Bayesian Statistics and Bayesian Cognitive Modeling (BayesCog)

Part 1

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https://github.com/lei-zhang/BayesCog_Part1

Schedule

cognitive model
statistics
computing

Part1	09–10 Jan.	Introduction R Basics Probability Basics Bayes' theorem MCMC and Stan Single-Parameter Model – Binomial Model Multiple-Parameter Model – Linear Regression Inference, Posterior Predictive Check
Part2	12–14 Feb.	Reinforcement Learning Model Hierarchical Models Optimizing Stan Codes Model Comparison Stan Style Tip and Debugging Model-Based fMRI Capstone Project: Delay Discounting Task

Introduction

What is your experience with…

- Statistics?
- R? (and / or Matlab?)
- Cognitive Modeling?

You would like to…

- gain knowledge of Bayesian stats?
- be able to read "computational modeling" section in papers?
- write your own model?

About me

- Current position: Postdoc @ SCAN-Unit, UNIVIE
- Ph.D. Cognitive/computational neuroscience @ Gläscher Lab, ISN UKE
- M.Sc. Cognitive neuroscience
- B.Sc. Psychology

- My journey through computational modeling
  - Started with MLE (@fminsearch in Matlab)
  - Switched to Bayesian: first JAGS, then Stan

Goal of this course

- Practical R programming

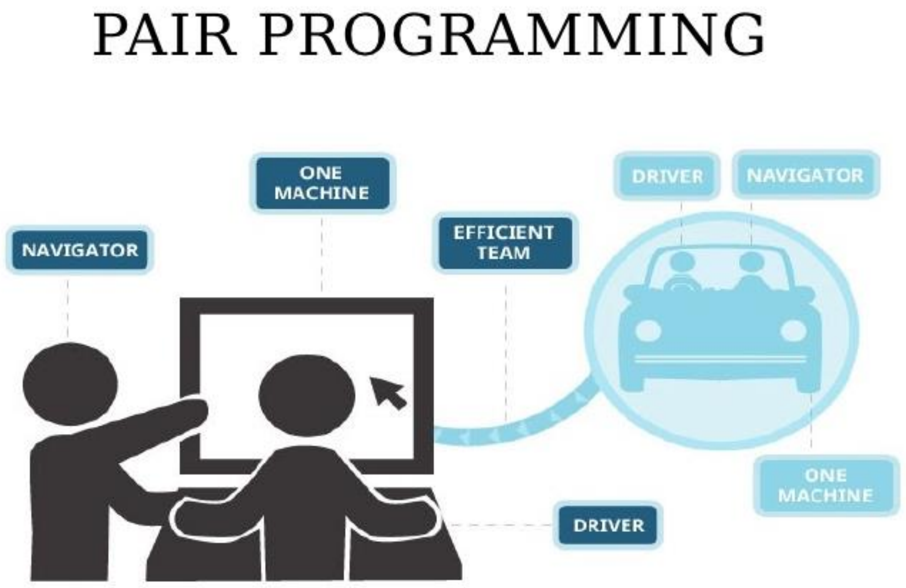
- Practical model-building in Stan, model diagnostics

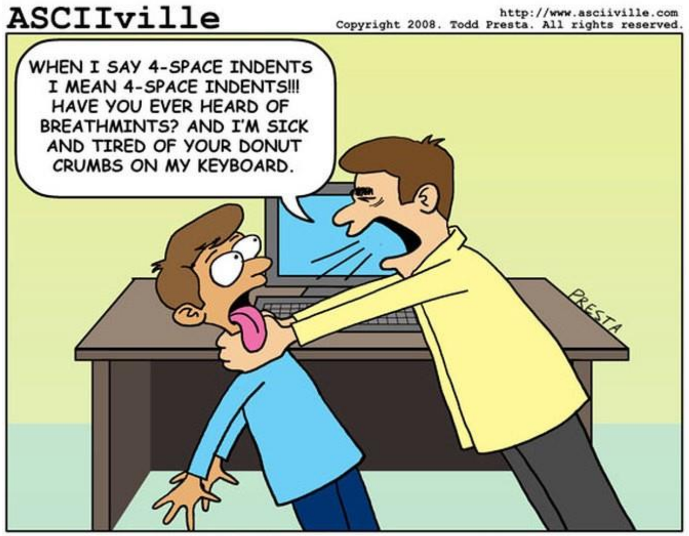
- (Enough) theory to ground you

- Be comfortable to use R/Stan for your own work: model building/estimating/testing/validating, and/or model-based fMRI/EEG

How to Get the Most out of the course

- Workshop structure: interleaved theory/demo + exercise
- Work in pairs: Talk to each other & help each other
- Ask questions
- Try the exercises





The dark side of pair programming.

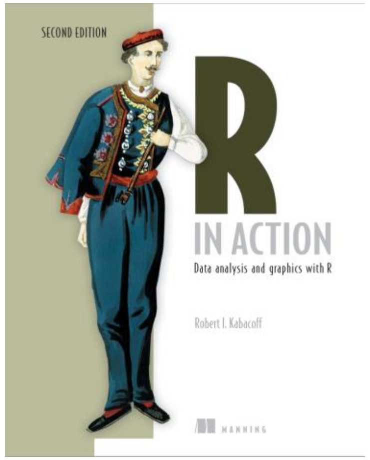
Richard McElreath
@rlmcelreath

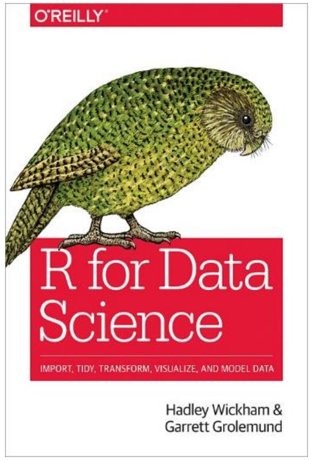
I say this a lot, bc I am also confused quite often.

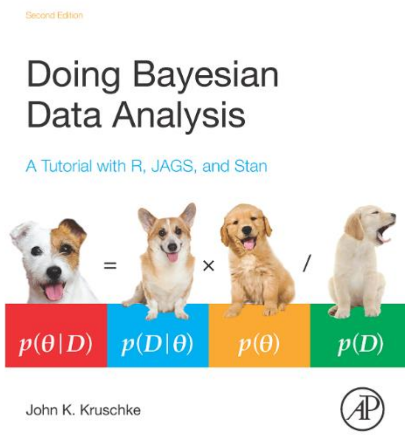
> **Anna Jacobson** @AnnaChingChing · Feb 21
>
> “If you are confused, it is only because you are trying to understand.” - @rlmcelreath in Statistical Rethinking

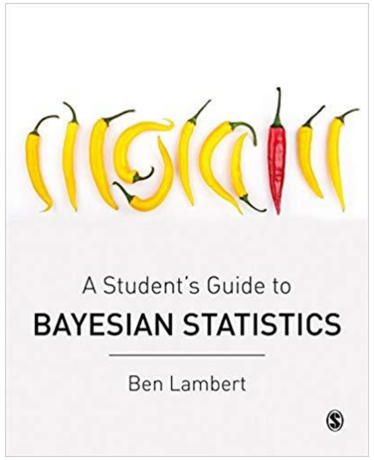
Now let’s begin!

Resources







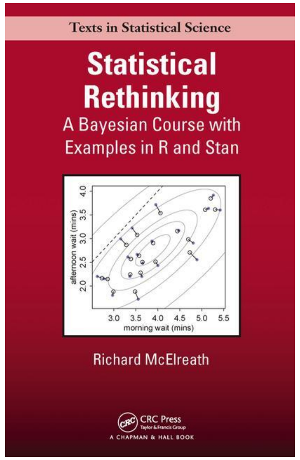


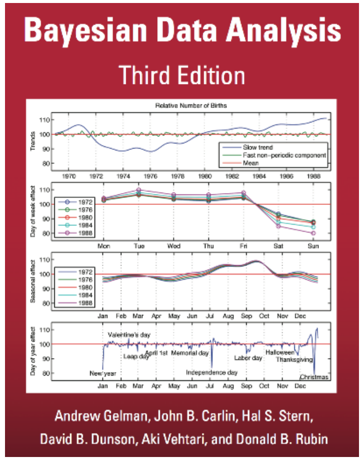
Statistical Thinking for the 21st Century

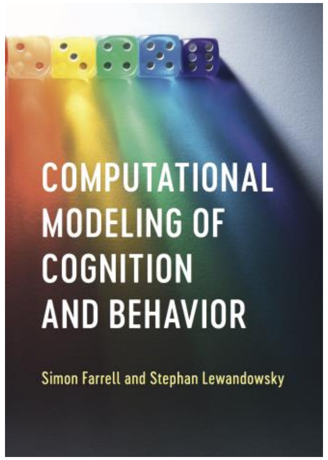
Copyright 2018 Russell A. Poldrack

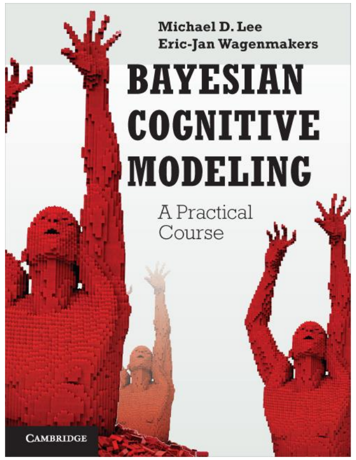
Draft: 2018-11-22

http://thinkstats.org/









DataCamp

https://www.datacamp.com/

JASP

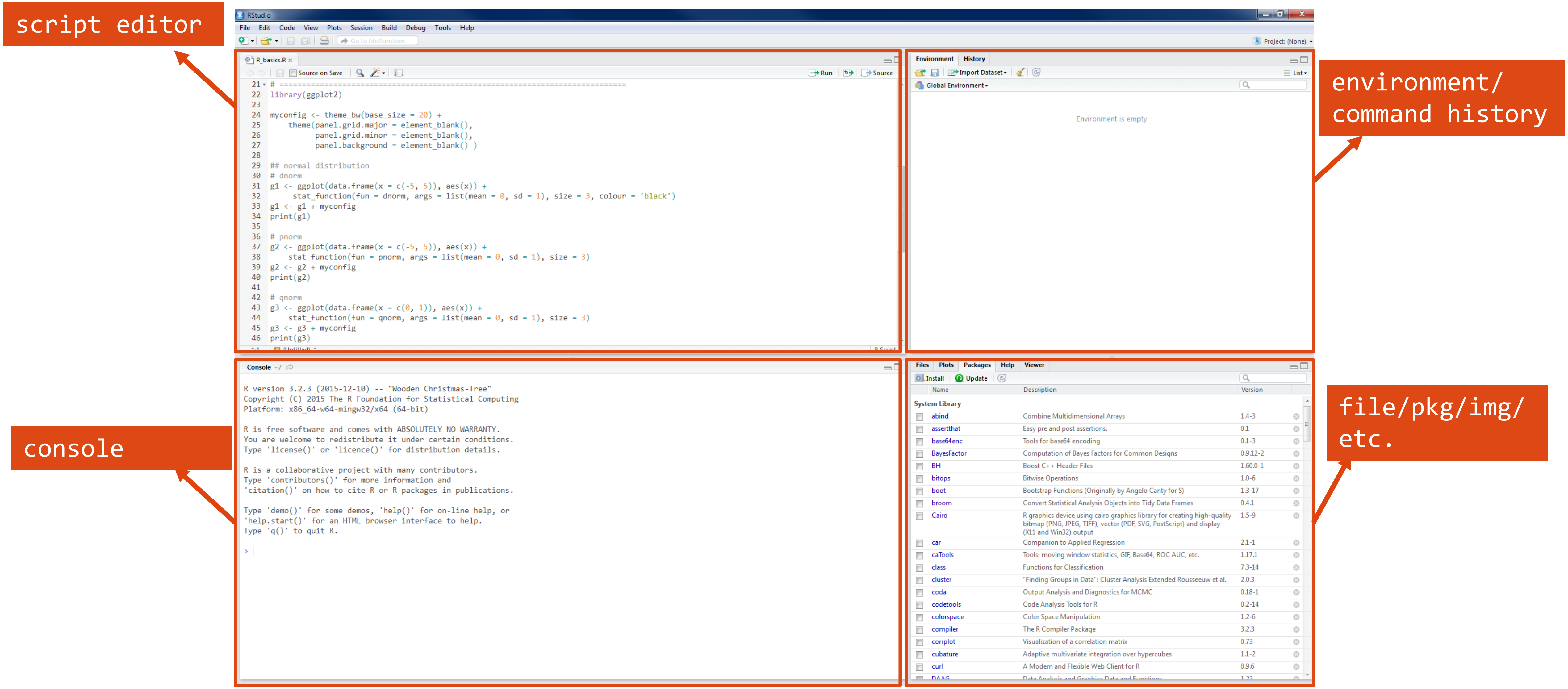
https://jasp-stats.org/

BASICS
OF
R
PROGRAMMING

R Basics

- R
  - a programming language for statistical computing
  - R has its own user interface
  - freely available on Windows, Mac, and Linux

- R Studio
  - integrated development environment (IDE) for R
  - a more sophisticated R-friendly editor, with helpful syntax highlight



script editor
environment/ command history
console
file/pkg/img/ etc.
RStudio
File Edit Code View Plots Session Build Debug Tools Help
Project: (None)
R_basics.R
Source on Save
Run
Source
# ==
library(ggplot2)
myconfig <- theme_bw(base_size = 20) +
theme(panel.grid.major = element_blank(),
panel.grid.minor = element_blank(),
panel.background = element_blank())
normal distribution
# dnorm
g1 <- ggplot(data.frame(x = c(-5, 5)), aes(x)) +
stat_function(fun = dnorm, args = list(mean = 0, sd = 1), size = 3, colour = 'black')
g1 <- g1 + myconfig
print(g1)
# pnorm
g2 <- ggplot(data.frame(x = c(-5, 5)), aes(x)) +
stat_function(fun = pnorm, args = list(mean = 0, sd = 1), size = 3)
g2 <- g2 + myconfig
print(g2)
# qnorm
g3 <- ggplot(data.frame(x = c(0, 1)), aes(x)) +
stat_function(fun = qnorm, args = list(mean = 0, sd = 1), size = 3)
g3 <- g3 + myconfig
print(g3)
Environment
History
Import Dataset
List
Global Environment
Environment is empty
Console ~/
R version 3.2.3 (2015-12-10) -- "Wooden Christmas-Tree"
Copyright (C) 2015 The R Foundation for Statistical Computing
Platform: x86_64-w64-mingw32/x64 (64-bit)
R is free software and comes with ABSOLUTELY NO WARRANTY.
You are welcome to redistribute it under certain conditions.
Type 'license()' or 'licence()' for distribution details.
R is a collaborative project with many contributors.
Type 'contributors()' for more information and
'citation()' on how to cite R or R packages in publications.
Type 'demo()' for some demos, 'help()' for on-line help, or
'help.start()' for an HTML browser interface to help.
Type 'q()' to quit R.
Files Plots Packages Help Viewer
Install
Update
Name Description Version
System Library
abind Combine Multidimensional Arrays 1.4-3
assertthat Easy pre and post assertions. 0.1
base64enc Tools for base64 encoding 0.1-3
BayesFactor Computation of Bayes Factors for Common Designs 0.9.12-2
BH Boost C++ Header Files 1.60.0-1
bitops Bitwise Operations 1.0-6
boot Bootstrap Functions (Originally by Angelo Canty for S) 1.3-17
broom Convert Statistical Analysis Objects into Tidy Data Frames 0.4.1
Cairo R graphics device using cairo graphics library for creating high-quality bitmap (PNG, JPEG, TIFF), vector (PDF, SVG, PostScript) and display (X11 and Win32) output 1.5-9
car Companion to Applied Regression 2.1-1
caTools Tools: moving window statistics, GIF, Base64, ROC AUC, etc. 1.17.1
class Functions for Classification 7.3-14
cluster "Finding Groups in Data": Cluster Analysis Extended Rousseeuw et al. 2.0.3
coda Output Analysis and Diagnostics for MCMC 0.18-1
codetools Code Analysis Tools for R 0.2-14
colorspace Color Space Manipulation 1.2-6
compiler The R Compiler Package 3.2.3
corrplot Visualization of a correlation matrix 0.73
cubature Adaptive multivariate integration over hypercubes 1.1-2
curl A Modern and Flexible Web Client for R 0.9.6

Know your R

```
>R.version
               _
platform       x86_64-w64-mingw32
arch           x86_64
os             mingw32
system         x86_64, mingw32
status
major          3
minor          5.1
year           2018
month          07
day            02
svn rev        74947
language       R
version.string R version 3.5.1 (2018-07-02)
nickname       Feather Spray
```

R Console as a Calculator

Addition and Subtraction

```
> 3+2
[1] 5

> 3-2
[1] 1
```

Multiplication and Division

```
> 3*2
[1] 6

> 3/2
[1] 1.5
```

Exponents in R

```
> 3^2
[1] 9

> 2^3
[1] 8
```

Constants in R

```
> pi
[1] 3.141593
```

```
> exp(1)
```

base of the natural logarithm

```
[1] 2.718282
```

Special values

Infinite Values

```
> Inf
[1] Inf

> 1+Inf
[1] Inf
```

Machine Epsilon

```
> .Machine$double.eps
[1] 2.220446e-16

> 0>.Machine$double.eps
[1] FALSE
```

Empty Values

```
> NULL
NULL

> 1+NULL
numeric(0)
```

Missing Values

```
> NA
[1] NA

> 1+NA
[1] NA
```

Storing and manipulating variables

Define objects `x` and `y` with values of 3 and 2, respectively:

```
> x=3
> y=2
```

Some calculations with the defined objects `x` and `y`:

```
> x+y
[1] 5

> x*y
[1] 6
```

Warning: R is case sensitve, so `x` and `X` are not the same object.

Basic R functions

Combine

```
> c(1,3,-2)
[1]  1  3 -2
```

```
> c("a","a","b","b","a")
[1] "a" "a" "b" "b" "a"
```

Sum and Mean

```
> sum(c(1,3,-2))
[1] 2
```

```
> mean(c(1,3,-2))
[1] 0.6666667
```

Variance and Std. Dev.

```
> var(c(1,3,-2))
[1] 6.333333
```

```
> sd(c(1,3,-2))
[1] 2.516611
```

Minimum and Maximum

```
> min(c(1,3,-2))
[1] -2
```

```
> max(c(1,3,-2))
[1] 3
```

Basic R functions (cont.)

Define objects x and y:

```
> x=c(1,3,4,6,8)
> y=c(2,3,5,7,9)
```

Calculate the correlation:

```
> cor(x,y)
[1] 0.988765
```

Calculate the covariance:

```
> cov(x,y)
[1] 7.65
```

Combine as columns

```
> cbind(x,y)
     x y
[1,] 1 2
[2,] 3 3
[3,] 4 5
[4,] 6 7
[5,] 8 9
```

Combine as rows

```
> rbind(x,y)
  [,1] [,2] [,3] [,4] [,5]
x    1    3    4    6    8
y    2    3    5    7    9
```

Basic Commands

```
getwd()
setwd('E:/teaching/BayesCog_Wien/')
dir() # folders/files in the wd
ls()  # anything in the environment/workspace
print('Hello World!')
cat('Hello', 'World!')
paste0('C:/', 'Group1')
help(func)
? func # and Google!
a <- 5
a = 5
head(d) # first 6 entries
tail(d) # last 6 entries
save(varname, file = "pathname/varname.RData")
load("pathname/varname.RData")
rm(list = ls())
q()
```

RStudio - Shortcuts

`Ctrl + L`: clean console

`Ctrl + Shift + N`: create a new script

`↑`: command history

`Ctrl(hold) + ↑`: command history with certain starts

`Ctrl + Enter`: execute selected codes (in a script)

Editor (WIN general) - Shortcuts

<u>`Ctrl + home/Pos`</u>`: go to the very top of a script`

<u>`Ctrl + end/Ende`</u>`: go to the very end of a script`

<u>`Shift(hold) + ↑/↓`</u>`: select line(s)`

<u>`Ctrl(hold) + ←/→`</u>`: select word(s)`

Data Classes

`numeric: 1.1 2.0`

`integer: 1 2 3`

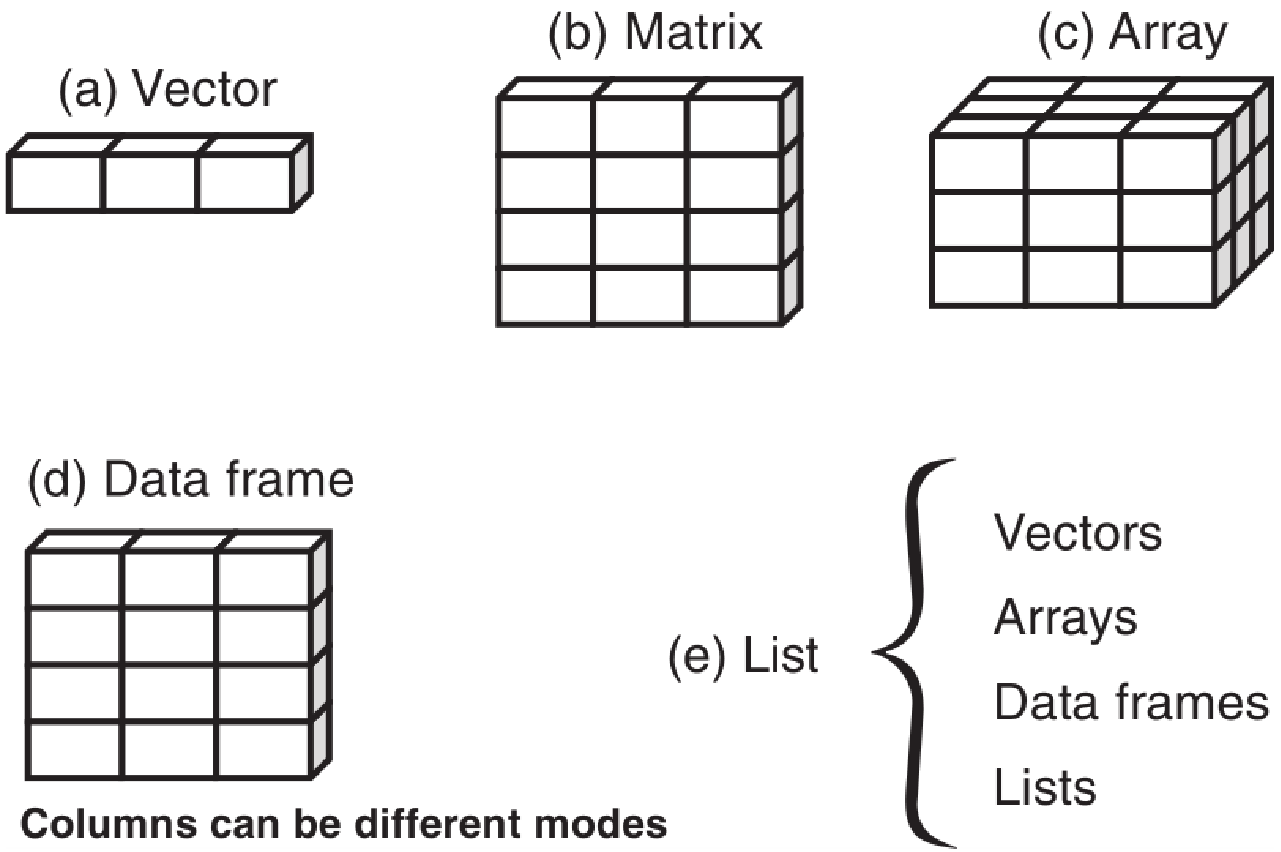
`character / string: "hello world!"`

`logical: TRUE FALSE`

`factors: "male" / "female"`

`(complex: 1+2i)`

Data Types



Kabacoff (2015)

Exercise I

```
…/01.R_basics/_scripts/R_basics.R

up to "Control Flow"
```

TASK: practise basic R commands and data type

TIP: `class()`, `str()`

Side note: folder structure

- .Rproj.user
- _plots
- _scripts
  - R_basics.R
  - R_basics_master.R
- .Rhistory
- R_basics.Rproj

click this to start each exercise,
then no need to set directory

Logical Operators

Operator	Summary
$<$	Less than
$>$	Greater than
$<=$	Less than or equal to
$>=$	Greater than or equal to
$==$	Equal to
$! =$	Not equal to
$!x$	NOT x
$x \vert y$	x OR y
$x\&y$	x AND y

Control Flow

- if-else

```
if (cond) {
    ..statement..
}
```

```
if (cond) {
    ..statement..
} else {
    ..statement..
}
```

```
if (cond) {
    ..statement..
} else if (cond) {
    ..statement..
} else {
    ..statement..
}
```

- for-loop

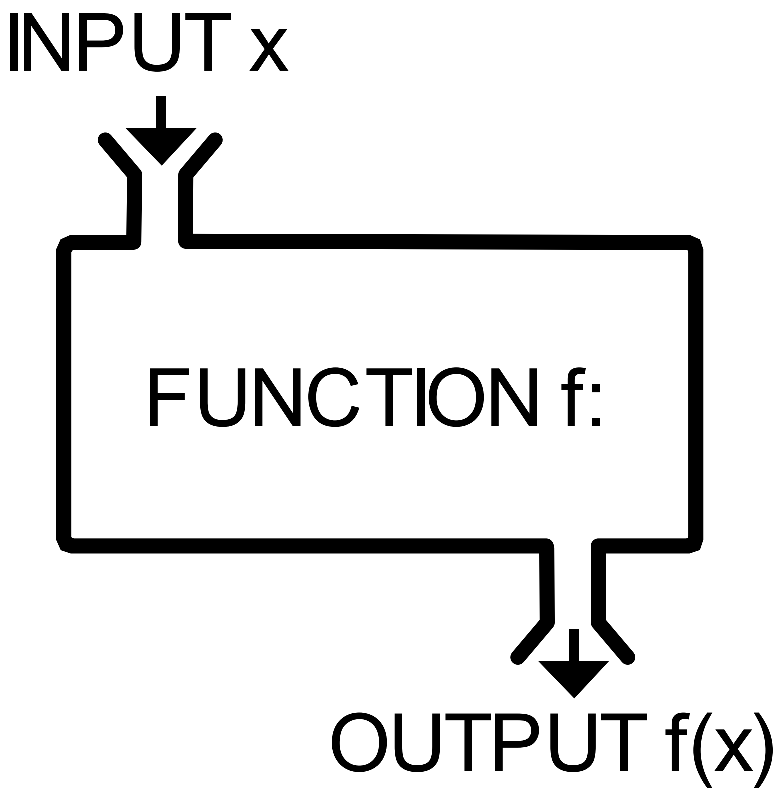
```
for ( j in 1:J) {
    ..statement..
}
```

```
for ( j in 1:J ) {
    for ( k in 1:K ) {
        ..statement..
    }
}
```

Functions

The operation(s) to obtain some quantity, based on another quantity.

