Probabilistic Machine Learning

Exercise Sheet #2

due on Monday, 8 November 2021, 10am sharp

1. **EXAMple:** The lecture introduced the notion of expected values of functions f under probability distributions with density $p(x)$. Use the definition from the lecture to show the following widely-known property of the *variance*:

$$\mathbb{E}_p[(x - \mathbb{E}_p(x))^2] = \mathbb{E}_p[x^2] - (\mathbb{E}_p[x])^2.$$

2. **Theory Question:** Suppose we play a game where we start with c_0 Euro. On each play of the game you either double or halve your money, with equal probability. What is your *expected* fortune after n trials? What is the *variance* of your fortune? Can you write a closed-form expression for the p-th non-central *moment* of the distribution of your fortune after n trials?
Hint: Note that the number of wins m and losses $n - m$ has a binomial distribution:

$$p(m \mid n) = \binom{n}{m} \left(\frac{1}{2}\right)^m \left(1 - \frac{1}{2}\right)^{n-m} = \binom{n}{m} 2^{-n}$$

In your derivation, you might find the binomial formula helpful:

$$\sum_{m=0}^{n} \binom{n}{m} y^m x^{n-m} = (x + y)^n$$

An extra thought (not necessary to complete this exercise): Once you have solved this exercise, convince yourself that you can now also solve it for the more general case where the probability of a win is $p \in [0, 1]$, and the effect of winning or losing is multiplying the current fortune by a factor a and b, respectively.

3. **Practical Question:** You can find this week's sheet on Ilias as `Exercise_02.ipynb`