

6435 Network Packet Ordering

Some of us sometimes overestimated the speed of light. In fact, light at vacuum travels at “only” 299,492,758 m/s, which is actually pretty slow for transferring network packets through fiber optic cables.

Imagine transferring network packets from New York to Jakarta through 20,000 km-long fiber optic cables. The photons in the fiber optics will need 66.7 msec at a minimum to travel such distance, and this assumes the cable is very optimally arranged to minimize its length, light travels with minimum reflections inside the cable, and there is no other delays caused by network routing, switching, relaying, etc.

For some large internet companies, like Gogololo, Inc. which deals with data centers across the globe, the delays are actually quite annoying. Suppose Gogololo’s server in New York sends packets of data labelled a_1, a_2, \dots, a_N to a database server in Dublin, and its Jakarta’s server also sends packets of data labelled b_1, b_2, \dots, b_M to the same Dublin database. The data a_1, a_2, \dots, a_N is expected to arrive respectively at time $t_{a_1}, t_{a_2}, \dots, t_{a_N}$ and the data b_1, b_2, \dots, b_M is expected to arrive respectively at time $t_{b_1}, t_{b_2}, \dots, t_{b_M}$ (all timestamps are in milliseconds)

However, due to the “slow” speed of light and uncertainty in the network delays, each packet from the two sources may experiences delay up to D milliseconds, so that a packet that is expected to arrive at time t can arrive at a time t' where:

$$t \leq t' < t + D \quad (\text{note: } t' \text{ can be fractional})$$

However, ordering protocol on the packets guarantee that all the packets from New York will arrive in the order which is relative to other packets from New York, and all the packets from Jakarta will arrive in the order which is relative to other packets from Jakarta. But it does not guarantee any fixed ordering between the packets from New York and the packets from Jakarta. Assume that any two packets’ arrival times must differ by at least some infinitesimal fraction of millisecond, so there is no confusion in their ordering.

As an engineer of Gogololo, Inc. you are tasked with calculating the number of possible final orderings of the packets arriving at the Dublin database.

Input

The first line of input contains an integer T ($T \leq 15$) denoting the number of cases. Each case begins with three integers N , M and D ($1 \leq N, M \leq 50,000$; $1 \leq D \leq 100$) denoting the number of packet from New York, the number of packet from Jakarta, and the uncertainty delay respectively. The next line contains N integers t_{a_i} ($1 \leq t_{a_i} \leq 1,000,000$; $t_{a_i} < t_{a_j}$ for all $i < j$) denoting the expected arrival time of all packets from New York. The following line contains M integers t_{b_i} ($1 \leq t_{b_i} \leq 1,000,000$; $t_{b_i} < t_{b_j}$ for all $i < j$) denoting the expected arrival time of all packets from Jakarta.

Output

For each case, output ‘Case # X : Y ’, where X is the case number starts from 1 and Y is the number of possible ordering modulo by 1,000,000,009.

Notes:

- Explanation for 1st sample case

There are only two variations of the final ordering: b_3 before a_1 or b_3 after a_1 .

- Explanation for 2nd sample case

All possible interleavings between the New York packets and the Jakarta packets are possible, forming 10 possible final orderings.

Sample Input

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2
3 3 1
5 6 7
3 4 5
3 2 4
1 2 3
1 2
```

Sample Output

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Case #1: 2
Case #2: 10
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