Bayesian Statistics and Hierarchical Bayesian Modeling for Psychological Science

Lecture 07

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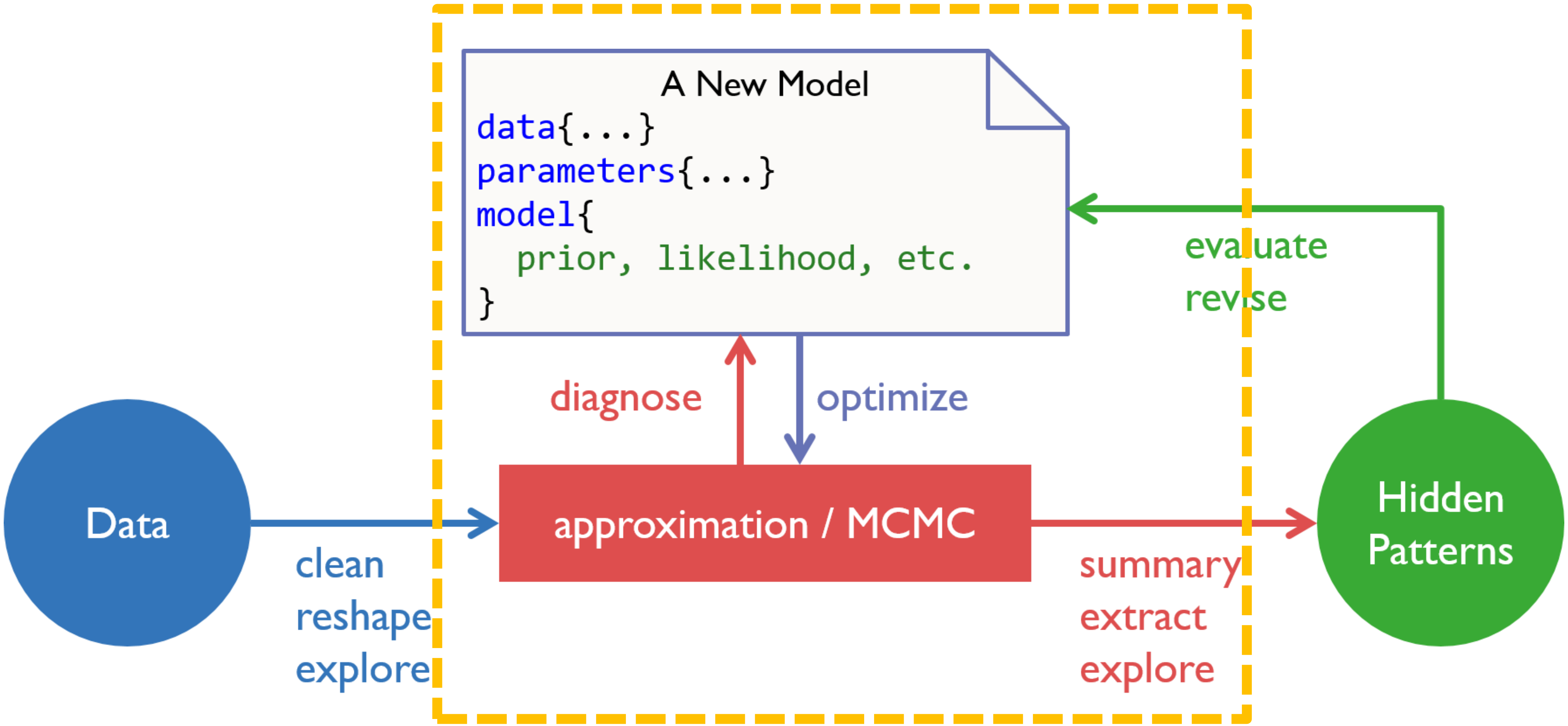
https://github.com/lei-zhang/BayesCog_Wien

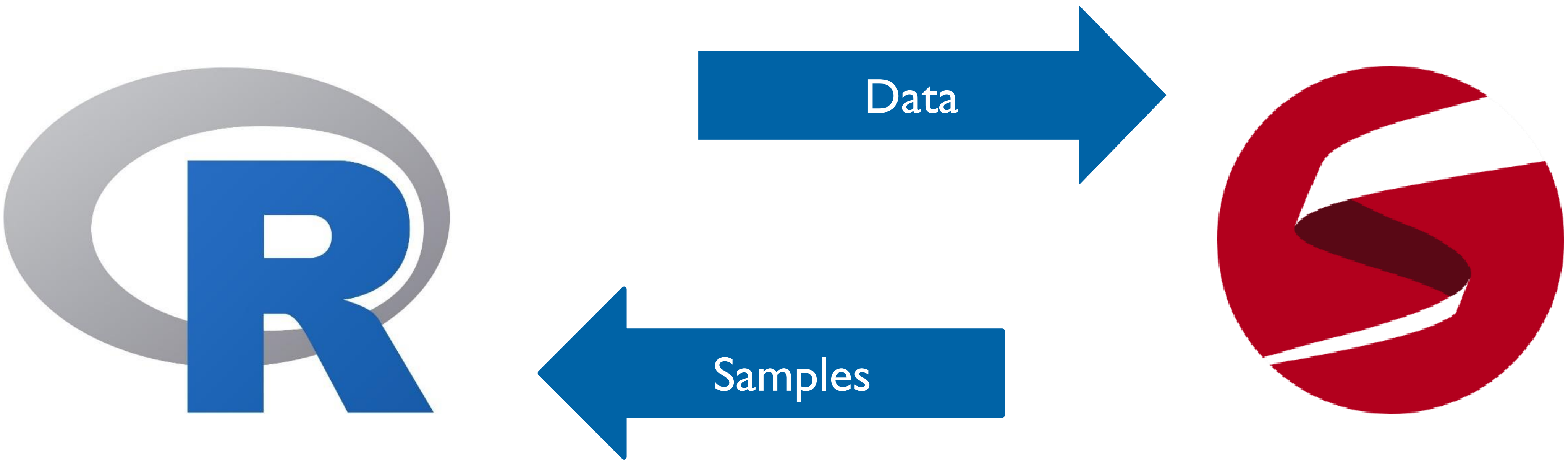
Bayesian warm-up?

