An Introduction to Hash Functions

Christophe Clavier - Florent Bruguier

University of Limoges - University of Montpellier

Definition and properties

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\mathcal{H} : \{0,1\}^* & \longrightarrow & \{0,1\}^n \\
m & \longmapsto & \mathcal{H}(m)
\end{array}
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- keyless: while used as an important cryptographic primitive

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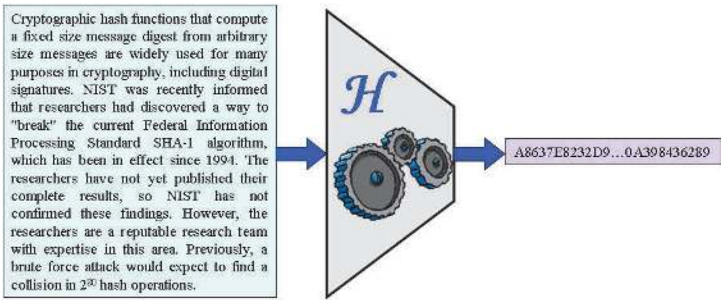

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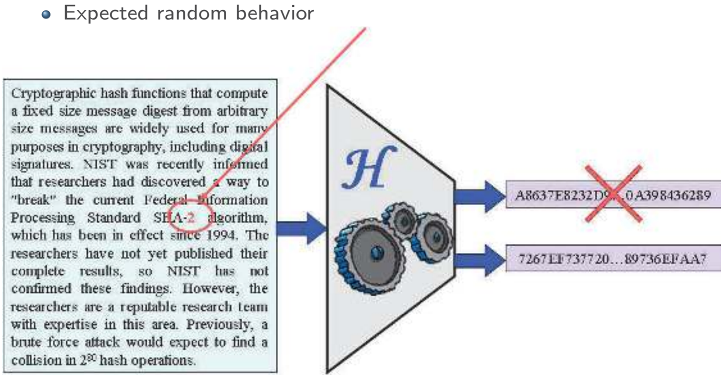

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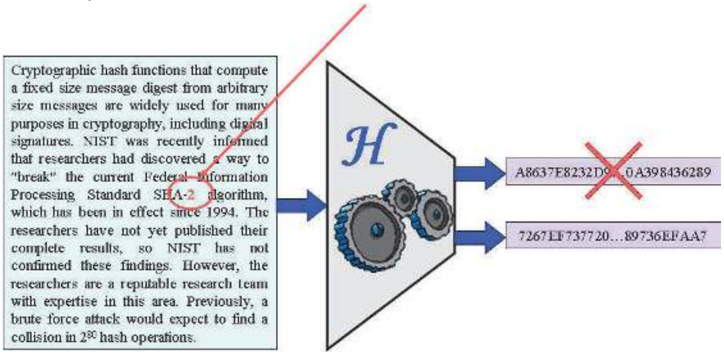

The hash value can be regarded as a fingerprint of the message

Examples of hash functions

Example (some hash functions)

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- MD4, MD5, RIPEMD have 128-bit hash values

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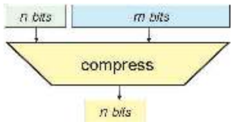
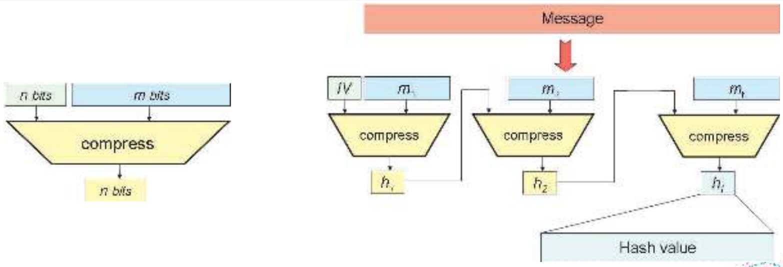


The Merkle-Damgård construction

How to tackle with arbitrarily long inputs ?

