$$|f(x-\tfrac{\delta}{2}) - f(x+\tfrac{\delta}{2})|$$
$$= |z_i - z_{i+1}| = 1 > \epsilon$$

for any $\epsilon < 1$ so that

$$\mathbb{P}(f \text{ is continuous})$$
$$= \mathbb{P}(z_1 = z_2 = \dots = z_m)$$
$$= \boxed{2^{-m+1}}$$

(b) $$\mathbb{P}\left(X_1 \in \left(\tfrac{i-1}{m}, \tfrac{i}{m}\right]\right) = \frac{i}{m} - \frac{i-1}{m} = \boxed{\frac{1}{m}}$$

(c) $$\mathbb{P}\left(\{X_1, X_2, \dots, X_n\} \cap \left(\tfrac{i-1}{m}, \tfrac{i}{m}\right] \neq \emptyset\right)$$
$$= 1 - \mathbb{P}\left(\bigcap_{j=1}^{n} \left\{X_j \notin \left(\tfrac{i-1}{m}, \tfrac{i}{m}\right]\right\}\right)$$
$$= 1 - \prod_{j=1}^{n} \mathbb{P}\left(X_j \notin \left(\tfrac{i-1}{m}, \tfrac{i}{m}\right]\right)$$
$$= 1 - \prod_{j=1}^{n} \left(1 - \frac{1}{m}\right) = \boxed{1 - \left(1 - \frac{1}{m}\right)^n}$$

(d) $$\mathbb{P}\left(\{X_1, X_2, \dots, X_n\} \cap \left(\tfrac{\lceil mX_{n+1}\rceil - 1}{m}, \tfrac{\lceil mX_{n+1}\rceil}{m}\right] \neq \emptyset\right)$$
$$= 1 - \mathbb{P}\left(\bigcap_{j=1}^{n} \left\{X_j \notin \left(\tfrac{\lceil mX_{n+1}\rceil - 1}{m}, \tfrac{\lceil mX_{n+1}\rceil}{m}\right]\right\}\right)$$
$$= 1 - \prod_{j=1}^{n} \mathbb{P}\left(X_j \notin \left(\tfrac{\lceil mX_{n+1}\rceil - 1}{m}, \tfrac{\lceil mX_{n+1}\rceil}{m}\right]\right)$$
$$= 1 - \prod_{j=1}^{n} \left(1 - \frac{1}{m}\right) = \boxed{1 - \left(1 - \frac{1}{m}\right)^n}$$

(e) $$\boxed{1 - \left(1 - \frac{1}{m}\right)^n}$$

(c) $$\mathbb{P}(\{X_1, X_2, \dots, X_n\} \cap (i-1, i])$$

2. (a) $C_{t+1} = C_t\left(1+\alpha(w^T R_t)^+ - \beta(w^T R_t)^-\right)$

$$C_{t+1} = \prod_{i=1}^{t}\left(1+\alpha(w^T R_t)^+ - \beta(w^T R_t)^-\right)$$

$$\lim_n \frac{1}{n}\ln C_n = \lim_n \frac{1}{n}\sum_{i=1}^{t}\ln\left(1+\alpha(w^T R_t)^+ - \beta(w^T R_t)^-\right)$$

$$= \boxed{E\left[\ln\left(1+\alpha(w^T R_t)^+ - \beta(w^T R_t)^-\right)\right]}$$

(b)

$$\ell = p_1\ln(1+\alpha w r_1^+ - \beta w r_1^-) + (1-p_1)\ln(1+\alpha w r_2^+ - \beta w r_2^-)$$

Assume $r_2^-, r_1^+ > 0$

$$= p_1\ln(1+\alpha w r_1) + (1-p_1)\ln(1+\beta w r_2)$$

$$\frac{\partial \ell}{\partial w} = \frac{p_1 \alpha r_1}{1+\alpha w r_1} + \frac{(1-p_1)\beta r_2}{1+\beta w r_2} = 0$$

$$p_1\alpha r_1(1+\beta w r_2) + (1-p_1)\beta r_2(1+\alpha w r_1) = 0$$

$$(p_1\alpha r_1\beta r_2 + (1-p_1)\beta r_2 \alpha r_1)w = -(p_1\alpha r_1 + (1-p_1)\beta r_2)$$

$$\boxed{w = -\frac{p_1\alpha r_1 + (1-p_1)\beta r_2}{\alpha\beta r_1 r_2}}$$

(c)

$$\frac{\alpha + \beta/4}{\alpha\beta}$$

α	β	w^*
$\frac{1}{100}$	$\frac{1}{100}$	50
$\frac{1}{2}$	1	0
$\frac{1}{2}$	2	-0.5
1	1	0.5

