What to care when using Bayes Factors?

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Outline

- (needless) recap of Bayes Factor (BF)
- BF is a continuous entity!
- Is BF the same as posterior odds?
- What are actually the priors we are talking about?
- How to properly select features/predictors?
- What about finding the full posterior?
- What to do when we have more than 2 hypotheses to test?

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The Bayes rule

Likelihood
How plausible is the data given our parameter is true?

Prior
How plausible is our parameter before observing the data?

$$p(\theta|D) = \frac{p(D|\theta)p(\theta)}{p(D)}$$

Posterior
How plausible is our parameter given the observed data?

Evidence
How plausible is the data under all possible parameters?

Bayes Factor

$$p(H_0 \mid D) \propto p(D \mid H_0)\, p(H_0)$$

$$p(H_1 \mid D) \propto p(D \mid H_1)\, p(H_1)$$

$$\frac{p(H_0 \mid D)}{p(H_1 \mid D)} = \boxed{\frac{p(D \mid H_0)}{p(D \mid H_1)}} \cdot \frac{p(H_0)}{p(H_1)}$$

posterior odds = Bayes factor × prior odds

BF describes the compares the predictive performance of two hypotheses (H_0 vs. H_1) .

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BF is continuous

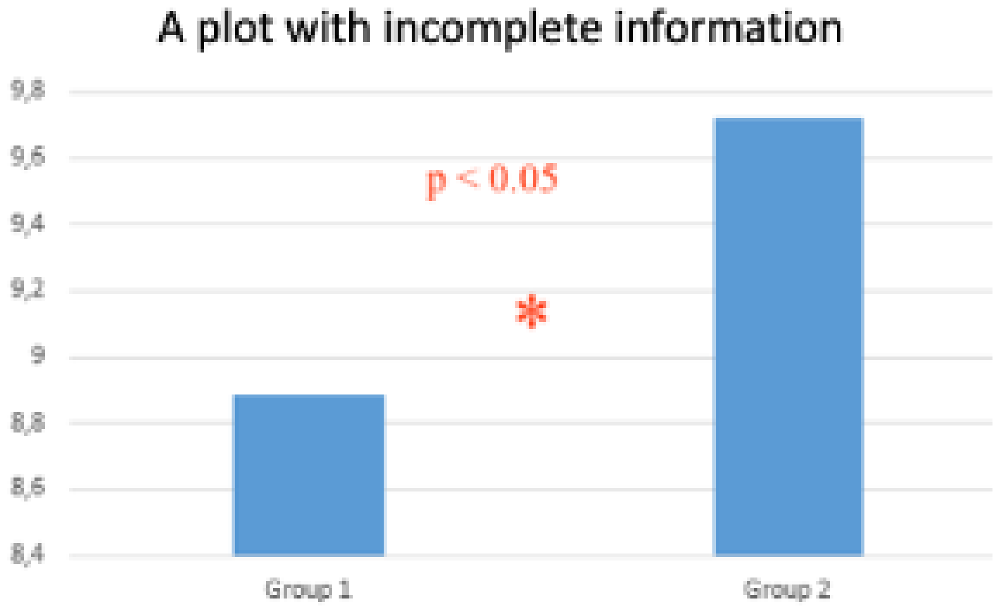
$$BF = \frac{p(D \mid H_0)}{p(D \mid H_1)}$$

Bayes factor	Interpretation
$B_f < 1/10$	Strong evidence for M_r
$1/10 \leq B_f < 1/3$	Moderate evidence for M_r
$1/3 \leq B_f < 1$	Weak evidence for M_r
$1 \leq B_f < 3$	Weak evidence for M_i
$3 \leq B_f < 10$	Moderate evidence for M_i
$B_f \geq 10$	Strong evidence for M_i

Source: Min et al. (2007).

These cut-offs are arbitrary!
The exact BFs need also to be reported.

Report the exact BF



Schmalz, Biurrun Manresa, & Zhang, *Psychological Methods* (in press)

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Is BF the posterior odds?

$$\frac{p(H_0 \mid D)}{p(H_1 \mid D)} = \frac{p(D \mid H_0)}{p(D \mid H_1)} \cdot \cancel{\frac{p(H_0)}{p(H_1)}}$$

$$p(H_0) = p(H_1) = 0.5$$

$$\frac{p(H_0 \mid D)}{p(H_1 \mid D)} = \frac{p(D \mid H_0)}{p(D \mid H_1)}$$

posterior odds = Bayes factor

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What are actually the priors we are talking about?

