Statistical Thinking with R

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Overview

What is your experience with…

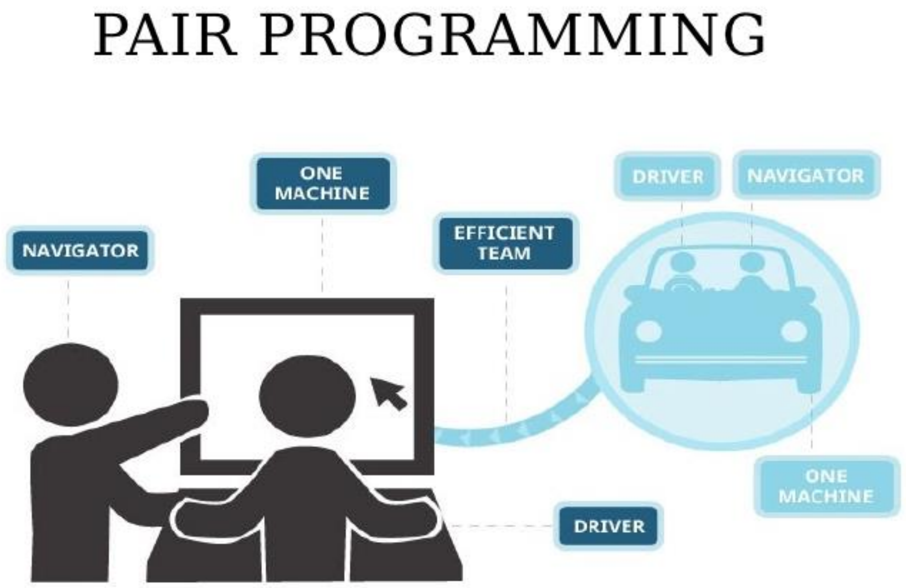
- Statistics?
- R? (and / or Matlab?)

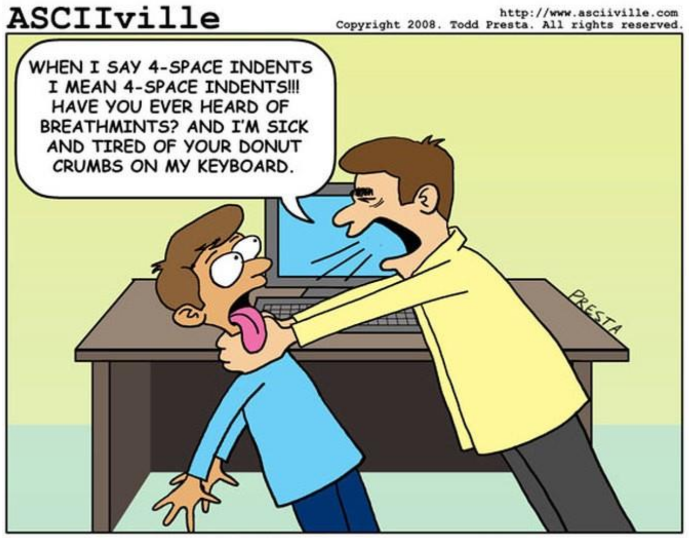
You would like to…

- know how to start and get help with R?
- gain knowledge of (Bayesian) stats?
- start here, know where you can go from it.

How to Get the Most out of the Workshop

- Work in pairs: Talk to each other & help each other
- Ask questions
- Try the exercises




ASCIIville
http://www.asciiville.com
Copyright 2008. Todd Presta. All rights reserved.
WHEN I SAY 4-SPACE INDENTS I MEAN 4-SPACE INDENTS!!! HAVE YOU EVER HEARD OF BREATHMINTS? AND I'M SICK AND TIRED OF YOUR DONUT CRUMBS ON MY KEYBOARD.
PRESTA

The dark side of pair programming.

R Basics

- R
  - a programming language for statistical computing
  - R has its own user interface
  - freely available on Windows, Mac, and Linux

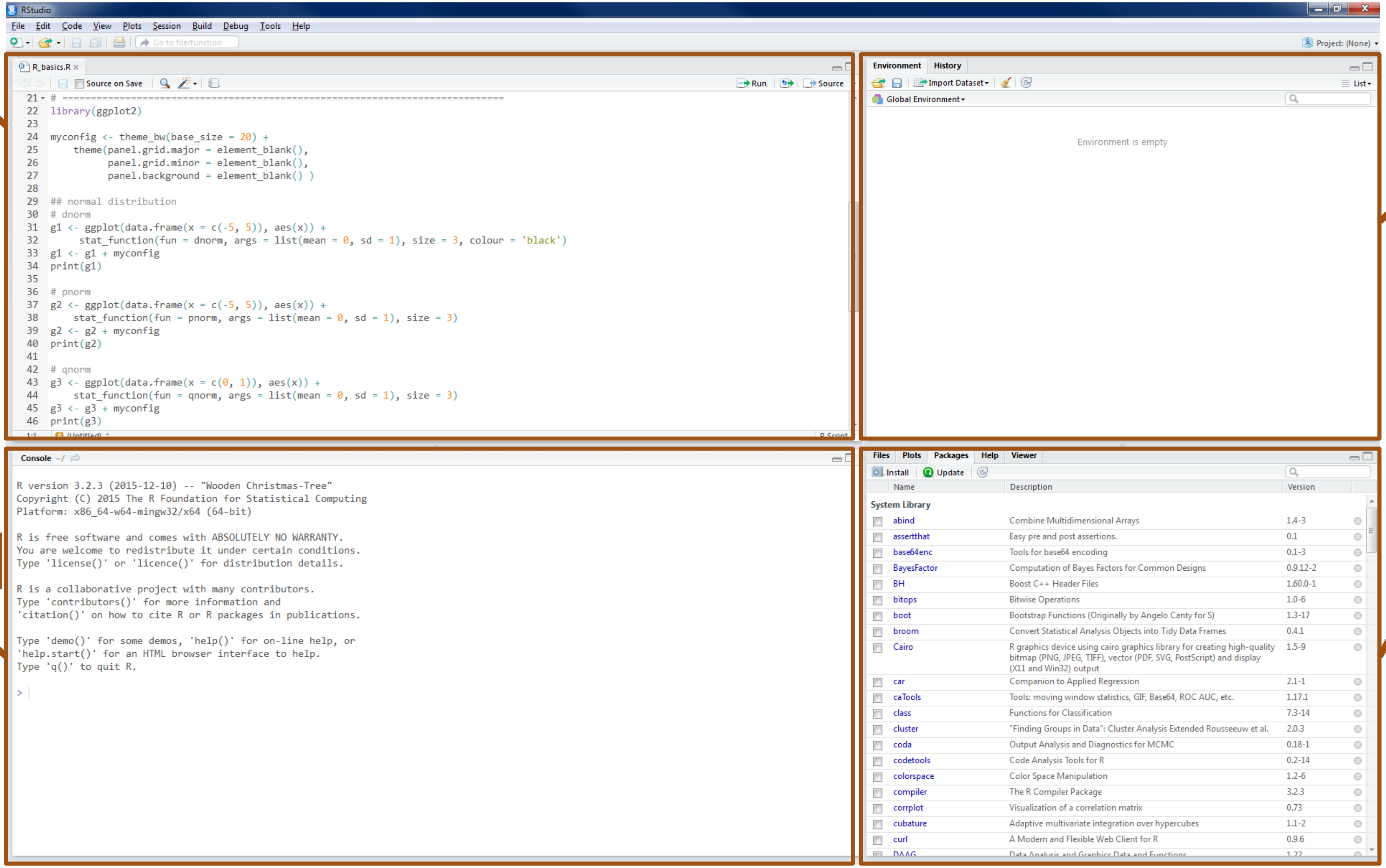
- R Studio
  - integrated development environment (IDE) for R
  - a more sophisticated R-friendly editor, with helpful syntax highlight

script editor

environment/ command history

console

file/pkg/img/ etc.



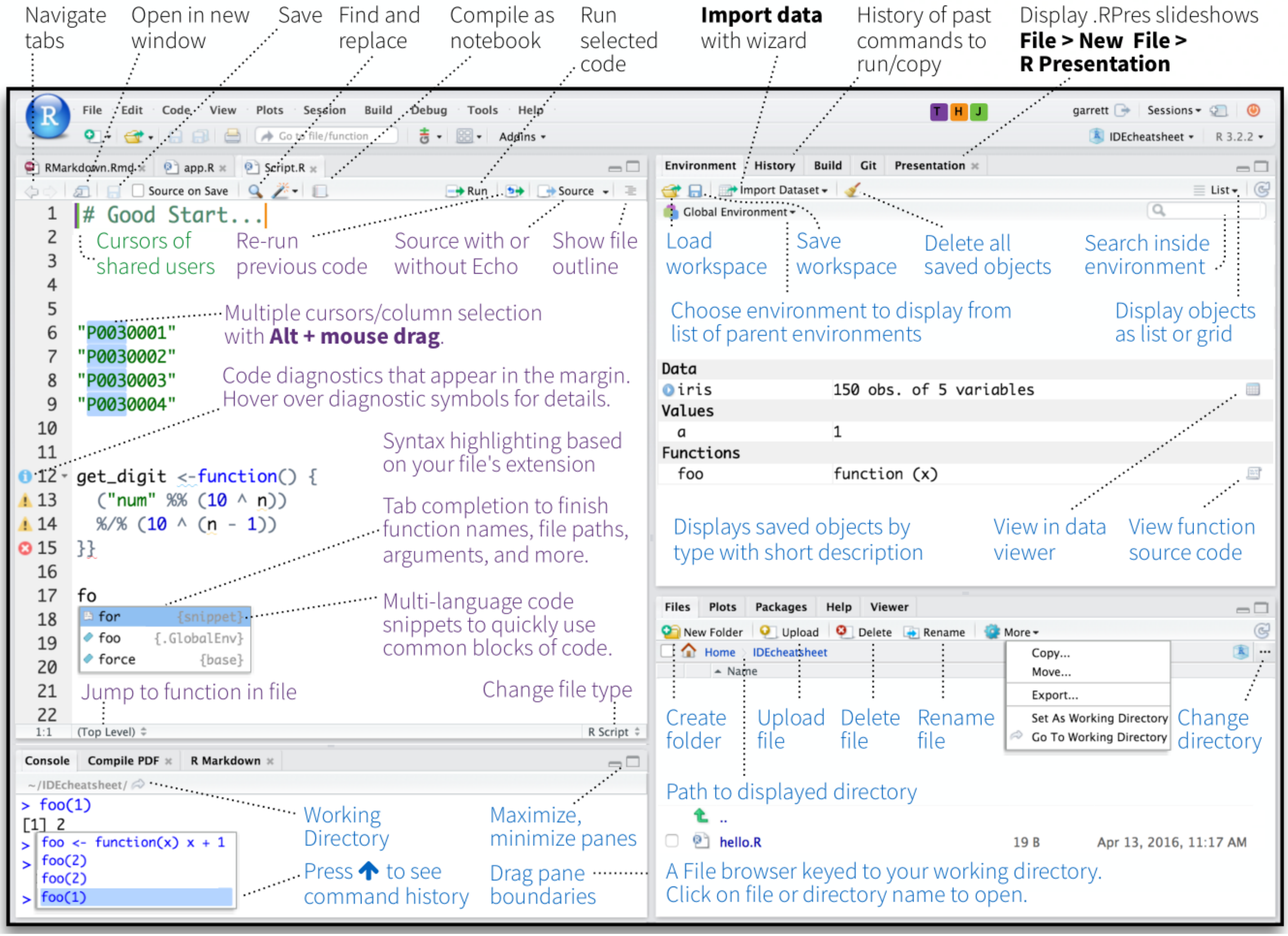
```r
# ==========================================================================
library(ggplot2)

myconfig <- theme_bw(base_size = 20) +
    theme(panel.grid.major = element_blank(),
          panel.grid.minor = element_blank(),
          panel.background = element_blank() )

## normal distribution
# dnorm
g1 <- ggplot(data.frame(x = c(-5, 5)), aes(x)) +
     stat_function(fun = dnorm, args = list(mean = 0, sd = 1), size = 3, colour = 'black')
g1 <- g1 + myconfig
print(g1)

# pnorm
g2 <- ggplot(data.frame(x = c(-5, 5)), aes(x)) +
    stat_function(fun = pnorm, args = list(mean = 0, sd = 1), size = 3)
g2 <- g2 + myconfig
print(g2)

# qnorm
g3 <- ggplot(data.frame(x = c(0, 1)), aes(x)) +
    stat_function(fun = qnorm, args = list(mean = 0, sd = 1), size = 3)
g3 <- g3 + myconfig
print(g3)
```

```
R version 3.2.3 (2015-12-10) -- "Wooden Christmas-Tree"
Copyright (C) 2015 The R Foundation for Statistical Computing
Platform: x86_64-w64-mingw32/x64 (64-bit)

R is free software and comes with ABSOLUTELY NO WARRANTY.
You are welcome to redistribute it under certain conditions.
Type 'license()' or 'licence()' for distribution details.

R is a collaborative project with many contributors.
Type 'contributors()' for more information and
'citation()' on how to cite R or R packages in publications.

Type 'demo()' for some demos, 'help()' for on-line help, or
'help.start()' for an HTML browser interface to help.
Type 'q()' to quit R.

>
```

