Lecture 1: Introduction to Reinforcement Learning

Idea: An automated agent learning through experience/data to make good decisions under uncertainty.

Reinforcement Learning (Generally) Involves:

- Optimization:
    - The goal is to find an optimal way to make decisions.
        - Yielding best outcomes or at lease very good outcomes
    - Explicit notion of decision utility.
    - Example: finding minimum distance route between two cities given network of roads.
- Delayed Consequences:
    - Decisions now can impact things much later…
    - Introduces two challenges:
        - When planning: decisions involve reasoning about not just immediate benefit of a decision but also its longer term ramifications.
        - When learning: temporal credit assignment is hard (what caused later high or low rewards?)
- Exploration:
    - Learning about the world by making decisions.
        - Agent as scientist.
        - Learn to ride a bike by trying and failing.
    - Decisions impact what we learn about.
        - Only get a reward for decision made.
        - Don't know what would have happened for other decision.
        - If we choose to go to Stanford instead of MIT, we will have different later experiences…
- Generalization:

- Policy is mapping from past experience to action (i.e. the rule of how to act in certain circumstances).
- Why not just pre-program a policy?
    - The scenario space is HUGE → We can't write rules to address each of these cases individually.

RL vs Other AI and Machine Learning:

	AI Planning	Supervised Learning	Unsupervised Learning	Reinforcement Learning	Imitation Learning
Optimization	X			X	X
Learns from experience		X	X	X	X
Generalization	X	X	X	X	X
Delayed Consequences	X			X	X
Exploration				X	

- Imitation learning typically assumes input demonstrations of good policies. ← learn only from the experts.
- Imitation Learning reduces Reinforcement Learning to Supervised Learning. For many good reasons, imitation learning is very popular.

Two Problem Categories where RL is Particularly Powerful

1. No examples of desired behavior.
    - e.g., because the goal is to go beyond human performance or there is no existing data for a task.
2. Enormous search or optimization problem with delayed outcomes.
    - For problem with really really large search space, even there, we may not have great techniques for solving them. → It's sort of a reduction that people can think of taking a planning problem and trying to reduce it to a reinforcement learning problem to make it more tractable.

Course Outline:

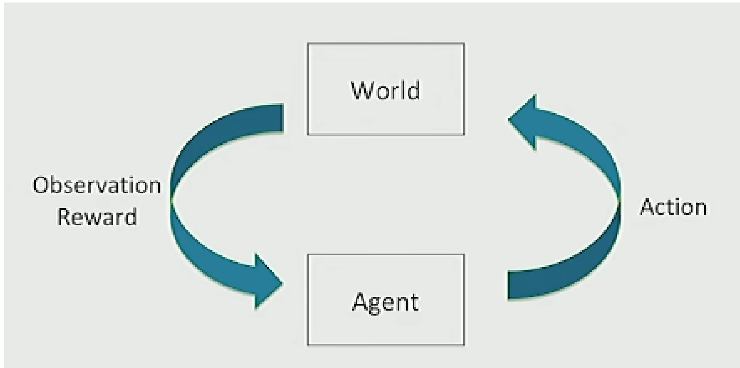
- Markov decision processes & planning.
- Model-free policy evaluation.

- Model-free control.
- Policy Search.
- Offline RL **including RL from Human Feedback and Direct Preference Optimization**.
- Exploration.
- Advanced Topics.

Refresher Exercise: AI tutor as a Decision Process:

