HW1

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1. **Union and intersection**:
   (a) $\{x \in \mathbb{N} : x \text{ is even}\} \cap \{x \in \mathbb{N} : x \text{ is odd}\} = \varnothing$
   (b) $\{x \in \mathbb{N} : x \text{ is even}\} \cup \{x \in \mathbb{N} : x \text{ is odd}\} = \{x \in \mathbb{N}\}$
   (c) $\{x \in \mathbb{N} : x \text{ is even}\} \cap \{x \in \mathbb{N} : x \text{ is odd}\} = \{x : x = 2\}$
2. **Disjoint and subsets**: $S \subseteq T$ if and only if $S \cap T = S$; At the same time, S and T are disjoint, i.e. $S \cap T = \varnothing$; $=>$ S is$\varnothing$
3. **Complement**: $(S_1 \cup S_2) \cap \tilde{S_1} = (S_1 \cap \tilde{S_1}) \cup (S_2 \cap \tilde{S_1}) = \varnothing \cup (S_2 \cap \tilde{S_1}) = S_2 - S_1$
4. **Cartesian product**: $S_1 \times (S_2 \cup S_3) = \{(a, b) : a \in S_1 \text{ and } b \in S_2 \text{ or } S_3\} = \{(a, b) : a \in S_1 \text{ and } b \in S_2\} \cup \{(a, b) : a \in S_1 \text{ and } b \in S_3\} = (S_1 \times S_2) \cup (S_1 \times S_3)$
5. **Cartesian product calculations**:
   (a) $\{1, 2, 3\} \times \{H, T\} = \{(1, H), (2, H), (3, H), (1, T), (2, T), (3, T)\}$
   (b) $S \times \varnothing = \{(a, b) : a \in S \text{ and } b \in \varnothing\} = \{(a, \varnothing)\} = \varnothing$
6. **Set algebras**: If $S_1, S_2 \in \mathcal{F}$, it is known from 2nd property of algebra definition that $S_1 \cup S_2 \in \mathcal{F}$. Since $(S_1 \cap S_2) \in (S_1 \cup S_2)$, then $(S_1 \cup S_2) \in \mathcal{F}$.
7. **Trivial set algebra**: $\{\varnothing, \Omega\}$ is an algebra:
   (a) $\Omega \in \{\varnothing, \Omega\}$
   (b) $F_1, F_2 \in \mathcal{F}$, then $(F_1, F_2) \in \{(\varnothing, \varnothing), (\varnothing, \Omega), (\Omega, \varnothing), (\Omega, \Omega)\}$, then $(F_1 \cup F_2) \in \{\varnothing, \Omega\}$ anyway.
   (c) If $F \in \{\varnothing, \Omega\}$, then F is $\varnothing$ or Ω, $\tilde{F}$ would be Ω or $\varnothing$.Either way,F $\in \{\varnothing, \Omega\}$ and $\tilde{F} \in \{\varnothing, \Omega\}$
8. **Power set and empty set**: $\mathcal{P}(\varnothing)$.
   (a) The power set of a set is the set that contains all of its subsets.
   (b) The only subset of $\varnothing$is$\varnothing$ itself.
   (c) As a result, the power set of $\varnothing$ is $\varnothing$
   (d) $\mathcal{P}(\varnothing) = \{\varnothing\}$.
9. **Empty set probability**:
   (a) $\mathbb{P}(\Omega) = 1$
   (b) When $S_1, S_2 \in \mathcal{F}$ are disjoint, $\mathbb{P}(S_1 \cup S_2) = \mathbb{P}(S_1) + \mathbb{P}(S_2)$. As a result, $\mathbb{P}(\varnothing \cup \Omega) = \mathbb{P}(\varnothing) + \mathbb{P}(\Omega)$.
   (c) $\mathbb{P}(\varnothing \cup \Omega) = \mathbb{P}(\Omega) = 1, \text{ then } \mathbb{P}(\varnothing) + \mathbb{P}(\Omega) = \mathbb{P}(\Omega) = 1, \text{ so } \mathbb{P}(\varnothing) = 0.$
10. **Probability measure calculations**: Given a,b,c,d, it is known that $\mathbb{P}(S_3) = 0.3, \mathbb{P}(S_1 - S_3) = 0.2, \mathbb{P}(S_2 - S_3) = 0.1$
    (a) $\mathbb{P}(S_1 \cup S_2) = 0.2 + 0.3 + 0.1 = 0.6$
    (b) $\mathbb{P}(S_1 - S_2) = \mathbb{P}(S_1 \cap \tilde{S_2}) = 0.2$
    (c) $\mathbb{P}(S_1 \tilde{\cup} S_2) = \mathbb{P}(x : x \notin S_1 \text{ nor } S_2) = 1 - (0.2 + 0.3 + 0.1) = 0.4$