	Name	Description	Version
	System Library		
□	abind	Combine Multidimensional Arrays	1.4-3
□	assertthat	Easy pre and post assertions.	0.1
□	base64enc	Tools for base64 encoding	0.1-3
□	BayesFactor	Computation of Bayes Factors for Common Designs	0.9.12-2
□	BH	Boost C++ Header Files	1.60.0-1
□	bitops	Bitwise Operations	1.0-6
□	boot	Bootstrap Functions (Originally by Angelo Canty for S)	1.3-17
□	broom	Convert Statistical Analysis Objects into Tidy Data Frames	0.4.1
□	Cairo	R graphics device using cairo graphics library for creating high-quality bitmap (PNG, JPEG, TIFF), vector (PDF, SVG, PostScript) and display (X11 and Win32) output	1.5-9
□	car	Companion to Applied Regression	2.1-1
□	caTools	Tools: moving window statistics, GIF, Base64, ROC AUC, etc.	1.17.1
□	class	Functions for Classification	7.3-14
□	cluster	"Finding Groups in Data": Cluster Analysis Extended Rousseeuw et al.	2.0.3
□	coda	Output Analysis and Diagnostics for MCMC	0.18-1
□	codetools	Code Analysis Tools for R	0.2-14
□	colorspace	Color Space Manipulation	1.2-6
□	compiler	The R Compiler Package	3.2.3
□	corrplot	Visualization of a correlation matrix	0.73
□	cubature	Adaptive multivariate integration over hypercubes	1.1-2
□	curl	A Modern and Flexible Web Client for R	0.9.6
□	DAAG	Data Analysis and Graphics Data and Functions	1.22

Write Code

R Support



Navigate tabs

Open in new window

Save

Find and replace

Compile as notebook

Run selected code

Cursors of shared users

Re-run previous code

Source with or without Echo

Show file outline

Multiple cursors/column selection with **Alt + mouse drag**.

Code diagnostics that appear in the margin. Hover over diagnostic symbols for details.

Syntax highlighting based on your file's extension

Tab completion to finish function names, file paths, arguments, and more.

Multi-language code snippets to quickly use common blocks of code.

Jump to function in file

Change file type

Working Directory

Maximize, minimize panes

Press ↑ to see command history

Drag pane boundaries

Import data with wizard

History of past commands to run/copy

Display .RPres slideshows **File > New File > R Presentation**

Load workspace

Save workspace

Delete all saved objects

Search inside environment

Choose environment to display from list of parent environments

Display objects as list or grid

Displays saved objects by type with short description

View in data viewer

View function source code

Create folder

Upload file

Delete file

Rename file

Change directory

Path to displayed directory

A File browser keyed to your working directory. Click on file or directory name to open.

```
# Good Start...

"P0030001"
"P0030002"
"P0030003"
"P0030004"

get_digit <-function() {
  ("num" %% (10 ^ n))
  %/% (10 ^ (n - 1))
}}

fo
```

```
~/IDEcheatsheet/
> foo(1)
[1] 2
> foo <- function(x) x + 1
> foo(2)
> foo(2)
> foo(1)
```

Data	
iris	150 obs. of 5 variables
Values	
a	1
Functions	
foo	function (x)

Name	Size	Modified
hello.R	19 B	Apr 13, 2016, 11:17 AM

https://github.com/rstudio/cheatsheets/raw/master/rstudio-ide.pdf

Know your R

```
>R.version

               _
platform       x86_64-w64-mingw32
arch           x86_64
os             mingw32
system         x86_64, mingw32
status
major          3
minor          5.1
year           2018
month          07
day            02
svn rev        74947
language       R
version.string R version 3.5.1 (2018-07-02)
nickname       Feather Spray
```

R Console as a Calculator

Addition and Subtraction

```
> 3+2
[1] 5

> 3-2
[1] 1
```

Multiplication and Division

```
> 3*2
[1] 6

> 3/2
[1] 1.5
```

Exponents in R

```
> 3^2
[1] 9

> 2^3
[1] 8
```

Constants in R

```
> pi
[1] 3.141593

> exp(1)
[1] 2.718282
```

base of the natural logarithm

Special values

Infinite Values

```
> Inf
[1] Inf

> 1+Inf
[1] Inf
```

Machine Epsilon

```
> .Machine$double.eps
[1] 2.220446e-16

> 0>.Machine$double.eps
[1] FALSE
```

Empty Values

```
> NULL
NULL

> 1+NULL
numeric(0)
```

Missing Values

```
> NA
[1] NA

> 1+NA
[1] NA
```

Storing and manipulating variables

Define objects `x` and `y` with values of 3 and 2, respectively:

```
> x=3
> y=2
```

Some calculations with the defined objects `x` and `y`:

```
> x+y
[1] 5
```

```
> x*y
[1] 6
```

Warning: R is case sensitve, so `x` and `X` are not the same object.

Basic R functions

Combine

```
> c(1,3,-2)
[1]  1  3 -2

> c("a","a","b","b","a")
[1] "a" "a" "b" "b" "a"
```

Sum and Mean

```
> sum(c(1,3,-2))
[1] 2

> mean(c(1,3,-2))
[1] 0.6666667
```

Variance and Std. Dev.

```
> var(c(1,3,-2))
[1] 6.333333

> sd(c(1,3,-2))
[1] 2.516611
```

Minimum and Maximum

```
> min(c(1,3,-2))
[1] -2

> max(c(1,3,-2))
[1] 3
```

Basic R functions (cont.)

Define objects `x` and `y`:

```
> x=c(1,3,4,6,8)
> y=c(2,3,5,7,9)
```

Calculate the correlation:

```
> cor(x,y)
[1] 0.988765
```

Calculate the covariance:

```
> cov(x,y)
[1] 7.65
```

Combine as columns

```
> cbind(x,y)
     x y
[1,] 1 2
[2,] 3 3
[3,] 4 5
[4,] 6 7
[5,] 8 9
```

Combine as rows

```
> rbind(x,y)
  [,1] [,2] [,3] [,4] [,5]
x    1    3    4    6    8
y    2    3    5    7    9
```

Basic Commands

```
getwd()
setwd('E:/teaching/BayesCog/')
dir() # folders/files in the wd
ls()  # anything in the environment/workspace
print('Hello World!')
cat('Hello', 'World!')
paste0('C:/', 'Group1')
help(func)
? func # and Google!
a <- 5
a = 5
head(d) # first 6 entries
tail(d) # last 6 entries
save(varname, file = "pathname/varname.RData")
load("pathname/varname.RData")
rm(list = ls())
q()
```

RStudio - Shortcuts

Ctrl + L: clean console

Ctrl + Shift + N: create a new script

↑: command history

Ctrl(hold) + ↑: command history with certain starts

Ctrl + Enter: execute selected codes (in a script)

Editor (WIN general) - Shortcuts

Ctrl + home/Pos: go to the very top of a script

Ctrl + end/Ende: go to the very end of a script

Shift(hold) + ↑/↓: select line(s)

Ctrl(hold) + ←/→: select word(s)

Data Classes

numeric: 1.1 2.0

integer: 1 2 3

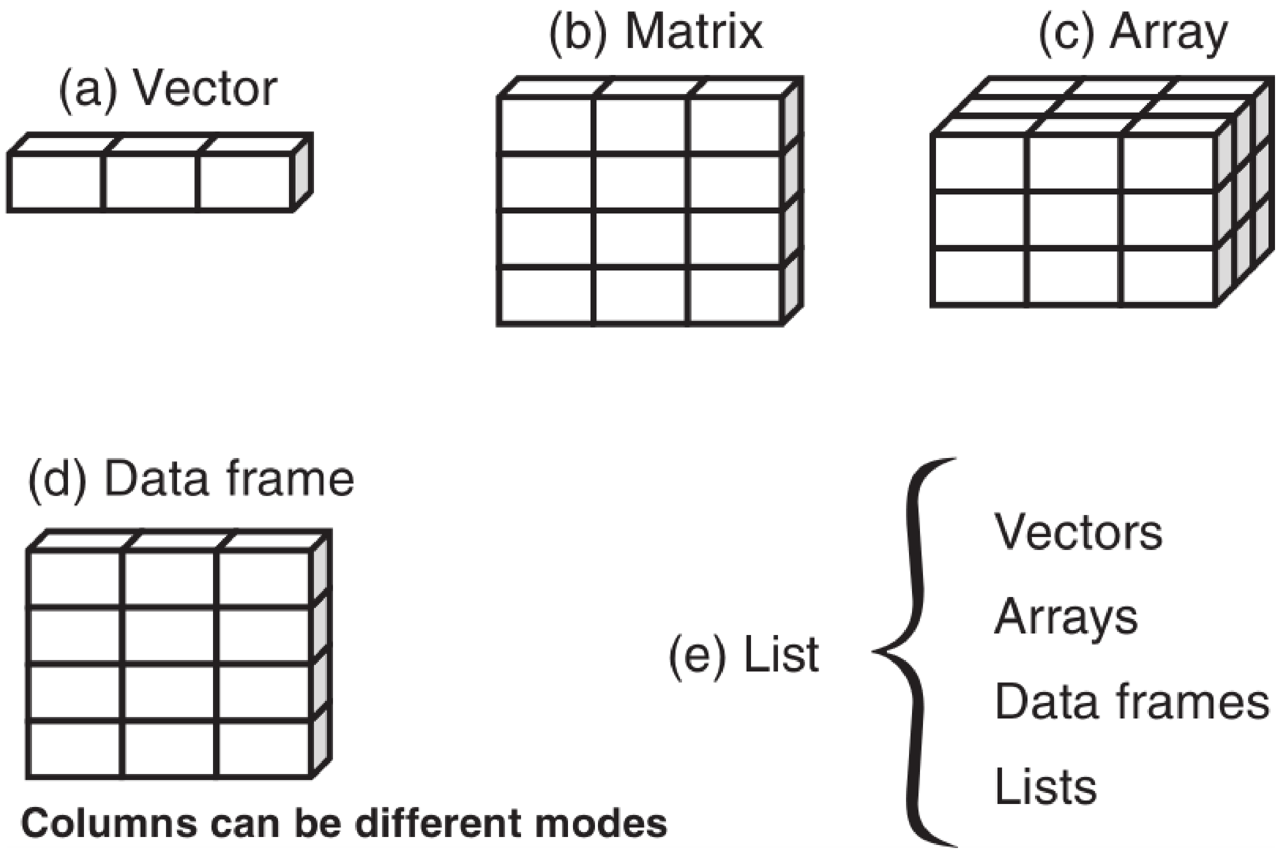
character / string: "hello world!"

logical: TRUE FALSE

factors: "male" / "female"

(complex: 1+2i)

Data Types



Kabacoff (2015)

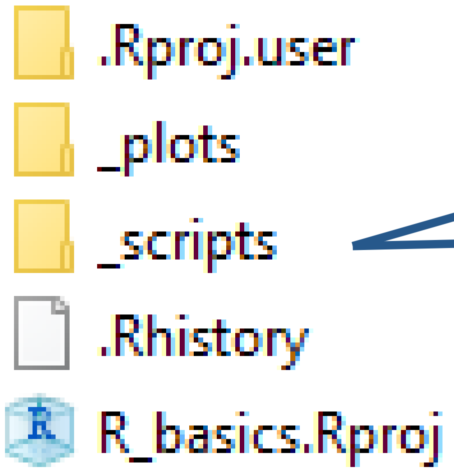
Exercise I

```
…/01.R_basics/_scripts/R_basics.R

up to “Control Flow”
```

TASK: practise basic R commands and data type

TIP: `class()`, `str()`

Side note: folder structure



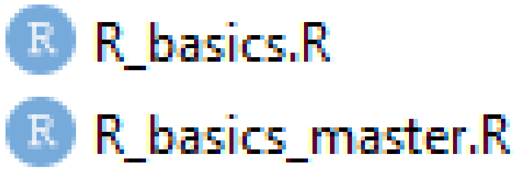
.Rproj.user

_plots

_scripts

.Rhistory

R_basics.Rproj



R_basics.R

R_basics_master.R

click this to start each exercise,
then no need to set directory

Logical Operators

Operator	Summary
$<$	Less than
$>$	Greater than
$<=$	Less than or equal to
$>=$	Greater than or equal to
$==$	Equal to
$! =$	Not equal to
$!x$	NOT x
$x \mid y$	x OR y
$x\&y$	x AND y

