Lucio Enzo Horie 21413

① Série de Fourier

a) $\boxed{V_M = \frac{1}{T}\int_0^T f(t)\,dt}$

I) $V_{M,I} = \frac{1}{T}\int_0^T \frac{At}{T}dt = \frac{1}{T}\cdot\frac{A}{T}\frac{T^2}{2}$

$\Rightarrow \boxed{V_{M,I} = \frac{A}{2}}$ //

II)

$$f_{II}(t) = \begin{cases} -A\cos\left(\frac{2\pi t}{T}\right), & \frac{T}{4} < t < \frac{3T}{4} \\ 0, & \text{caso contrário} \\ \text{Periódica de período } T \end{cases}$$

$$V_{M,II} = \frac{1}{T}\int_{\frac{T}{4}}^{\frac{3T}{4}} -A\cos\left(\frac{2\pi t}{T}\right)dt$$

$$= -\frac{A}{\cancel{T}}\frac{\cancel{T}}{2\pi}\,\text{sen}\left(\frac{2\pi t}{T}\right)\Bigg|_{\frac{T}{4}}^{\frac{3T}{4}}$$

$$= -\frac{A}{2\pi}\left(\text{sen}\left(\frac{3}{2}\pi\right) - \text{sen}\left(\frac{\pi}{2}\right)\right)$$

$\boxed{V_{M,II} = \frac{A}{\pi}}$ //

III)

$$f_{III}(t) = \begin{cases} -\frac{A}{\tau}t + A, & 0 < t < \tau \\ \frac{A}{\tau}t + A, & -\tau < t < 0 \\ 0, & \text{caso contrário} \\ \text{periódica de período } T \\ \tau < \frac{T}{2}. \end{cases}$$

$$V_{M,III} = \frac{1}{T}\left(\int_{-\tau}^{0}\left(\frac{A}{\tau}t + A\right)dt + \int_0^{\tau}\left(-\frac{A}{\tau}t + A\right)dt\right)$$

$$= \frac{1}{T}\left(\frac{A}{\cancel{\tau}}\left(-\frac{\tau^{\cancel{2}}}{2}\right) + A\tau + \left(\frac{A}{\cancel{\tau}}\left(-\frac{\tau^{\cancel{2}}}{2}\right) + A\tau\right)\right)$$

$$= \frac{1}{T}(A\tau)$$

$\boxed{V_{M,III} = \frac{A\tau}{T}}$ //

IV)

$$f_{IV}(t) = \begin{cases} \frac{2A}{\tau}t + 2A, & -\tau < t < -\frac{\tau}{2} \\ -\frac{2A}{\tau}t, & -\frac{\tau}{2} < t < \frac{\tau}{2} \\ \frac{2A}{\tau}t - 2A, & \frac{\tau}{2} < t < \tau \\ 0, & \text{caso contrário} \\ \text{Periódica de período } T \\ 0 < 2\tau < T \end{cases}$$

$$V_{M,IV} = \frac{1}{T}\left(\int_{-\tau}^{-\frac{\tau}{2}} \frac{2A}{\tau}t + 2A\,dt + \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} -\frac{2A}{\tau}t\,dt + \int_{\frac{\tau}{2}}^{\tau}\frac{2A}{\tau}t - 2A\,dt\right)$$

$\boxed{V_{M,IV} = 0}$

b) $V_{RMS} = \sqrt{\dfrac{\int_{-\frac{T}{2}}^{\frac{T}{2}} |f(t)|^2 \, dt}{T}}$

I) $V_{RMS,I} = \sqrt{\dfrac{\int_0^T \left|\frac{A}{T} t\right|^2 dt}{T}}$

$V_{RMS,I} = \sqrt{\dfrac{A^2}{T^3} \int_0^T t^2 \, dt}$

$$\boxed{V_{RMS,I} = \frac{A}{\sqrt{3}}}$$

II) $V_{RMS,II} = \sqrt{\dfrac{\int_0^T A^2 \cos^2\left(\frac{2\pi t}{T}\right) dt}{T}}$

$$\boxed{V_{RMS,II} = \frac{A}{2}}$$

III) $V_{RMS,III} = \sqrt{\dfrac{\int_{-\tau}^{0} \left(\frac{At}{\tau} + A\right)^2 dt + \int_0^{\tau} \left(-\frac{At}{\tau} + A\right)^2 dt}{T}}$

$$\boxed{V_{RMS,III} = A\sqrt{\frac{2\tau}{3T}}}$$

IV) $\boxed{V_{RMS,IV} = \sqrt{\frac{2\tau}{3T}}\, A}$

c) $C_n = \dfrac{1}{T} \int_0^T f(t) \cdot e^{-jn\omega t} \, dt$

I) $C_{n,I} = \dfrac{1}{T} \int_0^T \dfrac{At}{T} e^{-jn\omega t} \, dt$

$= -\dfrac{A}{T^2} \left(\dfrac{1}{n^2\omega^2} - \dfrac{e^{-n\omega Tj}(1+n\omega Tj)}{n^2\omega^2} \right)$

