to an end, my desire was to go home and rest, maybe cry a lot if I didn't receive the medal. After getting down and checking out, we waited for the other contestants to do the same and then we all went together to the junior contestants' hotel, where we would wait for a bus that would take us to the University of Algarve, where the event would take place the closing ceremony.

After a few minutes, we left for the other hotel. As I was walking, suddenly two boys from the same class came up to me, asking what had happened the night before. If I'm not mistaken, I said I just wasn't feeling well and immediately cut the conversation short. I didn't want to talk to anyone.

Finally arriving at the little ones' hotel, we stayed in the lobby waiting for the bus to arrive. The delfics seemed to be talking at full steam, some other contestants seemed like they hadn't woken up and in short, there was a little bit of everything. While I was waiting, Doctor appeared and asked if everything was ok with me and said he was a little sick. Then he apologized for being so insistent that I go to game night yesterday. I just said that he didn't have to apologize for anything, being by his side made that night very special for me. I took advantage of the waiting time to take a look at MODS and see the chats, I didn't see anything interesting. After waiting for the bus for some time, it finally

arrived and it was very wide. I sat next to the window and one of those guys who were from the same class sat next to me, but I wasn't too bothered by it. Everyone had already gotten in and the bus left for the university. While the bus was moving, I watched the view through the window. The city was very beautiful and the sweet day's sun contributed to a sweet view of the houses and streets. After a few minutes, we had finally arrived at the university. It seemed to be quite large but I didn't know for sure.

We walked into a very spacious area of the university where we ended up staying for some time. There were a lot of people there, teachers and some of the contestants' parents, which I thought was quite beautiful. My parents couldn't come, so much so that they don't know much about mathematics, but they gave me comfort, food, education, which for me was a luxury of having parents like that. There are so many children around the world who don't have loving parents and who don't have the same opportunities I had. Shit of the world.

After a while, I noticed that some people were entering a room and I followed them. When I entered, it was a very large auditorium, with a very large screen over the olympiad and background music that gave the feeling that I had discovered the key to the universe xD. The contestants started to sit down, the delfics were further forward and I decided to be further back, so I had a cozy view. Suddenly Doctor appears behind me, where he was supposed to sit with his other colleagues. It felt like being at a gala event in front of all that environment, it was without a doubt an extraordinary experience.

Time passed and that music was already making my ears sick, until several people with important roles on the stage, sitting at the tables in front of us. My body shook and my heart beat faster. They turned off the music and the people on stage started talking, introducing themselves, each one explaining their role and telling a little about their stories, which I found quite curious.

The funniest part was the part where one of these people wanted to offer the book "flatland" to someone and as he didn't know who to offer it to, he said: "what are the first 5 digits..." and everyone raised their hands, including me xD. Then he mentioned

upcoming birthdays and almost said my birthday, because the day of the ceremony was April 2nd and my birthday is April 10th. It was incredible, but it would get even more incredible when a professor came to give a short talk on "Problems in mathematics" and I loved it. The lecture told the story of some problems and how the ancients solved them, which for me was something I was very curious about. I was surprised by the way mathematics had evolved throughout history and the way that what we have was the result of a lot of research and the search to satisfy our curiosity. When the talk ended, I clapped my hands with a smile on my face, I simply loved it, but that soon disappeared as the results approached.

Then Renata appeared, the person with the most authorship when it comes to mathematics olympiads in our country. She gave a small presentation about the Delfos project and emphasized that only medal winners can enter, which made me a little nervous. Then he talked about the upcoming olympiad, IMO, Ibero, etc. and finally he started announcing the results for the junior category. First all the non-medal winners went on stage, then the bronze, silver and gold medalists. I would have to wait for the junior category, category A and finally my category, category B. With every second that passed, it felt like I was going to have a heart attack, I wanted to know the results right away and I couldn't wait any longer.

After the results for the junior and A categories were announced, category B finally arrived and it is now that the suffering begins to be greater and greater. The teacher would call the 18 who did not receive medals, one by one, randomly, not in alphabetical order, saying first the school, the year of schooling and then the name. I started to panic inside myself, it felt like my body was being strangled without anyone touching me. Finally, the teacher began to say the names.

She said the first one, it wasn't mine. She said the second one, it wasn't mine. With each name the teacher said I became more and more distressed. I lowered my head and covered my face with my hands, only my sense of hearing mattered now. With each name that was said that wasn't mine, I felt more relieved, but it seemed like it never ended.

Until, when the teacher said the name of the school I almost went into despair, but when she said it was in the 11th year, I immediately saw that it was my delfic classmate, who didn't manage to win a medal. And she said one more name, and one more and one more, WON'T THIS DRUG NEVER END?!?!

Then she said one more name and it wasn't mine, then a fraction of silence filled the auditorium and then a quick jubilation took over the space. I looked at the stage, she stopped saying names, it was real, I had achieved it, I had achieved what I had been working so hard for months, I managed to join the elite. Doctor quickly patted me on the shoulder and said "It wasn't the end after all haha" with a smile on his face.