- built-in functions
- external functions (packages)
- user-defined functions



User-defined Function

```
funname <- function (input_arges) {
   .. function body ..
   .. function body ..
   return(output_arges)
}
```

$$sem = \sqrt{\frac{s^2}{n-1}}$$

```
sem <- function(x) {
   sqrt( var(x,na.rm=TRUE) / (length(na.omit(x))-1) )
}
```

Exercise II

```
…/01.R_basics/_scripts/R_basics.R
```

TASK: practise control flow and user-defined function

Exercise II

- Generate a random number between 0 and 1
- Compare it against 1/3 and 2/3
- Print the random number and its position relative to 1/3 and 2/3.

```
# if-else
t <- runif(1) # random number between 0 and 1
if (t <= 1/3) {
    cat("t =", , ", t <= 1/3. \n")
} else if () {
    cat("t =", t, ", t > 2/3. \n")
} else {
    cat("t =", t, ", 1/3 < t <= 2/3. \n")
}
```

Example outcome:

```
t = 0.895 , t > 2/3.
```

- Get the name of each month
- Print it one by one

```
# for-loop
month_name <- format(ISOdate(2018,1:12,1),"%B")
for (j in 1:length(month_name) ) {
    cat()
}
```

```
The month is January
The month is February
The month is March
The month is April
The month is May
The month is June
The month is July
The month is August
The month is September
The month is October
The month is November
The month is December
```

Packages in R

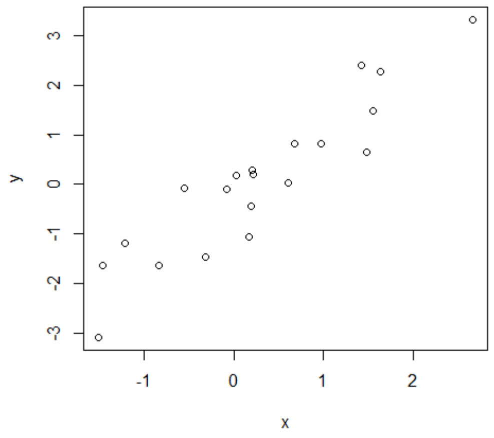
R packages are collections of functions and data sets developed by the community, to make your life a lot easier!

```r
install.packages('ggplot2')
library(ggplot2)
detach('package:ggplot2')
```

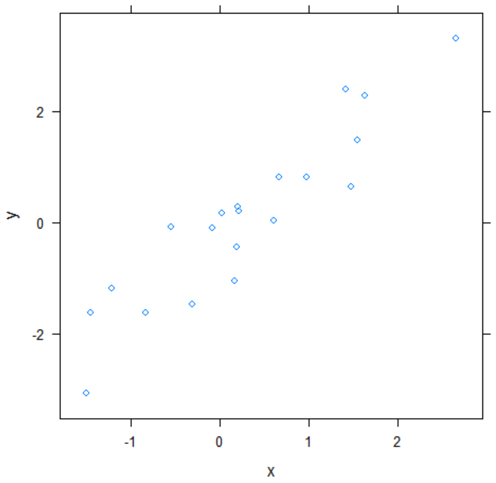
Visualization

- built-in plotting functions – first attempt / quick look / exploratory
- {lattice} – making nicer, similar to basic plotting functions (takes lm formulae)
- {ggplot2} – making nicer, a layering philosophy

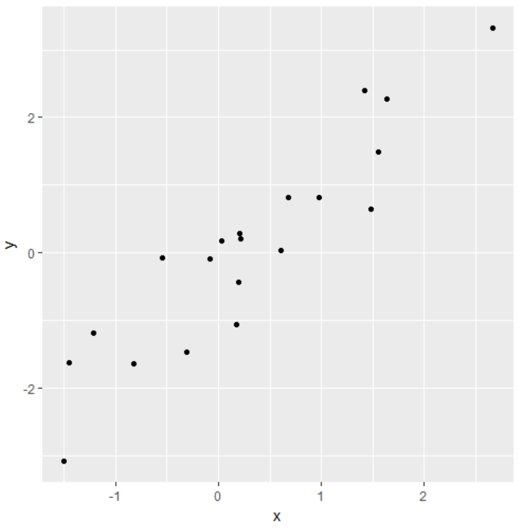
`plot(x,y)`



`lattice::xyplot(y~x)`



`ggplot2::qplot(x,y)`



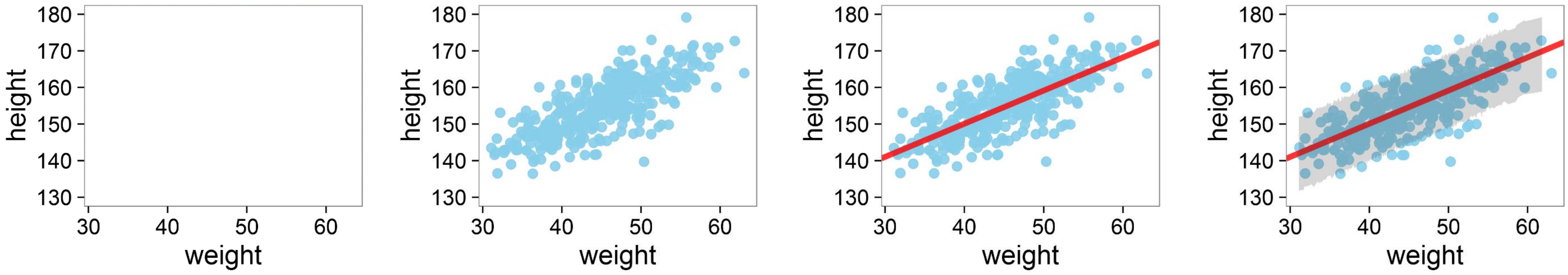
Brief Intro to ggplot2

cognitive model
statistics
computing

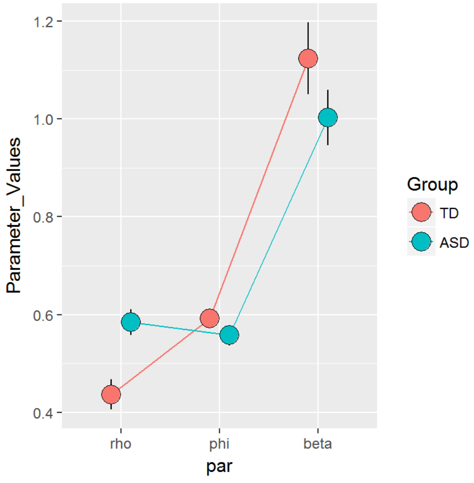
`plot = geometric (points, lines, bars) + aesthetic (color, shape, size)`

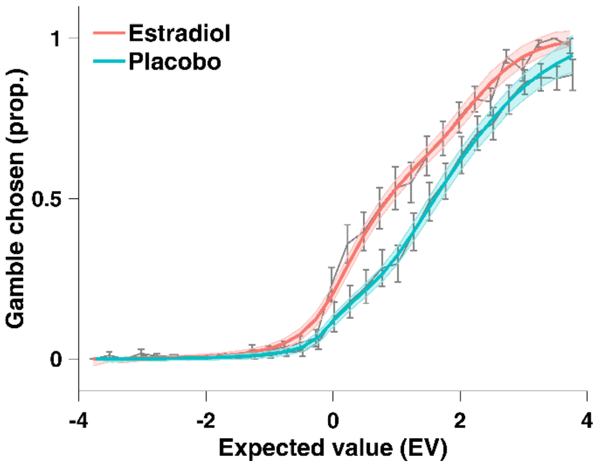
game of adding layers!

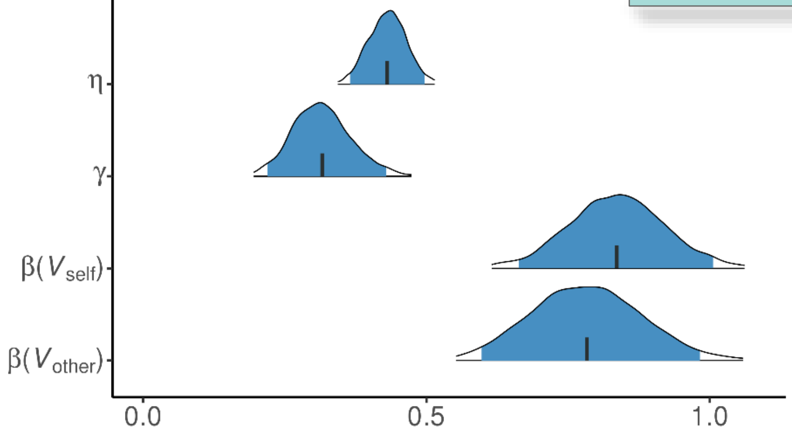
background | add scatters | add regression line | add uncertainty

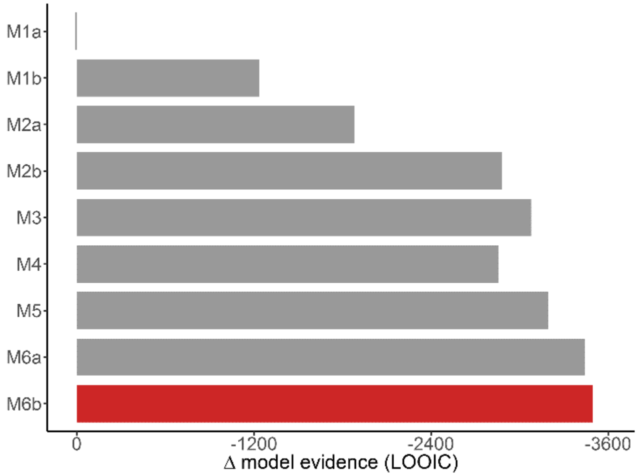


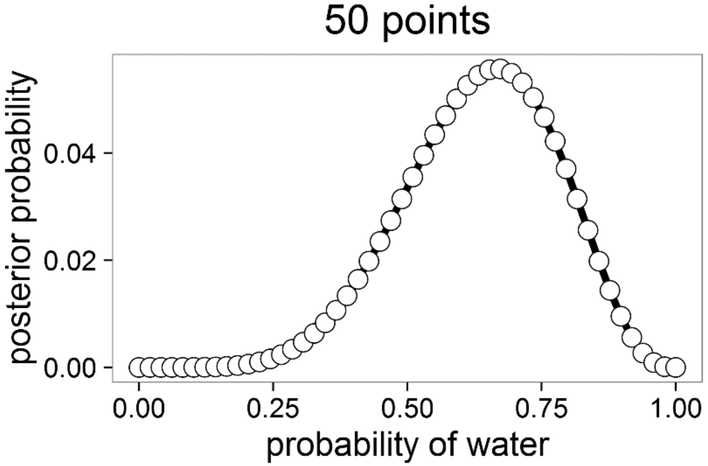
A taste of ggplot2

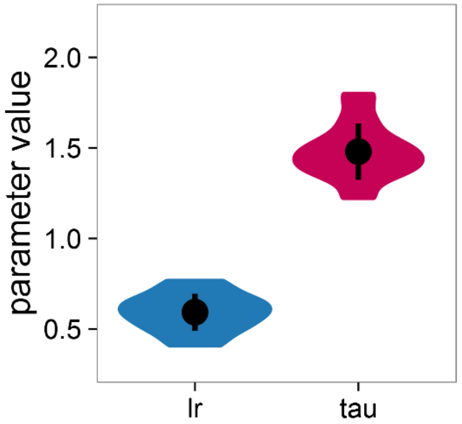


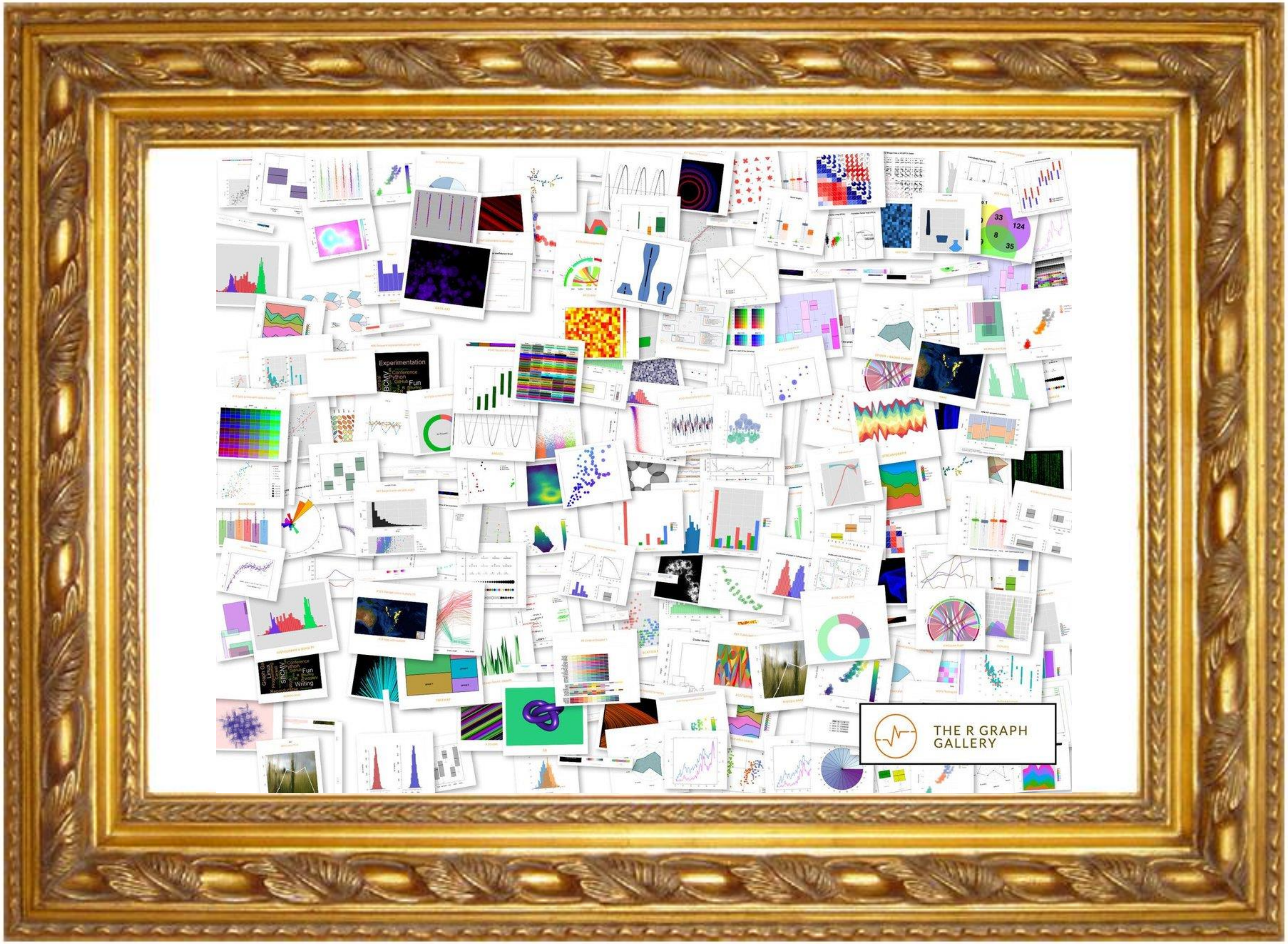






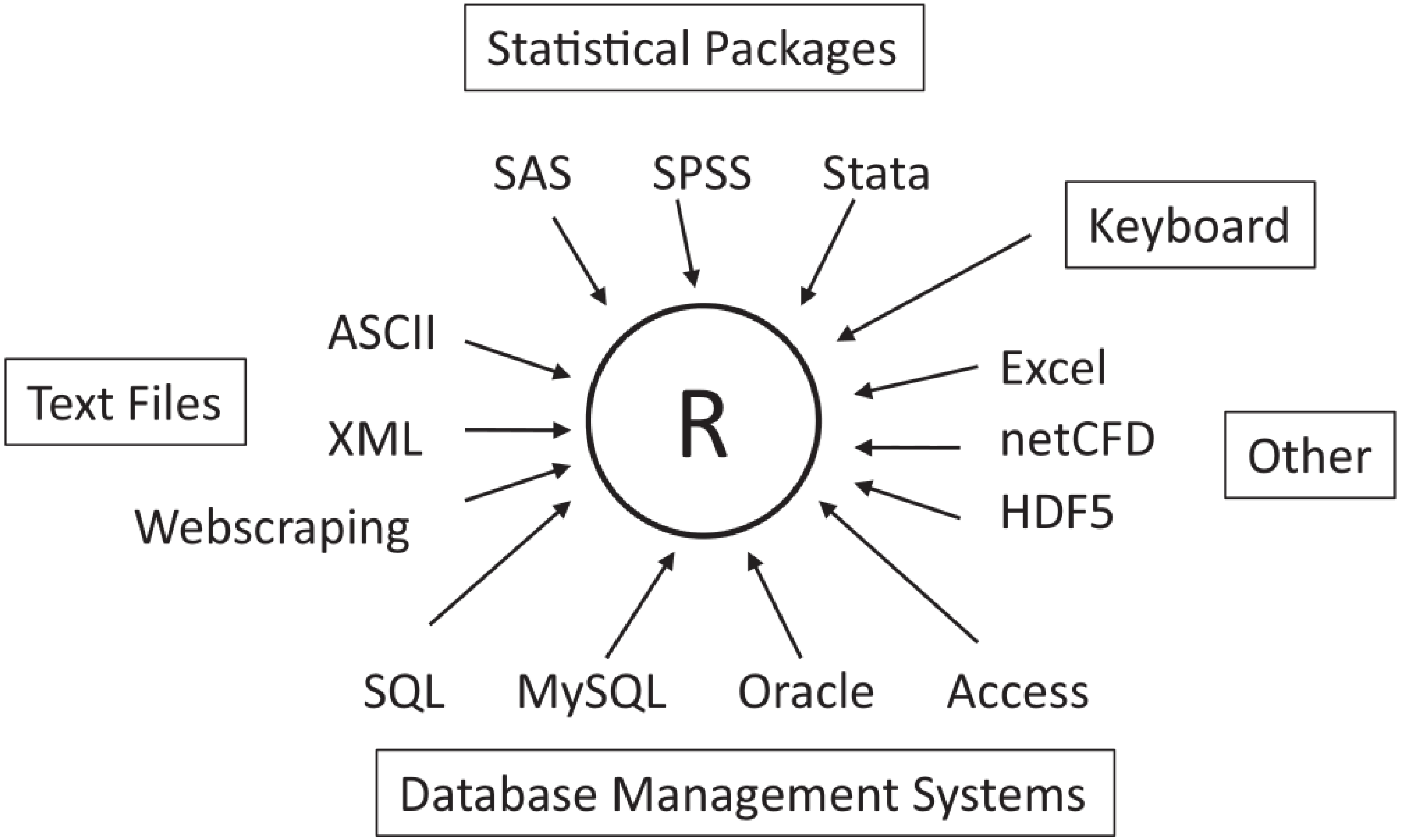






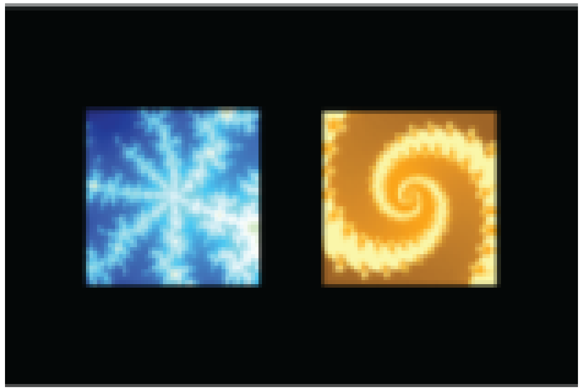
https://www.r-graph-gallery.com/

Data management

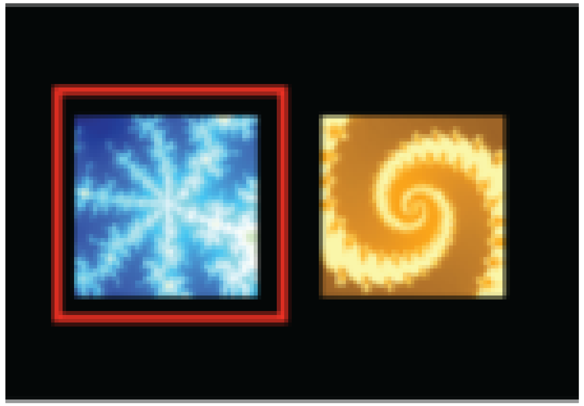


Kabacoff (2015)

One simple experiment



choice
presentation



action
selection

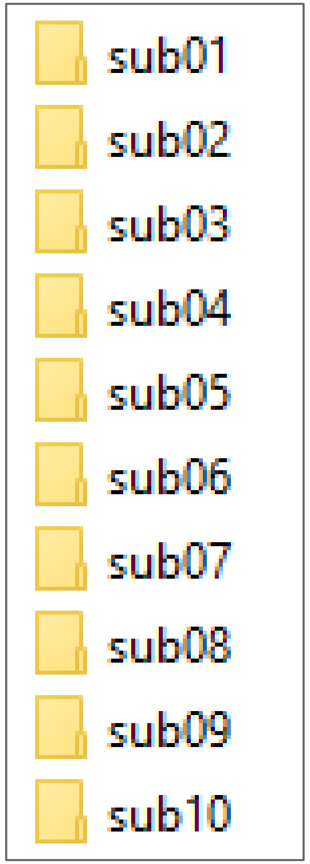
outcome

reward contingency – 80:20

The data

- nSub = 10
- nTrial = 80

```
./_data/_raw_data/sub01/raw
_data_sub01.txt
```




```
subjID, trialID, choice, outcome, correct
1,1,2,-1,1
1,2,1,1,1
1,3,1,1,1
1,4,1,1,1
1,5,2,-1,1
1,6,1,1,1
1,7,1,1,1
1,8,1,1,1
1,9,1,-1,1
1,10,2,-1,1
1,11,1,1,1
1,12,1,1,1
1,13,1,-1,2
```

Import some data!

```
data_dir = ('_data/RL_raw_data/sub01/raw_data_sub01.txt')
data = read.table(data_dir, header = T, sep = ",")
head(data)

  subjID trialID choice outcome correct
1      1       1      1       1       1
2      1       2      1       1       1
3      1       3      1       1       1
4      1       4     NA       1       1
5      1       5      1      -1       1
6      1       6      2      -1       1
```

Indexing

```
data[1,1]
data[1,]
data[,1]
data[1:10,]
data[,1:2]
data[1:10, 1:2]
data[c(1,3,5,6), c(2,4)]

data$choice
```

```
> data
   subjID trialID choice outcome correct
1       1       1      1       1       1
2       1       2      1       1       1
3       1       3      1       1       1
5       1       5      1      -1       1
6       1       6      2      -1       1
7       1       7      1       1       1
8       1       8      1       1       1
9       1       9      1       1       1
10      1      10      1       1       1
11      1      11      1       1       1
```

Import some data!

```
data_dir = ('_data/RL_raw_data/sub01/raw_data_sub01.txt')
data = read.table(data_dir, header = T, sep = ",")
head(data)

  subjID trialID choice outcome correct
1      1       1      1       1       1
2      1       2      1       1       1
3      1       3      1       1       1
4      1       4     NA       1       1
5      1       5      1      -1       1
6      1       6      2      -1       1

sum(complete.cases(data))   # number of valid trials
data = data[complete.cases(data),]
dim(data[complete.cases(data),])
```

Exercise III

```
…/01.R_basics/_scripts/R_basics.R
```

TASK:

write a for loop

… which reads in each participant's raw data

… and reshape it in the "long format" by subj

TIP: complete line 173; consider `sprint()`

```
for ( j in 1:n ) {
   read.table(file, header = T, sep = ",")
}
```

subID	Choice
sub01	1
sub01	2
…	
sub02	2
sub02	2
…	
sub10	2
sub10	1

Read all the data!

```
ns = 10
data_dir = '_data/RL_raw_data'

rawdata = c();
for (s in 1:ns) {
    sub_file = file.path(data_dir, sprintf('sub%02i/raw_data_sub%02i.txt',s,s))
    sub_data = read.table(sub_file, header = T, sep = ",")
    rawdata = rbind(rawdata, sub_data)
}
rawdata = rawdata[complete.cases(rawdata),]
rawdata$accuracy = (rawdata$choice == rawdata$correct) * 1.0

acc_mean = aggregate(rawdata$accuracy, by = list(rawdata$subjID), mean)[,2]
```

mean choice accuracy across trials, per participant.

Basic stats

```
mean(acc_mean)
sd(acc_mean)
sem(acc_mean)

t.test(acc_mean, mu = 0.5) # one sample t-test

        One Sample t-test

data:  acc_mean
t = 13.788, df = 9, p-value = 2.34e-07
alternative hypothesis: true mean is not equal to 0.5
95 percent confidence interval:
 0.6962988 0.7733565
sample estimates:
mean of x
0.7348277
```

```
> as.matrix(acc_mean, 10, 1)
           [,1]
 [1,] 0.8076923
 [2,] 0.7125000
 [3,] 0.6875000
 [4,] 0.6493506
 [5,] 0.7750000
 [6,] 0.7250000
 [7,] 0.7662338
 [8,] 0.8000000
 [9,] 0.7500000
[10,] 0.6750000
```

Basic correlation

```r
load('_data/RL_descriptive.RData')
descriptive$acc = acc_mean
df = descriptive

cor.test(df$IQ, df$acc)

        Pearson's product-moment correlation

data:  df$IQ and df$acc
t = 4.8347, df = 8, p-value = 0.001297
alternative hypothesis: true correlation is not equal to 0
95 percent confidence interval:
 0.5114810 0.9671586
sample estimates:
      cor
0.8631401
```

```
> descriptive
   subjID        IQ      Age    acc
1       1 123.98691 31.07218 0.8125
2       2  87.63187 30.13800 0.7125
3       3  89.39930 23.44219 0.6875
4       4  84.34607 27.44848 0.6500
5       5 134.72208 23.30624 0.7750
6       6  84.60797 25.67858 0.7250
7       7 111.10238 24.36375 0.7750
8       8 117.89599 32.74026 0.8000
9       9  96.88233 22.80211 0.7500
10     10  76.01652 30.44258 0.6750
```

Exercise IV

```
…/01.R_basics/_scripts/R_basics.R
```

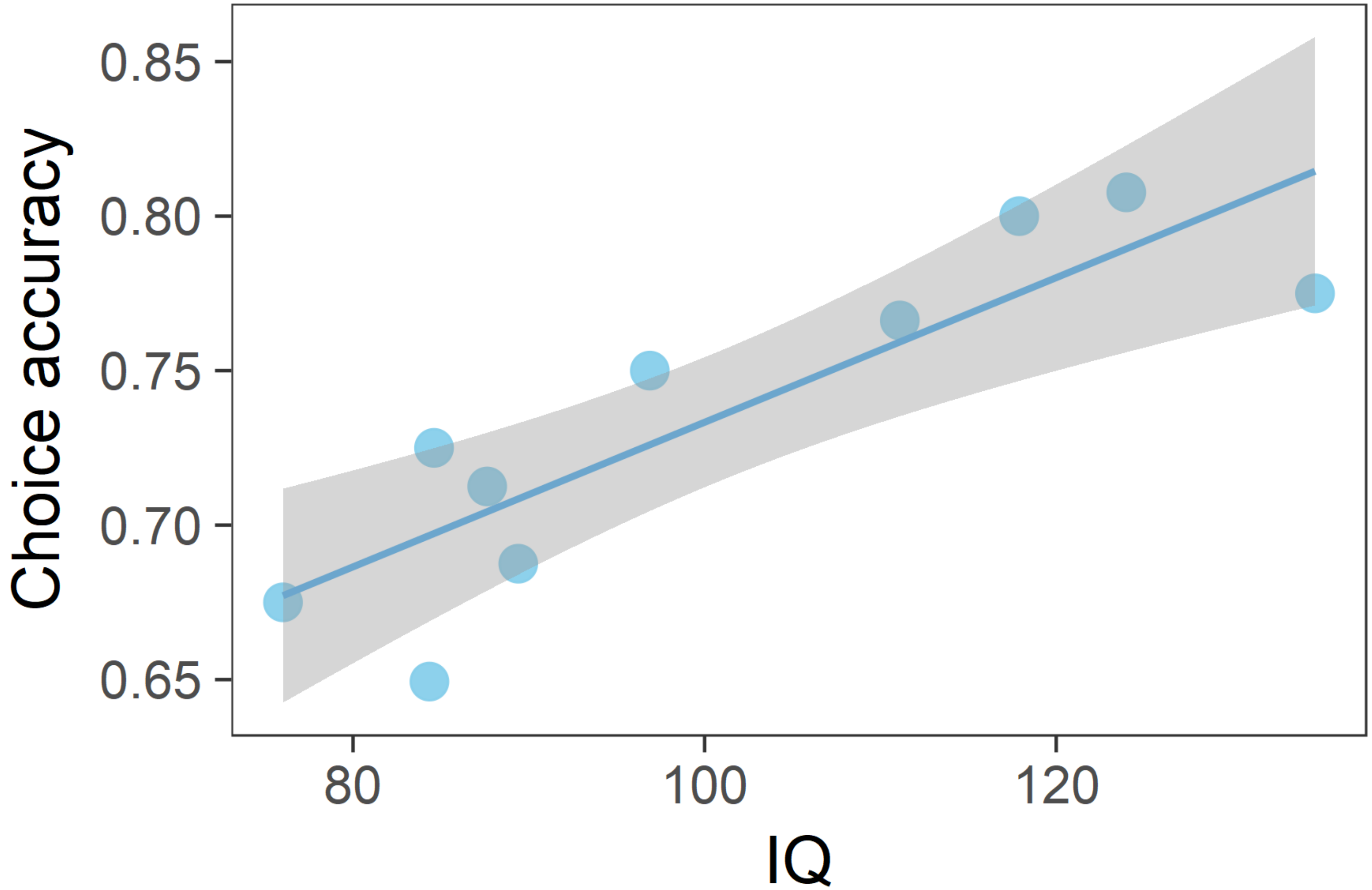
TASK:

Read in the descriptive data: _data/descriptive.RData
…include 'acc_mean' as a new column, and
…rename 'descriptive' as df.