- A compression function maintains an n-bit internal state while processing m-bit message blocks
- A chaining construction builds the hash function upon the compression function



Security requirements

Preimage resistance (one-wayness)

Given $y \in \{0,1\}^n$, it should be impossible to find x s.t. $\mathcal{H}(x) = y$

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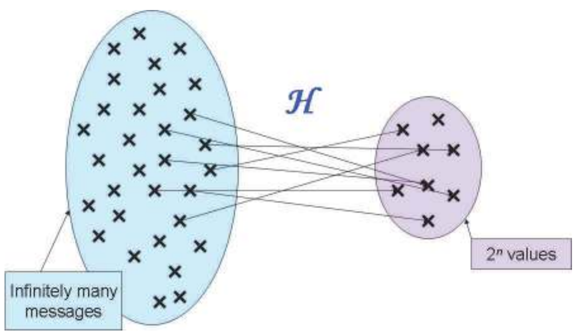
Question

What does impossible means ?

Security considerations



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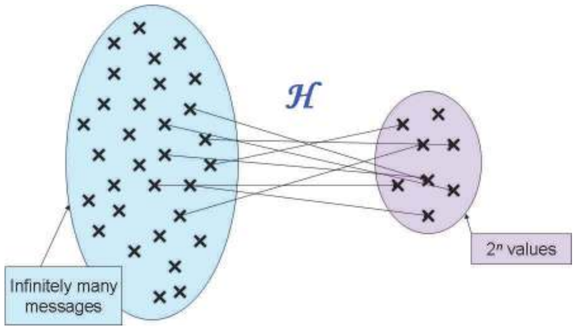

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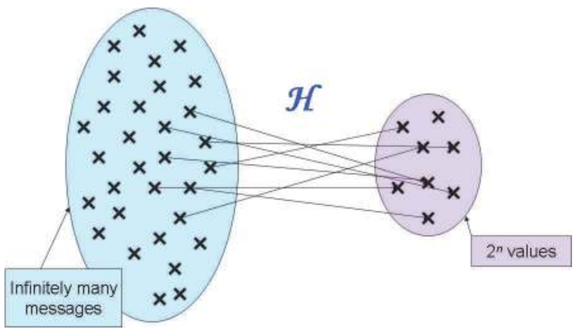

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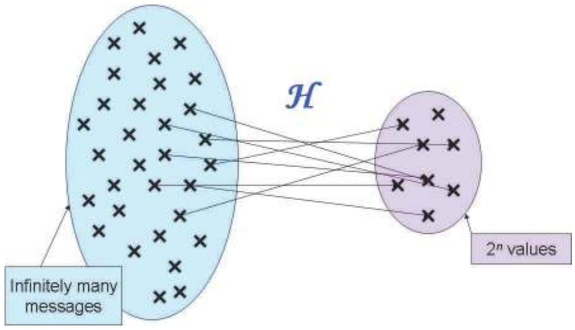

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  - Collisions are unavoidable
- Impossibility (absolute) so reduces to computational unfeasibility (relative)

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A secure hash function must not be vulnerable to better attacks

Complexity figures

- A machine able to perform 10^9 computations per second will perform:

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- $n = 256$ (SHA-256) $\longrightarrow$ $2^{n/2} = 2^{128}$ (highly secure)

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- To be granted access to her account, Alice must present a password which is to be compared with a previously stored value

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Only sign the hash of the message: $\mathsf{Sign}(\mathcal{H}(m))$

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  - Signing several megabytes is too slow
  - Signature algorithms may be malleable

Solution

Only sign the hash of the message: $\mathsf{Sign}(\mathcal{H}(m))$

- If the message is modified, the signature is not valid anymore ...