Control Flow

- if-else

```
if (cond) {
   ..statement..
} else {
   ..statement..
}
```

```
if (cond) {
   ..statement..
} else if (cond) {
   ..statement..
} else {
   ..statement..
}
```

- for-loop

```
for ( j in 1:n) {
   ..statement..
}
```

```
for ( j in 1:J ) {
   for ( k in 1:K ) {
      ..statement..
   }
}
```

User-defined Function

```
funname <- function (input_arges) {
    .. function body ..
    .. function body ..
    return(output_arges)
}
```

$$sem = \sqrt{\frac{s^2}{n-1}}$$

```
sem <- function(x) {
    sqrt( var(x,na.rm=TRUE) / (length(na.omit(x))-1) )
}
```

Exercise II

```
…/01.R_basics/_scripts/R_basics.R
```

TASK: practise control flow and user-defined function

Packages in R

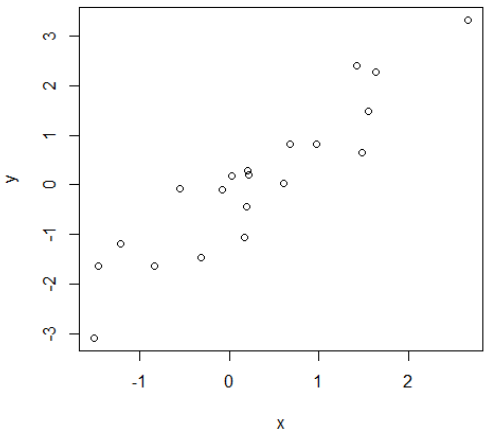
R packages are collections of functions and data sets developed by the community, to make your life a lot easier!

```
install.packages('ggplot2')
library(ggplot2)
detach('package:pkg')
```

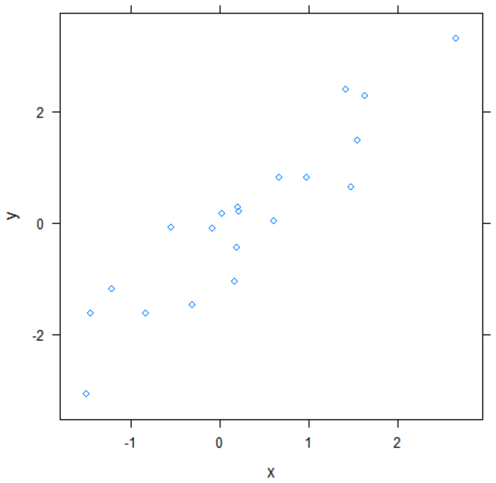
Visualization

- built-in plotting functions – first attempt / quick look / exploratory
- {lattice} – making nicer, similar to basic plotting functions
- {ggplot2} – making nicer, a layering philosophy

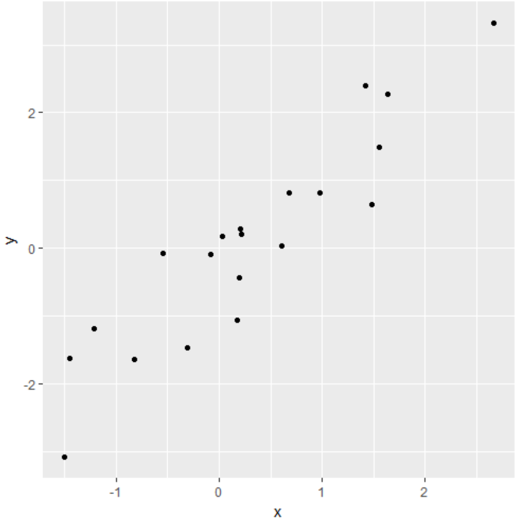
```
plot(x,y)
```



```
lattice::xyplot(y~x)
```



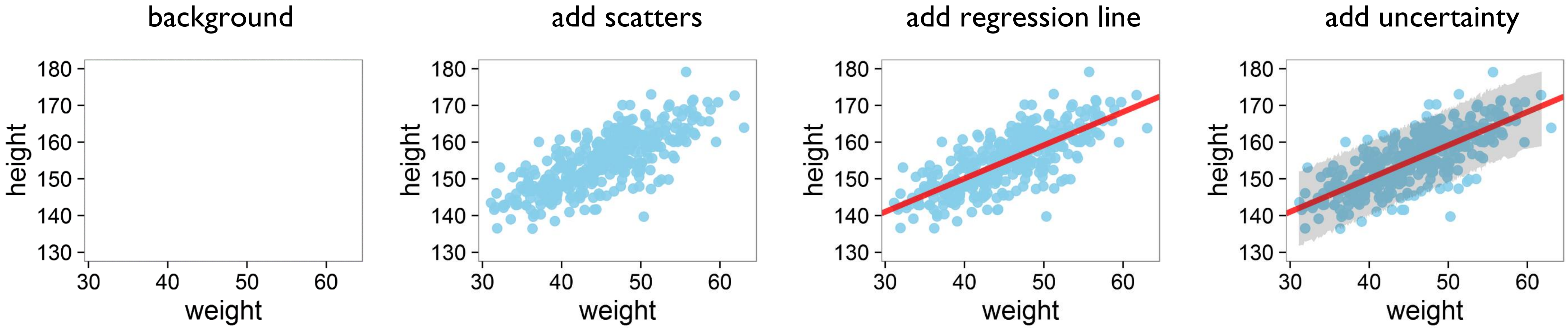
```
ggplot2::qplot(x,y)
```



Brief Intro to ggplot2

`plot = geometric (points, lines, bars) + aesthetic (color, shape, size)`

game of adding layers!