Practice all the basic stats.

```
df$new_Col = new_Col
```

A simple linear regression



What is exactly the regression line in R?

$$\mu_i = \alpha + \beta x_i$$

$$y_i = \mu_i + \varepsilon$$

```
fit1 = lm(acc ~ IQ, data = df)
summary(fit1)

Call:
lm(formula = acc ~ IQ, data = df)

Residuals:
      Min        1Q    Median        3Q       Max
-0.047305 -0.016277  0.007562  0.022577  0.027731

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.499292   0.049565  10.073 8.04e-06 ***
IQ          0.002340   0.000484   4.835   0.0013 **
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.02885 on 8 degrees of freedom
Multiple R-squared:  0.745,  Adjusted R-squared:  0.7131
F-statistic: 23.37 on 1 and 8 DF,  p-value: 0.001297
```

Exercise V

```
…/01.R_basics/_scripts/R_basics.R
```

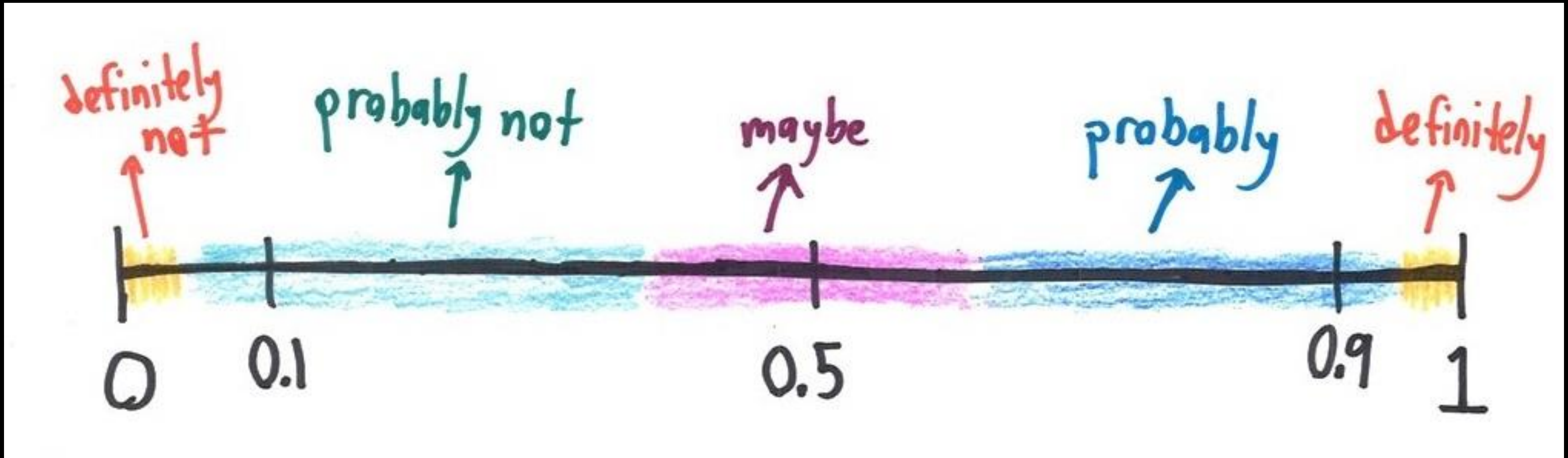
TASK:

Read and make sense of the ggplot functions,

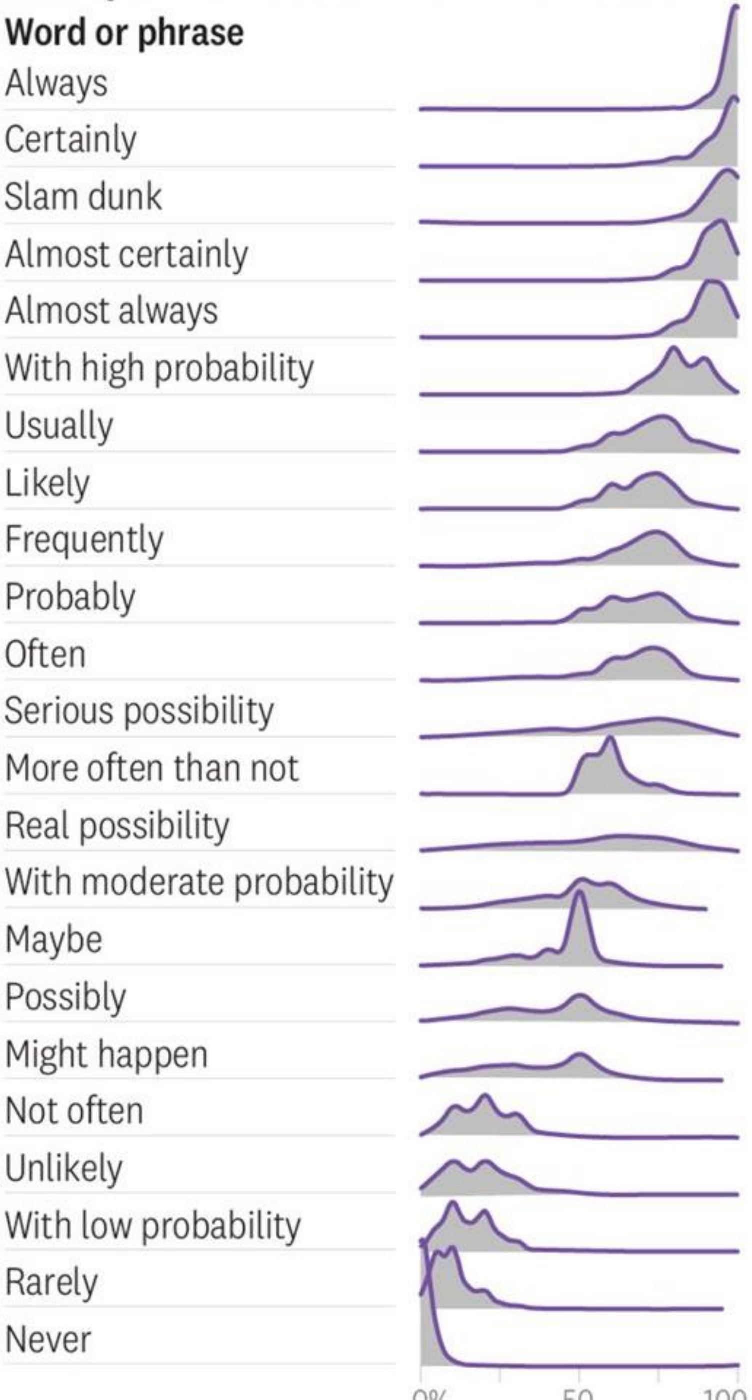
… experiment make some adjustments (color marker size etc.), and

… run the `lm(acc ~ IQ)`

BASICS OF PROBABILITY



to respondents' estimate of likelihood



Probability

…assigning numbers to a set of possibilities

Properties (Kolmogorov, 1956)

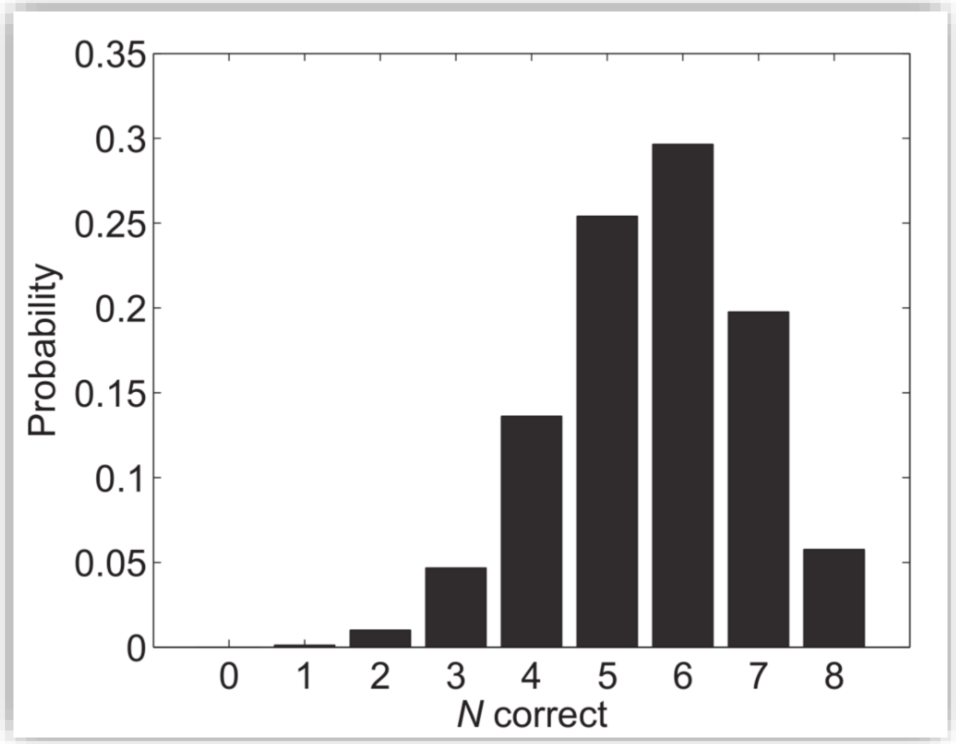
- $p \in [0,1]$
- $\Sigma p = 1$

Probabilities are used to express uncertainty.

Probability Functions

discrete events – we talk about mass

Run a test and record each student's correct responses

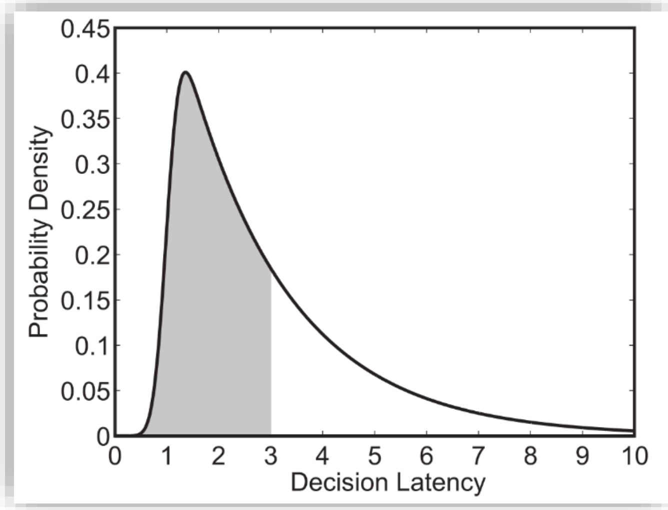


Lewandowsky & Farrell (2011)

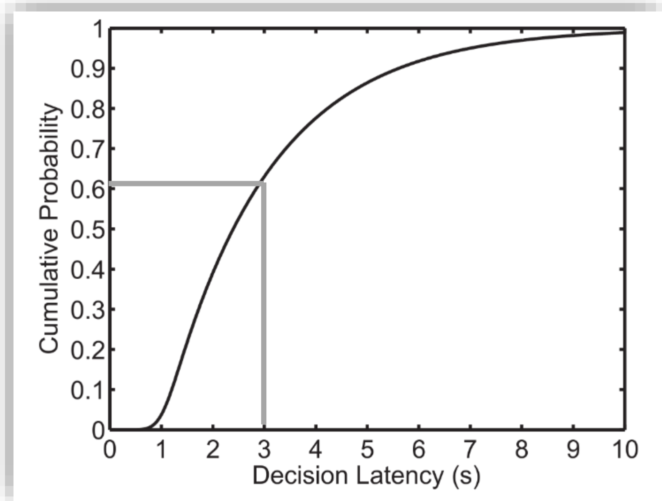
Probability Functions

continuous events – we talk about density

probability density function
(PDF)



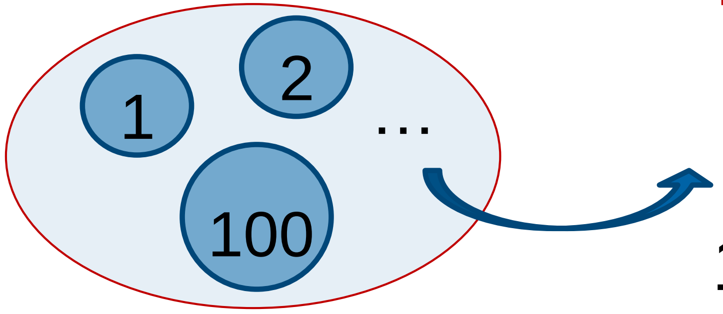
cumulative distribution function
(CDF)



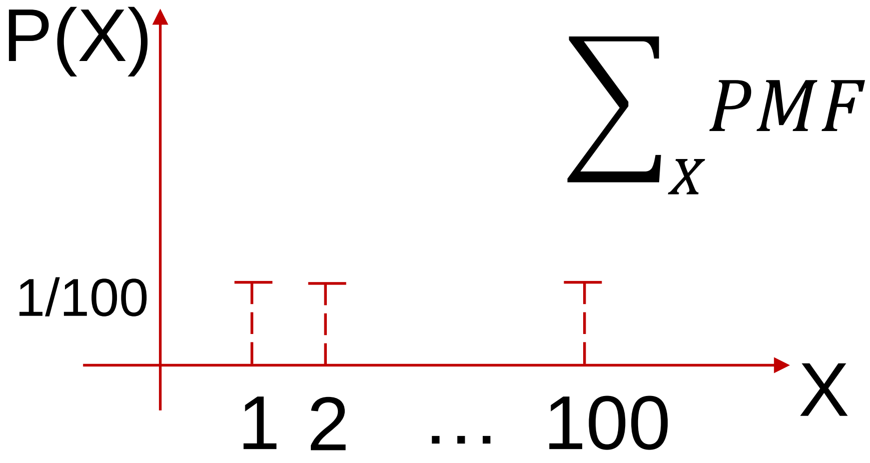
Lewandowsky & Farrell (2011)

Another example

Discrete



X	P(X)
1	1/100
2	1/100
…	…
100	1/100



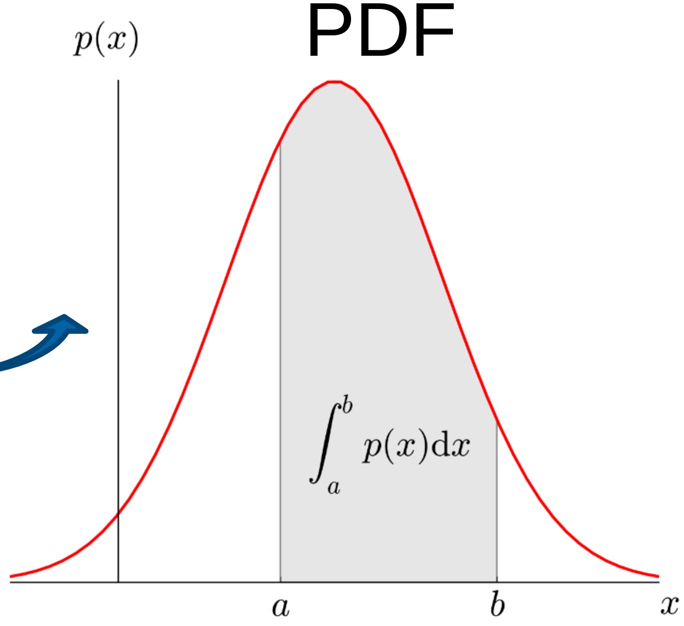
$$\sum_X PMF(X) = 1$$

Continuous

Height

X	P(X)
1.8 m	0

PDF



P given by the area

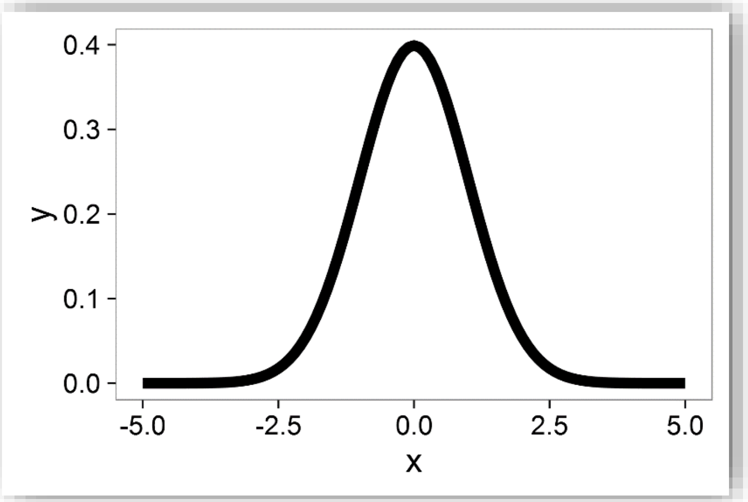
$$1.75 \le X \le 1.85$$

Playing with Probability Functions in R

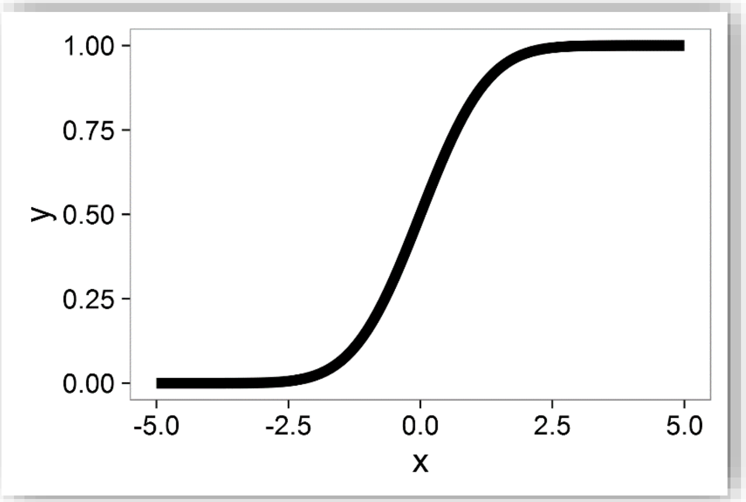

```
dnorm() - PDF
pnorm() - CDF
qnorm() - quantile, inverse cdf
rnorm() - random number generator
```

Example: Normal(0,1)

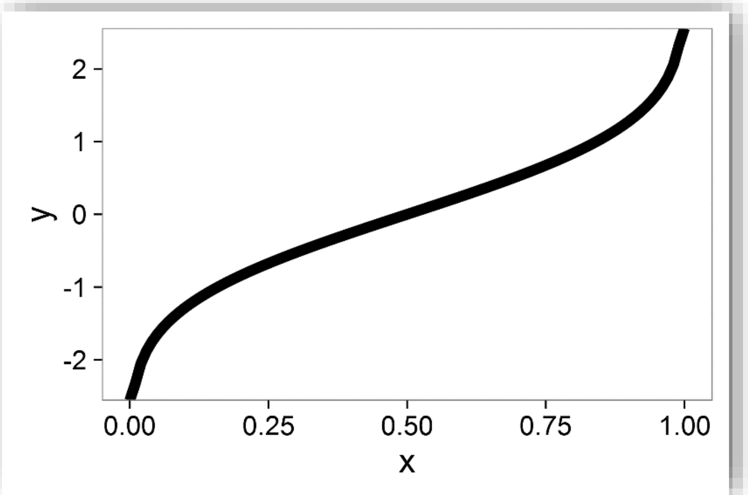
`dnorm()`



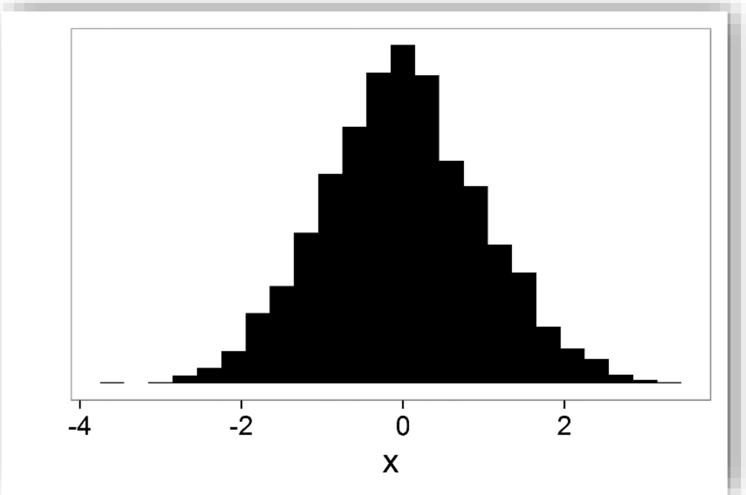
`pnorm()`



`qnorm()`



`rnorm()`



Joint Probability and Conditional Probability

Joint Probability

$p(A, B) = p(B, A)$

- e.g., $p(\text{raining, cold})$: $p(\text{raining})$ AND $p(\text{cold})$

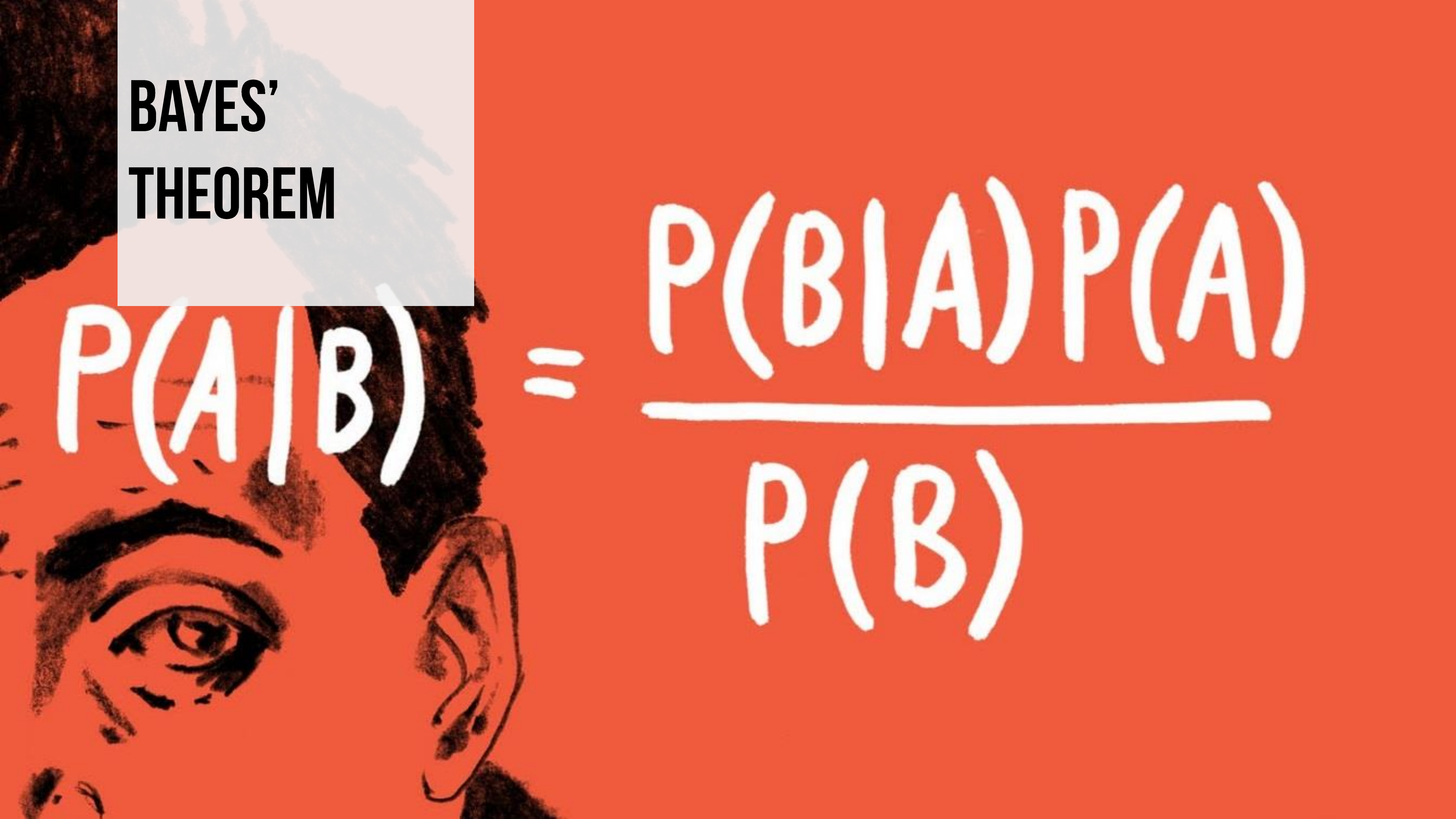
Conditional Probability

$p(A|B)$ – 'p of A given B' – event B is fixed, not uncertainty

$p(A,B) = p(A|B)p(B)$

- e.g., $p(\text{raining, cold}) = p(\text{raining}|\text{cold})p(\text{cold})$

BAYES' THEOREM



$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Bayes' theorem

$$p(A,B) = p(B,A)$$

$$p(A,B) = p(A|B)\,p(B)$$

$$p(B,A) = p(B|A)\,p(A)$$

$$p(A|B)\,p(B) = p(B|A)\,p(A)$$

$$\boxed{p(A \mid B) = \frac{p(B \mid A)\,p(A)}{p(B)}}$$

Example

disease X

symptoms Y	X = 0	X = 1
0	0.5	0.1
1	0.1	0.3

Joint probability : $P(X = 0, Y = 1) = 0.1$

$$\sum_{x,y} P(X = x, Y = y) = 1$$

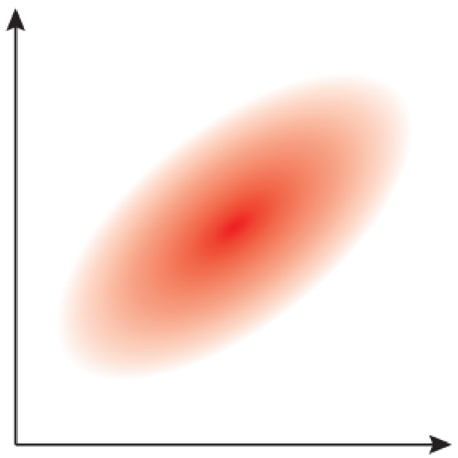
Marginal probability :

$$P(Y = 1) = 0.1 + 0.3 = 0.4$$

$$P(X = 0) = 0.1 + 0.5 = 0.6$$

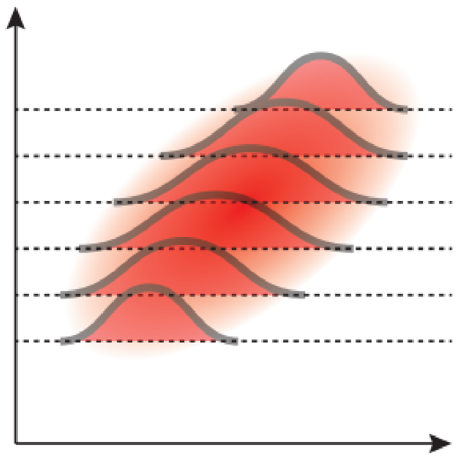
$$P(X = x) = \sum_{y} P(X = x, Y = y)$$

joint distribution



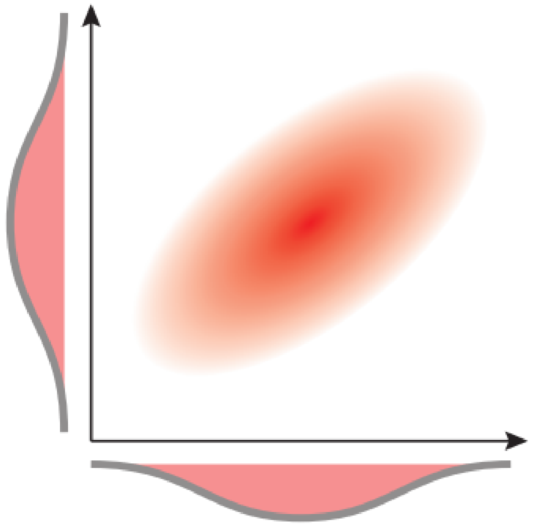
The “co-distribution” of x and y.

conditional distribution



The probability distribution of x,
given that we know the value of y.

mariginal distribution



The density of x- (or y-) values,
without knowing the other’s value.

Credit: [Eric Ma]

Second Example

Row	Column			Marginal
	...	c	...	
$\vdots$		$\vdots$		
r	$\cdots$	$p(r, c) = p(r\|c)\, p(c)$	$\cdots$	$p(r) = \sum_{c^*} p(r\|c^*)\, p(c^*)$
$\vdots$		$\vdots$		
Marginal		$p(c)$		

Kruschke (2015)

Second Example

Eye color	Hair color: Black	Brunette	Red	Blond	Marginal (Eye color)
Brown	0.11	0.20	0.04	0.01	0.37
Blue	0.03	0.14	0.03	0.16	0.36
Hazel	0.03	0.09	0.02	0.02	0.16
Green	0.01	0.05	0.02	0.03	0.11
Marginal (hair color)	0.18	0.48	0.12	0.21	1.0

$$p(A \mid B) = \frac{p(B \mid A)\, p(A)}{\sum_{A^*} p(B \mid A^*)\, p(A^*)}$$

Kruschke (2015)

Exercise VI

Suppose that in the general population, the probability of having a rare disease is 1/1000. We denote the true presence or absence of the disease as the value of a parameter, ϑ, that can have the value ϑ = ☹ if disease is present in a person, or the value ϑ = ☺ if the disease is absent. The base rate of the disease is therefore denoted $p(\vartheta = ☹) = 0.001$.

Suppose(1): a test for the disease that has a 99% hit rate: $p(T = + \mid \vartheta = ☹) = 0.99$

Suppose(2): the test has a false alarm rate of 5%: $p(T = + \mid \vartheta = ☺) = 0.05$

Q: Suppose we sample a person at random from the population, administer the test, and it comes up positive. What is the posterior probability that the person has the disease?

Kruschke (2015)

Exercise VI

Q: What is the posterior probability that the person has the disease?

→ $p(\vartheta = ☹ \mid T = +)$

Kruschke (2015)

Exercise VI

Test result	Disease: $\theta = \ddot{\frown}$ (present)	Disease: $\theta = \ddot{\smile}$ (absent)	Marginal (test result)
$T = +$	$p(+\mid\ddot{\frown})\,p(\ddot{\frown}) = 0.99 \cdot 0.001$	$p(+\mid\ddot{\smile})\,p(\ddot{\smile}) = 0.05 \cdot (1 - 0.001)$	$\sum_\theta p(+\mid\theta)\,p(\theta)$
$T = -$	$p(-\mid\ddot{\frown})\,p(\ddot{\frown}) = (1 - 0.99) \cdot 0.001$	$p(-\mid\ddot{\smile})\,p(\ddot{\smile}) = (1 - 0.05) \cdot (1 - 0.001)$	$\sum_\theta p(-\mid\theta)\,p(\theta)$
Marginal (disease)	$p(\ddot{\frown}) = 0.001$	$p(\ddot{\smile}) = 1 - 0.001$	1.0

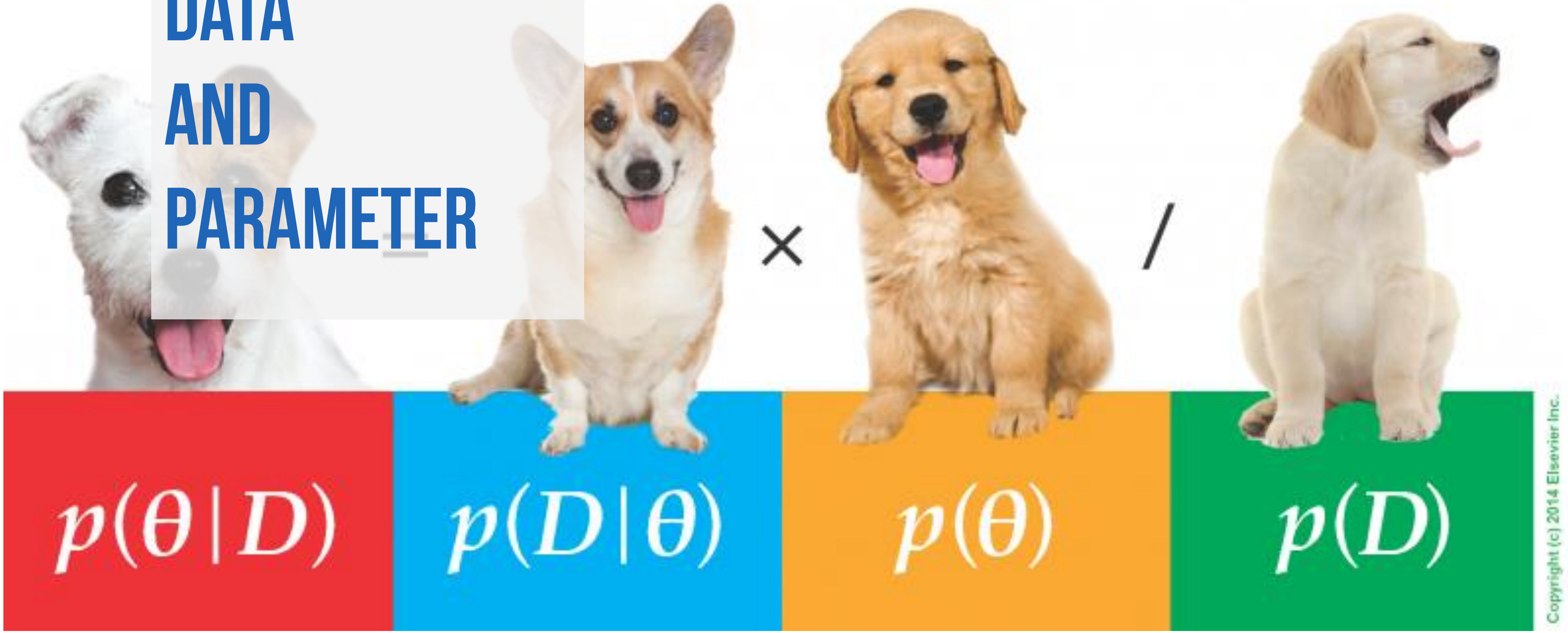
$$p(\theta = \ddot{\frown} \mid T = +) = \frac{p(T = + \mid \theta = \ddot{\frown})\,p(\theta = \ddot{\frown})}{\sum_\theta p(T = + \mid \theta)\,p(\theta)}$$

$$= \frac{0.99 \cdot 0.001}{0.99 \cdot 0.001 + 0.05 \cdot (1 - 0.001)}$$

$$= 0.019$$

Kruschke (2015)

LINKING DATA AND PARAMETER



Linking Data and Parameter

θ $\quad$ D

$$p(A|B) = \frac{p(B|A)p(A)}{p(B)}$$

Linking Data and Parameter

$$p(\theta|D) = \frac{p(D|\theta)p(\theta)}{p(D)}$$

Linking Data and Parameter

Likelihood
How plausible is the data given our parameter is true?

