

Vectorization

What is vectorization?

In logistic regression, you need to compute $z = w^T x + b$ where w and x are vectors (may be some large vector)

$$w = \begin{bmatrix} : \\ : \\ : \\ : \end{bmatrix} \quad x = \begin{bmatrix} : \\ : \\ : \\ : \end{bmatrix} \quad \begin{matrix} w \in \mathbb{R}^{n_x} \\ x \in \mathbb{R}^{n_x} \end{matrix}$$

To compute $z = w^T x + b$:

- Non-vectorized method:

```
z = 0
for i in range(n-x):
    z += w[i] * x[i]
z += b
```

-> very slow

- Vectorized method:

$$z = \underbrace{\text{np.dot}(w, x)}_{w^T x} + b$$

-> Much faster

Vectorization Code Demo:

```
[2]: import numpy as np
a = np.array([1, 2, 3, 4])
print(a)
[1 2 3 4]

[3]: import time
a = np.random.rand(1000000)
b = np.random.rand(1000000)
tic = time.time()
c = np.dot(a, b)
toc = time.time()
print("Vectorized version: " + str(1000*(toc - tic)) + "ms")
|
c = 0
tic = time.time()
for i in range(1000000):
    c += a[i] * b[i]
toc = time.time()
print("For loop version: " + str(1000 * (toc - tic)) + "ms")

Vectorized version: 0.9770393371582031ms
For loop version: 396.4099884033203ms
```

Neural network programming guideline

Whenever possible, avoid explicit for-loops.

Say you need to apply the exponential operation on every element of a matrix/vector.

$$v = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix} \rightarrow u = \begin{bmatrix} e^{v_1} \\ e^{v_2} \\ \vdots \\ e^{v_n} \end{bmatrix}$$

```
→ u = np.zeros((n,1))
→ for i in range(n):
    → u[i]=math.exp(v[i])
```

```
import numpy as np
u = np.exp(v) ←
```

Python numpy provides many different element-wise operators:

```
np.log(v)
np.abs(v)
np.maximum(v, 0)
v**2      v/v
```

Logistic regression derivatives

```

J = 0, dw1 = 0, dw2 = 0, db = 0
→ for i = 1 to 'm:
    z(i) = wTx(i) + b
    a(i) = σ(z(i))
    J += -[y(i) log a(i) + (1 - y(i)) log(1 - a(i))]
    dz(i) = a(i) - y(i)
    for j=1...nx
    dwj +=
     $\left. \begin{aligned} dw_1 &+= x_1^{(i)} dz^{(i)} \\ dw_2 &+= x_2^{(i)} dz^{(i)} \\ db &+= dz^{(i)} \end{aligned} \right| n_x=2$ 
J = J/m, dw1 = dw1/m, dw2 = dw2/m, db = db/m

```

Instead of using explicit for loop like the above example, we can do the following way:

```

J = 0, dw1 = 0, dw2 = 0, db = 0
→ for i = 1 to 'm:
    z(i) = wTx(i) + b
    a(i) = σ(z(i))
    J += -[y(i) log a(i) + (1 - y(i)) log(1 - a(i))]
    dz(i) = a(i) - y(i)
    for j=1...nx
    dwj +=
     $\left. \begin{aligned} dw_1 &+= x_1^{(i)} dz^{(i)} \\ dw_2 &+= x_2^{(i)} dz^{(i)} \\ db &+= dz^{(i)} \end{aligned} \right| n_x=2$ 
     $dw += x^{(i)} dz^{(i)}$ 
J = J/m, dw1 = dw1/m, dw2 = dw2/m, db = db/m
dw /= m.

```

dw = np.zeros((n_x, 1))

Vectorizing Logistic Regression

$$\begin{aligned}
 \rightarrow \underline{z^{(1)}} &= \underline{w^T x^{(1)} + b} & \underline{z^{(2)}} &= \underline{w^T x^{(2)} + b} & \underline{z^{(3)}} &= w^T x^{(3)} + b \\
 \rightarrow \underline{a^{(1)}} &= \sigma(z^{(1)}) & \underline{a^{(2)}} &= \sigma(z^{(2)}) & \underline{a^{(3)}} &= \sigma(z^{(3)})
 \end{aligned}$$

$$\begin{aligned}
 \underline{X} &= \begin{bmatrix} x^{(1)} & x^{(2)} & \dots & x^{(m)} \\ 1 & 1 & \dots & 1 \end{bmatrix} \quad \begin{matrix} (n_x, m) \\ \mathbb{R}^{n_x \times m} \end{matrix} & \xrightarrow{w^T} & \begin{bmatrix} x^{(1)} & x^{(2)} & \dots & x^{(m)} \\ 1 & 1 & \dots & 1 \end{bmatrix} \\
 \underline{Z} = [\underline{z^{(1)}} \quad \underline{z^{(2)}} \quad \dots \quad \underline{z^{(m)}}] &= \underbrace{w^T X}_{1 \times m} + \underbrace{[b \quad b \quad \dots \quad b]}_{1 \times m} = \underbrace{[w^T x^{(1)} + b \quad w^T x^{(2)} + b \quad \dots \quad w^T x^{(m)} + b]}_{1 \times m} \\
 & \quad \underline{Z} = \text{np.dot}(w.T, X) + \underline{\frac{b}{\mathcal{C}}}_{(1,1)} \quad \mathbb{R} \quad \text{"Broadcasting"}
 \end{aligned}$$

Z is going to be a 1xm matrix that contain all of the lowercase z's

To recap, what we've seen on this slide is that instead of needing to loop over M training examples to compute lowercase Z and lowercase A, one of the time, you can implement this one line of code, to compute all these Z's at the same time.

So this is how you implement a vectorize implementation of the four propagation for all M training examples at the same time.