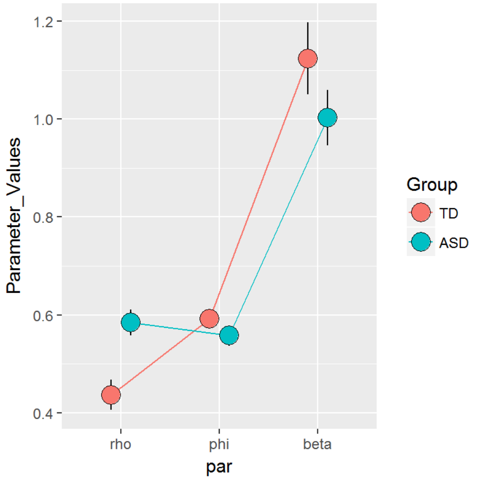
background

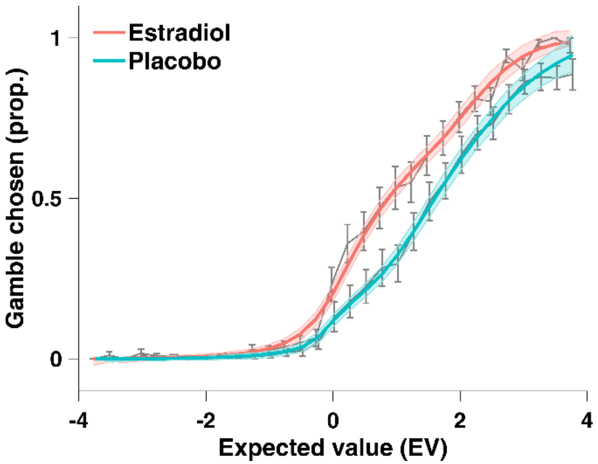
add scatters

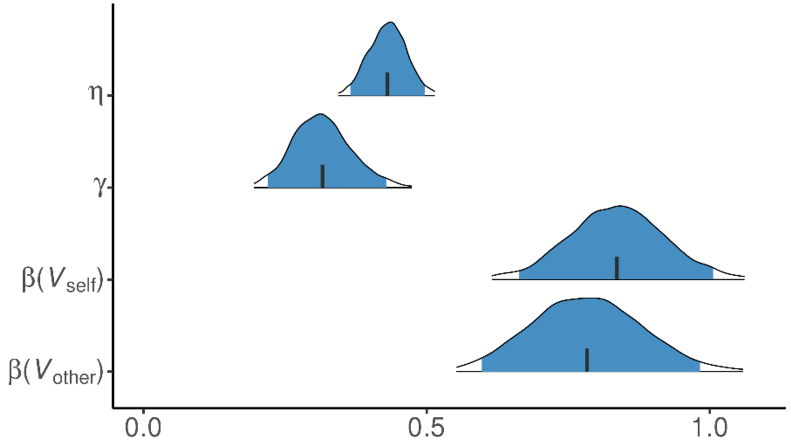
add regression line

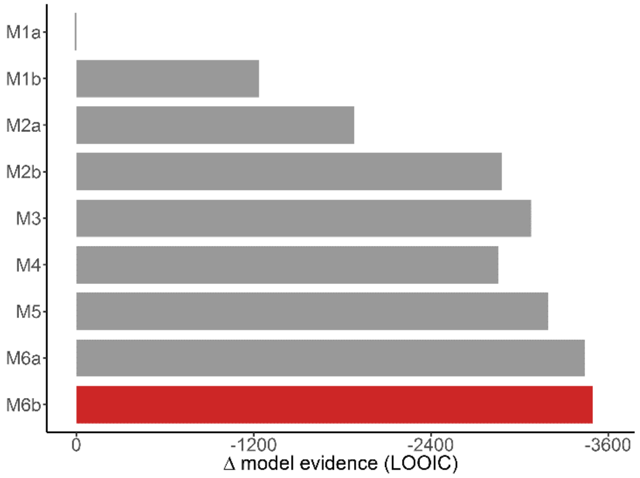
add uncertainty

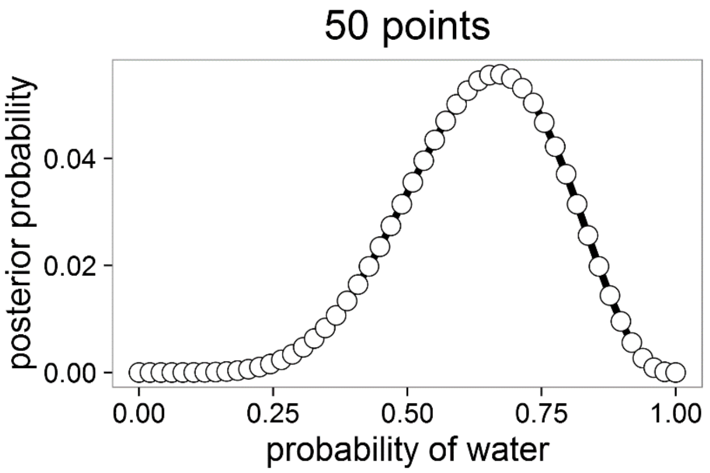
A taste of ggplot2

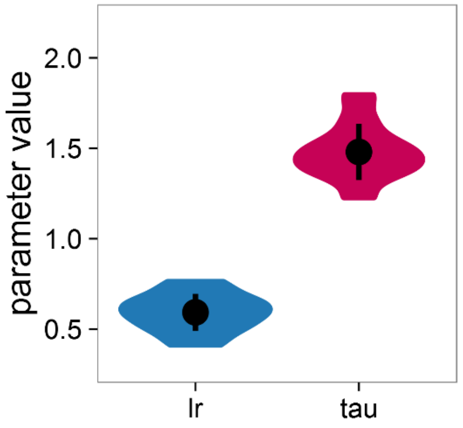


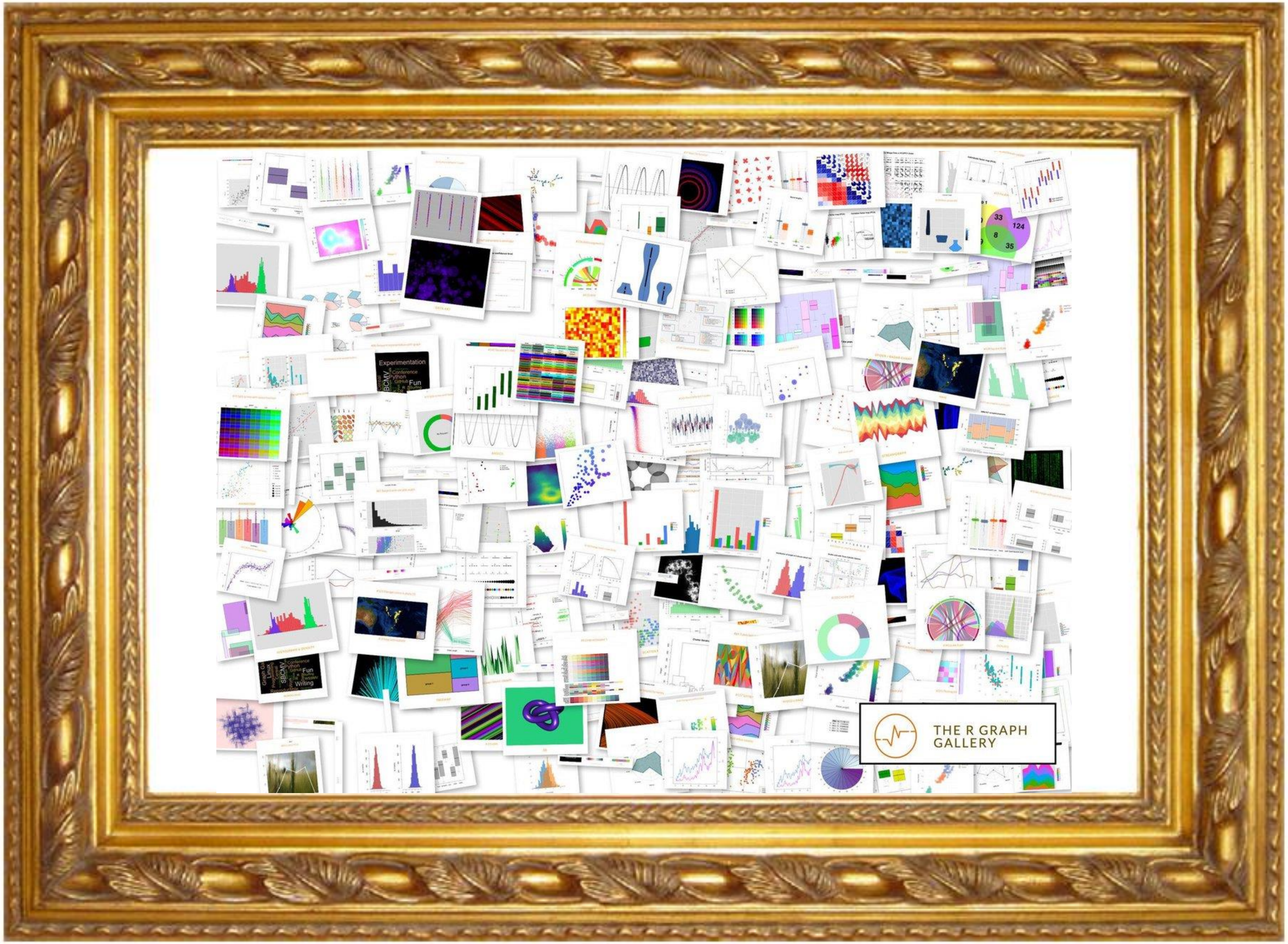






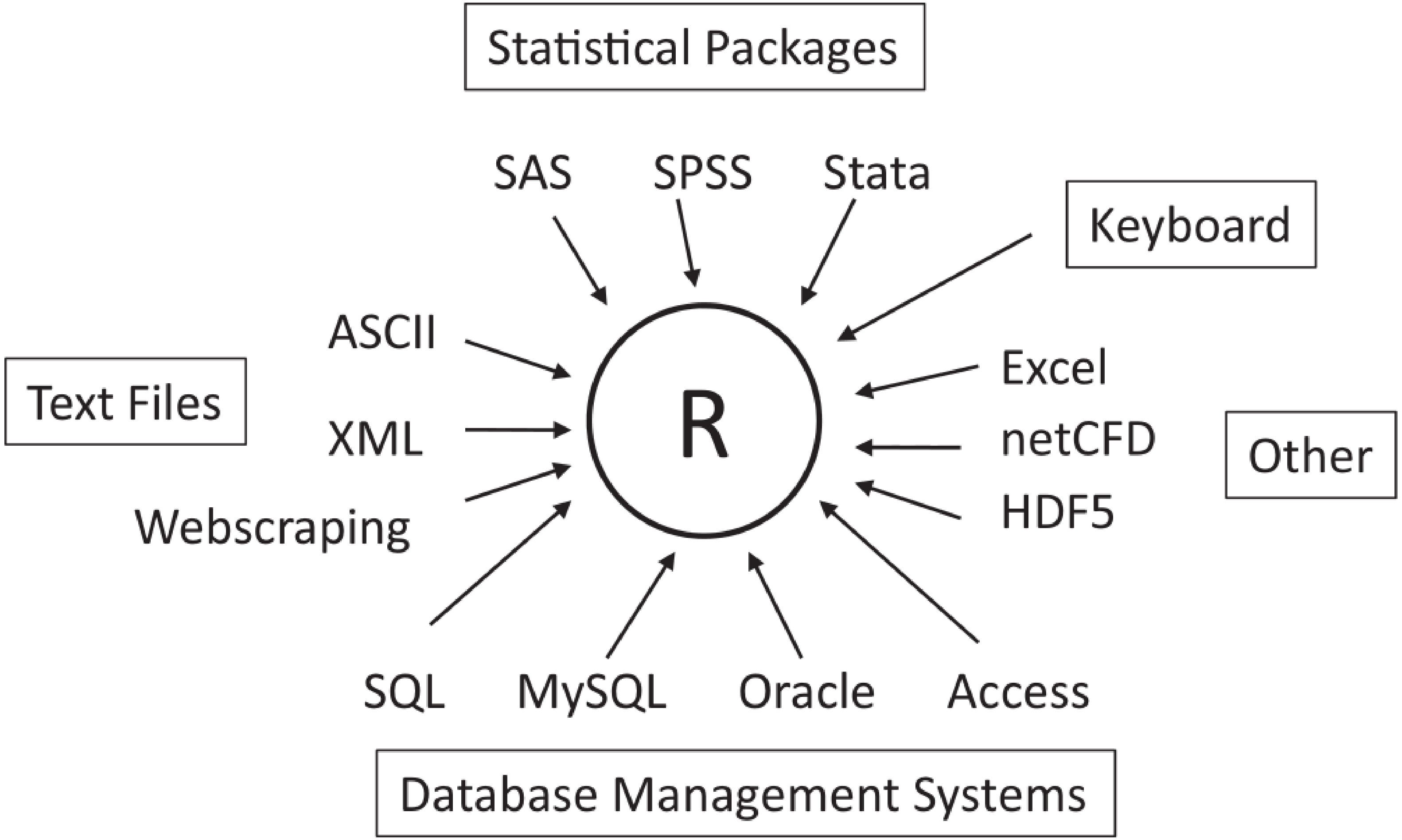






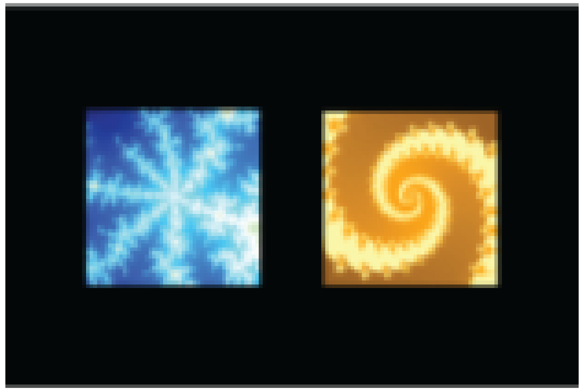
https://www.r-graph-gallery.com/

Data management

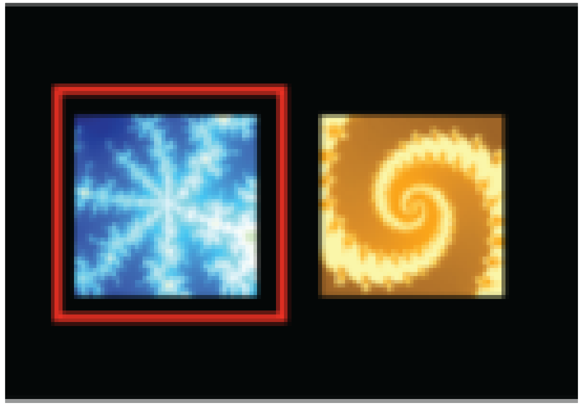


Kabacoff (2015)

One simple experiment



choice presentation



action selection

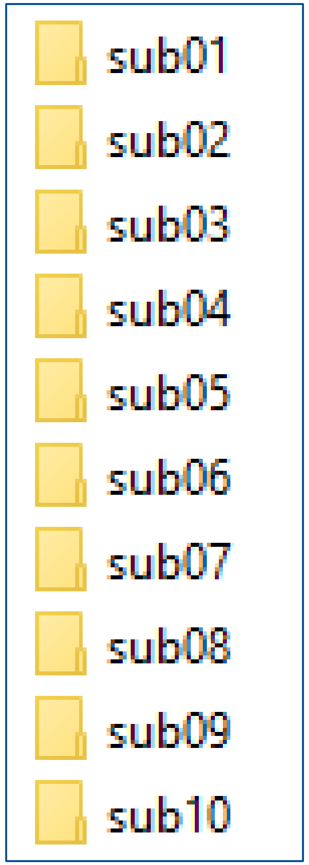
outcome

reward contingency – 80:20

The data

- nSub = 10
- nTrial = 80

```
./_data/_raw_data/sub01/raw
_data_sub01.txt
```



sub01
sub02
sub03
sub04
sub05
sub06
sub07
sub08
sub09
sub10

```
subjID, trialID, choice, outcome, correct
1,1,2,-1,1
1,2,1,1,1
1,3,1,1,1
1,4,1,1,1
1,5,2,-1,1
1,6,1,1,1
1,7,1,1,1
1,8,1,1,1
1,9,1,-1,1
1,10,2,-1,1
1,11,1,1,1
1,12,1,1,1
1,13,1,-1,2
```

Import some data!

```
data_dir = ('_data/RL_raw_data/sub01/raw_data_sub01.txt’)
data = read.table(data_dir, header = T, sep = ",")
head(data)

  subjID trialID choice outcome correct
1      1       1      1       1       1
2      1       2      1       1       1
3      1       3      1       1       1
4      1       4     NA       1       1
5      1       5      1      -1       1
6      1       6      2      -1       1

sum(complete.cases(data))
data = data[complete.cases(data),]
dim(data[complete.cases(data),])
```

Indexing

```
data[1,1]
data[1,]
data[,1]
data[1:10,]
data[,1:2]
data[1:10, 1:2]
data[c(1,3,5,6), c(2,4)]

data$choice
```

Exercise III

```
…/01.R_basics/_scripts/R_basics.R
```

TASK:

write a for loop

… which reads in each participant’s raw data

… and reshape it in the “long format”

```
for ( j in 1:n) {
   read.table(file, header = T, sep = ",")
}
```

Read all the data!

```
ns = 10
data_dir = '_data/RL_raw_data'

rawdata = c();
for (s in 1:ns) {
    sub_file = file.path(data_dir, sprintf('sub%02i/raw_data_sub%02i.txt',s,s))
    sub_data = read.table(sub_file, header = T, sep = ",")
    rawdata = rbind(rawdata, sub_data)
}
rawdata = rawdata[complete.cases(rawdata),]
rawdata$accuracy = (rawdata$choice == rawdata$correct) * 1.0

acc_mean = aggregate(rawdata$accuracy, by = list(rawdata$subjID), mean)[,2]
```

Basic stats

```
mean(acc_mean)
sd(acc_mean)
sem(acc_mean)

t.test(acc_mean, mu = 0.5) # one sample t-test

      One Sample t-test

data:  acc_mean
t = 13.788, df = 9, p-value = 2.34e-07
alternative hypothesis: true mean is not equal to 0.5
95 percent confidence interval:
 0.6962988 0.7733565
sample estimates:
mean of x
0.7348277
```

Basic correlation

```
load('_data/RL_descriptive.RData')
descriptive$acc = acc_mean
df = descriptive

cor.test(df$IQ, df$acc)

	Pearson's product-moment correlation

data:  df$IQ and df$acc
t = 4.8347, df = 8, p-value = 0.001297
alternative hypothesis: true correlation is not equal to 0
95 percent confidence interval:
 0.5114810 0.9671586
sample estimates:
      cor 
0.8631401
```