Prior
How plausible is our parameter before observing the data?

$$p(\theta|D) = \frac{p(D|\theta)p(\theta)}{p(D)}$$

Posterior
How plausible is our parameter given the observed data?

Evidence
How plausible is the data under all possible parameters?

What is $p(\text{Data}|\,\vartheta)$

- This is the "Model"
- Data is fixed, ϑ varies
- Not a probability distribution
  - the sum is not "one"

$$Pr(X=0\,|\,\theta)=Pr(T,T\,|\,\theta)=Pr(T\,|\,\theta)\times Pr(T\,|\,\theta)=(1-\theta)^2$$

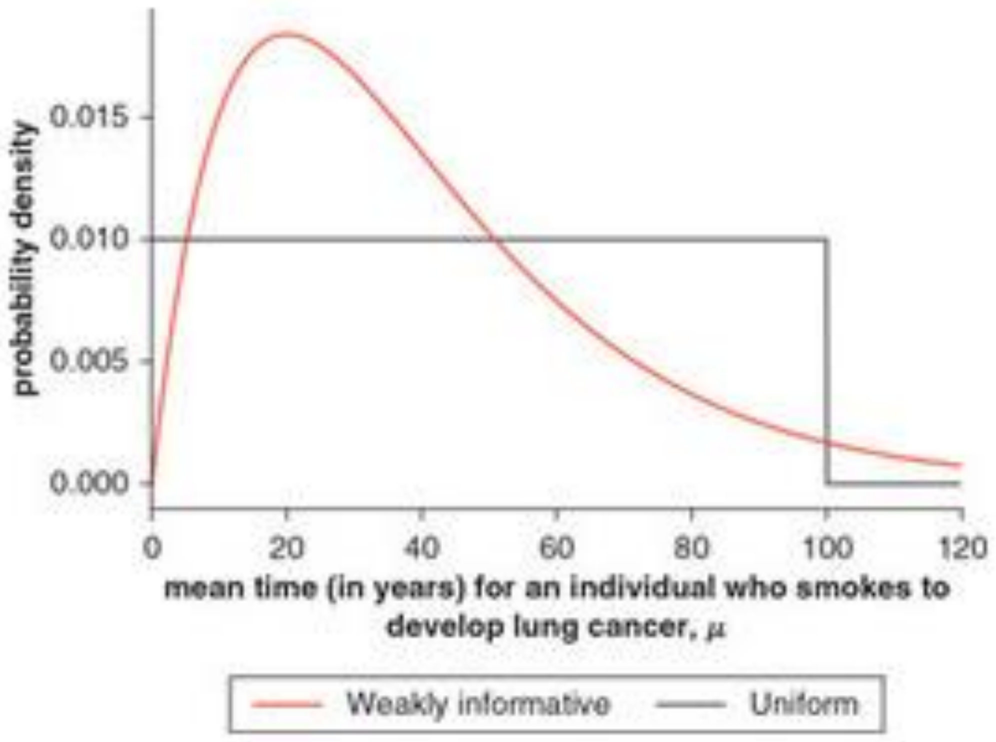
$$Pr(X=1\,|\,\theta)=Pr(H,T\,|\,\theta)+Pr(T,H\,|\,\theta)=2\times Pr(T\,|\,\theta)\times Pr(H\,|\,\theta)=2\theta(1-\theta)$$

$$Pr(X=2\,|\,\theta)=Pr(H,H\,|\,\theta)=Pr(H\,|\,\theta)\times Pr(H\,|\,\theta)=\theta^2.$$

Probability of coin landing heads up, θ	Number of heads, X: 0	1	2	Total
0.0	1.00	0.00	0.00	**1.00**
0.2	0.64	0.32	0.04	**1.00**
0.4	0.36	0.48	0.16	**1.00**
0.6	0.16	0.48	0.36	**1.00**
0.8	0.04	0.32	0.64	**1.00**
1.0	0.00	0.00	1.00	**1.00**
Total	**2.20**	**1.60**	**2.20**	

Lambert (2018)

What is $p(\vartheta)$?



Lambert (2018)

What is *p*(*Data*)?

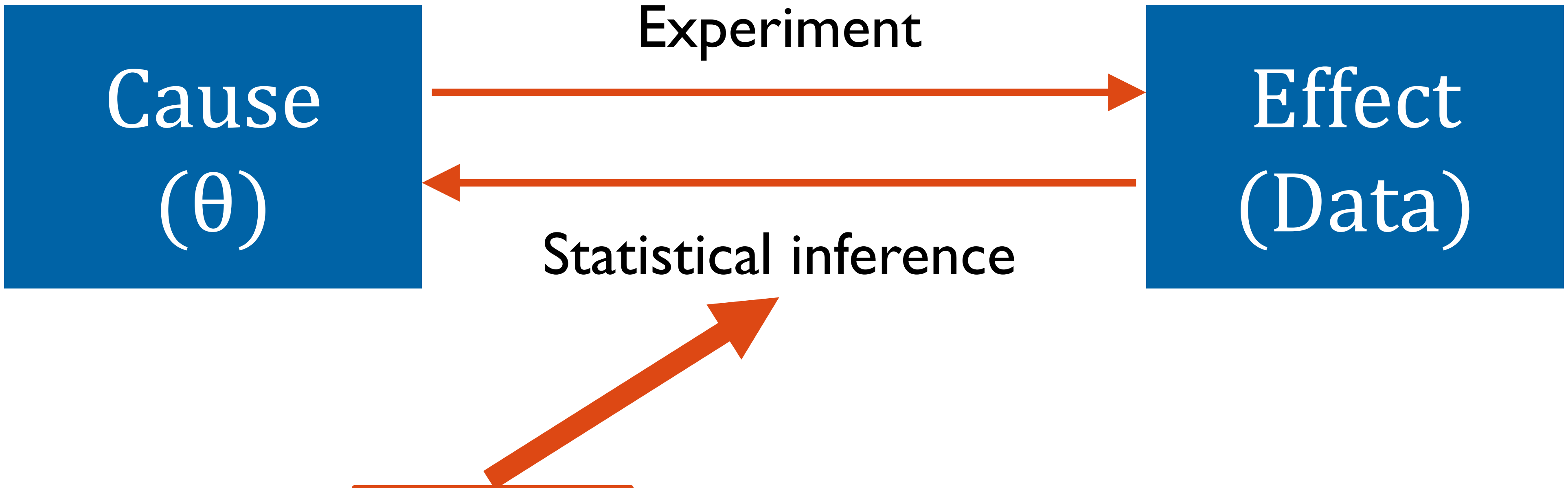
discrete parameters

$$p(\theta \mid D) = \frac{p(D \mid \theta)\, p(\theta)}{\boxed{\sum_{\theta^*} p(D \mid \theta^*)\, p(\theta^*)}}$$

continuous parameters

$$p(\theta \mid D) = \frac{p(D \mid \theta)\, p(\theta)}{\boxed{\int p(D \mid \theta^*)\, p(\theta^*)\, d\theta^*}}$$

Why the Bayes' theorem is important?



$$\boxed{p(\theta|D)} = \frac{p(D|\theta)p(\theta)}{p(D)}$$

BINOMIAL MODEL

Binomial Model

cognitive model

statistics

computing

- You are curious how much of the surface is covered in water.
- You will toss the globe up in the air.
- You will record whether or not the surface under your right index finger is water (W) or land (L).
- You might observe: W L W W W L W L W
- → 6/9 = 0.666667?
- Is it right? If not, what to do next?

McElreath (2016)

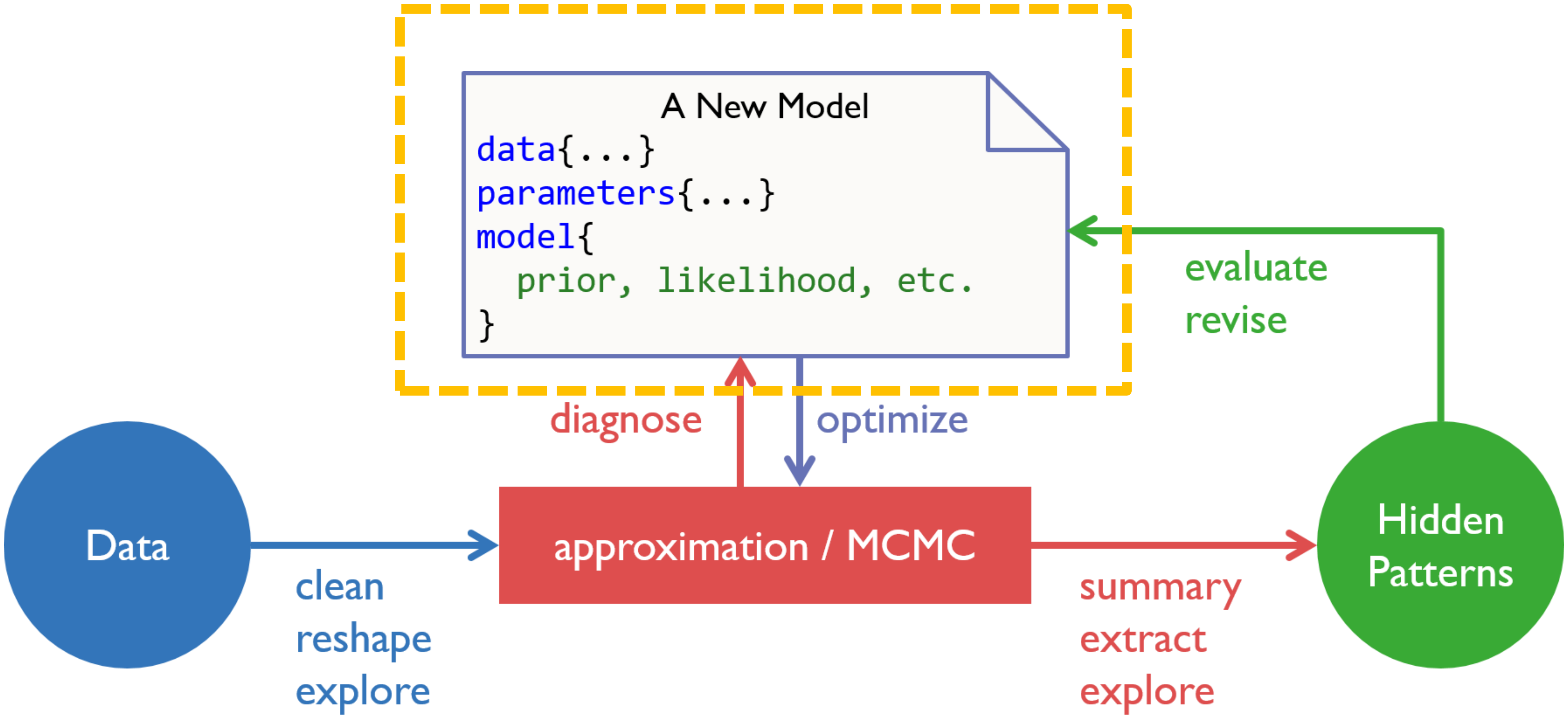
Steps of (Bayesian) Modeling?

cognitive model
statistics
computing

A data story	Think about how the data might arise. It can be *descriptive* or even *causal*.
Update	Educate your model by feeding it with data. Bayesian Update: update the prior, in light of data, to produce posterior the updated posterior then becomes the prior of next update
Evaluate	Compare model with reality. Revise your model.

McElreath (2016)

cognitive model
statistics
computing



A Data Story of the Globe

cognitive model
statistics
computing

- The true proportion of water covering the globe is ϑ.
- A single toss of the globe has a probability p of producing a water (W) observation.
- It has a probability $(1 - \vartheta)$ of producing a land (L) observation.
- Each toss of the globe is independent of the others.

McElreath (2016)

Components of a Model

think about the likelihood function (of Binomial):

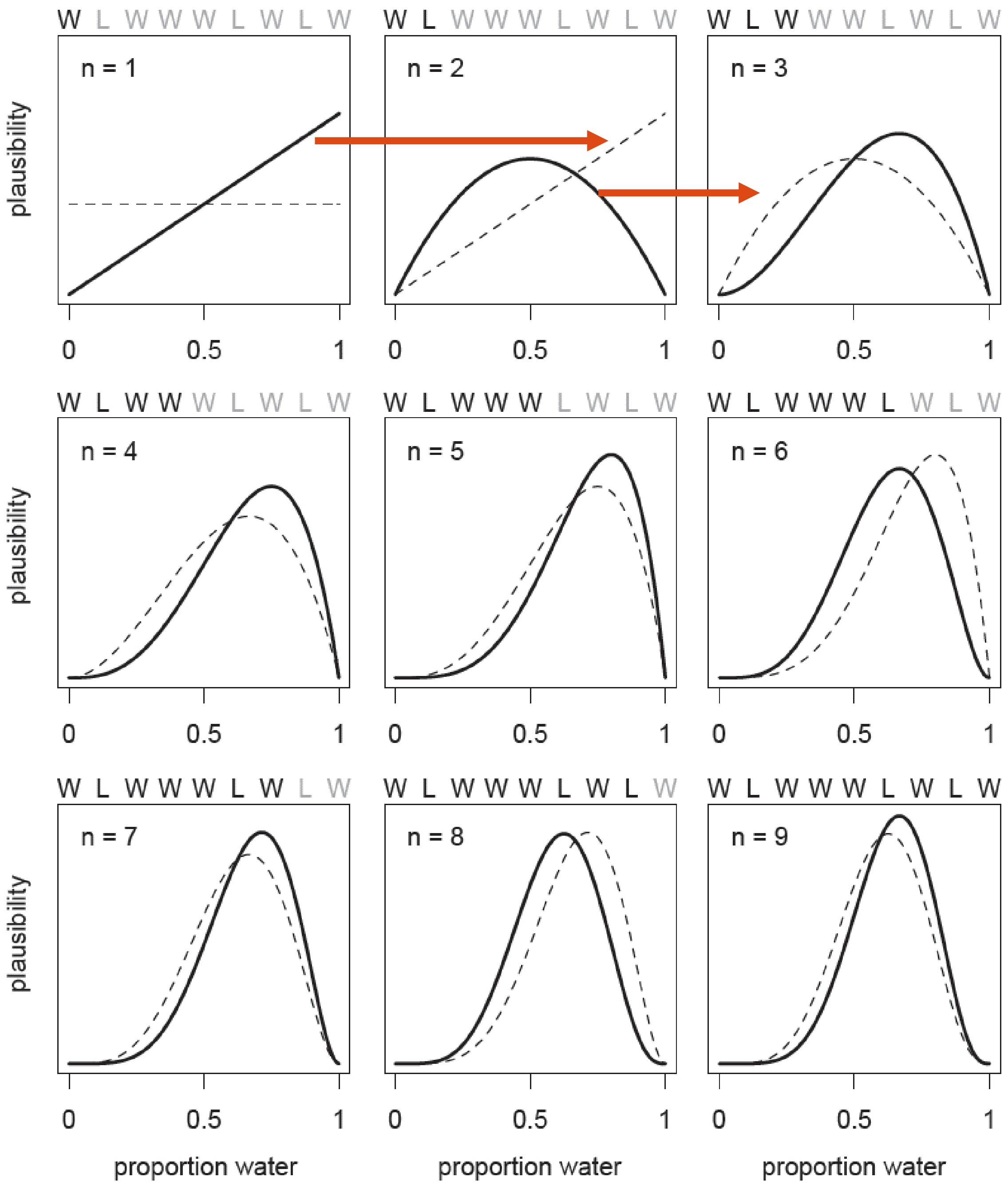
$$p(\theta \mid D) = \frac{\boxed{p(D \mid \theta)}\, p(\theta)}{\int p(D \mid \theta^*)\, p(\theta^*)\, d\theta^*}$$

$$p(w \mid N, \theta) = \binom{N}{w} \theta^w (1-\theta)^{N-w}$$

N: total number of observations
w: number of water
} known (data)

ϑ : proportion of water — unknown (parameter)

Update



cognitive model

statistics

computing

McElreath (2016)

Solve it by Grid Approximation

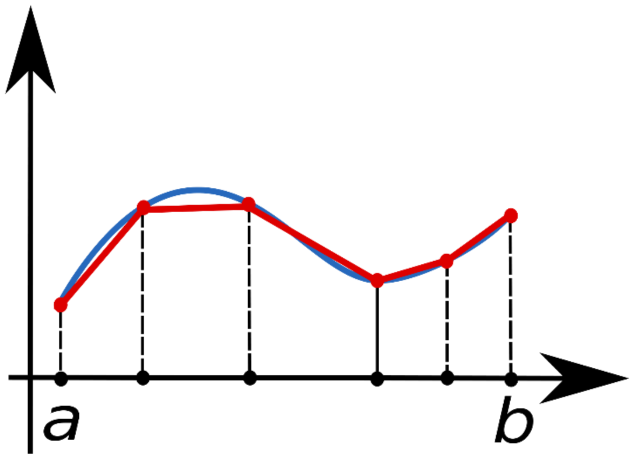
cognitive model
statistics
computing

discrete parameters

$$p(\theta \mid D) = \frac{p(D \mid \theta)\, p(\theta)}{\boxed{\sum_{\theta^*} p(D \mid \theta^*)\, p(\theta^*)}}$$

continuous parameters

$$p(\theta \mid D) = \frac{p(D \mid \theta)\, p(\theta)}{\boxed{\int p(D \mid \theta^*)\, p(\theta^*)\, d\theta^*}}$$



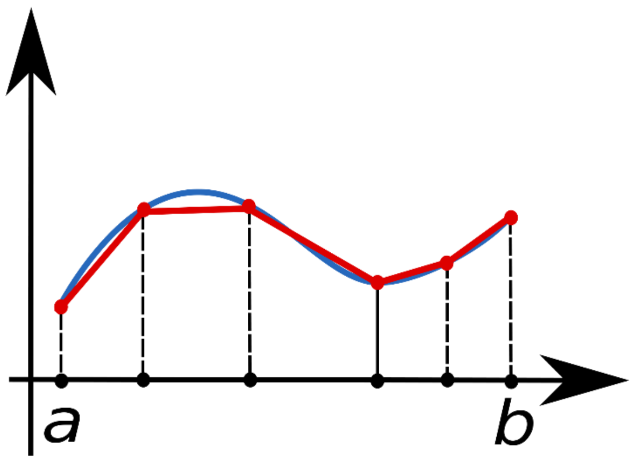
Binomial Model – Grid Approximation

cognitive model
statistics
computing

```
theta_start <- 0; theta_end <- 1; n_grid <- 20
w <- 6; N <- 9

# define grid
theta_grid <- seq(from = theta_start, to = theta_end,
                  length.out = n_grid)

# define prior
prior <- rep(1 , n_grid)

# compute likelihood at each value in grid
likelihood <- dbinom(w, size = N, prob = theta_grid)

# compute product of likelihood and prior
unstd.posterior <- likelihood * prior

# standardize the posterior, so it sums to 1
posterior <- unstd.posterior / sum(unstd.posterior)
```

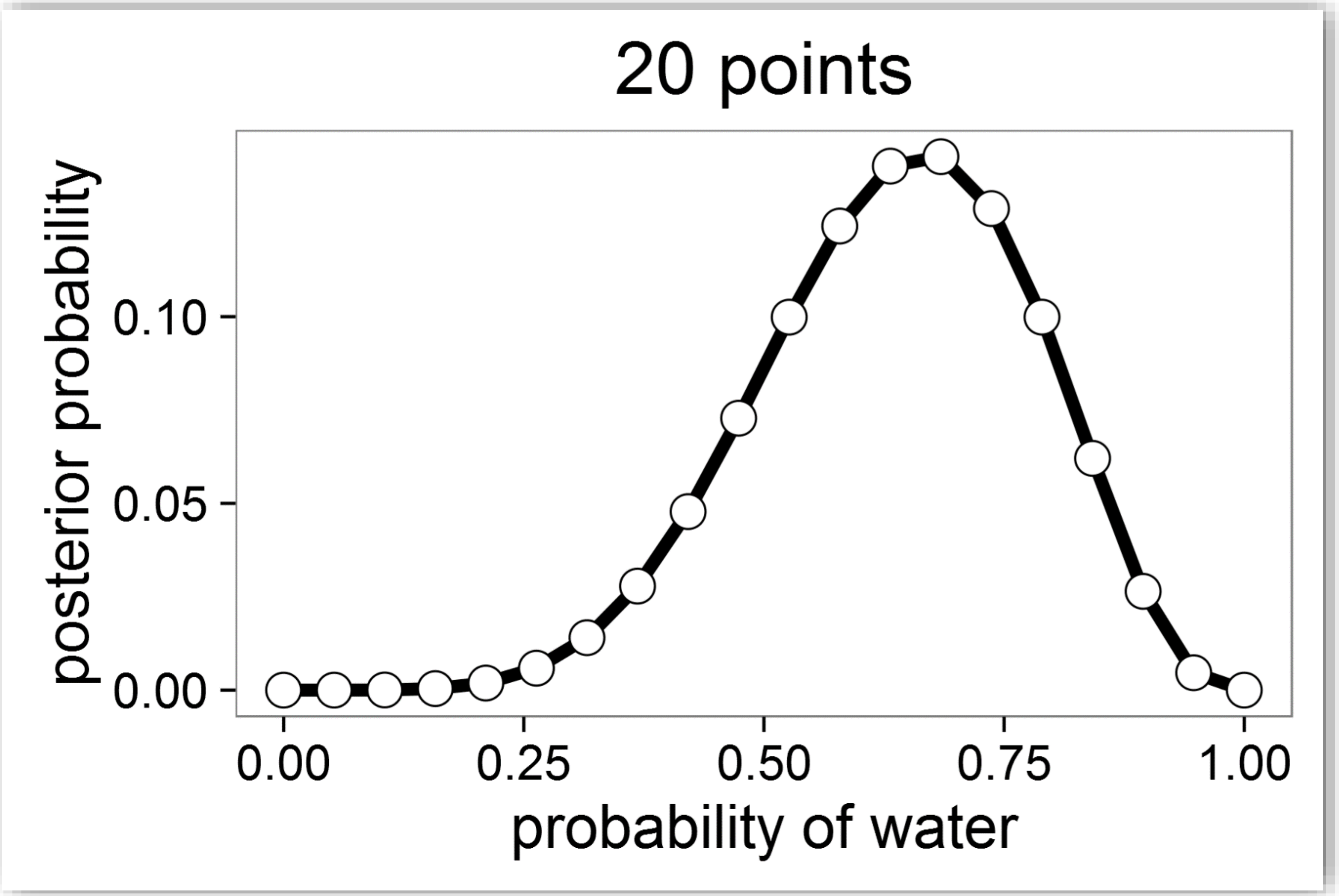
$$p(\theta \mid D) = \frac{p(D \mid \theta)\, p(\theta)}{\int p(D \mid \theta^*)\, p(\theta^*)\, d\theta^*}$$

$$p(w \mid N, \theta) = \binom{N}{w} \theta^w (1-\theta)^{N-w}$$

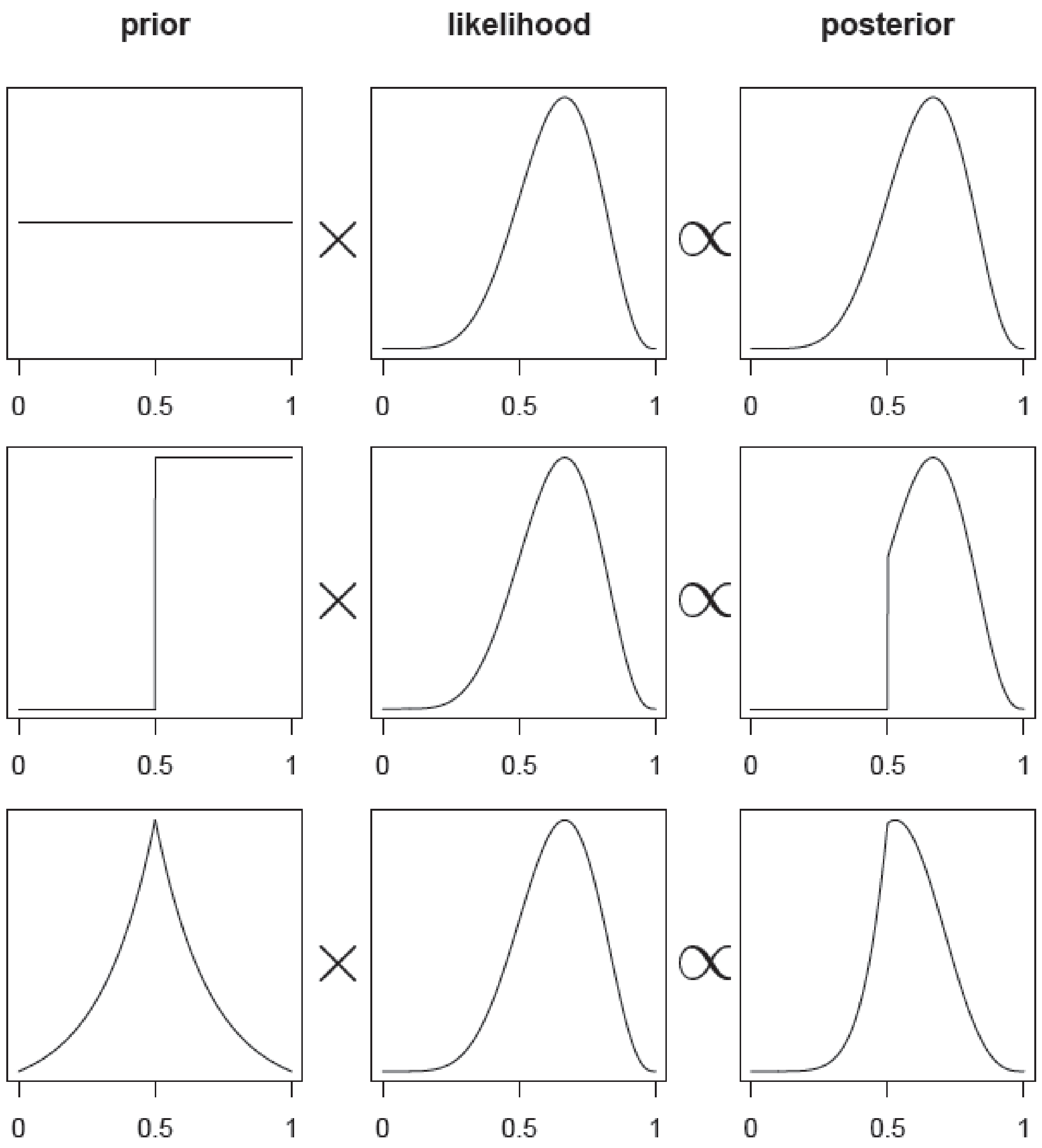


Binomial Model – Grid Approximation

cognitive model
statistics
computing



Impact of Prior



McElreath (2016)

Exercise VII


```
…/BayesCog/02.binomial_globe/_scripts/binomial_globe_grid.R
```

TASK: run a grid approximation with `grid_size = 50`

Components of a Model

cognitive model
statistics
computing

grid approximation for
2 parameters?
5 parameters?
10 parameters?