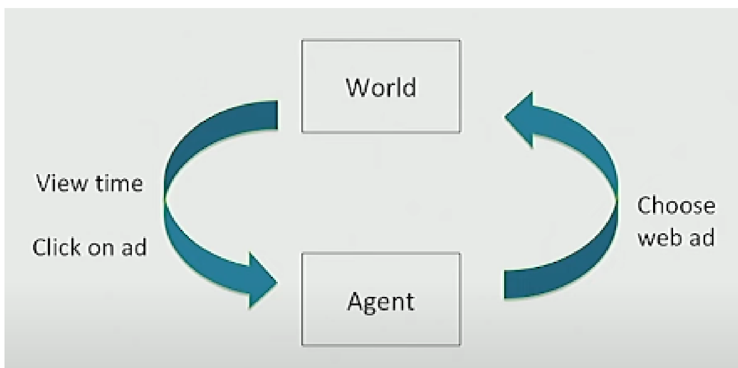
- Student initially does not know either addition (easier) nor subtraction (harder).
- AI tutor agent can provide practice problems about addition or subtraction.
- AI agent gets rewarded +1 if student gets problem right, -1 if gets problem wrong.
- Model this as a Decision Process.
    - Given the easier problem first, if student gets it right → move to the harder problem next, else give another easy problem.
- Define state space, action space, and reward model.
    - State space: Student get the easy problem right or wrong, the hard problem right or wrong.
    - Action space: AI tutor controls between the easier or harder problems.
    - Reward model: +1 if student gets problem right, -1 if gets problem wrong.
- What does the dynamics model represent?
    - The learning progress of the student for the easy or hard problem types.
- What would a policy to optimize the expected discounted sum of rewards yields?
    - The cumulative reward of either the addition or subtraction problem to be as high as possible.

Sequential Decision Making

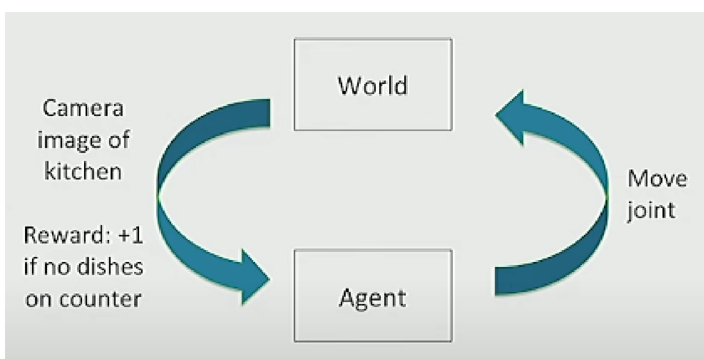


- Goal: Select actions to maximize total expected future reward.
- May require balancing immediate & long term rewards.

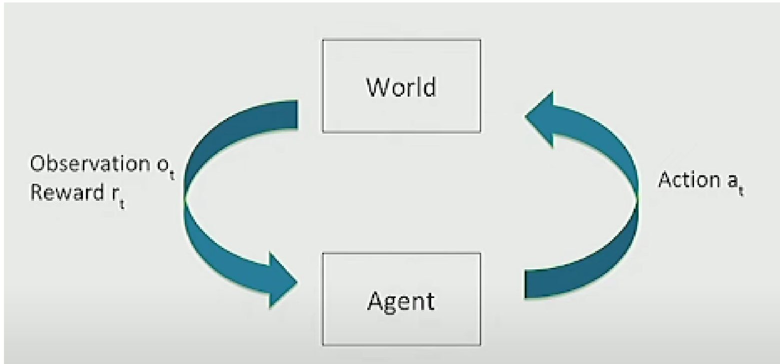
Example: Web Advertising



Example: Robot Unloading Dishwasher



Sequential Decision Process: Agent & the World (Discrete Time)



Assume that we have a finite series of time steps.

- Each time step t:
  - Agent takes an action a_t.
  - World updates given action a_t, emits observation o_t and reward r_t.
  - Agent receives observation o_t and reward r_t.

History: Sequence of Past Observations, Actions & Rewards

- History $h_t = (a_1, o_1, r_1, ..., a_t, o_t, r_t)$.
- Agent chooses action based on history.
- State is information assumed to determine what happens next.
  - Function of history: $s_t = (h_t)$

Markov Assumption

- Information state: sufficient statistic of history.
- State s_t is Markov if and only if:
  - $p(s_{t+1}|s_t, a_t) = p(s_{t+1}|h_t, a_t)$
  - → Sufficient if assume $s_t == h_t$.
- Future is independent of past given present.