3. (a)

$$f(x) = \frac{1}{1+\exp(-x)}$$

$$f'(x) = \frac{\exp(-x)}{(1+\exp(-x))^2}$$

$$\boxed{f'(x) \leq \frac{1}{2^2} = \frac{1}{4}}$$

$$f''(x) = \frac{-((1+\exp(-x))^2 \exp(-x) - \exp(-x)2(1+\exp(-x))(-\exp(-x))}{(1+\exp(-x))^4}$$

$$= \frac{-\exp(-x) - 2\exp(-2x) - \exp(-3x) + 2\exp(-2x) + 2\exp(-3x)}{(1+\exp(-x))^4}$$

$$= \frac{\exp(-3x) - \exp(-x)}{(1+\exp(-x))^4}$$

$$y^3 - y = 0$$

$$y = 0, \pm 1$$

(b)

$$h'(x) = g_1'(g_2(x))g_2'(x)$$

$$|h'(x)| \leq |g_1'(g_2(x))|\,|g_2'(x)| \leq \boxed{C_1 C_2}$$

(c)

$$\|X_{j+1,i}(x) - X_{j+1,i}(y)\|_\infty$$

$$= \|f(a_{i,j}^T X_j(x)) - f(a_{i,j}^T X_j(y))\|_\infty$$

$$\leq \frac{1}{4}\|a_{i,j}^T(X_j(x) - X_j(y))\|_\infty \leq \boxed{\frac{1}{4}\|a_{i,j}\|_\infty \|C_j\|_\infty}$$

(d)

$$\text{Let } \mathcal{L}(f) = \sup_{x,y} \|f(x) - f(y)\|_\infty$$

$$\mathcal{L}(X_{j,i}) \leq \mathcal{L}(f)\,\mathcal{L}(a_{j,i}^T X_{j-1})$$

$$= \frac{1}{4}\sup_{x,y} \|a_{j,i}^T(X_{j-1}(x) - X_{j-1}(y))\|_\infty$$

$$\leq \frac{1}{4}\sup_{x,y} \|a_{j,i}\|_\infty \|X_{j-1}(x) - X_{j-1}(y)\|_\infty$$

$$\leq \frac{1}{4}\, a \sup_i \mathcal{L}(X_{j,i})$$

$$\leq \boxed{\left(\frac{a}{4}\right)^m}$$

4. (a)

$$\mathbb{P}\left(\sup_{f\in F_C} \hat{E}_n[f(X)] - E[f(X)] > \epsilon\right)$$

$$\leq N\left(\frac{\epsilon}{3}, F_C, \|\cdot\|_\infty\right) \exp\left(-2n\left(\frac{\epsilon}{3}\right)^2\right)$$

$$\leq \boxed{\left(\frac{24}{\epsilon}\right)\left(\left(\frac{2C}{3\epsilon}\right)^d\right) \exp\left(-2n\left(\frac{\epsilon}{3}\right)^2\right)}$$

(b)

$$E[L(Y,\hat{f}(X))] - E[L(Y,f^*(X))]$$

$$= E[L(Y,\hat{f}(X))] - E_n[L(Y,\hat{f}(X))]$$

$$+ E_n[L(Y,\hat{f}(X))] - E_n[L(Y,f^*(X))]$$

$$+ E_n[L(Y,f^*(X))] - E[L(Y,f^*(X))]$$

$$\leq E[L(Y,\hat{f}(X))] - E_n[L(Y,\hat{f}(X))]$$

$$+ E_n[L(Y,f^*(X))] - E[L(Y,f^*(X))]$$

$$\mathbb{P}\left(E[L(Y,\hat{f}(X))] - E[L(Y,f^*(X))] \geq \epsilon \right)$$

$$\leq \mathbb{P}\left(E[L(Y,\hat{f}(X))] - E_n[L(Y,\hat{f}(X))] > \frac{\epsilon}{2} \right)$$

$$+ \mathbb{P}\left(E_n[L(Y,\hat{f}(X))] - E[L(Y,f^*(X))] > \frac{\epsilon}{2} \right)$$

$$\leq \boxed{2 \left(\frac{48}{\epsilon}\right)^{\left(\left(\frac{4c}{3\epsilon}\right)^d\right)} \exp\left(-2n\left(\frac{\epsilon}{6}\right)^2\right)}$$

(c)

$$2\left(\frac{48}{\epsilon}\right)^{\left(\left(\frac{4c}{3\epsilon}\right)^d\right)} \exp\left(-2n\left(\frac{\epsilon}{6}\right)^2\right) \leq \delta$$

$$\exp\left(-2n\left(\frac{\epsilon}{6}\right)^2\right) \leq \frac{\delta}{2}\left(\frac{48}{\epsilon}\right)^{-\left(\left(\frac{4c}{3\epsilon}\right)^d\right)}$$

$$-2n\left(\frac{\epsilon}{6}\right)^2 \leq \ln\left(\delta/2\right) - \left(\frac{4c}{2\epsilon}\right)^d \ln\left(\frac{48}{\epsilon}\right)$$

$$\boxed{n \geq \frac{\ln(2/\delta)}{2\left(\frac{\epsilon}{6}\right)^2} + \left(\frac{4c}{2\epsilon}\right)^d \frac{\ln\left(\frac{48}{\epsilon}\right)}{2\left(\frac{\epsilon}{8}\right)^2}}$$