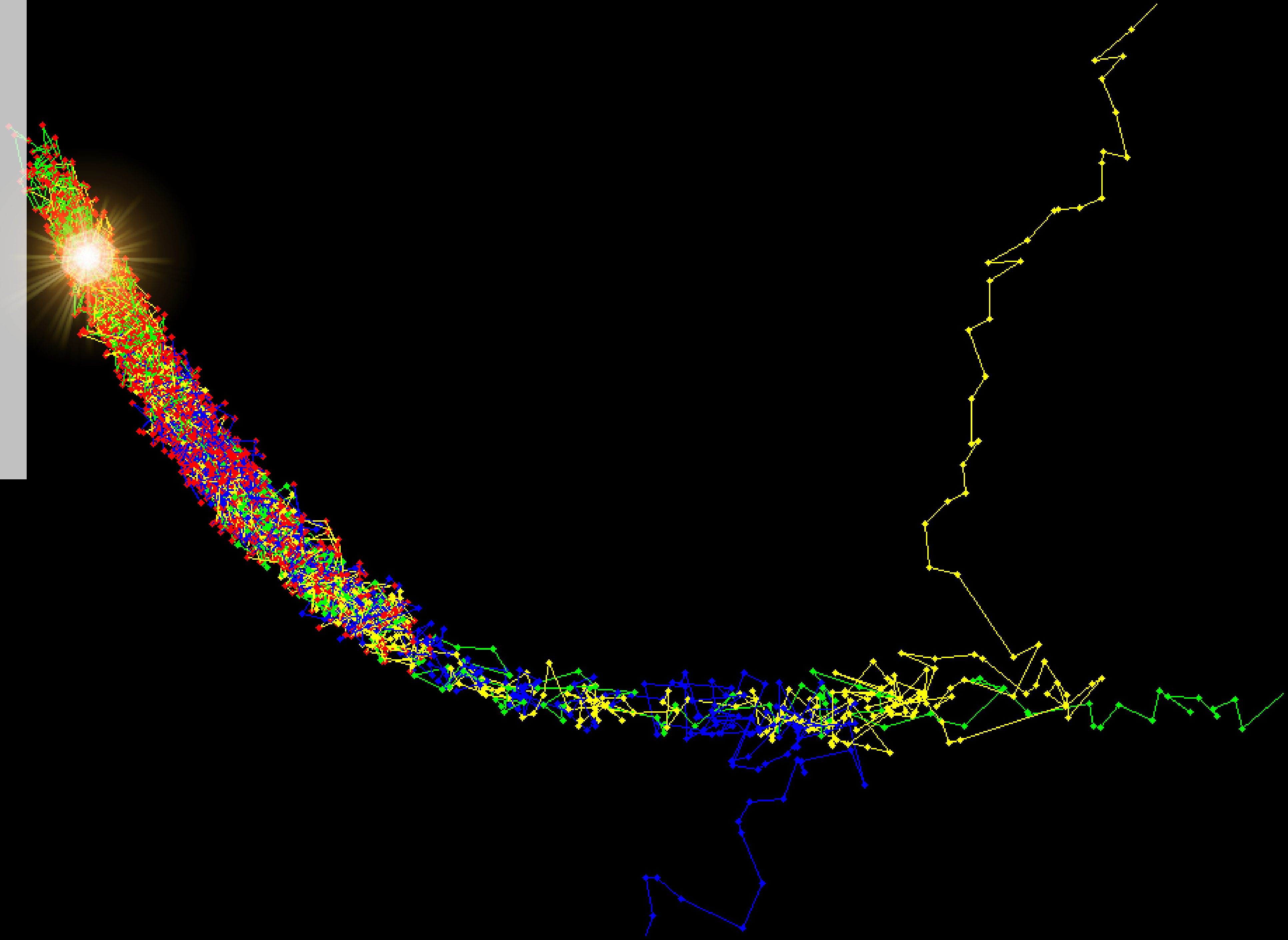
$$p(\theta \mid D) = \frac{p(D \mid \theta)\, p(\theta)}{\boxed{\int p(D \mid \theta^*)\, p(\theta^*)\, d\theta^*}}$$

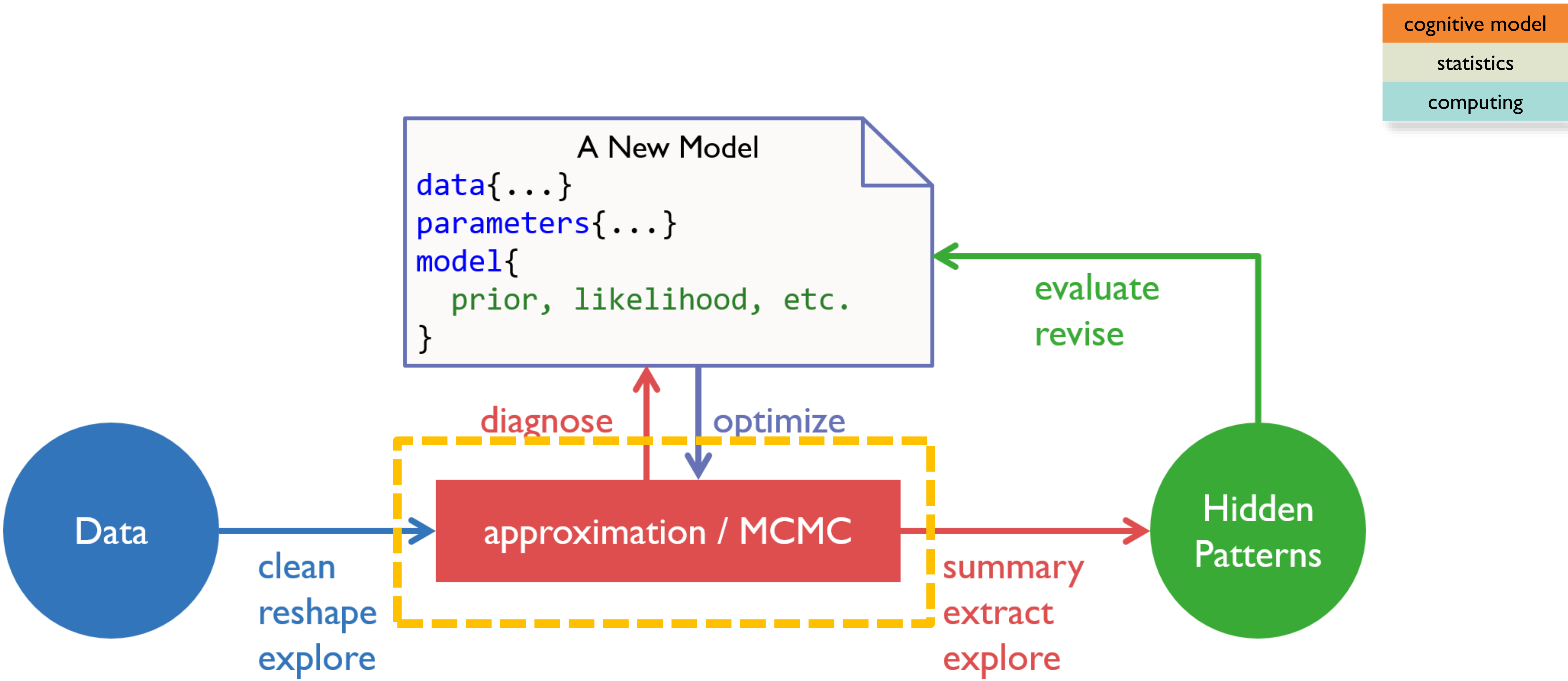
$$p(data) = \int_{\text{All}\theta_1} \int_{\text{All}\theta_2} p(data, \theta_1, \theta_2) \mathrm{d}\theta_1 \mathrm{d}\theta_2$$

$$p(data) = \int_{\mu_1} \int_{\sigma_1} \cdots \int_{\mu_{100}} \int_{\sigma_{100}} \underbrace{p(data \mid \mu_1, \sigma_1, \ldots, \mu_{100}, \sigma_{100})}_{\text{likelihood}} \times \underbrace{p(\mu_1, \sigma_1, \ldots, \mu_{100}, \sigma_{100})}_{\text{prior}} \mathrm{d}\mu_1 \mathrm{d}\sigma_1 \ldots \mathrm{d}\mu_{100} \mathrm{d}\sigma_{100},$$

$$\boxed{p(\theta \mid D) \propto p(D \mid \theta)\, p(\theta)}$$

MARKOV CHAIN MONTE CARLO




cognitive model
statistics
computing
A New Model
data{...}
parameters{...}
model{
prior, likelihood, etc.
}
evaluate
revise
diagnose
optimize
Data
approximation / MCMC
Hidden Patterns
clean
reshape
explore
summary
extract
explore

Solving the Problem by Approximation

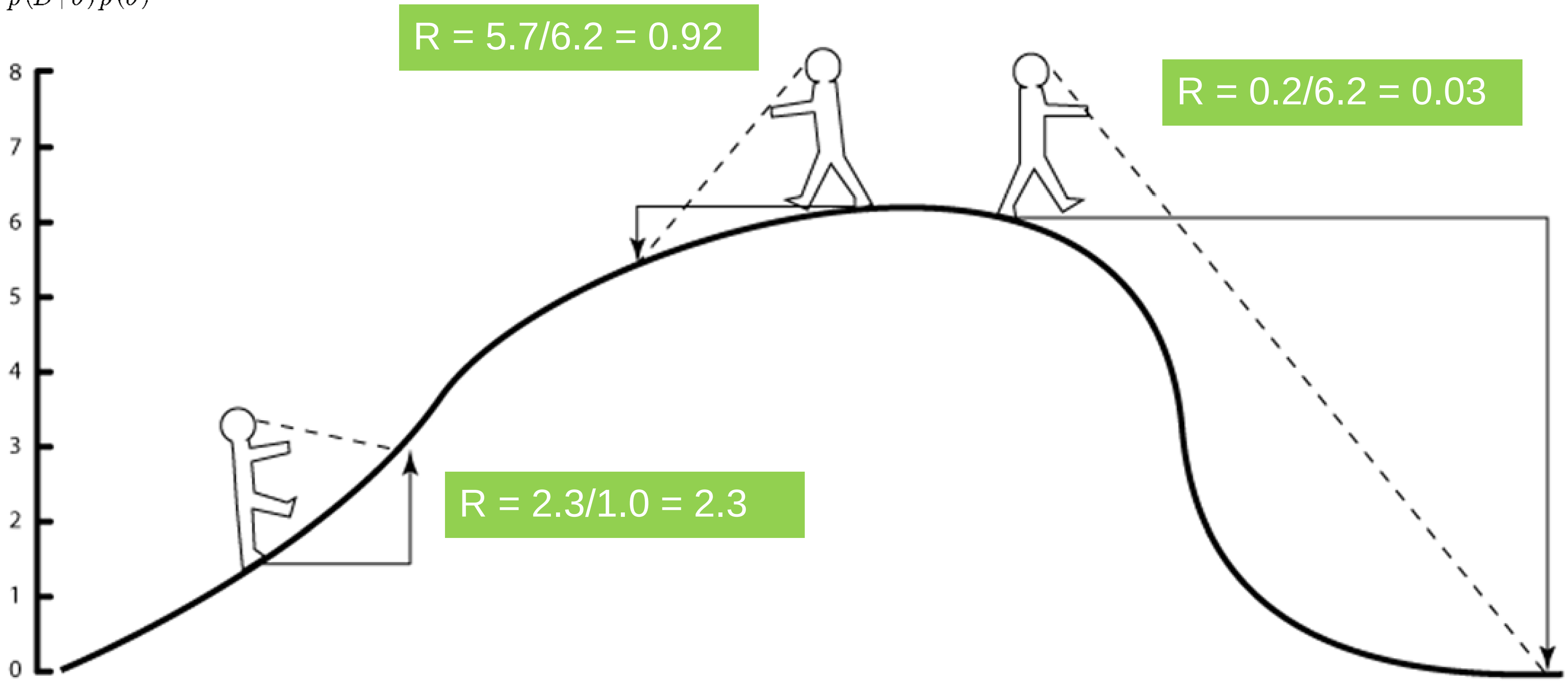
$$p(\theta \mid D) \propto p(D \mid \theta)\, p(\theta)$$

Deterministic Approximation	Stochastic Approximation
→ Variational Bayes	→ Sampling Methods

An MCMC Robot

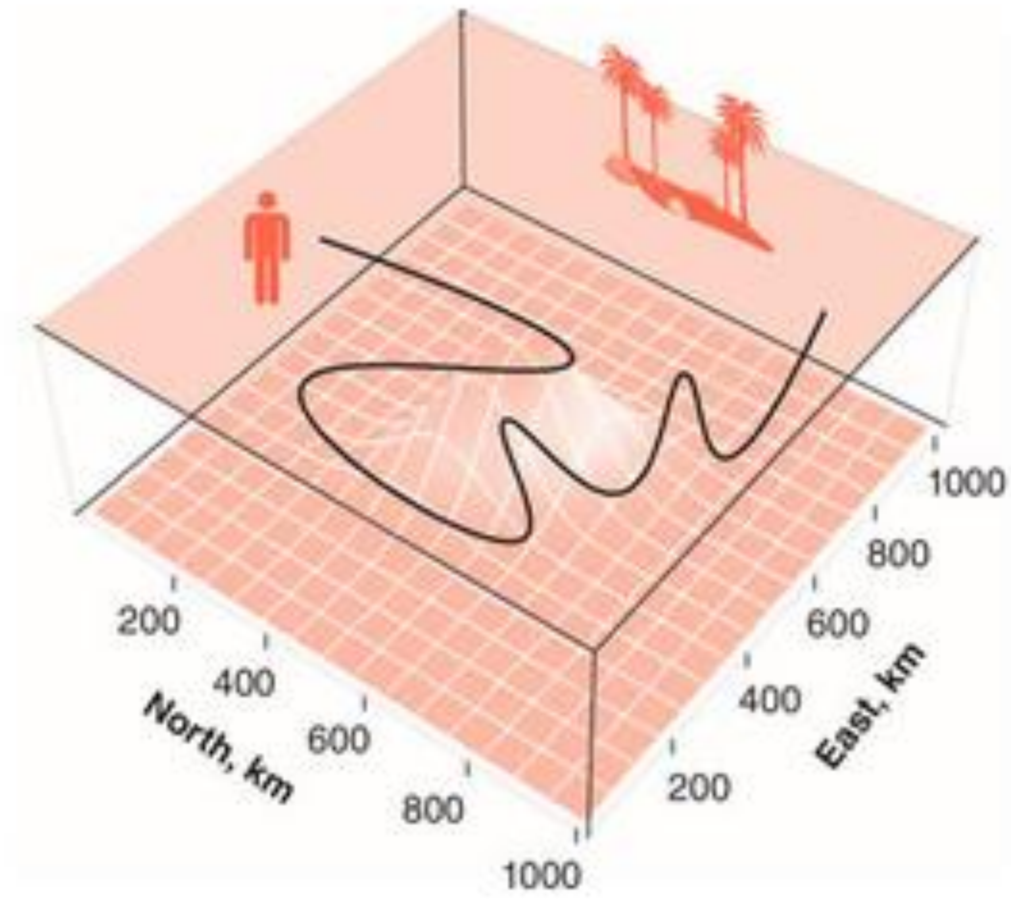
cognitive model
statistics
computing

$p(\theta \mid D) \propto p(D \mid \theta) p(\theta)$



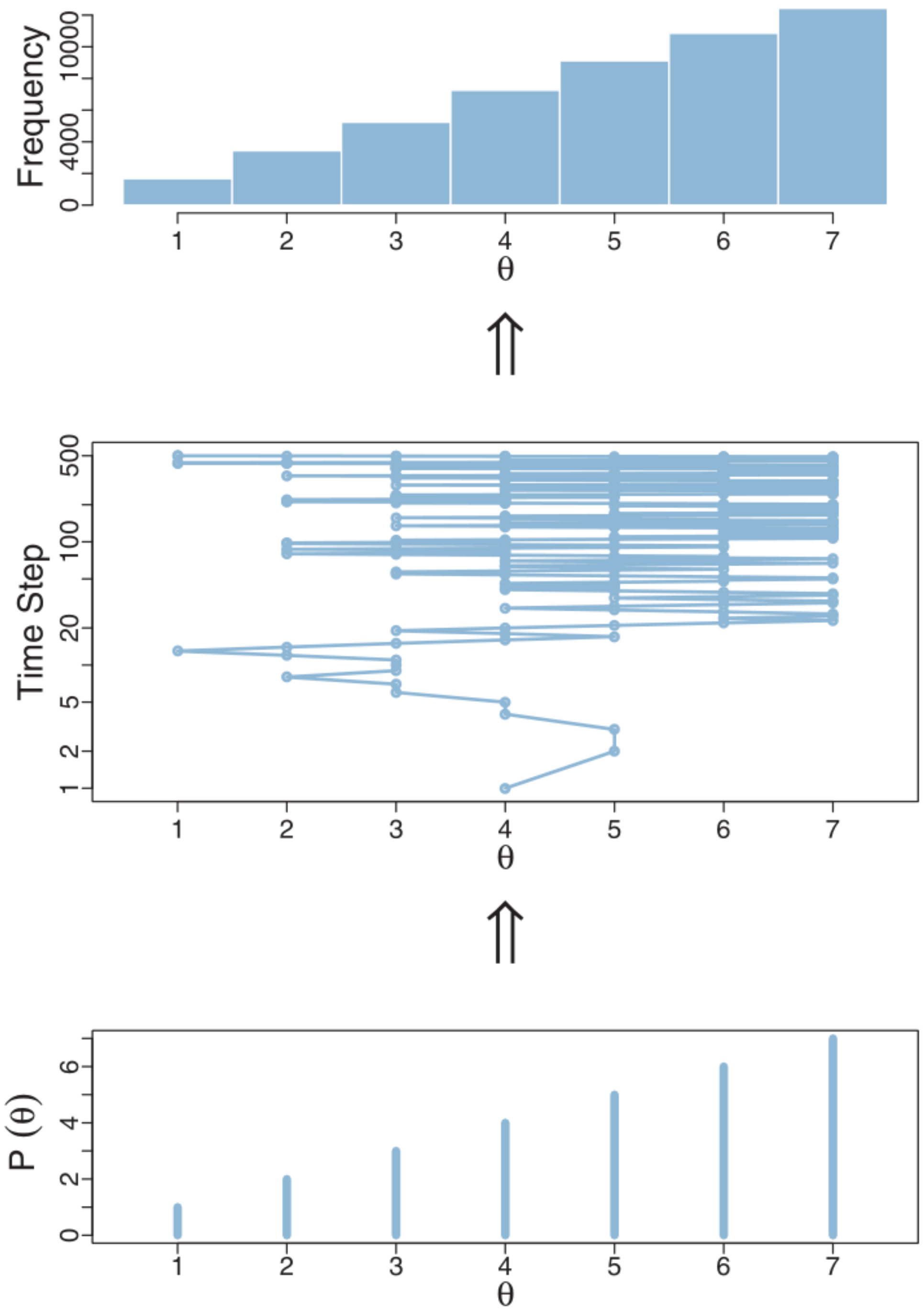
http://marple.eeb.uconn.edu/mcmcrobot/

An MCMC Robert in 3D



Lambert (2018)

Sampling Example: Discrete



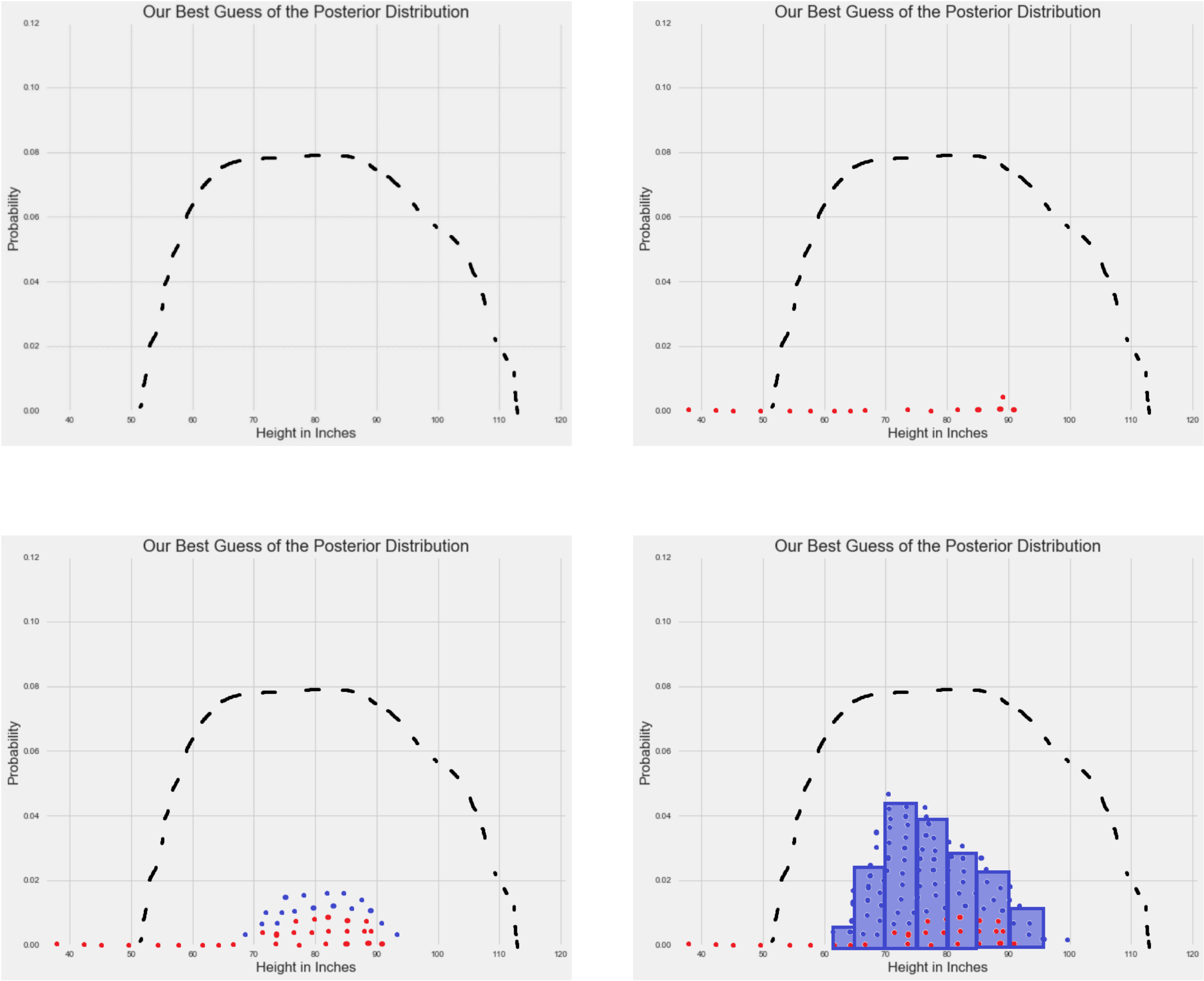
MCMC summary

MCMC trace

True distribution

Kruschke (2015)

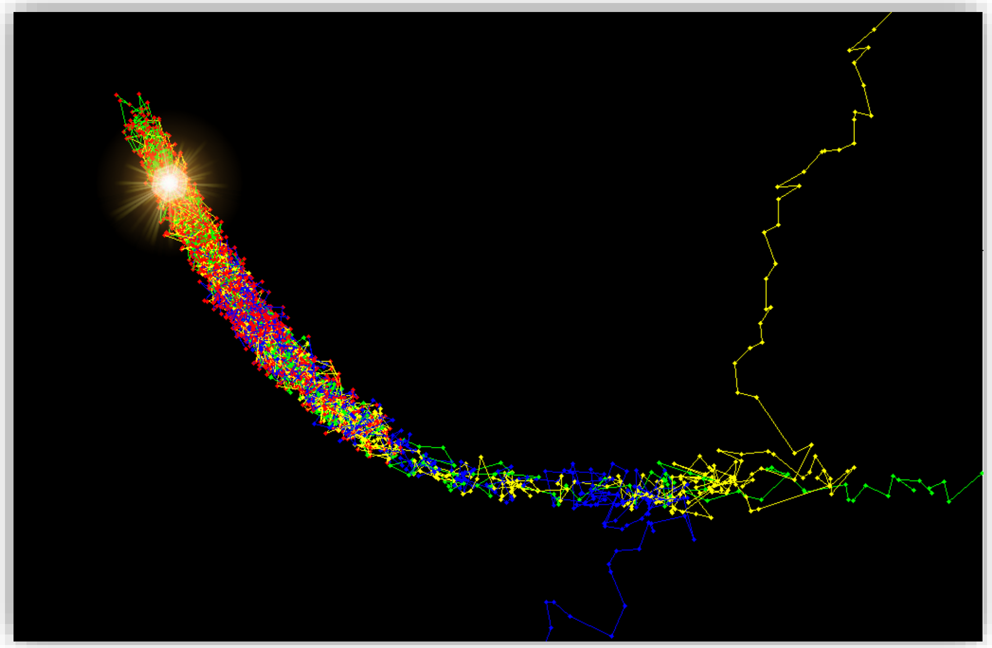
Sampling Example: Continuous

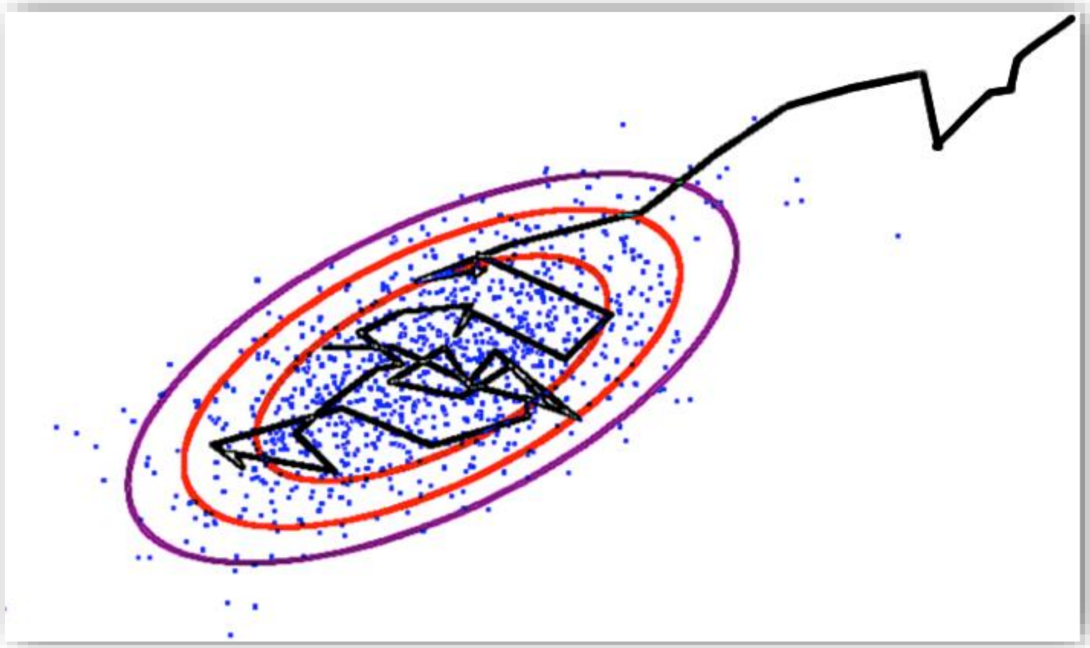


cognitive model
statistics
computing

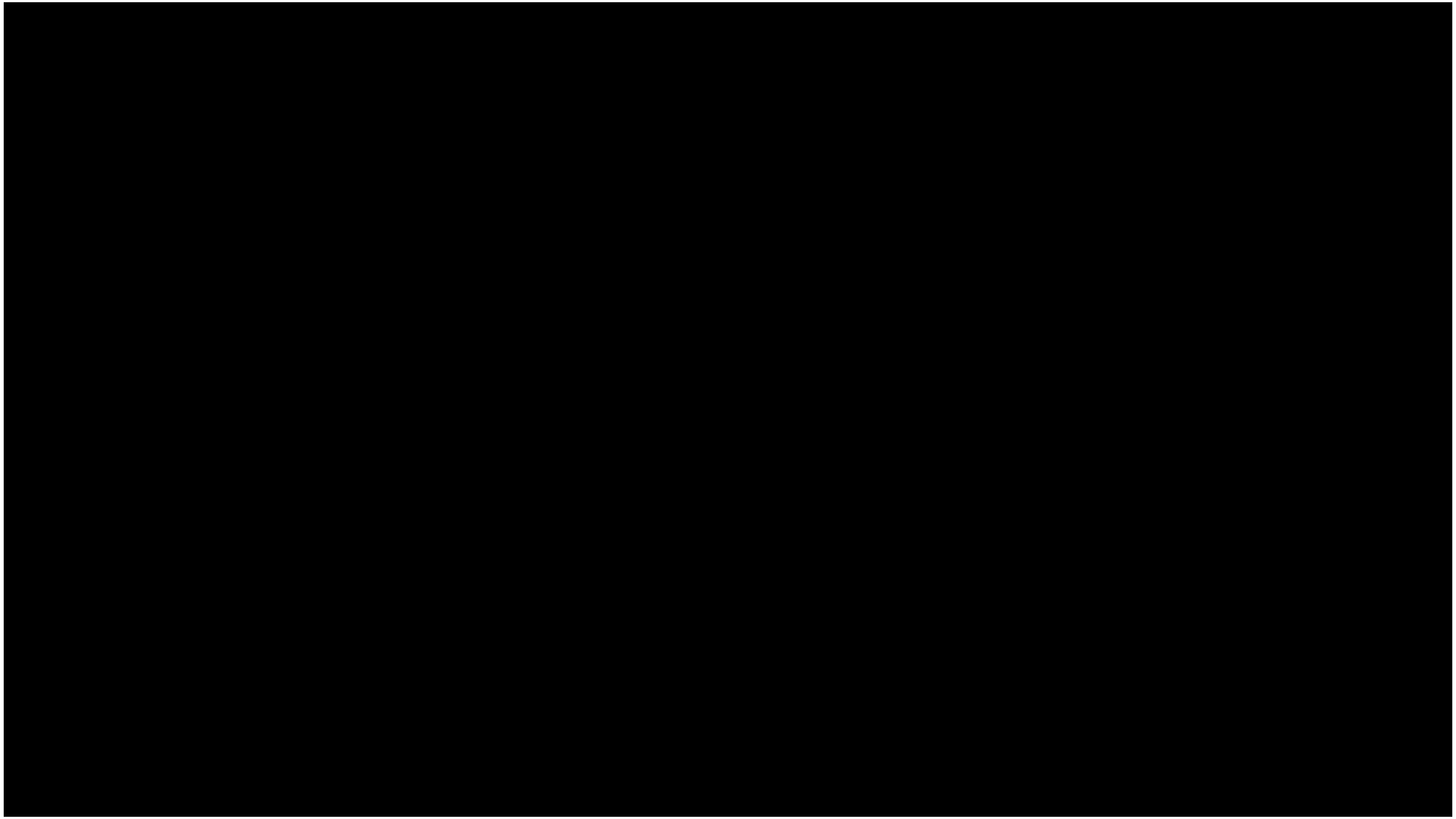
[A Zero-Math Introduction to Markov Chain Monte Carlo Methods]

Visual Example





Let's watch a video!