* Como $\omega = \dfrac{2\pi}{T}$

$$\Rightarrow \boxed{C_{n,I} = \frac{Aj}{2n\pi}}$$

II) $C_{n,II} = \dfrac{4\pi A e^{-\frac{n\omega Tj}{2}} \cos\left(\frac{n\omega T}{4}\right)}{4\pi^2 - T^2 n^2 \omega^2}$

* Como $\omega = \dfrac{2\pi}{T}$

$$\boxed{C_{n,II} = \frac{(-1)^{1-\frac{n}{2}} \cdot A((-1)^n + 1)}{2\pi(n^2-1)}}$$

III) Como $\omega = \dfrac{2\pi}{T}$

$C_{n,III} = -\dfrac{AT e^{-\frac{2n\pi\tau j}{T}} \left(e^{\frac{2n\pi\tau j}{T}} - 1\right)^2}{4n^2\tau\pi^2}$

IV) Como $\omega = \dfrac{2\pi}{T}$

$$\boxed{C_{n,IV} = -\frac{AT}{2n^2\tau\pi^2} \cdot e^{-\frac{2\pi n\tau j}{T}} \left(e^{\frac{n\pi\tau j}{T}} - 1\right)^3 \left(e^{\frac{n\pi\tau j}{T}} + 1\right)}$$

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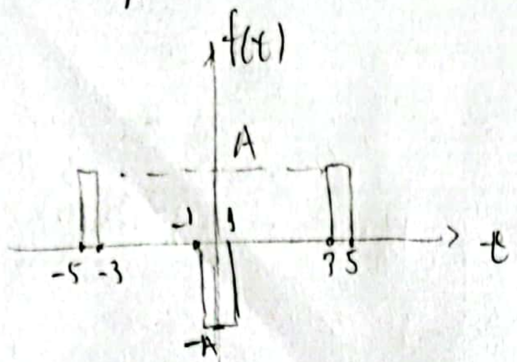
② Transformadas de Fourier

1)



O sinal é uma função definida por partes.

$$f(t) = \begin{cases} A, & -5 < t < -3; \\ -A, & -1 < t < 1; \\ A, & 3 < t < 5; \\ 0, & \text{caso contrário.} \end{cases}$$

$$\mathcal{F}[f(t)] = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt$$

$$F(\omega) = \underbrace{\int_{-5}^{-3} A e^{-j\omega t} dt}_{I_1} - \underbrace{\int_{-1}^{1} A e^{-j\omega t} dt}_{I_2} + \underbrace{\int_{3}^{5} A e^{-j\omega t} dt}_{I_3}$$

$$I_1 = A \cdot \frac{e^{-j\omega t}}{-j\omega}\Big|_{-5}^{-3} = \frac{-A}{j\omega}\left(e^{3j\omega} - e^{5j\omega}\right)$$

$$I_2 = A \frac{e^{-j\omega t}}{-j\omega}\Big|_{-1}^{1} = \frac{-A}{j\omega}\left(e^{-j\omega} - e^{j\omega}\right)$$

$$I_3 = \frac{A e^{-j\omega t}}{-j\omega}\Big|_{3}^{5} = \frac{-A}{j\omega}\left(e^{-5j\omega} - e^{-3j\omega}\right)$$

$$\Rightarrow F(\omega) = I_1 - I_2 + I_3$$

$$F(\omega) = \frac{A}{-j\omega}\left(e^{3j\omega} - e^{-3j\omega} - e^{-j\omega} + e^{j\omega} + -(e^{5j\omega} - e^{-5j\omega})\right)$$

$$F(\omega) = \frac{2A}{\omega}\left(\frac{e^{5j\omega} - e^{-5j\omega}}{2j} - \frac{e^{j\omega} - e^{-j\omega}}{2j} - \frac{e^{3j\omega} - e^{-3j\omega}}{2j}\right)$$

$$\boxed{F(\omega) = \frac{2A}{\omega}\left(\text{sen}(5\omega) - \text{sen}(\omega) - \text{sen}(3\omega)\right)}$$

