1. (a) $\emptyset$

(b) $\mathbb{N}$

(c) $\{2\}$

2. S and T disjoint

$$S \cap T = \emptyset$$

$$S \subseteq T$$

$$S \cap T = S$$

$$S = \emptyset$$

3. $(S_1 \cup S_2) \cap \tilde{S}_1$

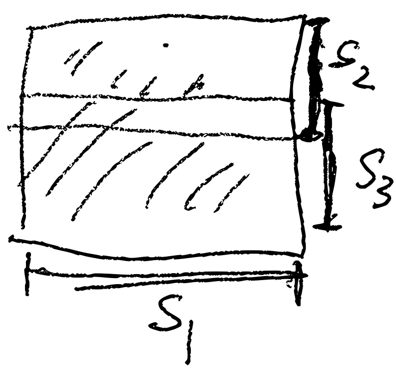
$$= (S_1 \cap \tilde{S}_1) \cup (S_2 \cap \tilde{S}_1)$$

$$= \emptyset \cup (S_2 \cap \tilde{S}_1)$$

$$= \underbrace{S_2 \cap \tilde{S}_1 = S_2 - S_1}_{\text{definition}}$$

4. WTS
$$S_1 \times (S_2 \cup S_3) = (S_1 \times S_2) \cup (S_1 \times S_3)$$

$$S_1 \times (S_2 \cup S_3)$$
$$= \{(x,y) : x \in S_1, y \in S_2 \cup S_3\}$$
$$= \{(x,y) : x \in S_1, (y \in S_2 \text{ or } y \in S_3)\}$$
$$= \{(x,y) : (x \in S_1, y \in S_2) \text{ or } (x \in S_1, y \in S_3)\}$$
$$= \{(x,y) : x \in S_1, y \in S_2\} \cup \{(x,y) : x \in S_1, y \in S_3\}$$
$$= (S_1 \times S_2) \cup (S_1 \times S_3)$$



5. a) $\{1, 2, 3\} \times \{H, T\}$

$= \{(1,H), (2,H), (3,H),$
$(1,T), (2,T), (3,T)\}$

b) $S \times \phi$

$= \{(x,y) : x \in S,\ y \in \phi\}$

no such y

$= \phi$

6. Axioms 1) $S_1 \in \mathcal{F} \Rightarrow \tilde{S}_1 \in \mathcal{F}$

2) $S_1, S_2 \in \mathcal{F} \Rightarrow S_1 \cup S_2 \in \mathcal{F}$

De Morgan's law

$S_1 \cap S_2 = \widetilde{\tilde{S}_1 \cup \tilde{S}_2}$

$S_1, S_2 \in \mathcal{F} \Rightarrow \tilde{S}_1 \in \mathcal{F}\ \ \tilde{S}_2 \in \mathcal{F}$ by 1

$\Rightarrow \tilde{S}_1 \cup \tilde{S}_2 \in \mathcal{F}$ by 2 $\Rightarrow \widetilde{\tilde{S}_1 \cup \tilde{S}_2} \in \mathcal{F}$ by 1

$= S_1 \cap S_2$ by Do Morgan

7. $\{\phi, \Omega\}$

Axioms

1) $\phi \in \mathcal{F}$

2) $S_1 \in \mathcal{F} \Rightarrow \tilde{S}_1 \in \mathcal{F}$

3) $S_1, S_2 \in \mathcal{F} \Rightarrow S_1 \cup S_2 \in \mathcal{F}$

Axiom 1 ✓

Axiom 2

$\tilde{\phi} = \Omega \in \mathcal{F}$

$\tilde{\Omega} = \phi \in \mathcal{F}$ ✓

Axiom 3

$\phi \cup \phi = \phi \in \mathcal{F}$

$\phi \cup \Omega = \Omega \in \mathcal{F}$

$\Omega \cup \phi = \Omega \in \mathcal{F}$

$\Omega \cup \Omega = \Omega \in \mathcal{F}$ ✓

8. ϕ (wrong) $\quad \mathcal{P}(\phi) = \{\phi\}$

$\phi \subseteq \phi$

9. $P(\tilde{S}) = 1 - P(S)$

$\underline{P(\phi)} = 1 - P(\tilde{\phi})$

$= 1 - \underline{P(\Omega)} = 1 - 1 = 0$

10. $P(S_1) = 0.5$

$P(S_2) = 0.4$

$P(S_3) = 0.3$

$S_1 \cap S_2 = S_2$

a) $P(S_1 \cup S_2) = P((S_1 - S_2) \cup (S_1 \cap S_2) \cup (S_2 - S_1))$

$= P(S_1 - S_2) + P(S_1 \cap S_2) + P(S_2 - S_1)$

$= P(S_1 - S_1 \cap S_2) + P(S_3) + P(S_2 - S_1 \cap S_2)$

$= P(S_1) - P(S_3) + P(S_3) + P(S_2) - P(S_3)$

$= 0.5 - 0.3 + 0.3 + 0.4 - 0.3 = 0.6$

10 b) $P(S_1 - S_2)$

$= P(S_1 - S_1 \cap S_2) = P(S_1 - S_3)$

$= P(S_1) - P(S_3) = 0.5 - 0.3 = 0.2$

c) $P(\overline{S_1 \cup S_2}) = 1 - P(S_1 \cup S_2)$

$= 1 - 0.6 = 0.4$