Didn't we say prior matters in BF?

But weren't they cancelled out?

Where are the priors we should care?

Two types of priors:

- priors of the hypothesis (model)
- priors of the parameter

$$p(H_0) = p(H_1) = 0.5$$

$$\frac{p(H_0 \mid D)}{p(H_1 \mid D)} = \frac{p(D \mid H_0)}{p(D \mid H_1)} \cdot \boxed{\frac{p(H_0)}{p(H_1)}}$$

model prior

$$p(D \mid H_1) = \int p(D \mid \mu, H_1) \boxed{p(\mu \mid H_1)} \, \mathrm{d}\mu$$

parameter prior

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select features/predictors vs. testing hypothesis

$$H_1: y \sim \beta_0 + \beta_1 * x_1$$

$$H_2: y \sim \beta_0 + \beta_1 * x_1 + \beta_2 * x_2$$

Scenario #1:

fitting H_1, β_1 is significant (95% CI > 0)
fitting H_2, β_1, β_2 are significant (95% CI > 0)

Conclusion:
both x_1 and x_2 are important predictors

Scenario #2:

compute BF with H_1 and H_2
BF favors H_1 (e.g., BF_{12} = 8.3)

Conclusion:
only x_1 is an important predictor

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Work with full posterior?

- BF is a relative measure. But, sometimes we want the exact posterior.

Example:
Given the data (D_1, D_2) from two groups, we are interested if their means differ (μ_1 vs. μ_2)?

<u>BF approach:</u>

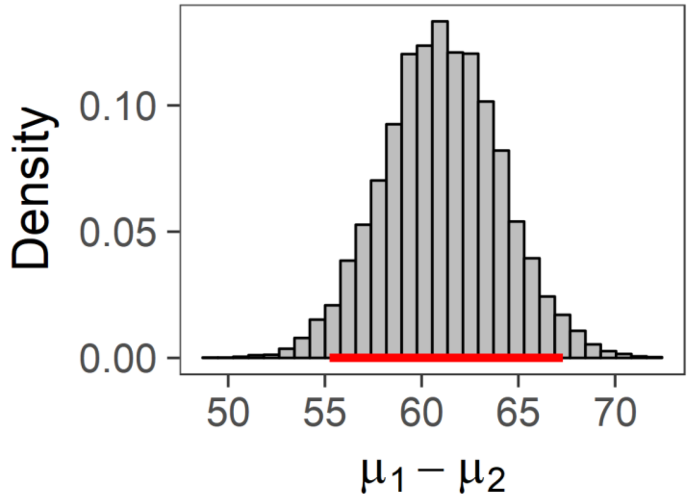
H_0: $\mu_1 = \mu_2$
H_1: $\mu_1 \neq \mu_2$

calculate BF_{10}

<u>full posterior approach:</u>

$p(\mu_1 - \mu_2 | D_1, D_2)$

calculate using tool: BEST



- mean: 61.06
- 95% HDI: [55.26 67.27]

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What to do when we have > 2 hypotheses?

$$H_1: y \sim \beta_0 + \beta_1 * x_1$$

$$H_2: y \sim \beta_0 + \beta_1 * x_1 + \beta_2 * x_2$$

$$H_3: y \sim \beta_0 + \beta_1 * x_1 + \beta_2 * x_2 + \beta_3 * x_3$$

<u>BF approach:</u>

calculate BF_{12}
calculate BF_{13}
calculate BF_{23}

<u>Model fitting and comparison approach:</u>

fit all three models using e.g., [BRMS]
then compare models using the proper information criterion, e.g., WAIC

Recommended reading

What is a Bayes Factor?

AUTHORS
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Schmalz, Biurrun Manresa, & Zhang, *Psychological Methods* (in press)
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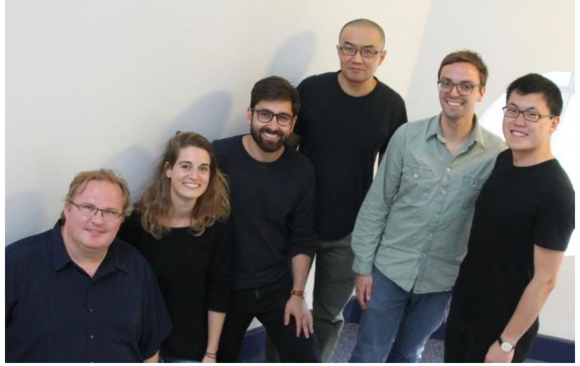

Thank you!

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QUESTIONS
?

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