Exercise IV

```
…/01.R_basics/_scripts/R_basics.R
```

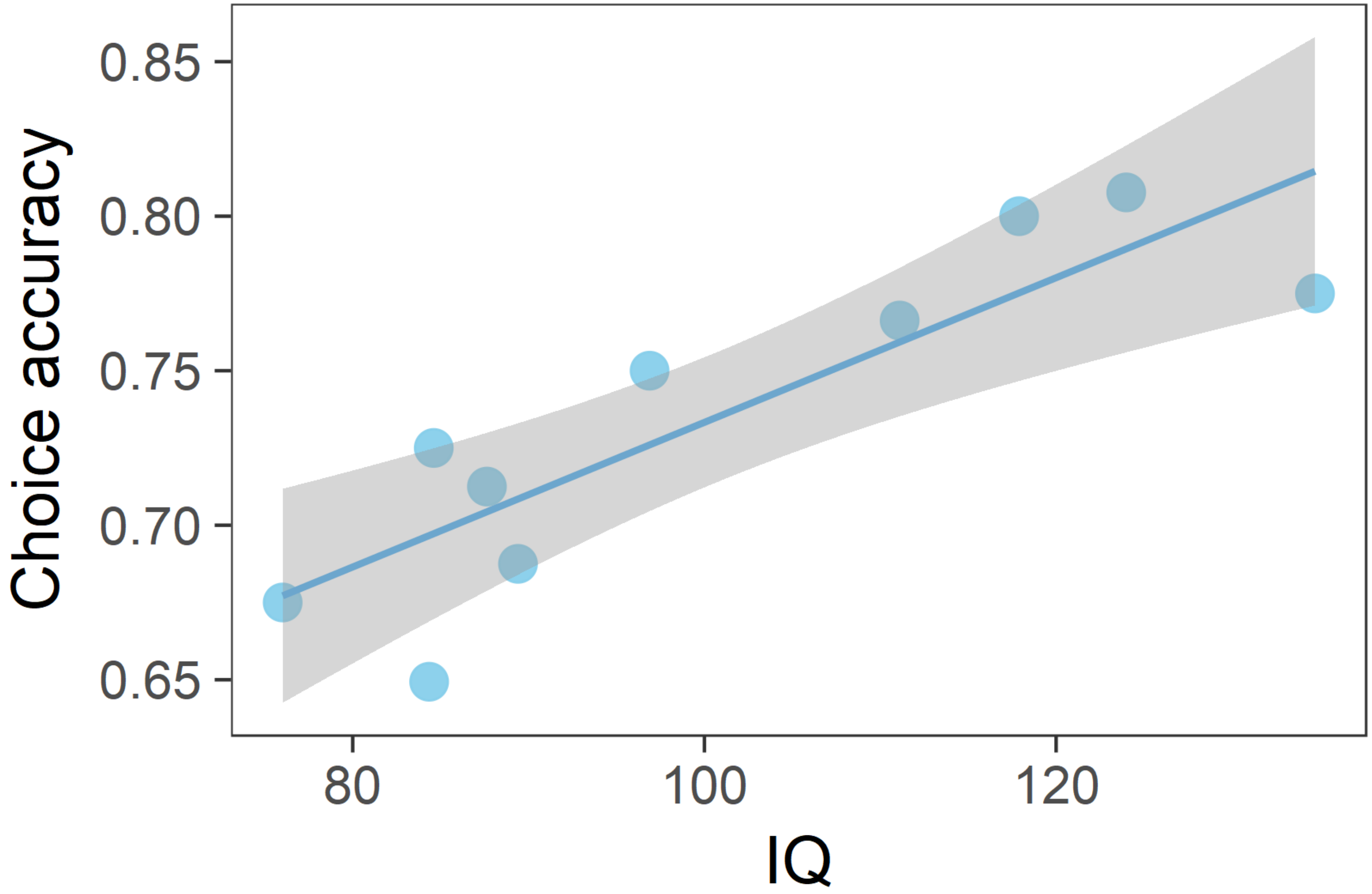
TASK:

Read in the descriptive data: _data/descriptive.RData
…include ‘acc_mean’ as a new column, and
…rename ‘descriptive’ as df.

Practice all the basic stats.

```
df$new_Col = new_Col
```

Plot the scatter and the regression line



Exercise V

```
…/01.R_basics/_scripts/R_basics.R
```

TASK:

Read and make sense of the ggplot functions,

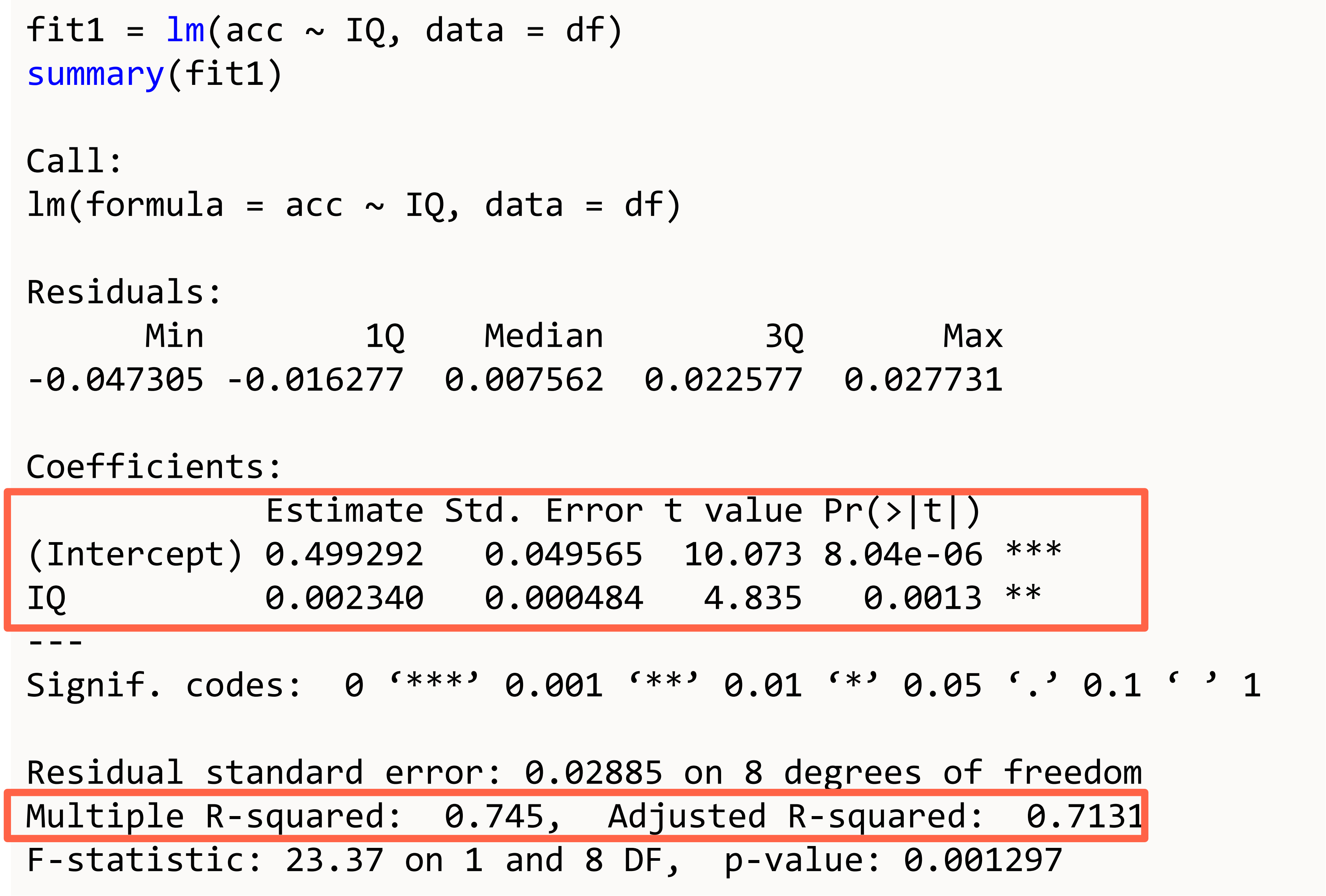
… experiment make some adjustments (color marker size etc.), and

… make a similar plot for acc ~ age.

What is exactly the regression line in R?

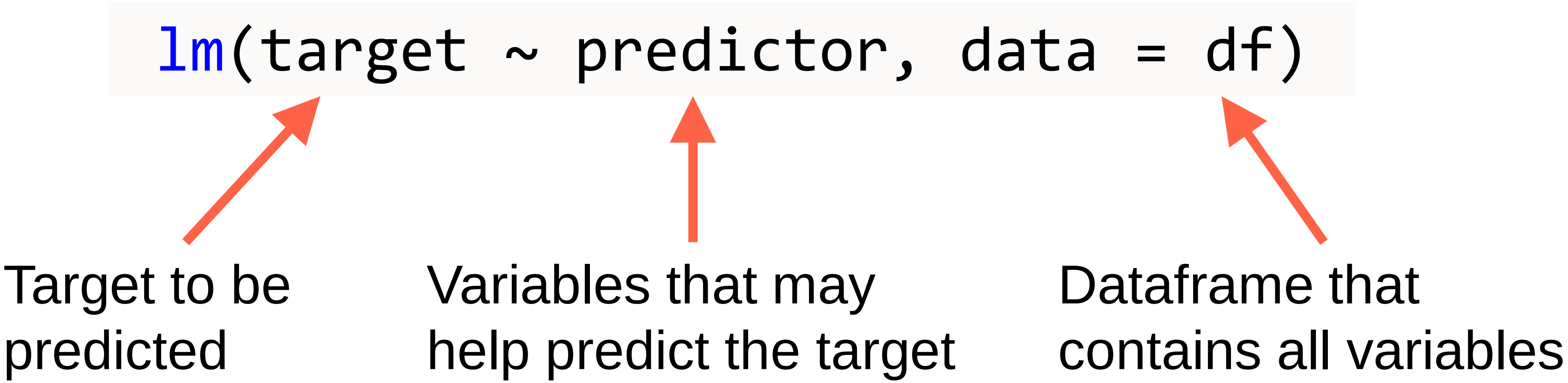
$$\mu_i = \alpha + \beta x_i$$

$$y_i = \mu_i + \varepsilon$$



```
fit1 = lm(acc ~ IQ, data = df)
summary(fit1)

Call:
lm(formula = acc ~ IQ, data = df)

Residuals:
      Min        1Q    Median        3Q       Max
-0.047305 -0.016277  0.007562  0.022577  0.027731

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.499292   0.049565  10.073 8.04e-06 ***
IQ          0.002340   0.000484   4.835   0.0013 **
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.02885 on 8 degrees of freedom
Multiple R-squared:  0.745,  Adjusted R-squared:  0.7131
F-statistic: 23.37 on 1 and 8 DF,  p-value: 0.001297
```

Multiple regression: more than one predictors



```
lm(target ~ predictor, data = df)
```

Target to be predicted

Variables that may help predict the target

Dataframe that contains all variables

Symbol	Example	Meaning
+	Y ~ X1	Include X (main effect of X)
:	Y ~ X1:X2	Interaction between X1 and X2
*	Y ~ X1*X2	Include both main affect and interaction

Y ~ X1 + X2 + X1:X2 <=> Y ~ X1*X2

Exercise VI

```
…/01.R_basics/_scripts/R_basics.R
```

TASK:

Construct the following regression models:

main effect of age

main effects of IQ and age

main effects and interaction between IQ and age

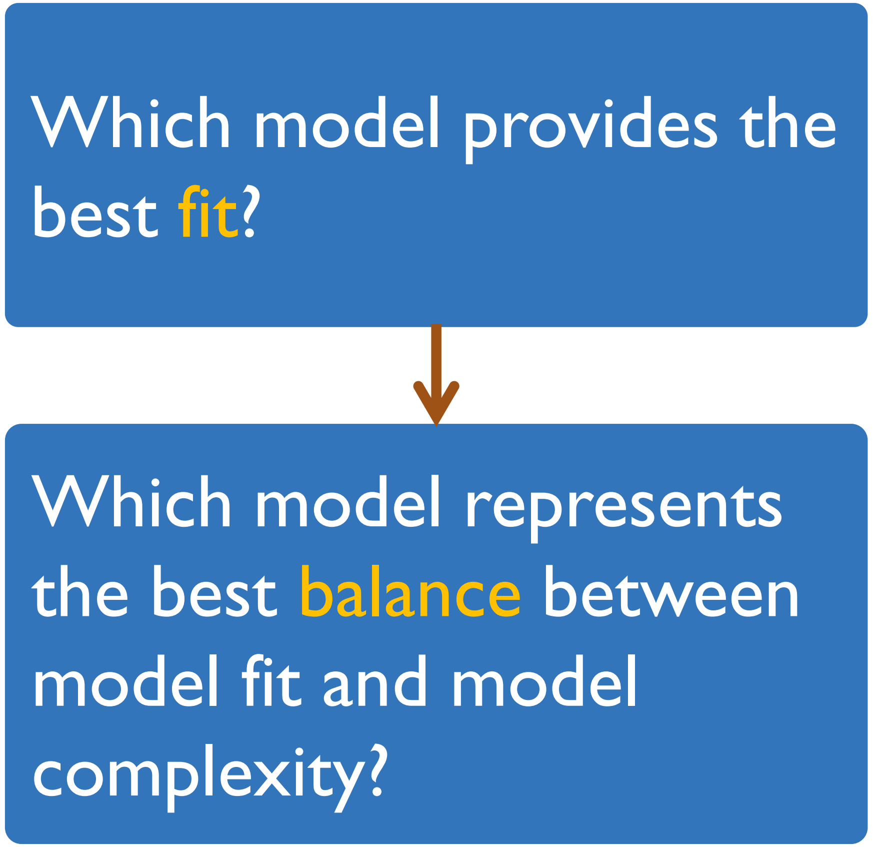
use `summary()` to check R^2 and adjusted-R^2