(a)

b) $g(t) = \int_{-\infty}^{t} f(\lambda)\, d\lambda$

Propriedade: $\mathcal{F}\left[\int_{-\infty}^{t} f(\lambda)\, d\lambda\right] = \frac{1}{j\omega} F(\omega) + \pi F(0)\delta(\omega)$

$$\Rightarrow G(\omega) = \mathcal{F}\left[\int_{-\infty}^{t} f(\lambda)\, d\lambda\right]$$

$$= \frac{1}{j\omega} F(\omega) + \pi F(0)\, \delta(\omega)$$

$$= \frac{2A}{j\omega^2}\left(\text{sen}(5\omega) - \text{sen}(\omega) - \text{sen}(3\omega)\right)$$

Lucio Enzo Thomé 21433

(2)

$$h(t) = \begin{cases} \cos^2(\omega_0 t), & -\frac{T}{4} < t < \frac{T}{4} \\ 0, & \text{caso contrário} \end{cases}$$

$$* \ \omega_0 = \frac{2\pi}{T}$$

$$\mathcal{F}[h(t)] = H(\omega) = \int_{-\infty}^{\infty} h(t) e^{-j\omega t} dt$$

$$H(\omega) = \int_{-\frac{T}{4}}^{\frac{T}{4}} \cos^2(\omega_0 t) e^{-j\omega t} dt$$

$$* \ \cos^2(\omega_0 t) = \frac{1 + \cos(2\omega_0 t)}{2}$$

$$\Rightarrow H(\omega) = \int_{-\frac{T}{4}}^{\frac{T}{4}} \left(\frac{1 + \cos(2\omega_0 t)}{2} \right) e^{-j\omega t} dt$$

$$= \frac{1}{2} \Big(\underbrace{\int_{-\frac{T}{4}}^{\frac{T}{4}} e^{-j\omega t} dt}_{I_1} + \underbrace{\int_{-\frac{T}{4}}^{\frac{T}{4}} e^{-j\omega t} \cos(2\omega_0 t) dt}_{I_2} \Big)$$

$$I_1 = \frac{e^{-j\omega t}}{-j\omega} \Big|_{-\frac{T}{4}}^{\frac{T}{4}} = \frac{1}{j\omega} \left(e^{\frac{j\omega T}{4}} - e^{-\frac{j\omega T}{4}} \right)$$

$$= \frac{2}{\omega} \operatorname{sen}\left(\frac{\omega T}{4} \right)$$

$$I_2 = \int_{-\frac{T}{4}}^{\frac{T}{4}} e^{-j\omega t} \cdot \cos(2\omega_0 t)\, dt$$

$$I_2 = \int_{-\frac{T}{4}}^{\frac{T}{4}} e^{-j\omega t} \left(\frac{e^{2j\omega_0 t} + e^{-2j\omega_0 t}}{2} \right) dt$$

$$I_2 \Rightarrow \frac{1}{2} \left(\int_{-\frac{T}{4}}^{\frac{T}{4}} e^{jt(2\omega_0 - \omega)} + e^{-jt(2\omega_0 + \omega)} dt \right)$$

$$= \frac{1}{2} \left(\frac{e^{jt(2\omega_0 - \omega)}}{j(2\omega_0 - \omega)} - \frac{e^{-jt(2\omega_0 + \omega)}}{j(2\omega_0 + \omega)} \right) \Bigg|_{-\frac{T}{4}}^{\frac{T}{4}}$$

$$I_2 = \frac{1}{2} \left(\frac{e^{j\frac{T}{4}(2\omega_0 - \omega)} - e^{-j\frac{T}{4}(2\omega_0 - \omega)}}{j(2\omega_0 - \omega)} + \frac{e^{-j\frac{T}{4}(2\omega_0 + \omega)} - e^{+j\frac{T}{4}(2\omega_0 + \omega)}}{j(2\omega_0 + \omega)} \right)$$

$$I_2 = \frac{\operatorname{sen}\left((2\omega_0 - \omega)\frac{T}{4} \right)}{2\omega_0 - \omega} + \frac{\operatorname{sen}\left((2\omega_0 + \omega)\frac{T}{4} \right)}{2\omega_0 + \omega}$$

$$= \frac{\operatorname{sen}\left(\frac{4\pi}{\cancel{T}} \cdot \frac{\cancel{T}}{\cancel{4}} - \frac{\omega T}{4} \right)}{2\omega_0 - \omega} + \frac{\operatorname{sen}\left(\frac{4\pi}{T} \cdot \frac{T}{4} + \frac{\omega T}{4} \right)}{2\omega_0 + \omega}$$

$$= \frac{\operatorname{sen}\left(\frac{\omega T}{4} \right)}{2\omega_0 - \omega} - \frac{\operatorname{sen}\left(\frac{\omega T}{4} \right)}{2\omega_0 + \omega}$$

$$\Rightarrow H(\omega) = \frac{\operatorname{sen}\left(\frac{\omega T}{4} \right)}{2} \left(\frac{2}{\omega} + \frac{1}{2\omega_0 - \omega} - \frac{1}{2\omega_0 + \omega} \right)$$

$$= \frac{\operatorname{sen}\left(\frac{\omega T}{4} \right)}{\cancel{2}} \cdot \frac{\cancel{8}^{4} \omega_0^2}{\omega(4\omega_0^2 - \omega^2)}$$

$$= \frac{4\omega_0^2}{\omega(4\omega_0^2 - \omega^2)} \operatorname{sen}\left(\frac{\omega T}{4} \right)$$