Digital signature

- The previous technique does not provide non-repudiation (John could claim Chris produced the MAC himself, using shared key K)
- To ensure non-repudiation, use digital signatures
  - Only John can sign the message using his private key
  - Anybody can verify the signature using John's public key
- But ...
  - Signing several megabytes is too slow
  - Signature algorithms may be malleable

Solution

Only sign the hash of the message: $\mathrm{Sign}(\mathcal{H}(m))$

- If the message is modified, the signature is not valid anymore ...
- ... provided the hash function is collision resistant

Birthday paradox

The classical problem

Question

How many persons are needed for having more than 50% chance that two of them share the same birthday ?

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Birthday paradox

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Let $p(365, m)$ the probability that, given m persons, at least two share the same birthday. Then ...

Birthday paradox
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Answer

$$\begin{array}{l} p(365, 22) = 0,476 \\ p(365, 23) = 0,507 \end{array} \longrightarrow m^* = 23$$

Birthday paradox

Generalization

- An urn contains t balls numbered 1 to t

Birthday paradox
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Birthday paradox

Generalization

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- m balls are drawn at random from the urn

What is the probability $p(t,m)$ of at least one coincidence (a ball drawn at least twice) ?

If $m = \mathcal{O}(\sqrt{t})$ and $t \to \infty$ then:

$$p(t,m) \longrightarrow 1 - exp\left(-\frac{m(m-1)}{2t} + \mathcal{O}\left(\frac{1}{\sqrt{t}}\right)\right) \approx 1 - exp\left(-\frac{m^2}{2t}\right)$$

Birthday paradox

Generalization

Proof

Let $q(t,m) = 1 - p(t,m)$ the probability that each m balls differ:

$$q(t,m) = \prod_{k=0}^{m-1}\left(1 - \frac{k}{t}\right)$$

$$ln(q(t,m)) = \sum_{k=0}^{m-1} ln\left(1 - \frac{k}{t}\right)$$

$$= \sum_{k=0}^{m-1}\left[-\frac{k}{t} + o\left(\frac{k}{t}\right)\right]$$

$$= -\frac{m(m-1)}{2t} + \mathcal{O}\left(\frac{m}{t}\right) \qquad \text{if } m = \mathcal{O}(\sqrt{t})$$

$$p(t,m) \overset{t\to\infty}{\longrightarrow} 1 - exp\left(-\frac{m(m-1)}{2t} + \mathcal{O}\left(\frac{1}{\sqrt{t}}\right)\right) \approx 1 - exp\left(-\frac{m^2}{2t}\right)$$

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$$p(t,m) \approx 1 - exp\left(-\frac{m^2}{2t}\right)$$

$$p(t,m) = \frac{1}{2} \iff m = \sqrt{2.ln(2).t} \simeq 1,18\sqrt{t}$$

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Birthday paradox

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$t = 365 \Rightarrow m = 22,54$
Among only 23 persons, you'd better bet on a birthday coincidence

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Among only 23 persons, you'd better bet on a birthday coincidence

Hash collisions

For an n-bit hash function ($t = 2^n$), a collision may be expected after having computed about $2^{n/2}$ hash values

Naive method

A first collision search algorithm:

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4. … and so on until the current hash is already present in the hash array

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This algorithm requires $\mathcal{O}(\sqrt{2^n})$ time and $\mathcal{O}(\sqrt{2^n})$ memory

Floyd's collision finding algorithm

- Starting from an arbitrary x_0, consider the sequence of iterated hashes

$$x_{i+1} = \mathcal{H}(x_i)$$

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$$\begin{cases} x_\alpha = x_\beta \\ x_{\alpha-1} \neq x_{\beta-1} \end{cases} \tag{1}$$

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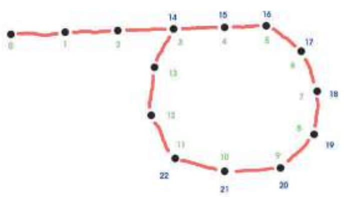
Problem ...

How to find two such equal sequence elements ?