Credit: Thomas Wiecki

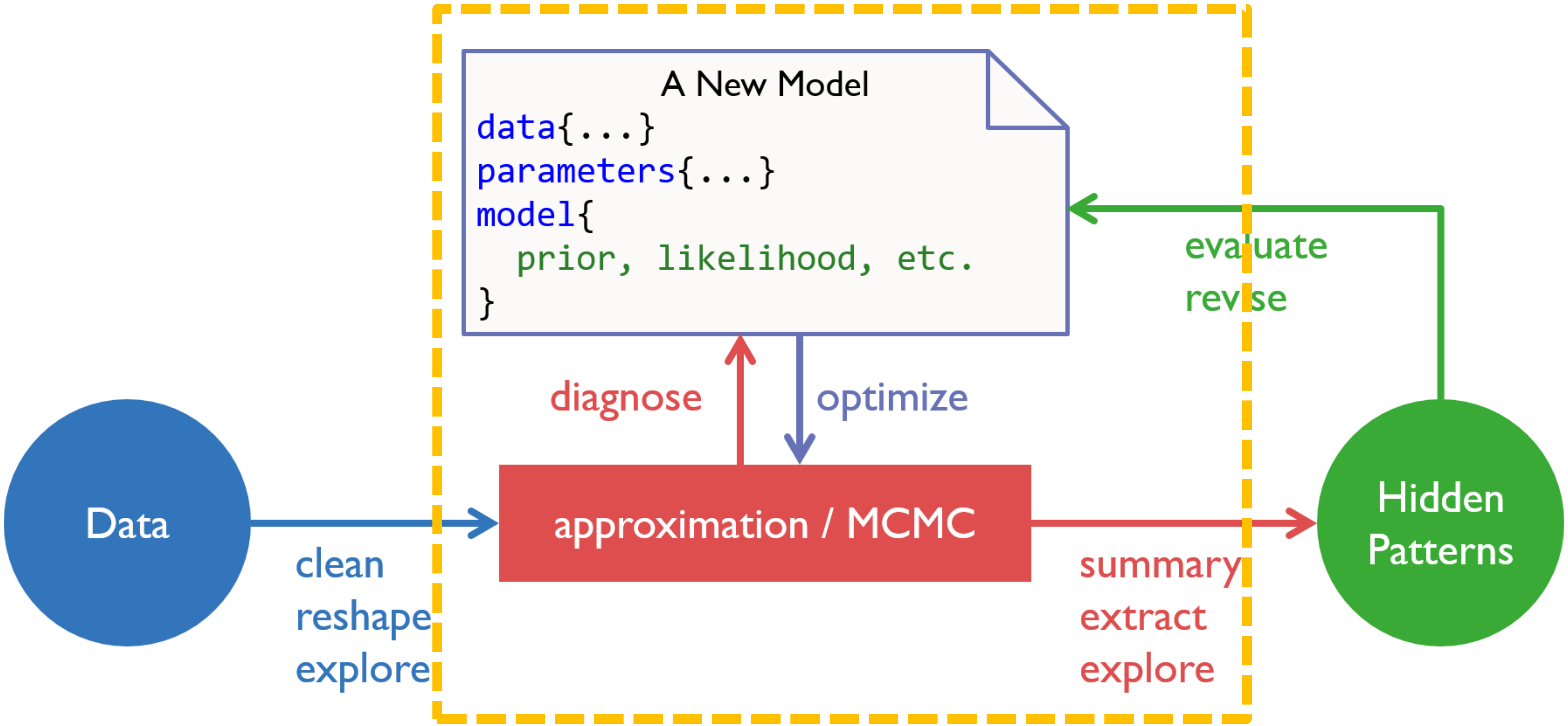
MCMC Sampling Algorithms

- Rejection sampling
- Importance sampling
- Metropolis algorithm
- Gibbs sampling → JAGS
- $\boxed{\text{HMC sampling*}}$

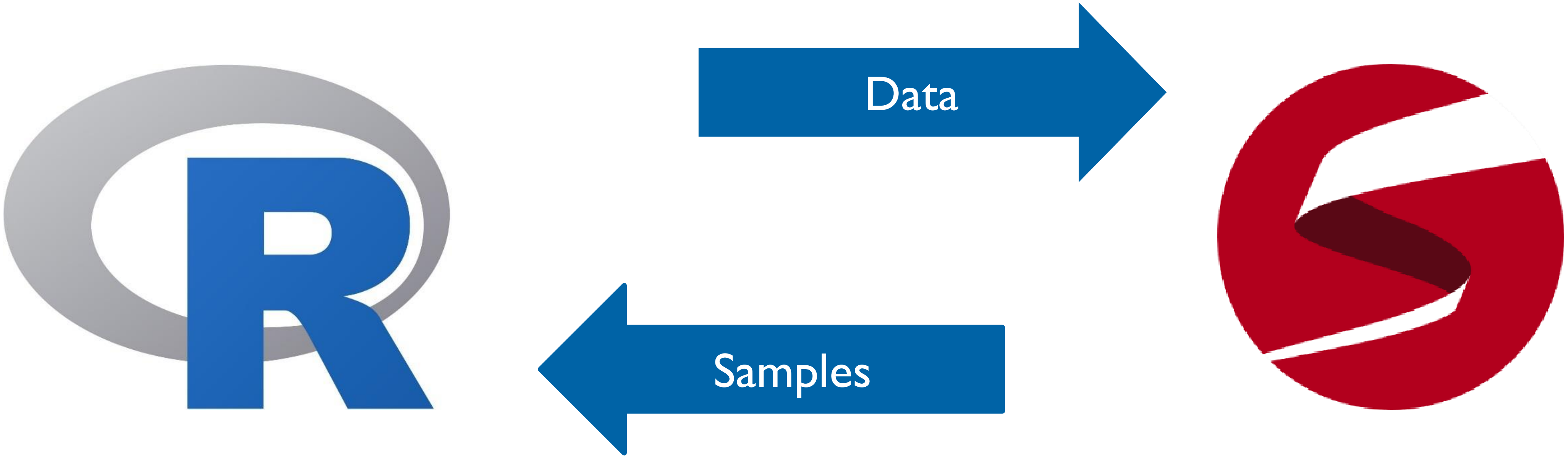
Stan!

*: also PyMC3 PyMC3

STAN PROGRAMMING LANGUAGE I


A New Model
data{...}
parameters{...}
model{
prior, likelihood, etc.
}
diagnose
optimize
evaluate
revise
Data
clean
reshape
explore
approximation / MCMC
summary
extract
explore
Hidden Patterns

Stan and RStan



Steps of Bayesian Modeling, with Stan

A data story	Think about how the data might arise. It can be *descriptive* or even *causal*. Write a Stan program (*.stan).
Update	Educate your model by feeding it the data. Bayesian Update: update the prior, in light of data, to produce posterior. Run Stan using RStan (PyStan, MatlabStan etc.)
Evaluate	Compare model with reality. Revise your model. Evaluate in RStan and ShinyStan.

McElreath (2016)

Steps of Using Stan

1. Stan program read into memory
2. Source-to-source transformation into C++
3. C++ compiled and linked (takes a while)
4. Run Stan program
5. Posterior analysis / interface

```
data {
  int<lower=0> N;
  int<lower=0,upper=1> y[N];
}
parameters {
  real<lower=0,upper=1> theta;
}
model {
  y ~ bernoulli(theta);
}
```

Stan Language

model blocks

```
data {
 //... read in external data...
}

transformed data {
 //... pre-processing of data ...
}

parameters {
 //... parameters to be sampled by HMC ...
}

transformed parameters {
 //... pre-processing of parameters ...
}

model {
 //... statistical/cognitive model ...
}

generated quantities {
 //... post-processing of the model ...
}
```

cognitive model

statistics

computing

REVISIT BINOMIAL MODEL

Binomial Model

cognitive model
statistics
computing

W L W W W L W L W

$$p(w \mid N, \theta) = \binom{N}{w} \theta^{w} (1-\theta)^{N-w}$$

$$w \sim \text{Binomial}(N, \theta)$$

reads as:
w is distributed as a binomial distribution, with number of trials *N*, and success rate *ϑ*.

McElreath (2016)

Graphical Model Notations

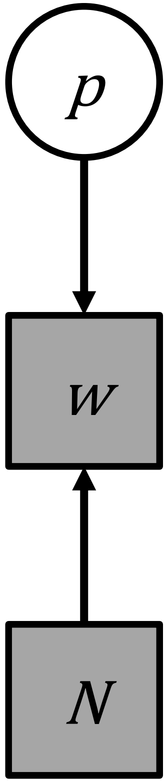
	continuous	discrete
unobserved	θ	δ
observed	y	N

Lee & Wagenmakers (2013)

Binomial Model

W L W W W L W L W

$$p(w \mid N, \theta) = \binom{N}{w} \theta^{w} (1-\theta)^{N-w}$$



$$\theta \sim \text{Uniform}(0, 1)$$

$$w \sim \text{Binomial}(N, \theta)$$

	continuous	discrete
unobserved	θ	δ
observed	y	N

McElreath (2016)

Binomial Model

W L W W W L W L W

$$p(w \mid N, \theta) = \binom{N}{w} \theta^{w} (1-\theta)^{N-w}$$

```
data {
    int<lower=0> w;
    int<lower=0> N;
}

parameters {
    real<lower=0,upper=1> theta;
}

model {
    w ~ binomial(N, theta);
}
```

McElreath (2016)

Running Binomial Model with Stan


```
…/BayesCog/02.binomial_globe/_scripts/binomial_globe_main.R
```

```
> R.version
R version 3.5.1 (2018-07-02)

> stan_version()
[1] "2.18.0"
```

Model Summary

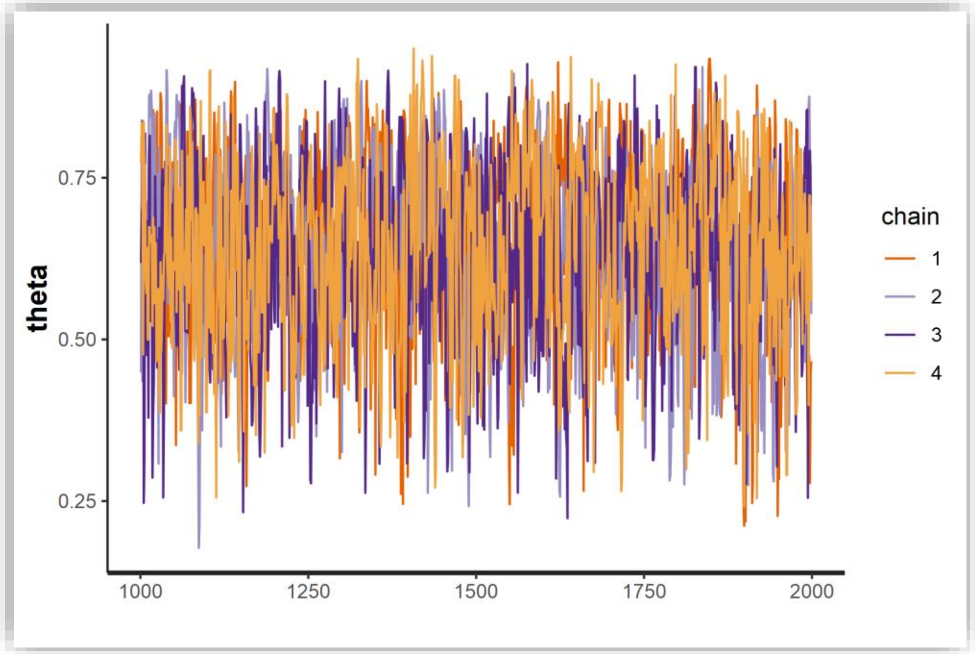
cognitive model
statistics
computing

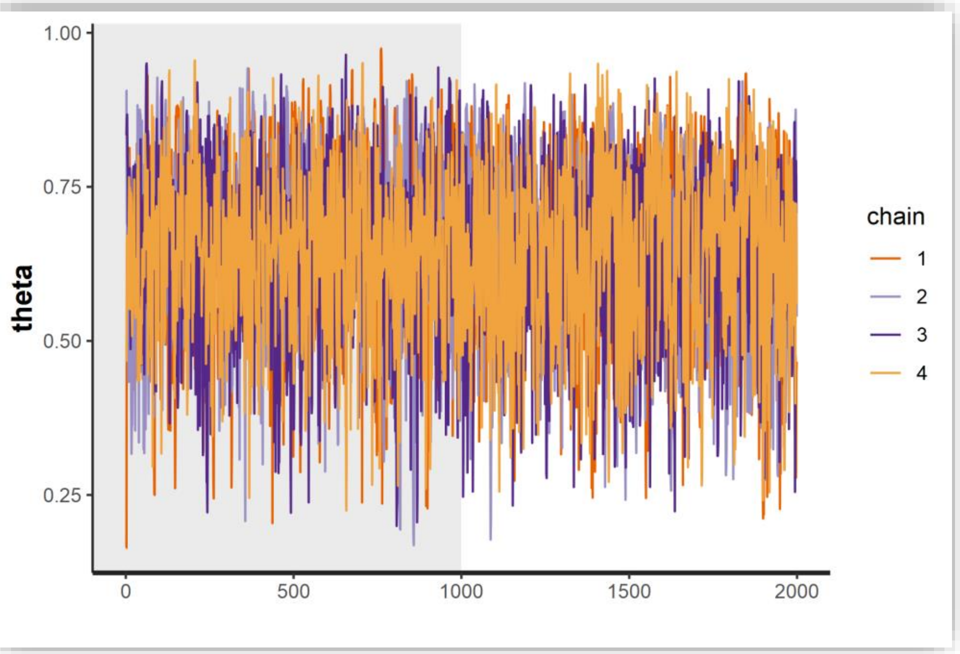
```
> print(fit_globe)
Inference for Stan model: binomial_globe_model.
4 chains, each with iter=2000; warmup=1000; thin=1;
post-warmup draws per chain=1000, total post-warmup draws=4000.

       mean se_mean   sd  2.5%   25%   50%   75% 97.5% n_eff Rhat
theta  0.64    0.00 0.14  0.35  0.54  0.65  0.74  0.87  1278    1
lp__  -7.72    0.02 0.69 -9.77 -7.89 -7.46 -7.27 -7.21  1824    1

Samples were drawn using NUTS(diag_e) at Tue Apr 09 12:44:04 2019.
For each parameter, n_eff is a crude measure of effective sample size,
and Rhat is the potential scale reduction factor on split chains (at
convergence, Rhat=1).
```

Gelman-Rubin convergence diagnostic (Gelman & Rubin, 1992)

Diagnostics - traceplot

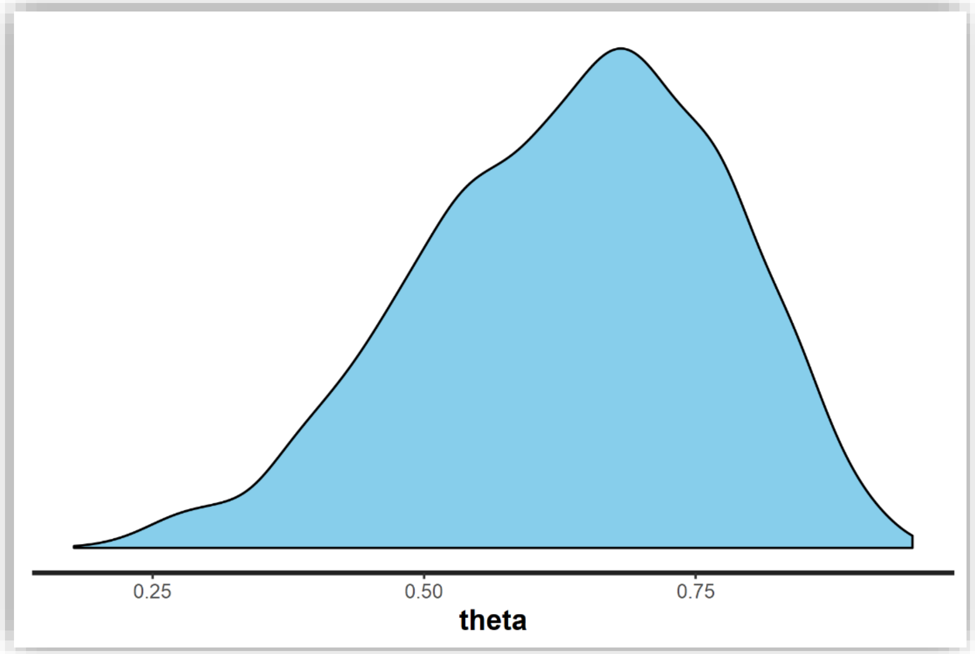
cognitive model
statistics
computing

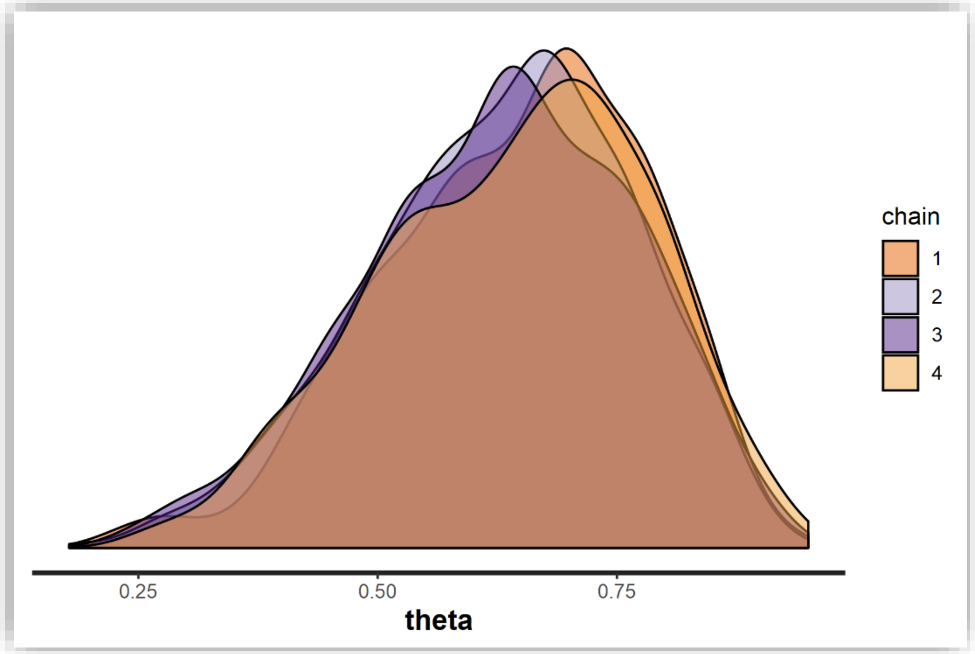




Diagnostics - density

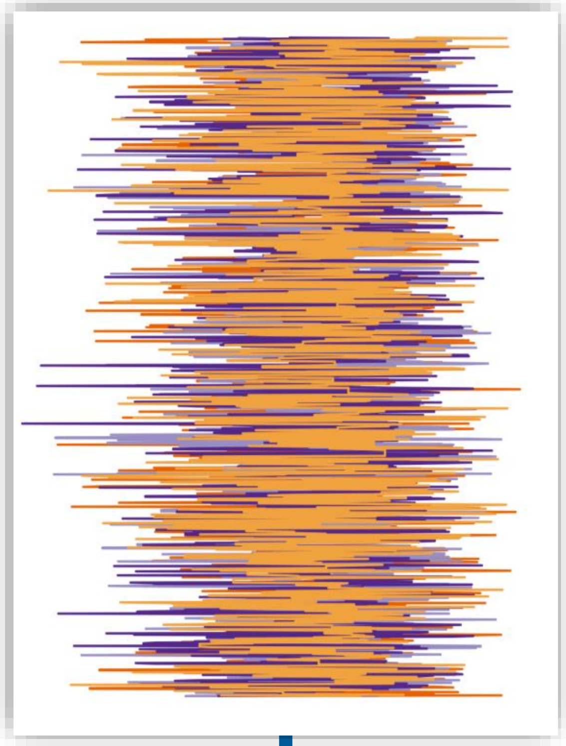
cognitive model
statistics
computing

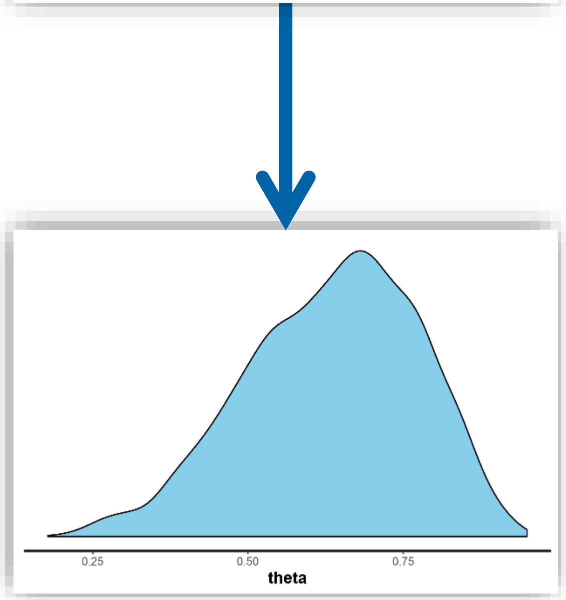




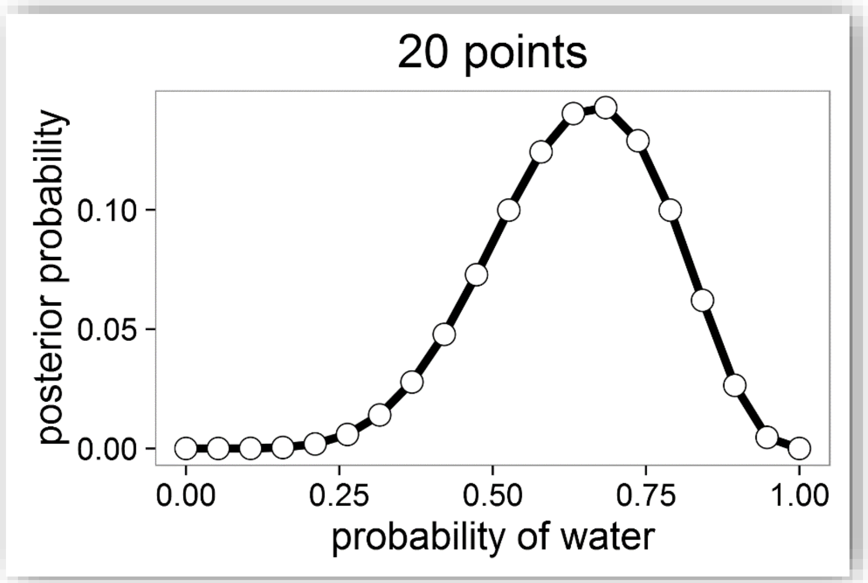
Diagnostics

MCMC



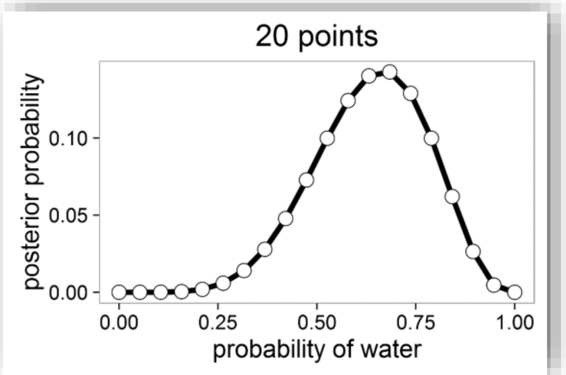


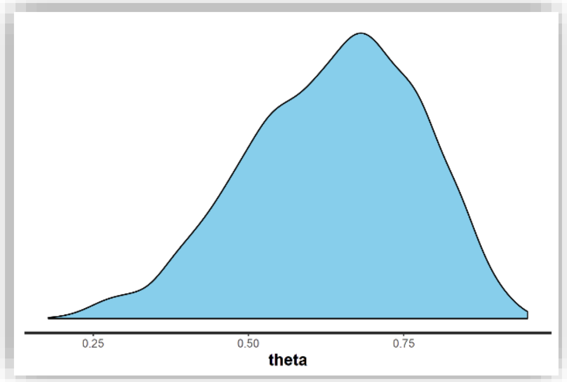
Grid Approximation



Draw a Conclusion?

- W = 6 out of N = 9
- uncertainty (relative plausibility) of all ϑ values
- the relative plausibility of $\vartheta = 0.64$ is the highest, but it never rules out the possibility of ϑ being other values, e.g., 0.5, 0.75
- → when $\vartheta = 0.5$, you may still observe 6W / 9 trials





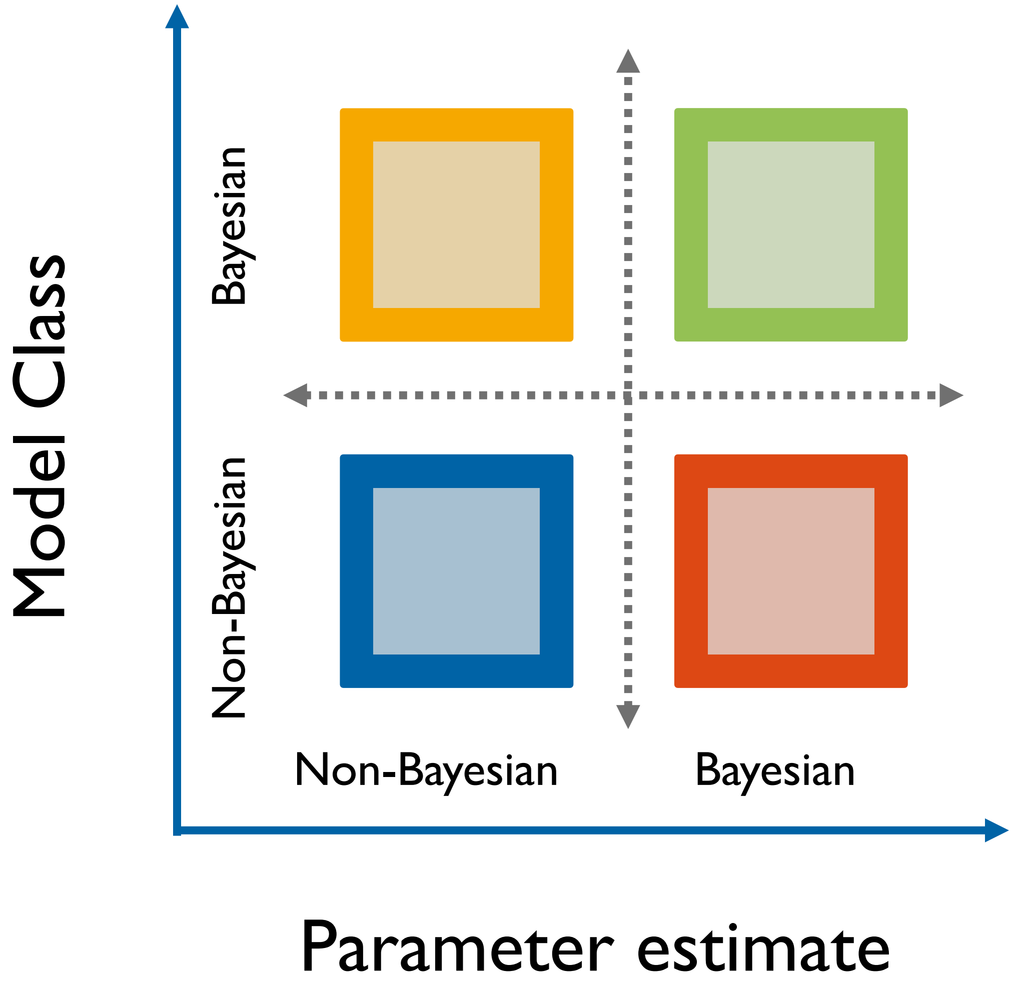
Is Anything Missing? – NO

cognitive model
statistics
computing

```
data {
    int<lower=0> w;
    int<lower=0> N;
}

parameters {
    real<lower=0,upper=1> theta;
}

model {
    theta ~ uniform(0,1);
    w ~ binomial(N, theta);
}
```

```
data {
    int<lower=0> w;
    int<lower=0> N;
}

parameters {
    real<lower=0,upper=1> theta;
}

model {
    w ~ binomial(N, theta);
}
```

What We Talk About When We Talk About "Bayesian" Models



STAN PROGRAMMING LANGUAGE II

Why Use Stan?

vs. BUGS and JAGS

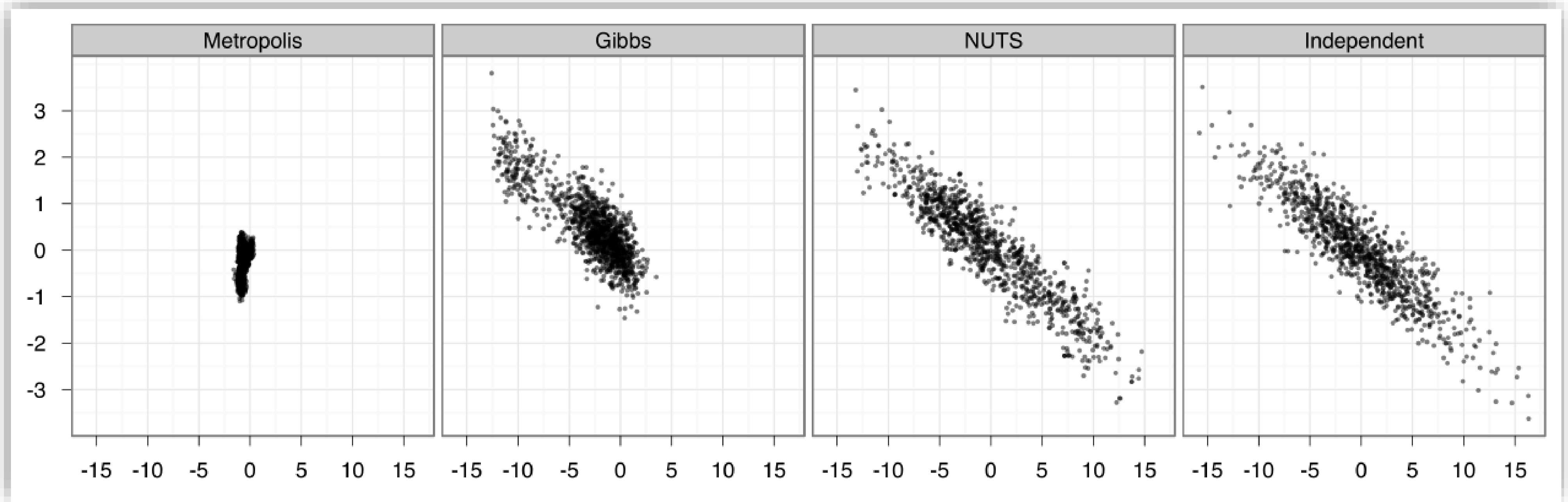
- Time to converge and per effective sample size:

  0.5 - ∞ times faster
- Memory usage: 1–10%
- Language features
  - variable overwrite: `a = 4, then a = 5`
  - formal control flow
  - full support of vectorizing

http://mc-stan.org/
https://discourse.mc-stan.org/t/selling-stan/3693/6

NUTS vs. Gibbs and Metropolis

Hamilton MC (HMC) implements No-U-Turn Sampler (NUTS)



- Two dimensions of highly correlated 250-dim normal
- 1,000,000 draws from Metropolis and Gibbs (thin to 1000)
- 1,000 draws from NUTS; 1000 independent draws

General Properties of Stan Language

- Whitespace does not matter
- Comments
  - //
  - /* … */
- Must use semicolon (;)
- Variables are typed and scoped

Variable Declaration

- Each variable has a type (static type; scalar, vector, matrix etc.)
- Only values of that type can be assigned to the variable
  - e.g. cannot assign [1 2 3] to a (declared as a scalar)
- Declaration of variables happen at the top of a block (including local blocks)

Scalar Variables

`real`

- scalar
- continuous

```
data {
  real y;
}
```

`int`

- scalar
- integer
- can't be used in `parameters` or `transformed parameters` blocks

```
data {
  int n;
}
```

Constraining Scalar Variables


```
data {
  int<lower=1> m;
  int<lower=0,upper=1> n;
  real<lower=0> x;
  real<upper=0> y;
  real<lower=-1,upper=1> rho;
}
```

Vector & Matrix

```
vector[3] a;
// column vector

row_vector[4] b;
// row vector

matrix[3,4] A;
// A is a 3x4 matrix
// A[1] returns a 4-element row vector

vector<lower=0,upper=1>[5] rhos;
row_vector<lower=0>[4] sigmas;
matrix<lower=-1, upper=1>[3,4] Sigma;
```

Control Flow

- if-else

```
if (cond) {
    ..statement..
}
```

```
if (cond) {
    ..statement..
} else {
    ..statement..
}
```

```
if (cond) {
    ..statement..
} else if (cond) {
    ..statement..
} else {
    ..statement..
}
```

- for-loop

```
for ( j in 1:J) {
    ..statement..
}
```

```
for ( j in 1:J ) {
    for ( k in 1:K ) {
        ..statement..
    }
}
```

same as the R syntax, but
terminate each line with ;

Variable's Scope

	data	transformed data	parameters	transformed parameters	model	generated quantities
Variable Declarations	Yes	Yes	Yes	Yes	Yes	Yes
Variable Scope	Global	Global	Global	Global	Local	Local
Variables Saved?	No	No	Yes	Yes	No	Yes
Modify Posterior?	No	No	No	No	Yes	No
Random Variables	No	No	No	No	No	Yes

BERNOULLI MODEL

Bernoulli Model

- You are interested in if a coin is biased.
- You will flip the coin.
- You will record whether it comes up a head (h) or a tail (t).
- You might observe 15 heads out of 20 flips.
- What is your degree of belief about the biased parameter ϑ?