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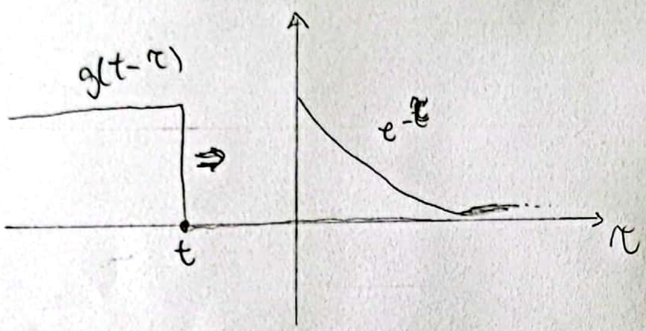
(3)

$$x(t) = \begin{cases} e^{-t}, & t>0 \\ 0, & t<0 \end{cases}$$

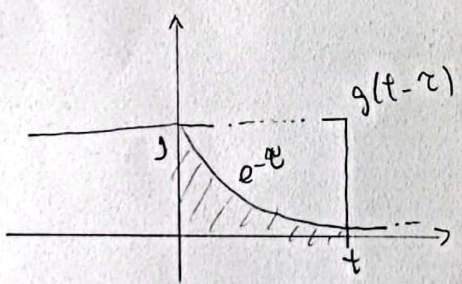
$$g(t) = \begin{cases} 0, & t<0 \\ 1, & t>0 \end{cases}$$

$$x(t) * g(t) = \int_{-\infty}^{\infty} x(\tau)\, g(t-\tau)\, d\tau$$



Para $t<0$, $y(t) = (x*g)(t) = 0$

Para $t>0$:



Podemos ver que para cada amplitude temos que $g(t-\tau)=1, \tau<t$ e $x(\tau) = e^{-\tau}, t>0$

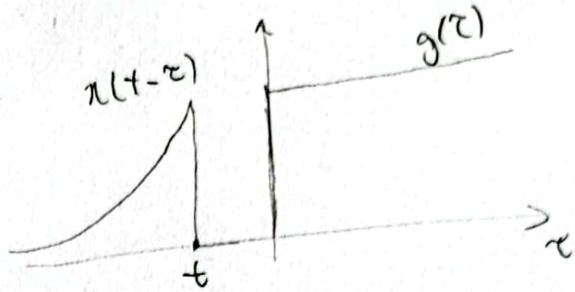
$$\int_0^t x(\tau)\cdot g(t-\tau)\, d\tau =$$

$$= \int_0^t e^{-\tau}\cdot 1\, dt = -e^{-\tau}\Big|_0^t$$

$$= 1 - e^{-t}$$

Assim: $\boxed{x(t) * g(t) = 1 - e^{-t}}$

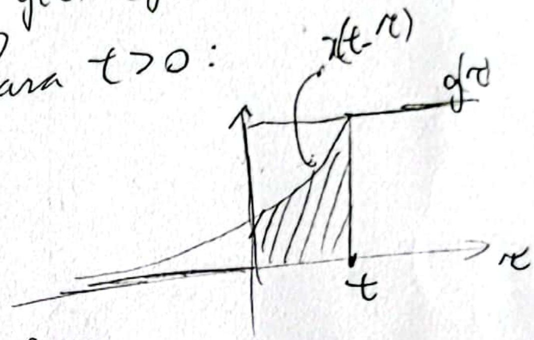
Agora, vamos calcular $g(t) * x(t)$



Novamente, para $t<0$:

$y(t) = (g * x)(t) = 0$

Para $t>0$:



$$\Rightarrow \int_{-\infty}^{\infty} g(\tau)\, x(t-\tau)\, d\tau =$$

$$= \int_0^t 1 \cdot e^{-t+\tau}\, d\tau$$

$$= e^{\tau - t}\Big|_0^t = 1 - e^{-t}$$

$$\Rightarrow g(t) * x(t) = 1 - e^{-t}$$

$$\Rightarrow x(t) * g(t) = g(t) * x(t)$$

c.q.d.