Multiple regression: more than one predictors

```
fit1 = lm(acc ~ IQ, data = df)
fit2 = lm(acc ~ Age, data = df)
fit3 = lm(acc ~ IQ + Age, data = df)
fit4 = lm(acc ~ IQ * Age, data = df) # IQ + Age + IQ:Age
```

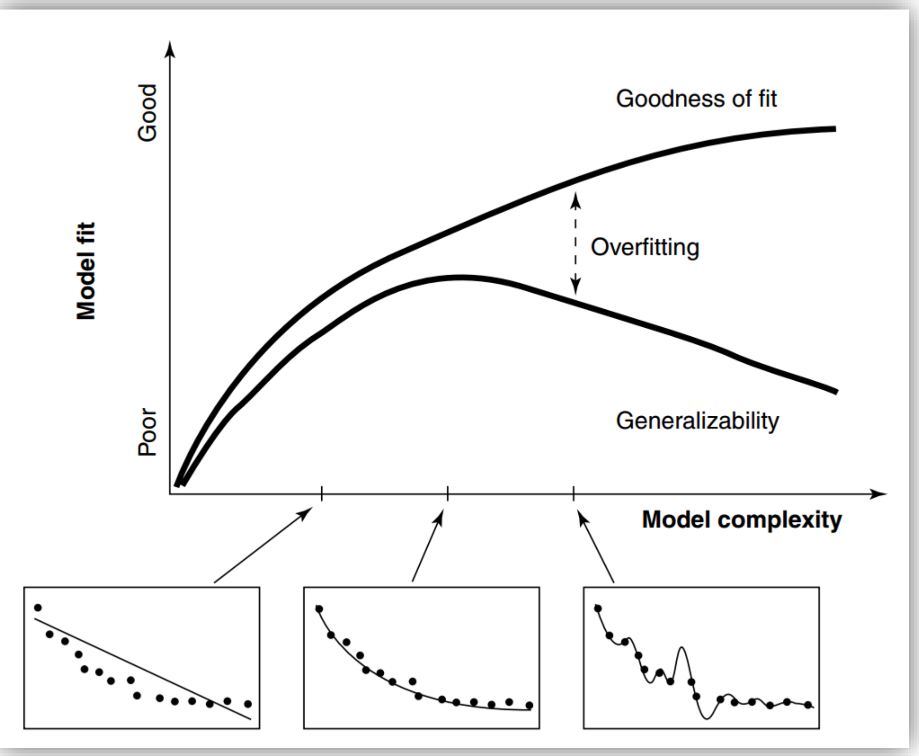
Model	Description	R^2	Adj-R^2	AIC
fit1	IQ only	0.75	0.71	-38.77
fit2	Age only	0.02	-0.10	-25.29
fit3	IQ Age additive	0.77	0.70	-37.76
fit4	IQ Age interactive	0.82	0.73	-38.31

Model Comparison



Which model provides the best fit?

Which model represents the best balance between model fit and model complexity?

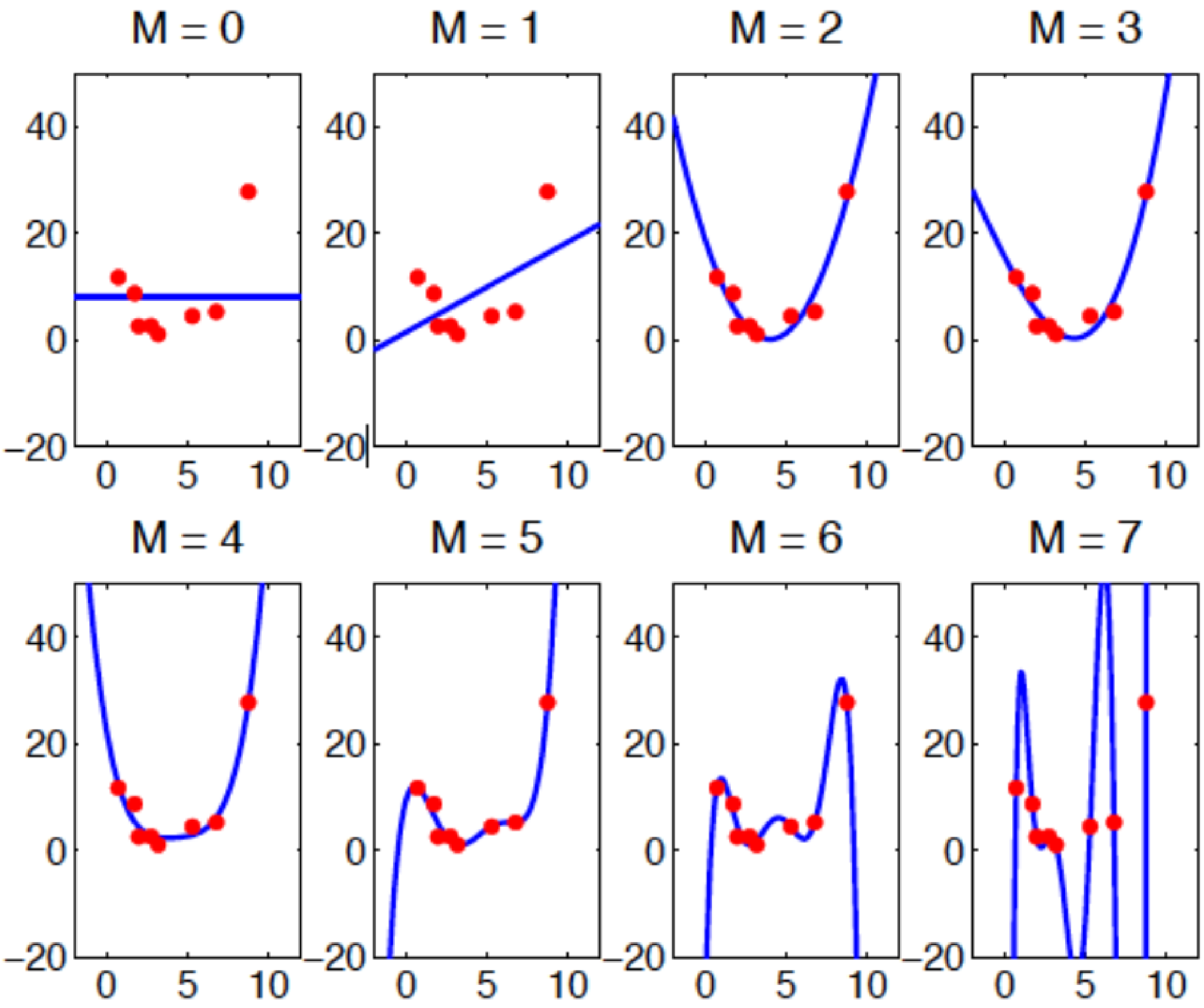


Ockham's razor:
Models with fewer assumptions are to be preferred

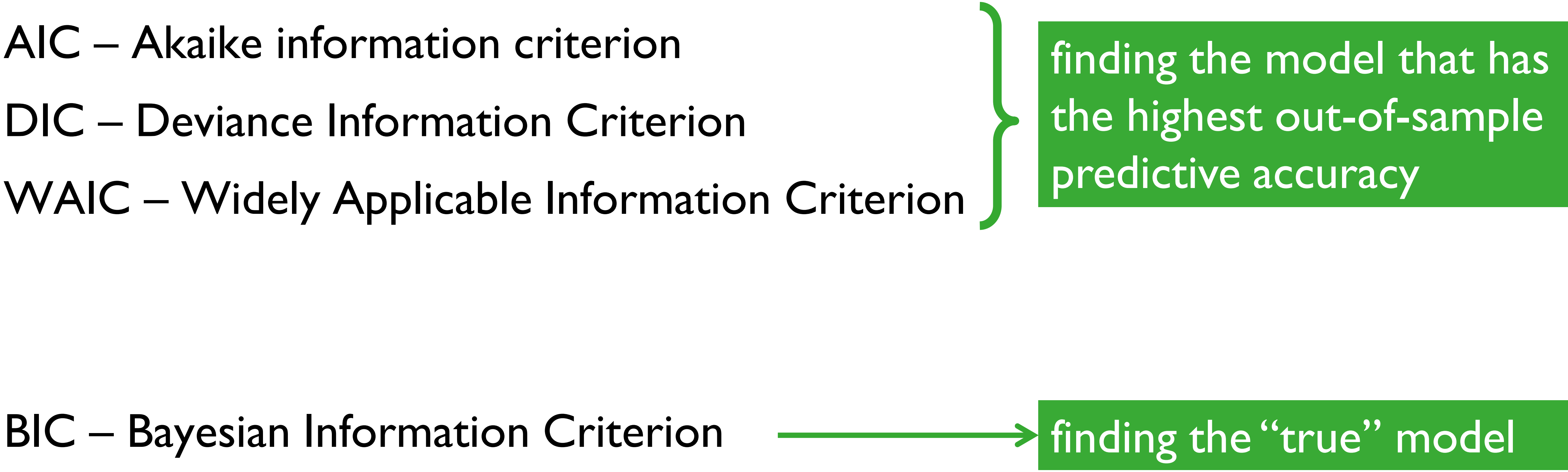
- overfitting: learn too much from the data
- underfitting: learn too little from the data

Pitt & Miyung (2002)

Which model has the highest predictive power?



Information Criteria



AIC – Akaike information criterion

DIC – Deviance Information Criterion

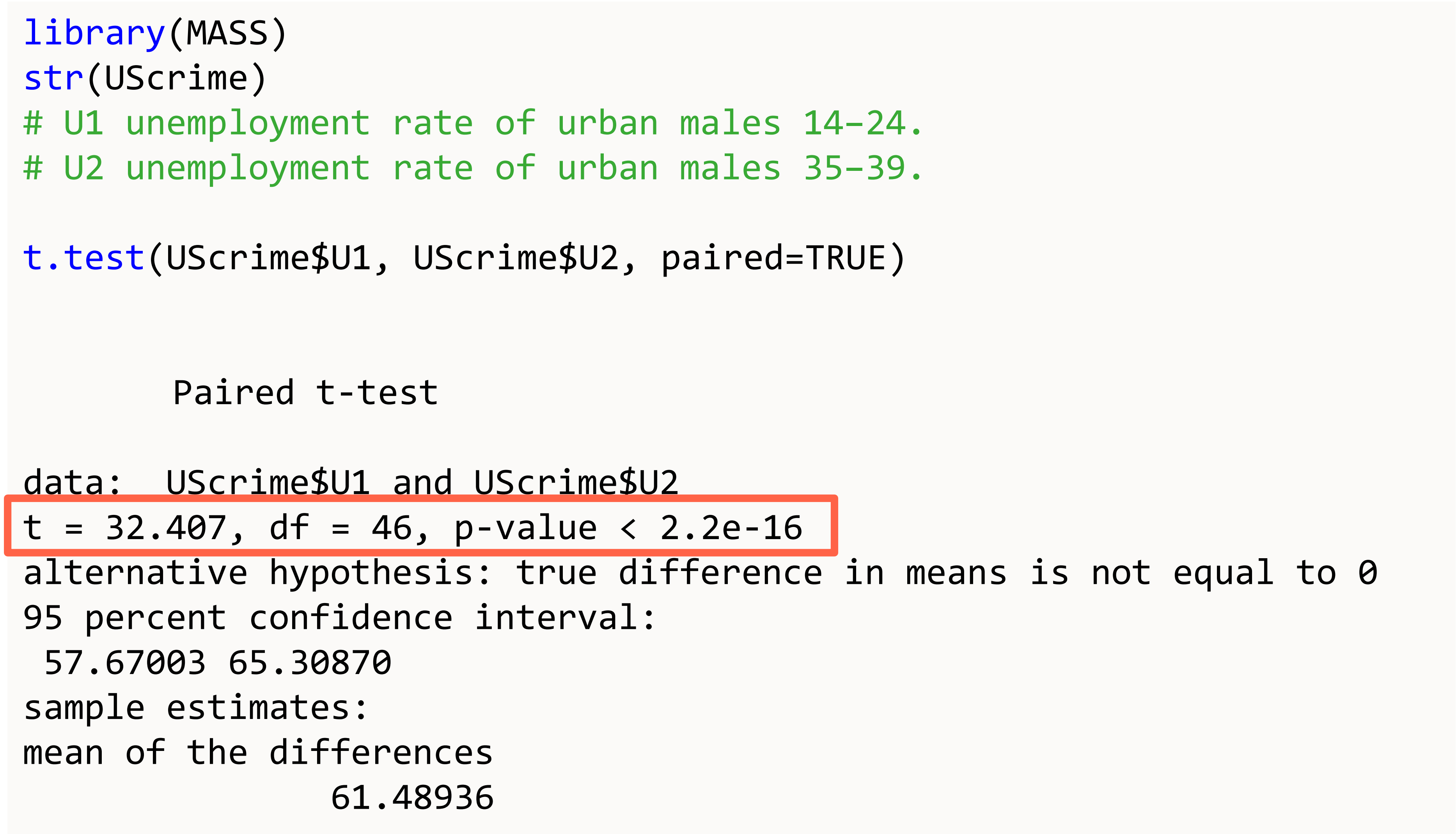
WAIC – Widely Applicable Information Criterion