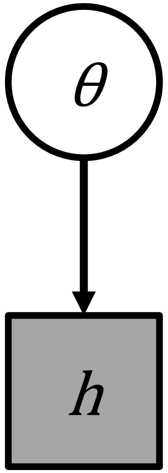
Kruschke (2015)

Bernoulli Model

$$p(w \mid N, p) = \binom{N}{w} p^{w} (1-p)^{N-w}$$

$N = 1$

$$p(h \mid \theta) = \theta^{h} (1-\theta)^{1-h}$$



$$\theta \sim \text{Uniform}(0, 1)$$

$$h \sim \text{Bernoulli}\,(\theta)$$

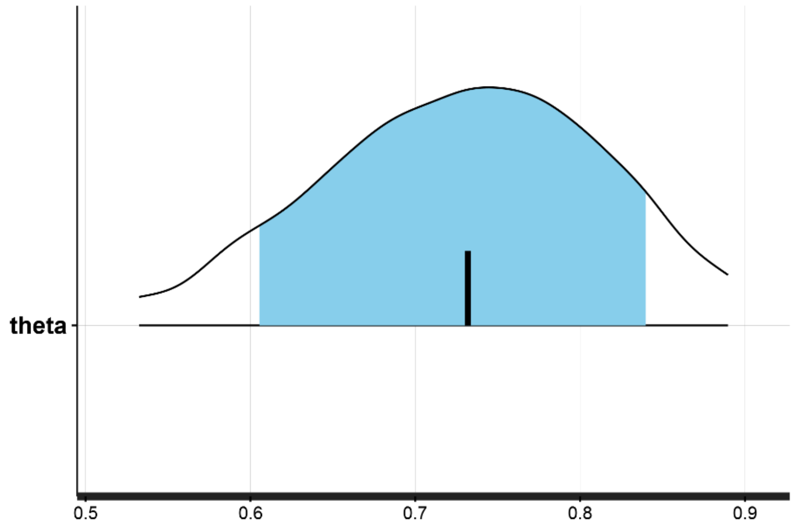
Exercise VIII

cognitive model
statistics
computing

```
…/BayesCog/03.bernoulli_coin/_scripts/bernoulli_coin_main.R
```

TASK: fit the Bernoulli model

```
> dataList
$`flip`
 [1] 1 1 1 0 1 1 1 1 1 0 0 1 1 0 1 1 1 1 0 1

$N
[1] 20
```



Possible Optimization?

```
model {
  for (n in 1:N) {
    flip[n] ~ bernoulli(theta);
  }
}
```

61.59 secs*

```
model {
  flip ~ bernoulli(theta);
}
```

53.25 secs*

Thinking before looping!

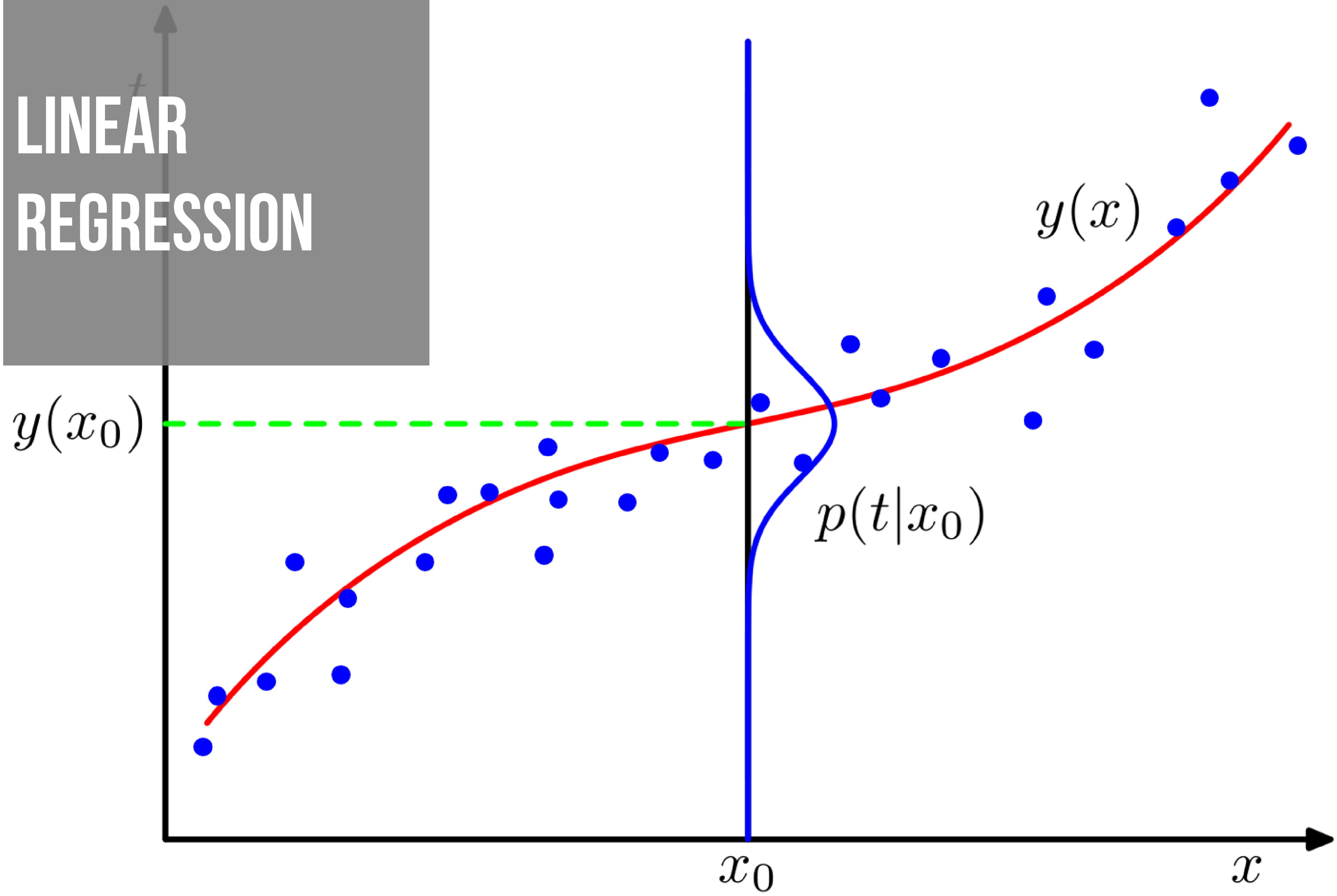
* compiling time included

Recap

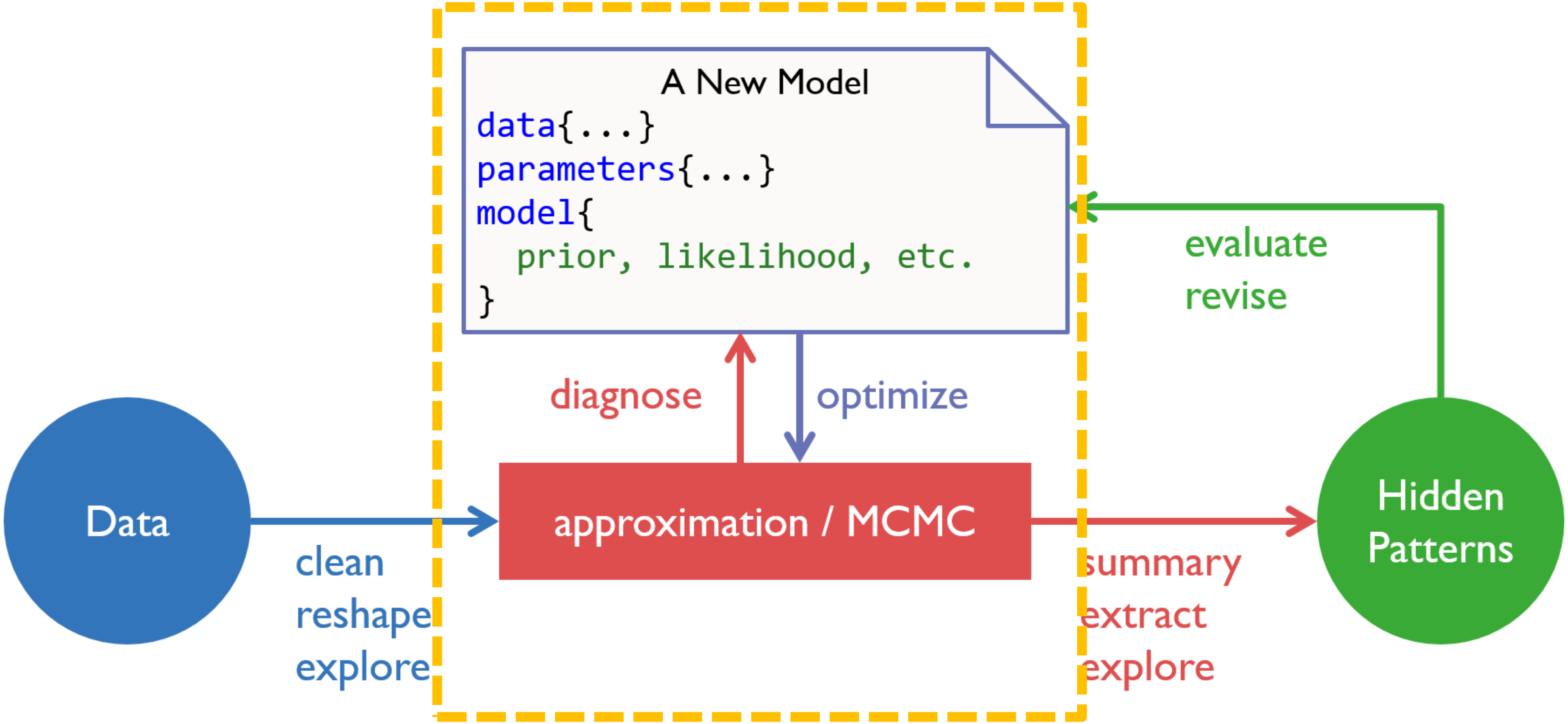
What we've learned…

- R Basics
- probability distributions
- Bayes' theorem, $p(\theta|D)$
- Binomial model
- MCMC and Stan

LINEAR REGRESSION



cognitive model
statistics
computing

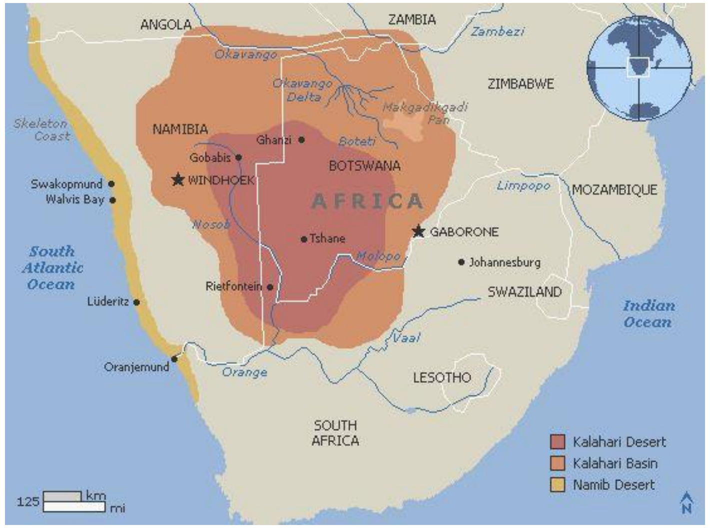


Linear Regression: height ~ weight

```
…/04.regression_height/_scripts/regression_height_main.R
```

make scatter plot and fit the model with `lm()`

```
>load('_data/height.RData')
>d <- Howell1
>d <- d[ d$age >= 18 , ]
>head(d)
height   weight age male
1 151.765 47.82561  63    1
2 139.700 36.48581  63    0
3 136.525 31.86484  65    0
4 156.845 53.04191  41    1
5 145.415 41.27687  51    0
6 163.830 62.99259  35    1
```

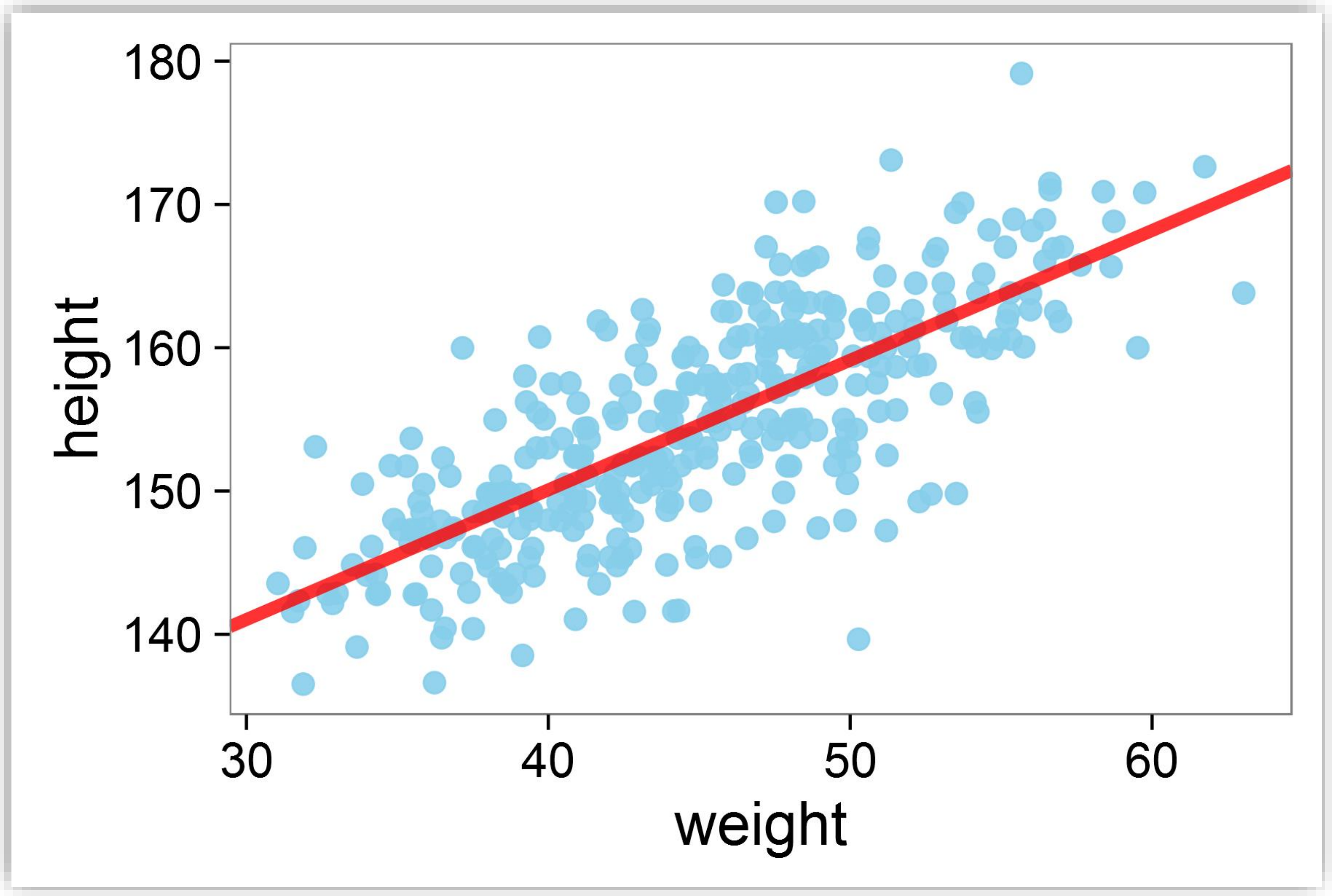


Results with lm()

cognitive model
statistics
computing

```
> L <- lm( height ~ weight, d) # estimate model by minimizing least squares errors
> summary(L)

Call:
lm(formula = height ~ weight, data = d)

Residuals:
     Min       1Q   Median       3Q      Max 
-19.7464  -2.8835   0.0222   3.1424  14.7744 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept) 113.87939    1.91107   59.59   <2e-16 ***
weight        0.90503    0.04205   21.52   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 5.086 on 350 degrees of freedom
Multiple R-squared:  0.5696, Adjusted R-squared:  0.5684 
F-statistic: 463.3 on 1 and 350 DF,  p-value: < 2.2e-16
```

height ~ weight



McElreath (2016)

Rethinking Regression Model

$$\mu_i = \alpha + \beta x_i$$

$$\cancel{y_i = \mu_i + \varepsilon}$$

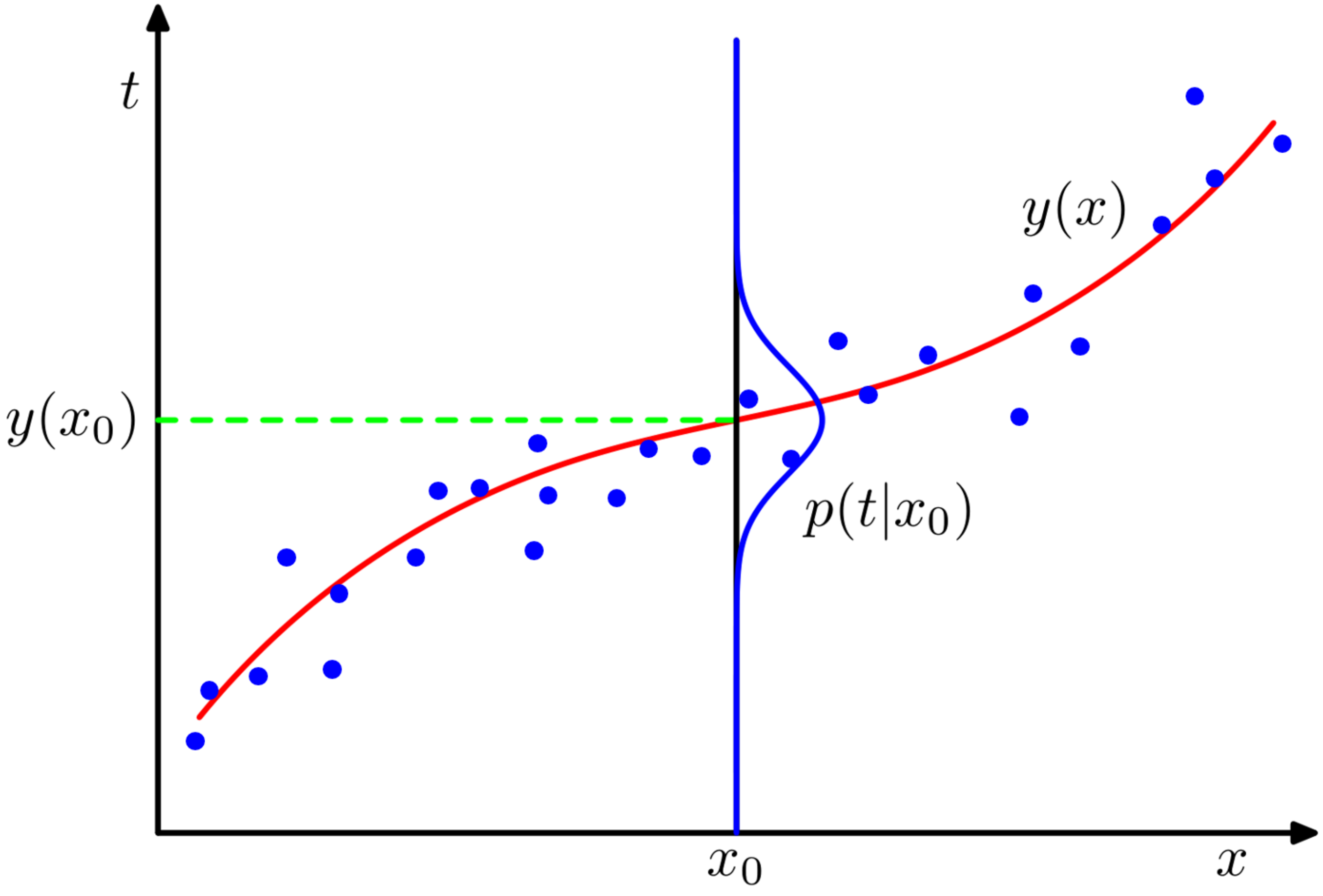
$$\cancel{\varepsilon \sim Normal(0, \sigma)}$$

$$\boxed{y_i \sim Normal(\mu_i, \sigma)}$$

Rethinking Regression Model

$$\mu_i = \alpha + \beta x_i$$

$$y_i \sim Normal(\mu_i, \sigma)$$

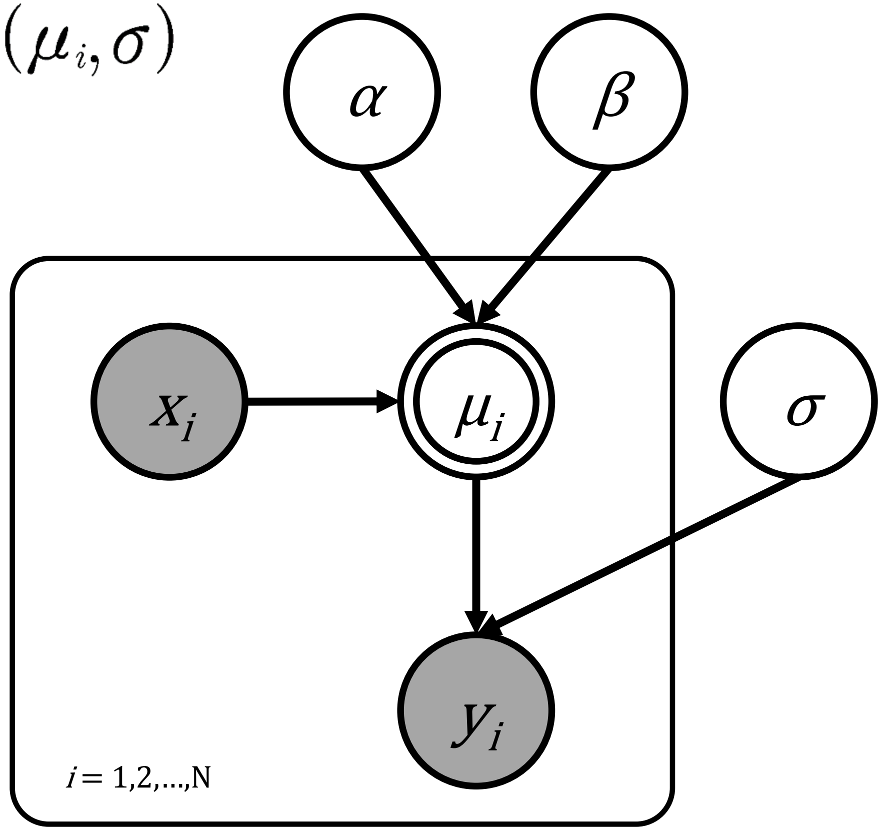


Bishop (2006)

Rethinking Regression Model

cognitive model
statistics
computing

$$\mu_i = \alpha + \beta x_i$$
$$y_i \sim Normal(\mu_i, \sigma)$$




```
model {
  vector[N] mu;
  for (i in 1:N) {
    mu[i] = alpha + beta * weight[i];
    height[i] ~ normal(mu[i], sigma);
  }
}
```

```
model {
  vector[N] mu;
  mu = alpha + beta * weight;
  height ~ normal(mu, sigma);
}
```

```
model {
  height ~ normal(alpha + beta * weight, sigma);
}
```

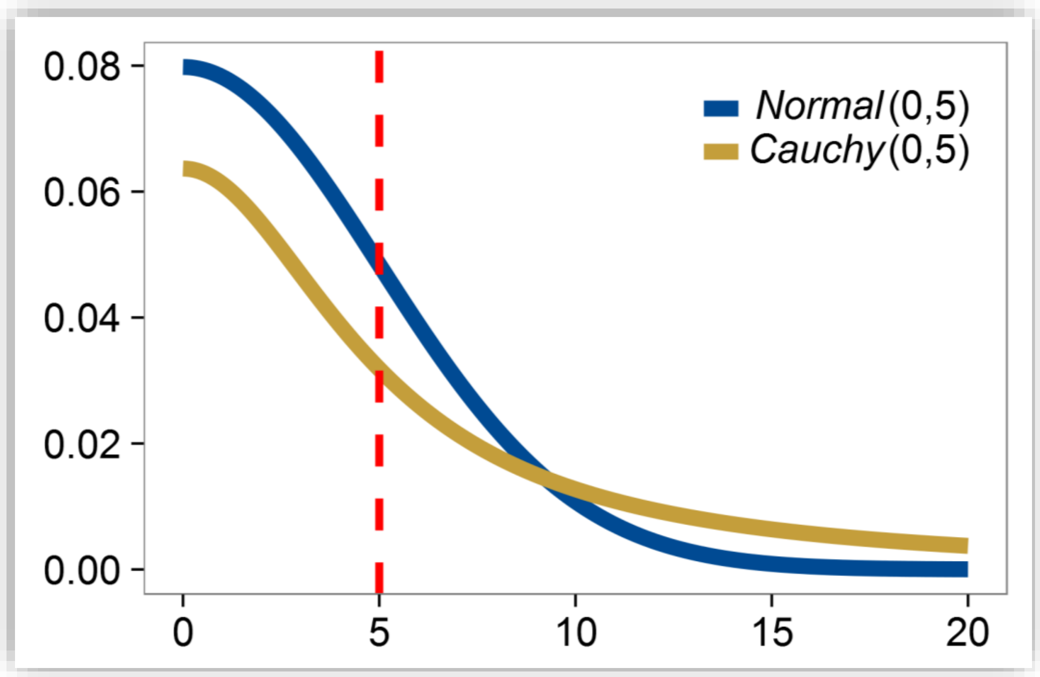
◎ deterministic variable

Thinking about Priors?