STAN PROGRAMMING LANGUAGE I

cognitive model
statistics
computing



Stan and RStan



Steps of Bayesian Modeling, with Stan

cognitive model
statistics
computing

A data story	Think about how the data might arise. It can be *descriptive* or even *causal*. Write a Stan program (*.stan).
Update	Educate your model by feeding it the data. Bayesian Update: update the prior, in light of data, to produce posterior. Run Stan using RStan (PyStan, MatlabStan etc.)
Evaluate	Compare model with reality. Revise your model. Evaluate in RStan and ShinyStan.

McElreath (2016)

Steps of Using Stan

1. Stan program read into memory
2. Source-to-source transformation into C++
3. C++ compiled and linked (takes a while)
4. Run Stan program
5. Posterior analysis / interface

```
data {
  int<lower=0> N;
  int<lower=0,upper=1> y[N];
}
parameters {
  real<lower=0,upper=1> theta;
}
model {
  y ~ bernoulli(theta);
}
```

Stan Language

model blocks

```
data {
 //... read in external data...
}

transformed data {
 //... pre-processing of data ...
}

parameters {
 //... parameters to be sampled by HMC ...
}

transformed parameters {
 //... pre-processing of parameters ...
}

model {
 //... statistical/cognitive model ...
}

generated quantities {
 //... post-processing of the model ...
}
```

REVISIT BINOMIAL MODEL

Binomial Model

cognitive model
statistics
computing

W L W W W L W L W

$$p(w \mid N, \theta) = \binom{N}{w} \theta^{w} (1-\theta)^{N-w}$$

$$w \sim \text{Binomial}(N, \theta)$$

reads as:
w is distributed as a binomial distribution, with number of trials *N*, and success rate ϑ.

Graphical Model Notations

	continuous	discrete
unobserved	θ	δ
observed	y	N

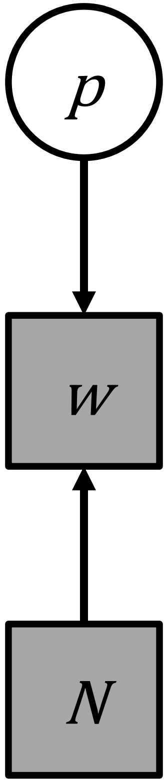
Lee & Wagenmakers (2013)

Binomial Model

cognitive model
statistics
computing

W L W W W L W L W

$$p(w \mid N, \theta) = \binom{N}{w} \theta^{w} (1-\theta)^{N-w}$$



$$\theta \sim \text{Uniform}(0, 1)$$

$$w \sim \text{Binomial}(N, \theta)$$

	continuous	discrete
unobserved	θ	δ
observed	y	N

McElreath (2016)

Binomial Model

W L W W W L W L W

$$p(w \mid N, \theta) = \binom{N}{w} \theta^{w} (1-\theta)^{N-w}$$

```
data {
    int<lower=0> w;
    int<lower=0> N;
}

parameters {
    real<lower=0,upper=1> theta;
}

model {
    w ~ binomial(N, theta);
}
```

McElreath (2016)

Running Binomial Model with Stan


```
…/BayesCog/02.binomial_globe/_scripts/binomial_globe_main.R
```

```
> R.version
R version 3.5.1 (2018-07-02)

> stan_version()
[1] "2.18.0"
```

Model Summary


```
> print(fit_globe)
Inference for Stan model: binomial_globe_model.
4 chains, each with iter=2000; warmup=1000; thin=1;
post-warmup draws per chain=1000, total post-warmup draws=4000.

       mean se_mean   sd  2.5%   25%   50%   75% 97.5% n_eff Rhat
theta  0.64    0.00 0.14  0.35  0.54  0.65  0.74  0.87  1278    1
lp__  -7.72    0.02 0.69 -9.77 -7.89 -7.46 -7.27 -7.21  1824    1

Samples were drawn using NUTS(diag_e) at Tue Apr 09 12:44:04 2019.
For each parameter, n_eff is a crude measure of effective sample size,
and Rhat is the potential scale reduction factor on split chains (at
convergence, Rhat=1).
```

Gelman-Rubin convergence diagnostic
(Gelman & Rubin, 1992)


ANY
QUESTIONS
?

Happy Computing!