Example 7.3

Suppose that the term structure of risk-free interest rates is flat in both Japan and the United States. The Japanese rate is 1.5% per annum and the U.S. rate is 2.5% per annum (both with continuous compounding). A financial institution has entered into a currency swap in which it receives 3% per annum in yen and pays 4% per annum in dollars once a year. The principals in the two currencies are \$10 million and 1,200 million yen. The swap will last for another three years, and the current exchange rate is 110 yen per dollar. The calculations for valuing the swap as the sum of forward foreign exchange contracts are summarized in the

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following table (all amounts are in millions):

Time (years)	Dollar cash flow	Yen cash flow	Forward exchange rate	Dollar value of yen cash flow	Net cash flow	Present value
1	-0.4	+36	0.009182	0.3306	-0.0694	-0.0677
2	-0.4	+36	0.009275	0.3339	-0.0661	-0.0629
3	-10.4	+1236	0.009368	11.5786	+1.1786	+1.0934
Total						+0.9629

The financial institution pays $0.04 \times 10 = \$0.4$ million dollars and receives $1,200\times0.03=36$ million yen each year. In addition, the dollar principal of \$10 million is paid and the yen principal of 1,200 is received at the end of year 3. The current spot rate is 1/110 = 0.009091 dollar per yen. In this case, r = 2.5%and $r_f = 1.5\%$ so that the one-year forward exchange rate is, from equation (5.9), $0.009091e^{(0.025-0.015)\times 1} = 0.009182$. The two- and three-year forward exchange rates in the table are calculated similarly. The forward contracts underlying the swap can be valued by assuming that the forward exchange rates are realized. If the one-year forward exchange rate is realized, the value of yen cash flow in year 1 will be $36 \times 0.009182 = 0.3306$ million dollars and the net cash flow at the end of year 1 will be 0.3306 - 0.4 = -0.0694 million dollars. This has a present value of $-0.0.0694e^{-0.025\times1} = -0.0677$ million dollars. This is the value of the forward contract corresponding to the exchange of cash flows at the end of year 1. The value of the other forward contracts are calculated similarly. As shown in the table, the total value of the forward contracts is \$0.9629 million.

¹⁰ Usually it makes sense for the financial institution to bear the foreign exchange risk, because it is in the best position to hedge the risk.