The Mathematics of Machine Learning
Homework Assignment 1

September 11, 2020

This assignment is due on September 23rd. Please make sure to show all work so that you can get partial credit. Also, please list any parts of definitions or properties required for the problems. The assignment was designed to be done without programming. There are 10 problems, each worth 10 points, for a total of 100 points. This assignment will count as 10% of your grade.

1. **Union and intersection:** Recall that $\mathbb{N} = \{1, 2, 3, \ldots\}$ is the set of natural numbers. Please answer the following questions:

   (a) (3 points) What is $\{x \in \mathbb{N} : x \text{ is even}\} \cap \{x : x \text{ is odd}\}$?

   (b) (3 points) What is $\{x \in \mathbb{N} : x \text{ is even}\} \cup \{x : x \text{ is odd}\}$?

   (c) (4 points) What is $\{x \in \mathbb{N} : x \text{ is even}\} \cap \{x : x \text{ is prime}\}$?

2. **Disjointness and subset:** (10 points) If S and T are disjoint and $S \subseteq T$, what is S? You can use the fact that $S \subseteq T$ if and only if $S \cap T = S$.

3. **Complement:** (10 points) Show that $(S_1 \cup S_2) \cap \tilde{S}_1 = S_2 - S_1$.

4. **Cartesian product:** (10 points) Show that Cartesian product distributes over unions, that is, $S_1 \times (S_2 \cup S_3) = (S_1 \times S_2) \cup (S_1 \times S_3)$.

5. **Cartesian product calculations:** (10 points)

   (a) (5 points) What is $\{1, 2, 3\} \times \{H, T\}$?

   (b) (5 points) Show that $S \times \varnothing = \varnothing$.

6. **Set algebras:** (10 points) For any algebra $\mathcal{F}$, show that if $S_1, S_2 \in \mathcal{F}$ then $S_1 \cap S_2 \in \mathcal{F}$.

7. **Trivial set algebra:** (10 points) Show that $\{\varnothing, \Omega\}$ is an algebra.

8. **Power set and empty set:** (10 points) What is $\mathcal{P}(\varnothing)$?

9. **Empty set probability:** (10 points) Show that $\mathbb{P}(\varnothing) = 0$

10. **Probability measure calculations:** Given the following facts related to a probability measure $\mathbb{P}$ on a universal set Ω:

(a) $\mathbb{P}(S_1) = 0.5$

(b) $\mathbb{P}(S_2) = 0.4$

(c) $\mathbb{P}(S_3) = 0.3$

(d) $S_1 \cap S_2 = S_3$

Calculate the following:

(a) (3 points) $\mathbb{P}(S_1 \cup S_2)$

(b) (3 points) $\mathbb{P}(S_1 - S_2)$

(c) (4 points) $\mathbb{P}\left(\widetilde{S_1 \cup S_2}\right)$