finding the model that has the highest out-of-sample predictive accuracy

BIC – Bayesian Information Criterion

finding the “true” model

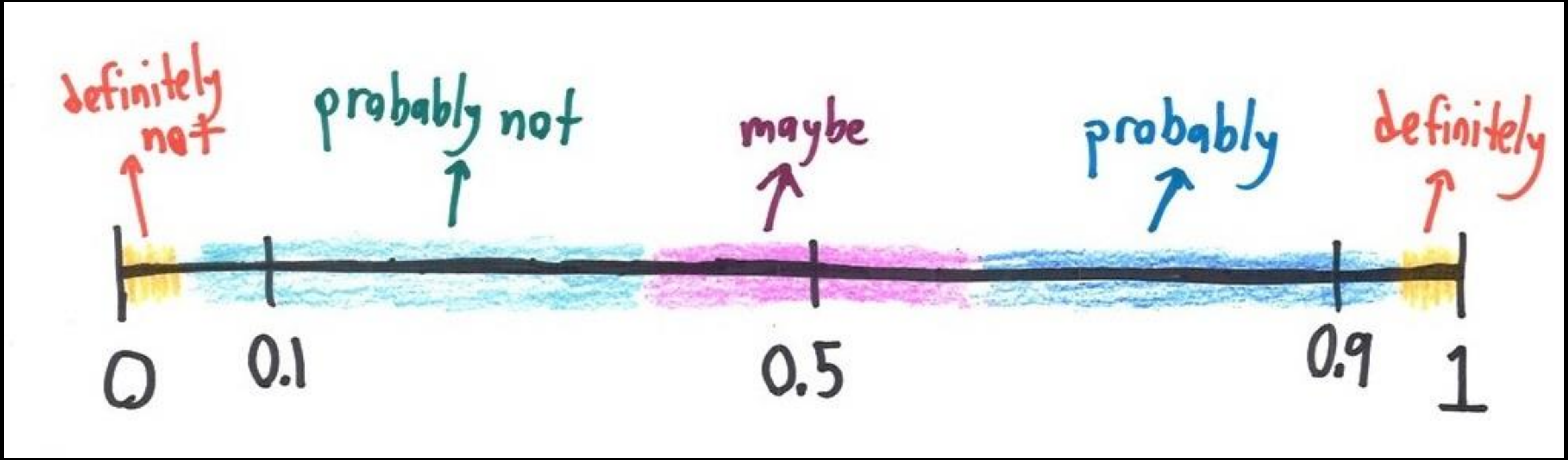
Compare Two Means



```r
library(MASS)
str(UScrime)
# U1 unemployment rate of urban males 14–24.
# U2 unemployment rate of urban males 35–39.

t.test(UScrime$U1, UScrime$U2, paired=TRUE)
```

```
        Paired t-test

data:  UScrime$U1 and UScrime$U2
t = 32.407, df = 46, p-value < 2.2e-16
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 57.67003 65.30870
sample estimates:
mean of the differences
               61.48936
```

BASICS OF PROBABILITY



Probability

…assigning numbers to a set of possibilities

Properties (Kolmogorov, 1956)

- $p \in [0,1]$
- $\Sigma p = 1$
- $p(\mathrm{A} \cup \mathrm{B}) = p(\mathrm{A}) + p(\mathrm{B})$, when A and B are *mutually exclusive*

Joint Probability and Conditional Probability

Joint Probability

$p(A, B) = p(B, A)$

- e.g., $p(\text{raining})$ and $p(\text{cold})$

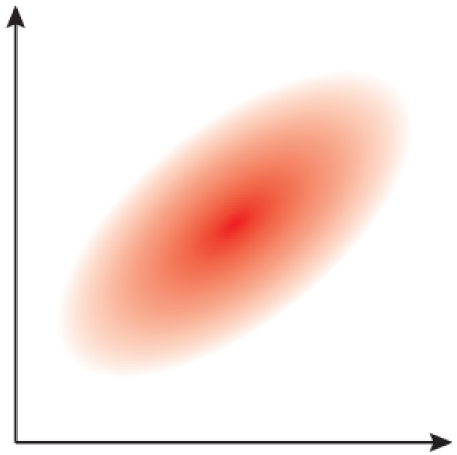
Conditional Probability

$p(A|B)$ – ‘p of A given B’

$p(A,B) = p(A|B)\,p(B)$

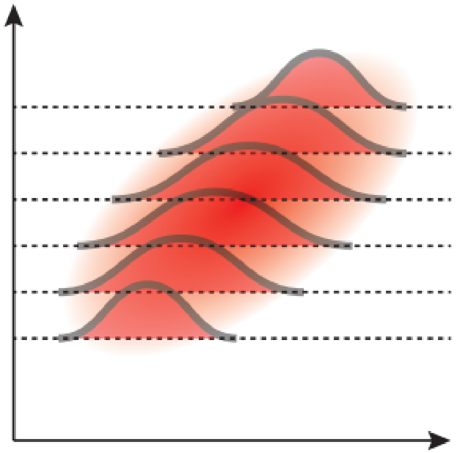
- e.g., $p(\text{raining, cold}) = p(\text{raining}|\text{cold})\,p(\text{cold})$

joint distribution



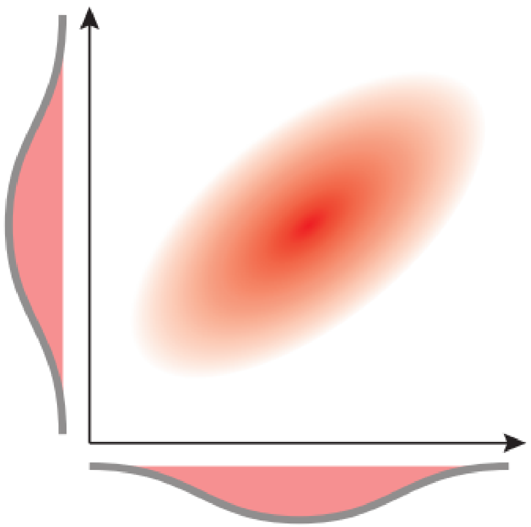
The "co-distribution" of x and y.

conditional distribution



The probability distribution of x, given that we know the value of y.

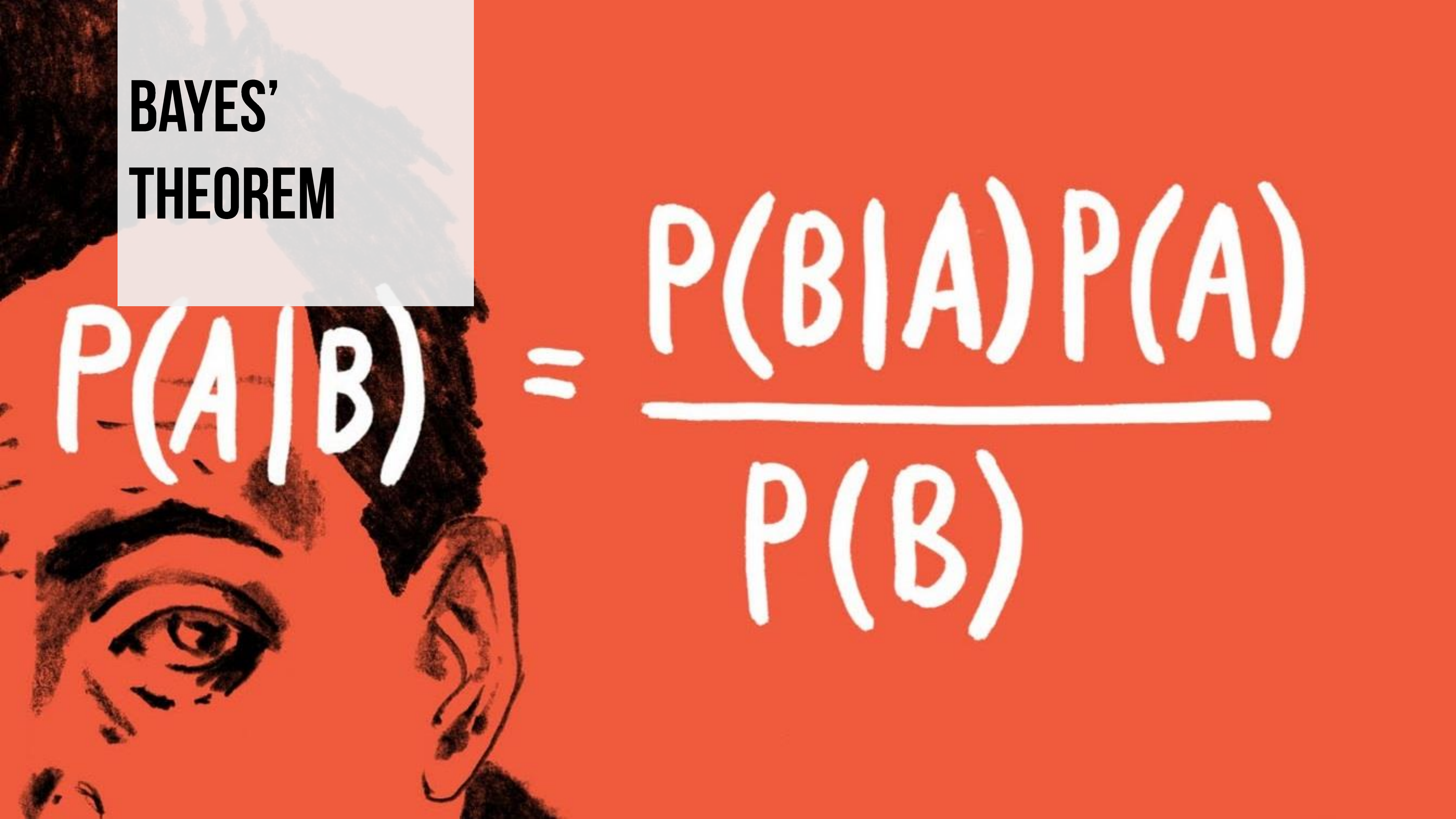
mariginal distribution



The density of x- (or y-) values, without knowing the other's value.

Credit: Eric Ma

BAYES' THEOREM



$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Bayes' theorem

$$p(A,B) = p(B,A)$$

$$p(A,B) = p(A|B)p(B)$$

$$p(B,A) = p(B|A)p(A)$$

$$p(A|B)p(B) = p(B|A)p(A)$$

$$\boxed{p\left(A \mid B\right) = \frac{p\left(B \mid A\right) p\left(A\right)}{p\left(B\right)}}$$

Linking Data and Parameter

$$p(\overset{\theta}{\overset{\uparrow}{A}}|\overset{D}{\overset{\uparrow}{B}}) = \frac{p(B|A)p(A)}{p(B)}$$

Linking Data and Parameter

$$p(\theta|D) = \frac{p(D|\theta)p(\theta)}{p(D)}$$

Linking Data and Parameter

Likelihood

How plausible is the data given our parameter is true?

Prior

How plausible is our parameter before observing the data?

$$p(\theta|D) = \frac{p(D|\theta)p(\theta)}{p(D)}$$

Posterior

How plausible is our parameter given the observed data?