Note: `observation` is independent from `state`.

Example: In atari games, the `state` of the agent is the history of the last 4 consecutive frames (to capture the temporal dependencies or momentum of the objects) while the `observation` is only the current frame.

Why is Markov Assumption Popular?

- Simple and often can be satisfied if include some history as part of the state.
- In practice often assume most recent observation is sufficient statistic of history: $s_t = o_t$.
- State representation has big implications for:
  - Computational complexity.
  - Data required.
  - Resulting performance.

Types of Sequential Decision Processes

- Is the state Markov? Is the world partially observed? (Partially Observed Markov Decision Process)
- Are dynamics deterministic or stochastic?
- Do actions influence only immediate reward (bandits) or reward and next state?

MDP Model

- **Agent's representation** of how world changes given agent's action (not the actual model of the world).
- Transition / dynamics model predicts next agent state.
  - $p(s_{t+1} = s' | s_t = s, a_t = a)$ or $p(s'|s, a)$
- Reward model predicts immediate reward (if I'm in this state, and I take this action, what will be my immediate reward?).
  - $r(s_t = s, a_t = a) = \mathbb{E}[r_t | s_t = s, a_t = a]$ or $r(s, a) = \mathbb{E}[r_t | s, a]$

Policy

- Policy π determines how the agent chooses actions in a specific state.
- $\pi : S \to A$, mapping from states to actions.
- Two types of policy:
  - Deterministic policy (we only have one action correspond to a state):

- $\pi(s) = a$

- Stochastic policy:
  - $\pi(a|s) = Pr(a_t = a|s_t = s)$

Note: Our model might start with a particular policy. Over time, it will explore lots of different policies in trying to search for something that's good.

Evaluation and Control

- Evaluation:
  - (*Someone give you a fixed policy and you want to know how good it is*)
  - Estimate/predict the expected rewards from following a given policy.
- Control:
  - Optimization: find the best policy.

Markov Process or Markov chain

- **Memoryless random process.**
  - A Markov Process is a type of random process where the future state only depends on the current state and not on the sequence of states that came before. This is called the **Markov Property**.
  - Mathematically:
    - $P(s_{t+1}|s_t, s_{t-1}, ..., s_0) = P(s_{t+1}|s_t)$
    - This means that the probability of moving to the next state s_{t+1} depends only on the current state s_t, not the entire history.
- **Definition of Markov Process:**
  - S is a (finite) set of states $(s \in S)$.
  - P is **dynamics / transition model** that specifies the probability of transitioning from one state to another, expressed as $p(s_{t+1} = s'|s_t = s)$, which means the probability of transitioning to state s' from state s.
- **Note: no rewards, no actions.**
  - A Markov Process is purely focused on state transitions. It does not involve:
    - **Reward**: Unlike Reinforcement Learning, where states and actions are associated with rewards.

- **Actions**: It doesn't model decisions that lead to different transitions - just the probabilistic behavior of the process.

- **Transition Matrix Representation (P):**
  - When a number of states N is finite, the transition probabilities between states can be represented as a matrix.
    - $P = \begin{bmatrix} P(s_1|s_1) & P(s_2|s_1) & ... & P(s_N|s_1) \\ P(s_1|s_2) & P(s_2|s_2) & ... & P(s_N|s_2) \\ \vdots & \vdots & \ddots & \vdots \\ P(s_1|s_N) & P(s_2|s_N) & ... & P(s_N|s_N) \end{bmatrix}$
    - Each row represents the probabilities of transitioning from a particular state to all other states.
    - Each row has the sum of 1.
  - ▼ Example to illustrate:
    - Imagine a weather system with 3 states:
      - $s_!$: Sunny.
      - s_2: Cloudy.
      - s_3: Rainy.
    - The transition probabilities might look like this:
      - $P = \begin{bmatrix} 0.8 & 0.1 & 0.1 \\ 0.3 & 0.4 & 0.3 \\ 0.2 & 0.3 & 0.5 \end{bmatrix}$
      - Row 1: If it's sunny (s_1), there's an 80% chance it will stay sunny ($P(s_1|s_1)$), 10% chance it becomes cloudy ($P(s_2|s_1)$), and 10% chance it will rain ($P(s_3|s_1)$).
      - Row 2: If it's cloudy (s_2), there's an 30% chance it becomes sunny ($P(s_1|s_2)$), 40% chance it stays cloudy ($P(s_2|s_2)$), and 30% chance it will rain ($P(s_3|s_2)$).
      - Row 3: If it's rainy (s_3), there's an 20% chance it will become sunny ($P(s_1|s_3)$), 30% chance it becomes cloudy ($P(s_2|s_3)$), and 50% chance it will rain ($P(s_3|s_3)$).