I was almost crying, I never thought that mathematics would take me there, so far, on an exciting journey full of ups and downs, but it was a lot of fun, I loved doing mathematics and practicing with other people at MODS. If I hadn't won, I would be very sad, but the sweet path I took to get here is what made me the happiest, continuing to do what I loved most.

After a few seconds, the teacher said the names of the bronze medalists, with each name, the audience burst into jubilation and excitement, full of applause and screams. By the third name, I already knew it was me because of the name of the school. So I left my seat, and walked to the stage, looking straight ahead and focused. After a few seconds, the teacher said the names of the bronze medalists, with each name, the audience burst into jubilation and excitement, full of applause and screams. By the third name, I already knew it was me because of the name of the school. So I left my seat, and walked to the stage, looking straight ahead and focused. The teacher handed me the diploma and a bag with some prizes and a little to the side, there was a lady with the medal in her hand. I greeted, I bent down and I could feel the sweet medal around my neck, then I walked to an area of the stage where we were going to take a photo. As I walked slowly, I picked up the medal. It was big, heavy and very beautiful. All my work was there, all the people who helped me were there, all the people from MODS who helped me were there. After admiring the medal for a few seconds, I went to the stage area and we took a photo. My hair was a little

messy but it didn't matter, I got what I wanted and I didn't need anything else, I already had the ticket to Delfos.

After the photo, I went to sit down. I was relaxed, relieved, happy and almost crying with joy. Afterwards they called out the names of the silver medalists, I could see EricXYX not very happy with the medal, which was understandable because he had solved everything, but not with a good presentation. After the photo, I went to sit down. I was relaxed, relieved, happy and almost crying with joy. Afterwards they called out the names of the silver medalists, I could see EricXYX not very happy with the medal, which was understandable because he had resolved everything, but not with a good presentation. Then the golden ones were called and finally the closing ceremony came to an end.

Everyone started leaving the auditorium to meet their parents, take photos and celebrate a lot. I felt a little lost, not knowing what to do, until those two boys from the same class came up to me to congratulate me and wanted to see the medal, which I allowed without hesitation. Then they asked what was inside the bag. I hadn't even noticed the bag, I was so happy with the medal that the prizes didn't matter much to me.

Inside was a book, a tablet, a card to spend in stores and a piece of paper saying that I was a member of the Portuguese mathematics society (which allowed you to buy books at a lower price). We had a little chat and after a while everyone started to go outside, because of course, it was time for the big lunch!

We walked to a queue that was stretching to the university canteen and waited. Suddenly, Doctor appeared, saying he had my ticket to Delfos and the boys next to me said they were impressed by the fact that I had studied since the summer break to get here. I told them that if you're passionate and curious, you can go very far.

After a brief talk, we finally went inside the canteen, which was beautiful inside, with decorated tables, delicious food and very luxurious, it felt like being at a gala event again haha. As I didn't know where to sit, my delfic colleague said I could sit next to him, along with his parents who were there. I was a little sad that he hadn't won a medal. That race was so easy that the cutoffs were very high, which made me very angry.

I got my food and savored the luxury of being there. It was like a dream come true.

After eating, I sat for a while just observing my surroundings. It looked like the delfics were having a jubilant time at their table, very cheerful and as social as ever. Time passed and it was time to leave. As I left the cafeteria, I met up with Doctor and we chatted for a while. The next stage at delfos would be in 20 days' time, that is, 20 days for the 1st TST for the IMO and I told Doctor that I would train for them, although I thought the likelihood of going to the IMO was very low because of my lack of experience in the math olympiads. Finally, we said goodbye, I was sad to have to say goodbye to Doctor, probably if it hadn't been for the support he had given me up to that point, I would have been worse off psychologically.

After saying goodbye to Doctor, I followed a lady who was going to take me and some other contestants by car to the hotel so that we could collect our bags and then go to the train station. After a while, we had already picked up our bags and were at the train station waiting for our train. The guide gave us snacks to try to survive this long journey to the north of the country and when the train finally arrived, we got on it. Finally, I could rest, relieved, with the feeling that I had completed my mission.

Approximately 6 hours had passed and we were almost close to our final destination, there were still 4 more contestants with me and as I felt bored, I decided to sit next to them, which for me was a decision that cost me a little at first. They were talking about the test and I took the opportunity to intervene in the conversation, which made me feel a little included. After a few minutes, the train stopped, I took my bags and got off the train. The station was quite busy at that time of night, when we left, we said goodbye and I managed to see my mother and stepfather waiting to pick me up.

The two were very happy for the great victory that day. Finally, we drove home. As the car drove, I looked out the window at the beautiful dark sky. Very satisfied with what I had already done, but I knew there was a lot of battle ahead, now I was fighting for something even higher and more glorious: a place on the IMO team. I knew it wouldn't be easy, but I knew I would have a lot of fun on this journey. I didn't think I could do it, but

why not try, right? After all, I'm here not for the reward, but to my olympiad journey doesn't end and for me have the most fun doing what I love most: playing with the deductive reasoning that can create the most beautiful things.

Thank you Mathematics for existing.