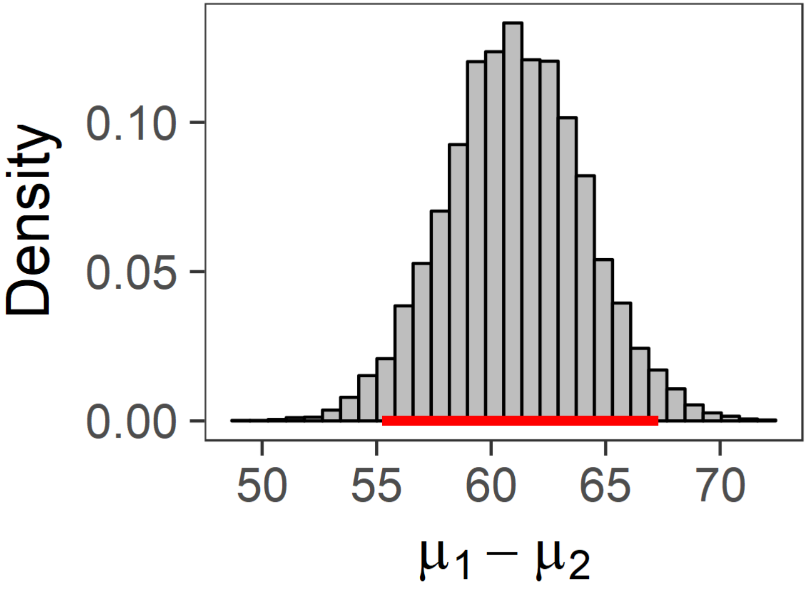
Evidence

How plausible is the data under all possible parameters?

How does that matter?

Given the data from two groups, we are interested if their means differ:

→ $p(\mu_1 - \mu_2 | D_1, D_2)$

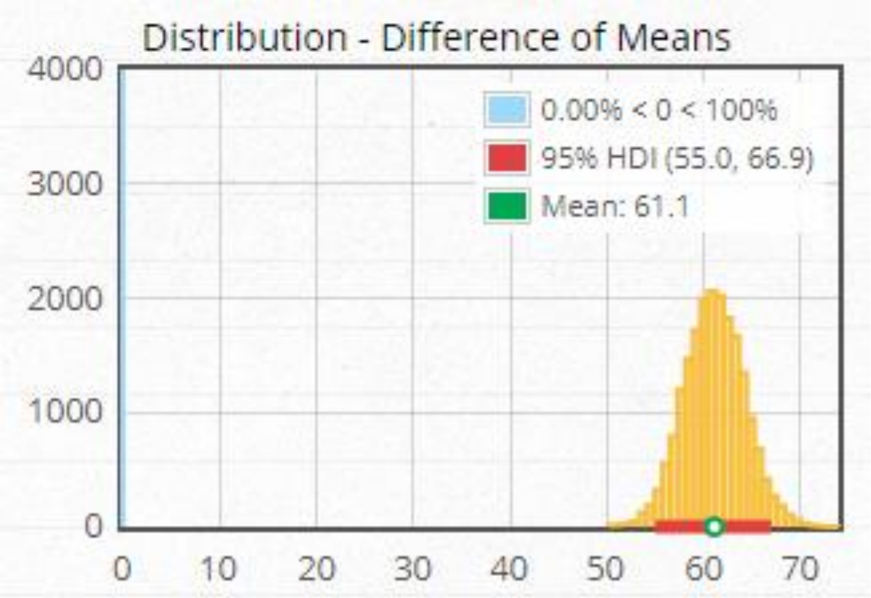


- mean: 61.06
- 95% HDI: [55.26 67.27]

Exercise VII

TASK:

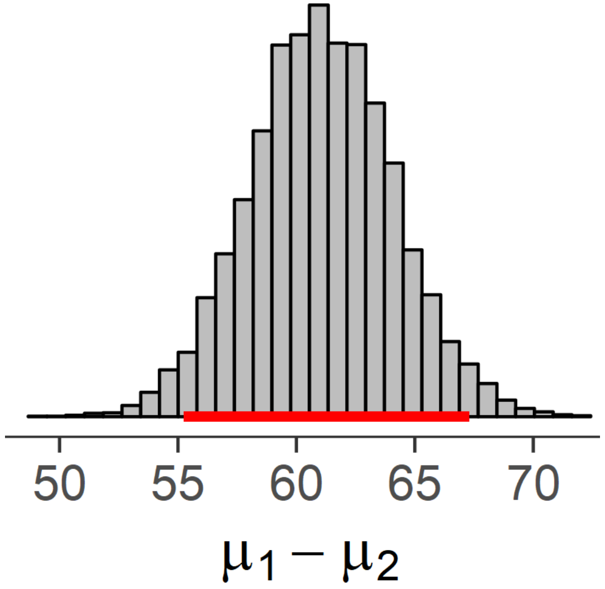
Use the online tool to compute the posterior mean difference (U1 vs. U2) in the UScrime dataset.



Online calculation tool: BEST

Why bother?

- Incorporate prior knowledge of $(\mu_1 - \mu_2)$
- Obtain belief (uncertainty of the estimate)
- Able to accept H_0 (null hypothesis)
  - frequentist: p value is $p(D|H_0)$
- Could test more than H_1, e.g., a bimodal distribution of the mean difference
- Have fewer assumptions



Bayes Factor

$$p(H_0 \mid D) \propto p(D \mid H_0)\, p(H_0)$$

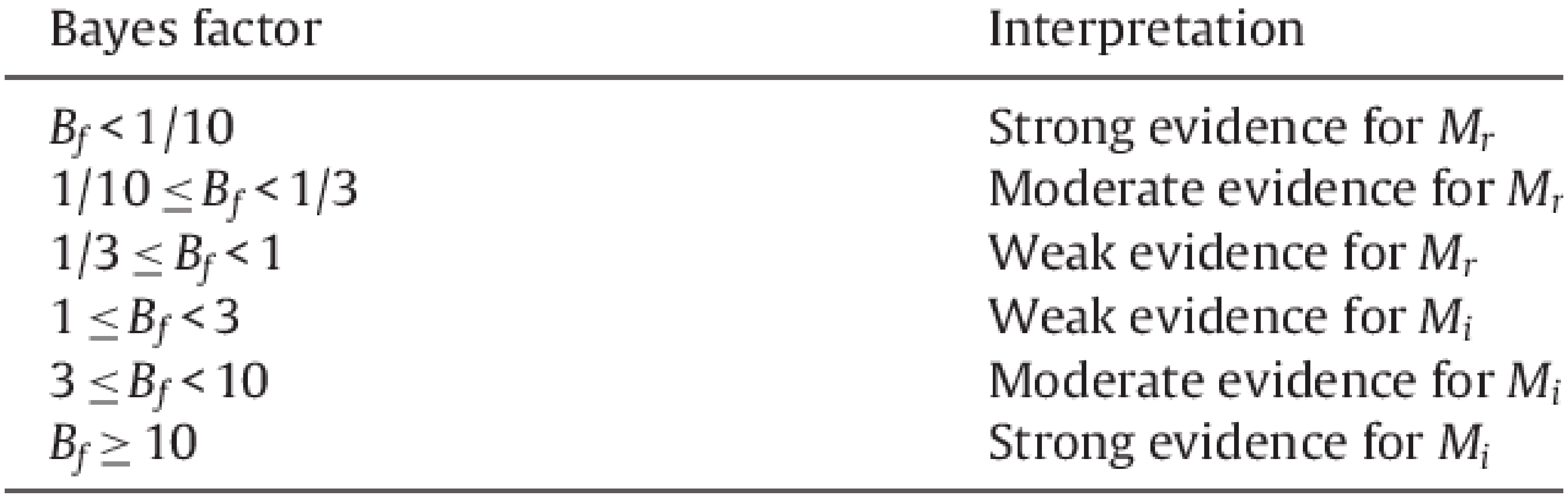
$$p(H_1 \mid D) \propto p(D \mid H_1)\, p(H_1)$$

$$\frac{p(H_0 \mid D)}{p(H_1 \mid D)} = \boxed{\frac{p(D \mid H_0)}{p(D \mid H_1)}} \cdot \frac{p(H_0)}{p(H_1)}$$

posterior odds = Bayes factor × prior odds

Bayes Factor

$$\mathrm{BF} = \frac{p(D \mid H_0)}{p(D \mid H_1)}$$

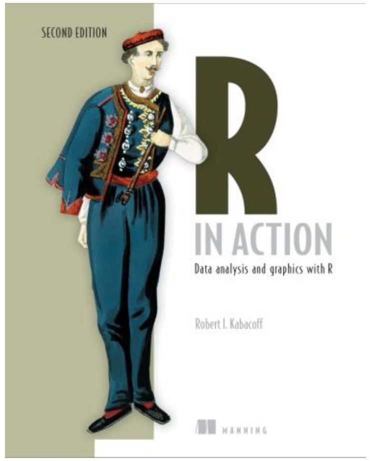


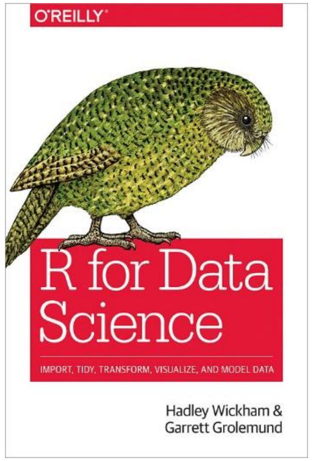
Bayes factor	Interpretation
$B_f < 1/10$	Strong evidence for M_r
$1/10 \leq B_f < 1/3$	Moderate evidence for M_r
$1/3 \leq B_f < 1$	Weak evidence for M_r
$1 \leq B_f < 3$	Weak evidence for M_i
$3 \leq B_f < 10$	Moderate evidence for M_i
$B_f \geq 10$	Strong evidence for M_i

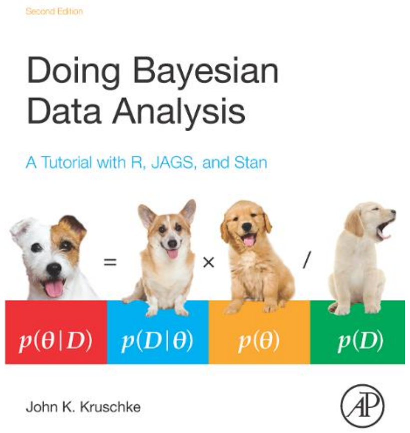
Source: Min et al. (2007).

BF tool: {BayesFactor}

Resources






Second Edition
Doing Bayesian Data Analysis
A Tutorial with R, JAGS, and Stan
=
×
/
p(θ|D)
p(D|θ)
p(θ)
p(D)
John K. Kruschke

Statistical Thinking for the 21st Century

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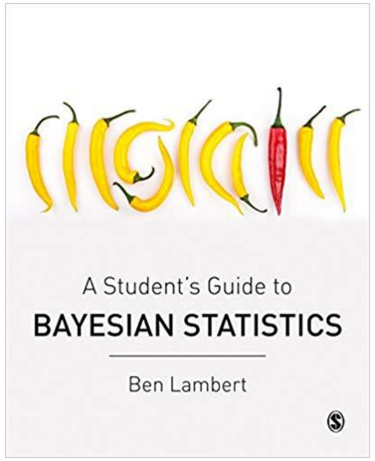

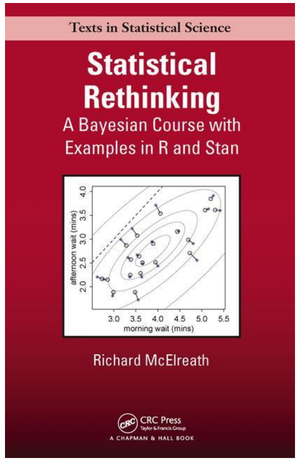

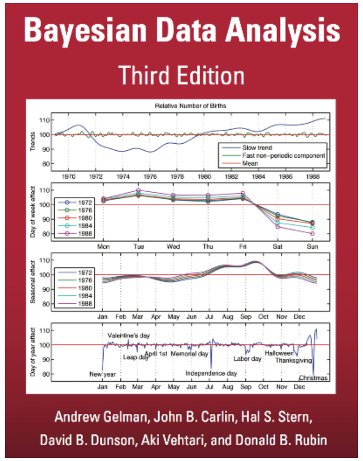
Bayesian Data Analysis
Third Edition
Relative Number of Births
Andrew Gelman, John B. Carlin, Hal S. Stern,
David B. Dunson, Aki Vehtari, and Donald B. Rubin

JASP

https://jasp-stats.org/

ANY QUESTIONS
?

Happy R Computing!