Floyd's collision finding algorithm (example)

- A collision occurred at $\alpha = 3$ and $\beta = 14$

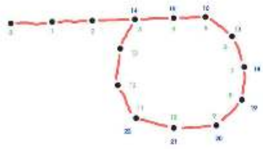
Floyd's collision finding algorithm (example)



- A collision occurred at $\alpha = 3$ and $\beta = 14$
- The cycle length is $\delta = 11$

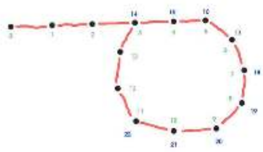
Floyd's collision finding algorithm (description)

- $\alpha = 3$ (tail) and $\delta = 11$ (cycle)



Floyd's collision finding algorithm (description)

- $\alpha = 3$ (tail) and $\delta = 11$ (cycle)
- $x_i = x_{i+k.\delta}$ for all $i \geqslant \alpha$ and $k \in \mathbb{N}$



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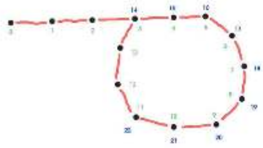

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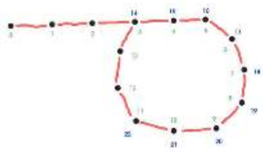

Floyd's algorithm (step 1)

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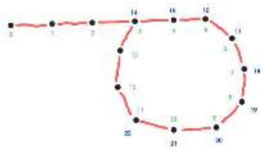

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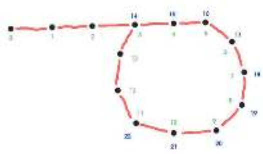

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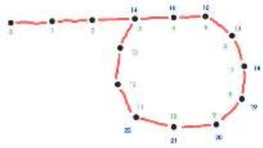

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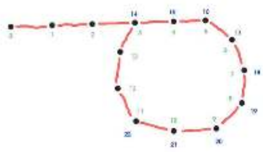

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Cycle length found

A multiple $i_0 = k.\delta$ of the cycle length is obtained

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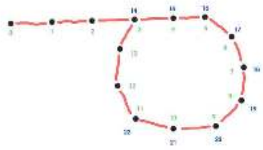

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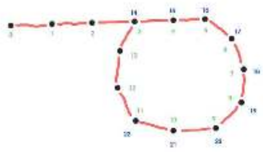

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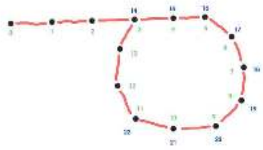

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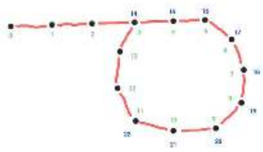

Floyd's algorithm (step 2)

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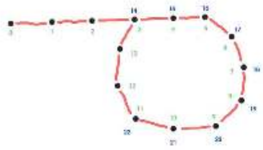

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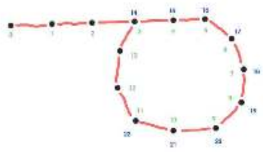

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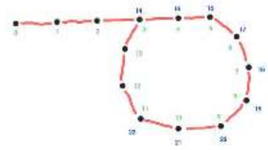

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6. Stop whenever $c_{i_1} = d_{i_1}$ (note that $i_1 = \alpha$)

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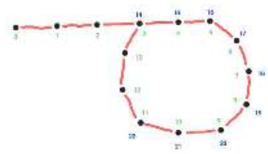

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Collision found !

The collision is given by $\mathcal{H}(x_{\alpha-1}) = \mathcal{H}(x_{\alpha-1+k\delta})$ with $x_{\alpha-1} \neq x_{\alpha-1+k\delta}$

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- This algorithm finds a collision in $3\left\lceil\frac{\alpha}{\delta}\right\rceil.\delta + 2\alpha$ hash evaluations

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Collision search

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- Nothing announced (yet) on recent SHA-2 hash functions family

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- Proclamation of the winner on October 2, 2012:
  (standardized on August 5, 2015)

KECCAK

An Introduction to Hash Functions

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