Markov Reward Process (MRP)

- Markov Reward Process is an extension of a Markov Process where rewards are added to the states.

- It combines:
  1. The state transitions from a **Markov Process.**
  2. A **Reward function** that assigns expected reward to states.
- Key components of an MRP:
  - S is a finite set of states ($s \in S$).
  - P is dynamics/transition model that specifies $P(s_{t+1} = s' | s_t = s)$, which defines the probability of transitioning to state s' from state s.
  - R is a reward function $R(s_t = s) = \mathbb{E}[r_t | s_t = s]$ for being in state s.
  - Discount factor $\gamma \in [0, 1]$ that determines how future rewards are valued relative to immediate rewards.
    - If $\gamma = 0$, only immediate rewards are considered.
    - If $\gamma = 1$, future rewards are valued equally as immediate rewards.
- Note: no actions.
  - Similar to Markov Process, an MRP doesn't involve actions. It models the system's behavior and rewards but doesn't account for decisions or strategies.
- If finite number (N) of states, can express P as a matrix like in Markov Process, and R as a vector.
- Discounted Return:
  - The goal in an MRP is often to compute the **discounted return**, which is the total reward received over time, discounted by γ:
    - $G_t = r_t = \gamma r_{t+1} + \gamma^2 r_{t+2} + ...$
    - This can be rewritten as: $G_t = \sum_{k=0}^{\infty} \gamma^k r_{t+k}$

Return & Value Function

- Horizon (H):
  - **Definition**: The horizon refers to the number of time steps in an episode, which can either be finite or infinite:
    - **Finite horizon**: The process ends after a certain number of steps (H is a finite number).
    - **Infinite horizon**: The process continues indefinitely ($H = \infty$).
  - Otherwise called **finite** Markov reward process.
- Definition of Return, G_t (for a Markov Reward Process)

- **Definition**: The return at time t is the **discounted sum of rewards** from time t up to the horizon H.
    - $G_t = \sum_{k=0}^{H-1} \gamma^k r_{t+k}$
- Key points:
    - If $\gamma = 1$, no discounting occurs, and all rewards are treated equally.
    - If $\gamma < 1$, rewards further in the future are given less weight.

- Definition of State Value Function, $V(s)$ (for a Markov Reward Process):
    - **Definition**: The state value function measures the **expected return** from starting from a particular state s. It accounts for all possible transitions and rewards from that state onward:
        - $$\begin{aligned} V(s) &= \mathbb{E}[G_t | s_t = s] \\ &= \mathbb{E}[\sum_{k=0}^{H-1} \gamma^k r_{t+k} | s_t = s] \\ &= \mathbb{E}[r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + ... + \gamma^{H-1} r_{t+H-1} | s_t = s] \end{aligned}$$

Discount Factor

- Mathematically convenient (avoid infinite returns and values)
- Humans often act as if there's a discount factor < 1.
- If episode lengths are always finite ($H < \infty$), can use $\gamma = 1$.

Quick Check Your Understanding

In a Markov decision process, a large discount factor γ means that short term rewards are much more influential than long term rewards.

- [ ] True
- [x] ~~False~~
- [ ] Don't know