$\alpha \sim Normal(170, 100)$ $\beta \sim Normal(0{,}20)$

$$\overline{\text{height}} = \alpha + \beta * \text{weight}$$

$\sigma \sim halfCauchy(0{,}20)$



$$\text{height} \sim Normal(\overline{\text{height}}, \sigma)$$

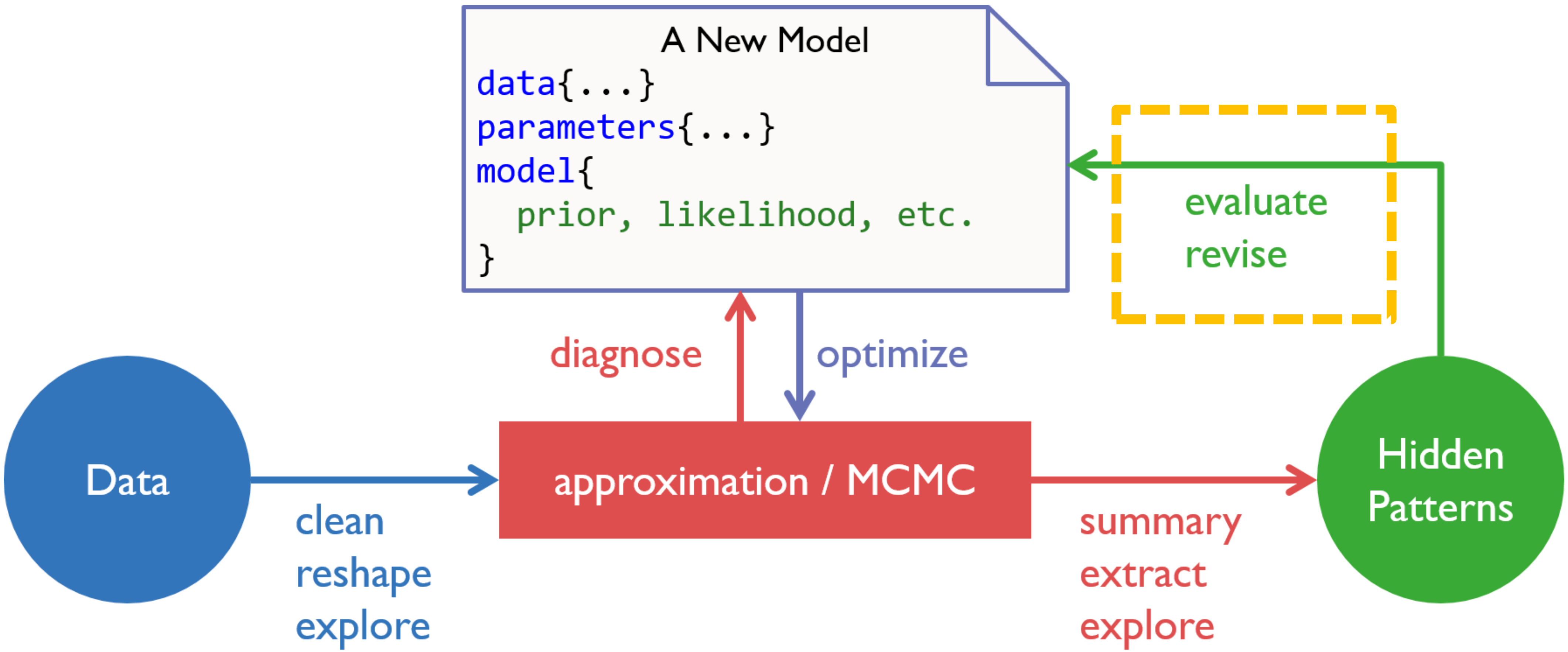
Exercise VIII

cognitive model
statistics
computing

```
…/04.regression_height/_scripts/regression_height_main.R
```

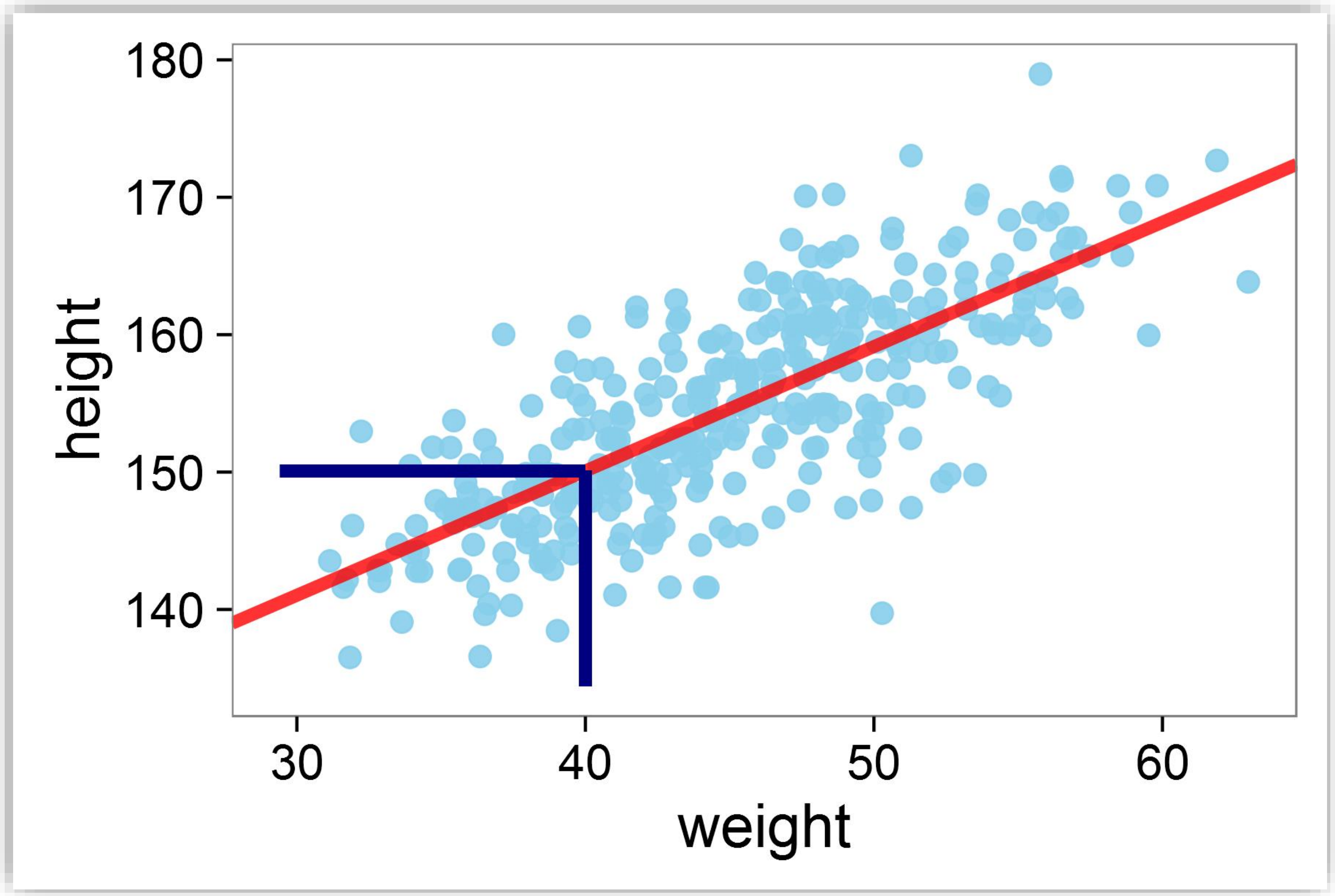
TASK: estimate the model and produce the results

```
Inference for Stan model: regression_height_model.
4 chains, each with iter=2000; warmup=1000; thin=1;
post-warmup draws per chain=1000, total post-warmup draws=4000.

          mean se_mean   sd    2.5%     25%     50%     75%   97.5% n_eff Rhat
alpha   113.97    0.06 1.86  110.27  112.76  113.93  115.20  117.66   934    1
beta      0.90    0.00 0.04    0.82    0.88    0.90    0.93    0.99   922    1
sigma     5.11    0.01 0.19    4.74    4.97    5.10    5.24    5.50  1437    1
lp__   -747.61    0.04 1.23 -750.80 -748.15 -747.28 -746.72 -746.24   993    1
```



A New Model
data{...}
parameters{...}
model{
prior, likelihood, etc.
}
evaluate
revise
diagnose
optimize
Data
clean
reshape
explore
approximation / MCMC
summary
extract
explore
Hidden Patterns

What does the Model Predict?



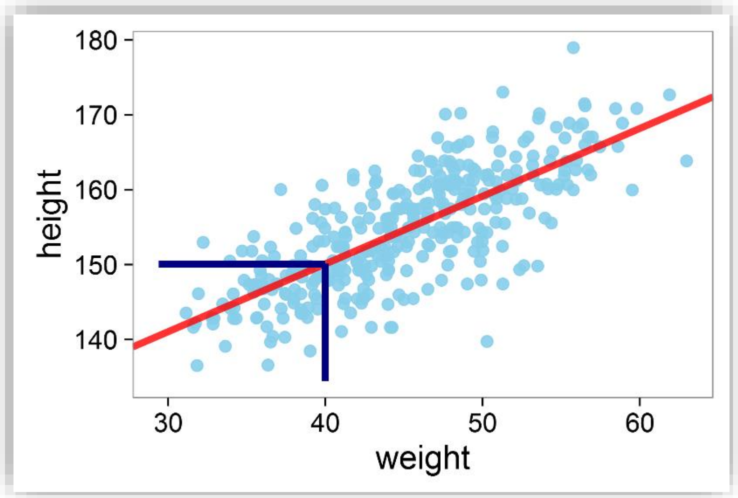
$$p\left(y_{rep} \mid y\right) = \int p\left(y_{rep} \mid \theta\right) p\left(\theta \mid y\right) d\left(\theta\right)$$

McElreath (2016)

Posterior Predictive Check (PPC)

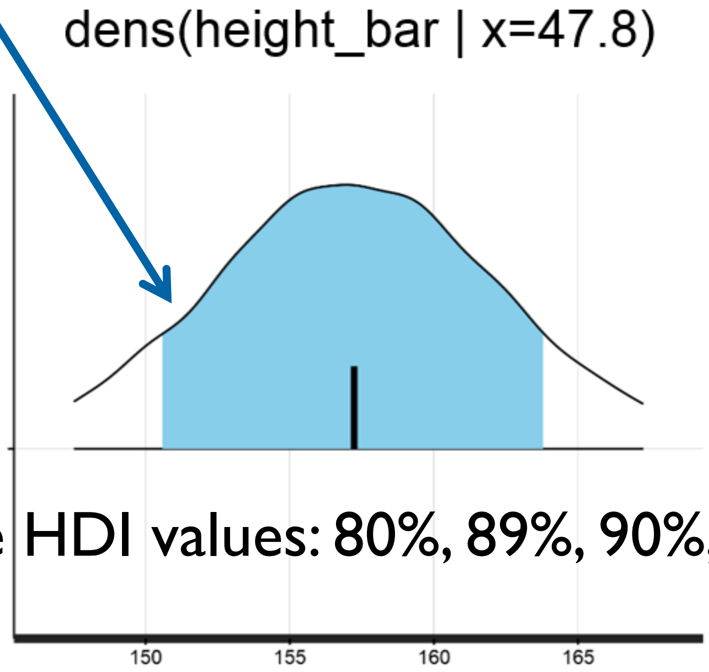

```
generated quantities {
  vector[N] height_bar;
  for (n in 1:N) {
      height_bar[n] = normal_rng(alpha + beta * weight[n], sigma);
  }
}
```

the generated quantities block runs only AFTER the sampling, and the time it costs can be essentially ignored!

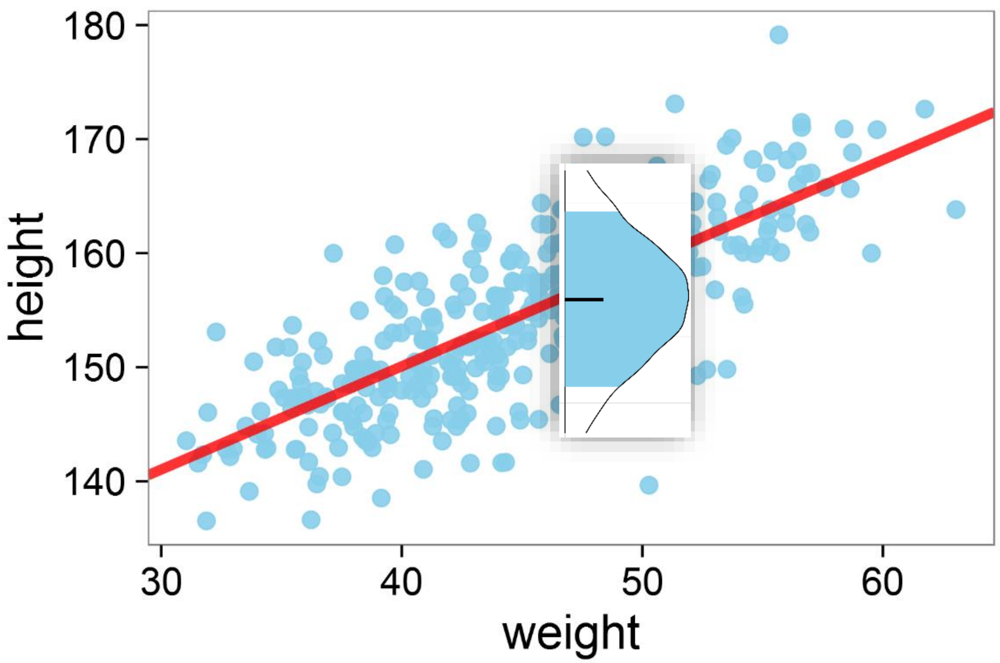


Posterior Predictive Check (PPC)

Highest density interval (HDI)

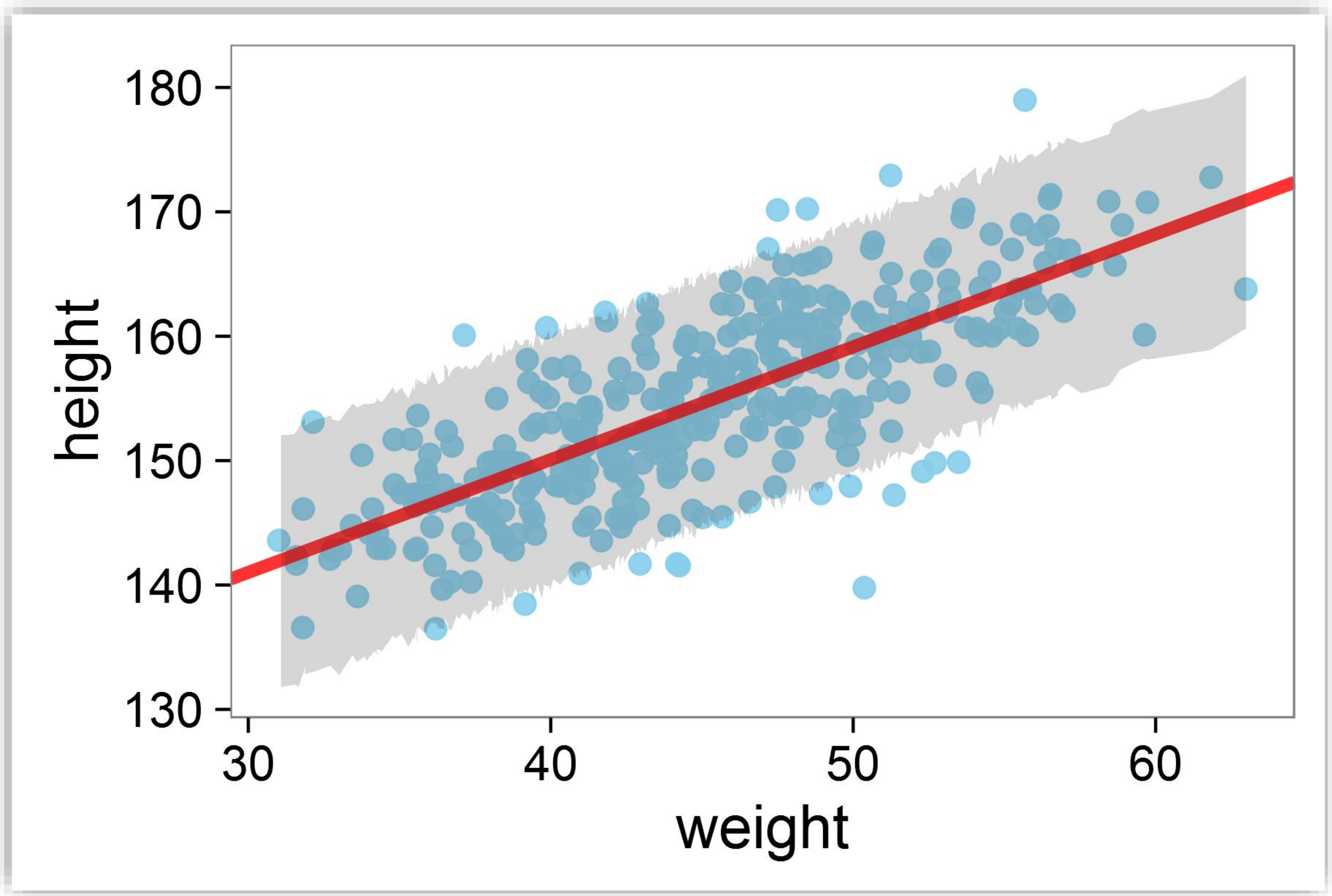


possible HDI values: 80%, 89%, 90%, 95%




```
height_bar <- extract(fit_reg_ppc, pars = 'height_bar',
                      permuted = FALSE)$height_bar
height_HDI <- apply(height_bar, 2, HDIofMCMC)
```

Posterior Predictive Check (PPC)

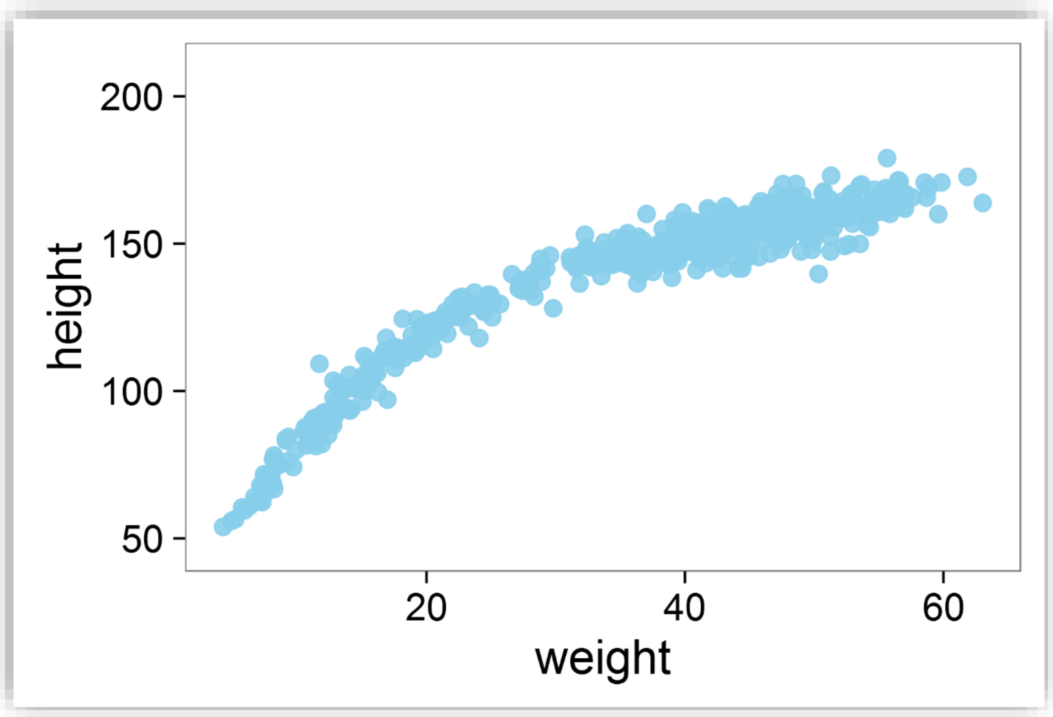


Exercise IX

```
.../05.regression_height_poly/_scripts
/regression_height_poly_main.R
```

TASK: (1) Complete “regression_height_poly2_model.stan”

(2) produce PPC plot for both 1st order and 2nd order polynomial fit



Exercise IX – Tips

cognitive model
statistics
computing

```
> source('_scripts/regression_height_poly_main.R')

> out1 <- reg_poly(poly_order = 1)
```

$$\overline{\text{height}} = \alpha + \beta 1 * \text{weight} + \beta 2 * \text{weight}^2$$

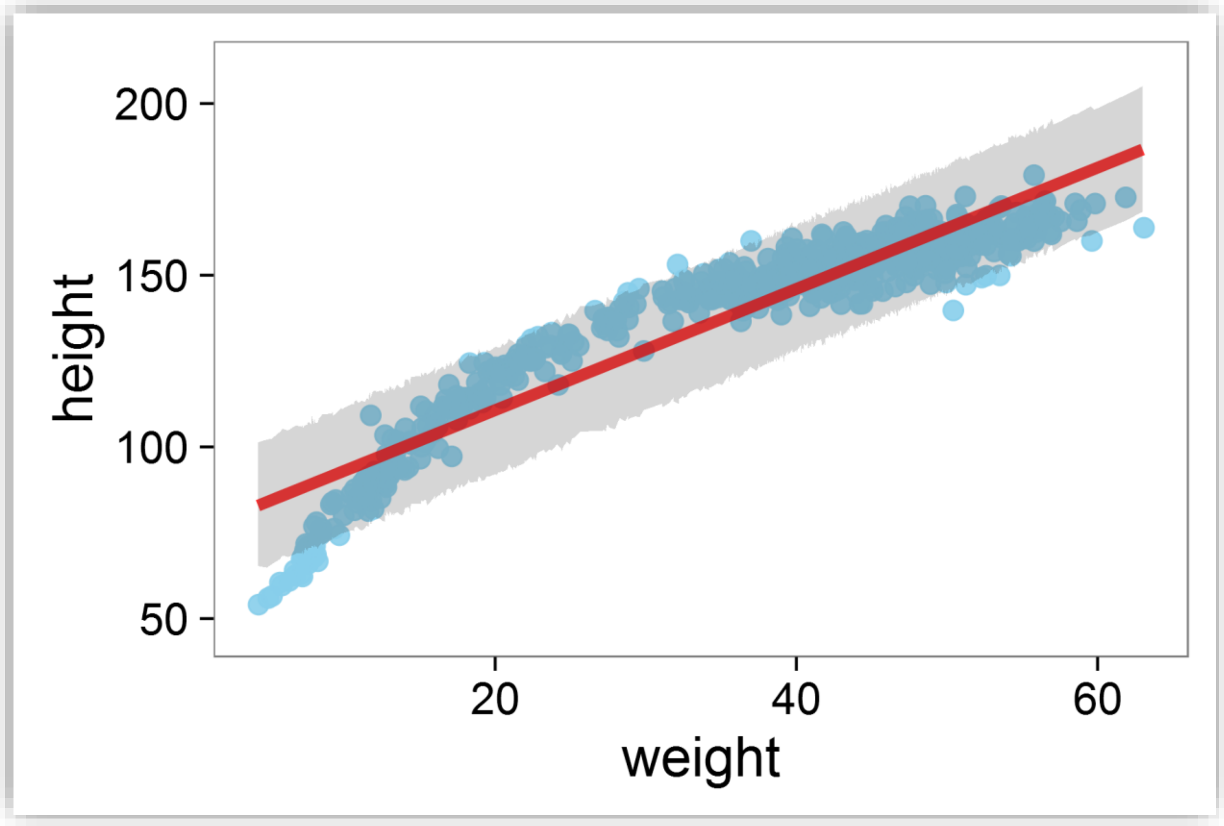
$$\text{height} \sim Normal(\overline{\text{height}}, \sigma)$$

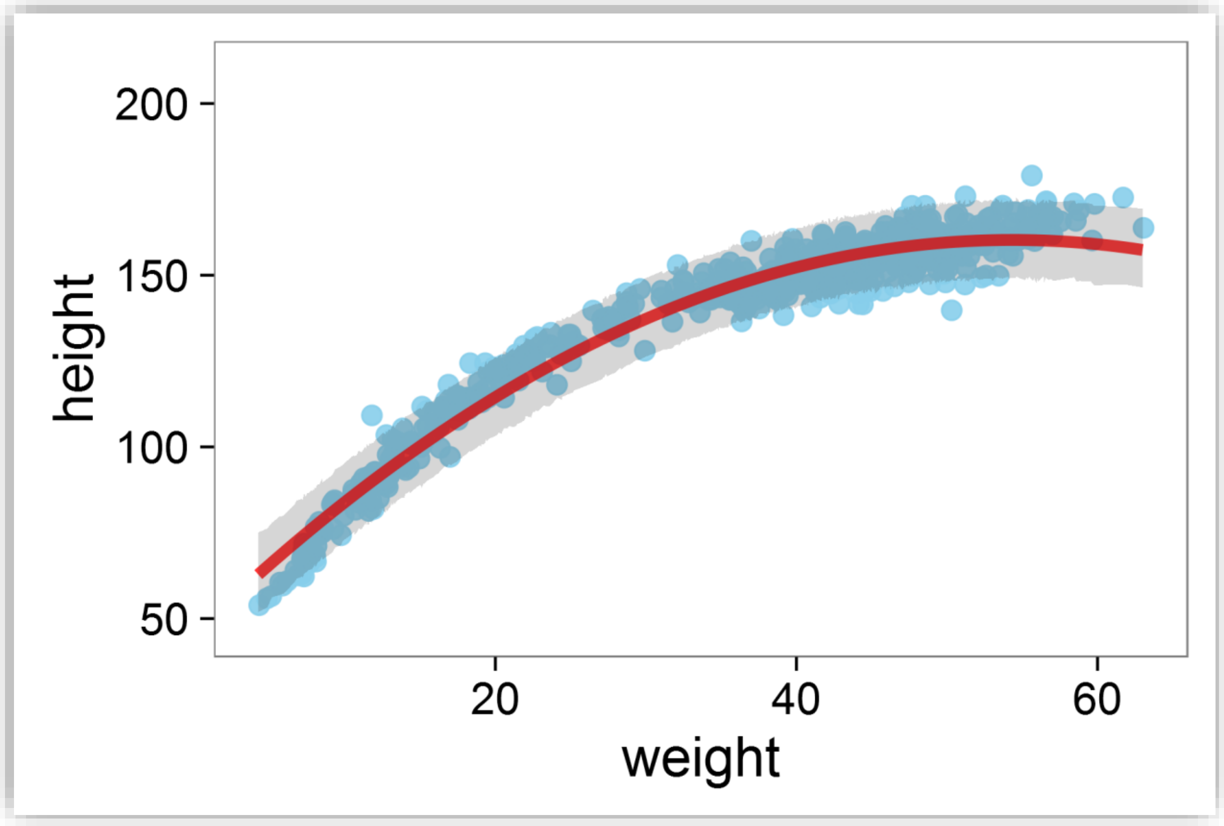
```
data {
  int<lower=0> N;
  vector<lower=0>[N] height;
  vector<lower=0>[N] weight;
  vector<lower=0>[N] weight_sq;
}
```

```
height ~ normal(alpha + beta1 * weight + beta2 * weight_sq, sigma);
```

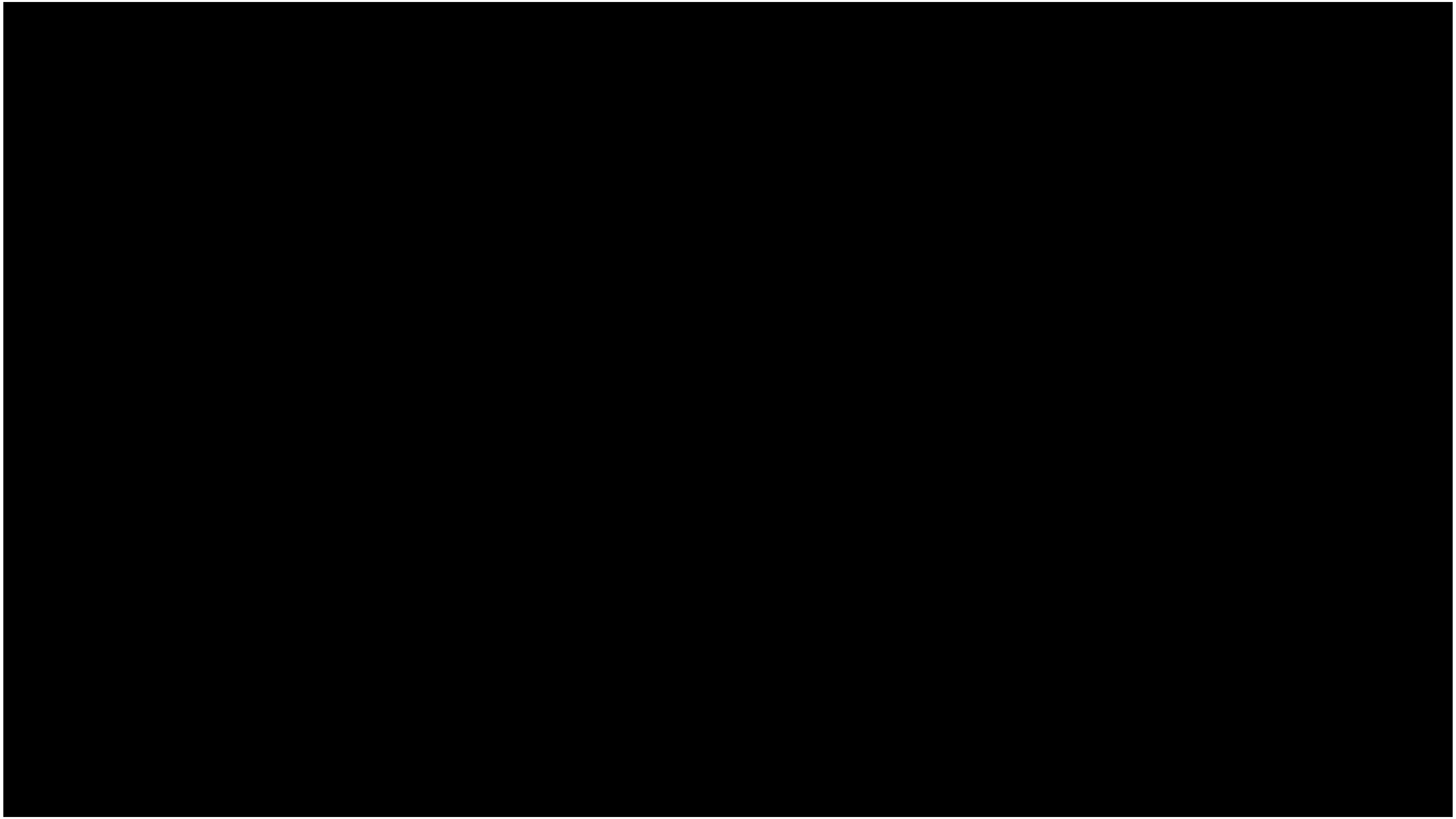
Exercise IX – output2

cognitive model
statistics
computing





Complex GLMs in Stan?



[Link]


ANY
QUESTIONS
?

Happy Computing!