IMONST-MRSM:
Algebra Junior Problems

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Contents

1 Big Numbers

Not discussed in class: 4, 5, 7

1. Find the value of $2024^2 - (2023)(2025)$.
2. Find the value of $\sqrt{111113^2 - 888888}$.
3. (CJR) Find the value of $\sqrt{499^2 + 999}$.
4. (CJR) Find the value of $\sqrt[3]{2022 \times 2024 \times 2026 + 4 \times 2024}$.
5. (IMONST 1) This sequence lists the perfect squares in increasing order:

   $$0, 1, 4, 9, 16, ..., a, 10^8, b, \ldots$$

   Determine the value of $b - a$.
6. (OMK) Find the value of

   $$\frac{66666666 \times 44444445 - 33333333 \times 88888888}{6666 \times 4445 - 3333 \times 8888}.$$

7. (SMO) Find the value of

   $$\frac{(2020^2 - 20100)(20100^2 - 100^2)(2000^2 + 20100)}{2010^6 - 10^6}.$$

2 Sums and Products

Not discussed in class: 4, 6, 12, 14, 15

1. Find the value of $1 + 2 + 3 + \cdots + 2024$.
2. Find the value of $2 + 4 + 6 + \cdots + 2024$
3. Find the value of $1 + 3 + 5 + \cdots + 99$.
4. Find the value of $1000^2 - 999^2 + 998^2 - 997^2 + \cdots + 2^2 - 1^2$.
5. Find the value of $1 - 2 + 3 - 4 + 5 - 6 + \cdots + 2023 - 2024$.
6. (IMONST 1) Find the value of

   $$+1 + 2 + 3 - 4 - 5 - 6 + 7 + 8 + 9 - 10 - 11 - 12 + \cdots - 2020,$$

   where the sign alternates between + and − after every three numbers.

7. Find the value of $2^0 + 2^1 + 2^2 + \cdots + 2^n$.

8. Find the value of $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$.

9. Find the value of

$$\left(1 + \frac{1}{2}\right)\left(1 + \frac{1}{3}\right)\left(1 + \frac{1}{4}\right)\ldots\left(1 + \frac{1}{2022}\right)\left(1 + \frac{1}{2023}\right).$$

10. (AMC) What is the value of the product

$$\left(\frac{1 \times 3}{2 \times 2}\right)\left(\frac{2 \times 4}{3 \times 3}\right)\left(\frac{3 \times 5}{4 \times 4}\right)\ldots\left(\frac{97 \times 99}{98 \times 98}\right)\left(\frac{98 \times 100}{99 \times 99}\right)?$$

11. Find the value of

$$\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \cdots + \frac{1}{2023 \times 2024}.$$

12. Find the value of

$$\frac{1}{1 \times 3} + \frac{1}{3 \times 5} + \frac{1}{5 \times 7} + \cdots + \frac{1}{2021 \times 2023}.$$

13. Find the value of

$$\frac{1}{\sqrt{1} + \sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{3}} + \cdots + \frac{1}{\sqrt{99} + \sqrt{100}}.$$

14. Find the value of

$$\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \cdots + \frac{2023}{2024!}$$

15. Find the value of

$$\frac{2^3 - 1}{2^3 + 1} \times \frac{3^3 - 1}{3^3 + 1} \times \frac{4^3 - 1}{4^3 + 1} \times \ldots.$$

3 Algebraic Manipulations

Not discussed in class: 5, 6, 10, 12, 13, 14

1. Given that $\frac{a}{b} = 2$ and $\frac{b}{c} = 4$, find $\frac{a}{c}$.
2. Given that $xy = 2$ and $x + y = 4$, find $x^2 + y^2$.
3. Given that $\frac{a}{1+a} + \frac{b}{1+b} + \frac{c}{1+c} = 1$, find
$$\frac{1}{1+a} + \frac{1}{1+b} + \frac{1}{1+c}.$$
4. (CJR) If $143x - 77y = 451$, find the value of $299x - 161y$.
5. (IMONST 1) Given that $x^2yz^3 = 2^3$, and $xy^2 = 2^9$. If $xyz = 2^n$, find the value of n.
6. (CJR) Given that $\frac{x}{x+y+z} = \frac{1}{3}$ and $\frac{y}{x+y+z} = \frac{1}{4}$, find $\frac{24x+36y+48z}{x+y+z}$.
7. Given that $x + \frac{1}{x} = 5$, find $x^2 + \frac{1}{x^2}$, $x^3 + \frac{1}{x^3}$ and $x^4 + \frac{1}{x^4}$.
8. (CJR) Let x, y be two non-zero real numbers such that $x \neq y$. If $x + \frac{9}{x} = y + \frac{9}{y}$, find xy.
9. (CJR) Given that x and y are real numbers such that $x > y$, $x + y = 14$ and $xy = 12$, find the value of $x^2 + \frac{168}{x}$.
10. (SMO) If $x+\sqrt{xy}+y = 9$ and $x^2+xy+y^2 = 27$, find the value of $x-\sqrt{xy}+y$.
11. Find the value of $\sqrt[3]{2+\sqrt{5}} + \sqrt[3]{2-\sqrt{5}}$.
12. (CJR) Given that a, b, c are real numbers and
$$(b - 196a)^2 - 128(b - 14c)(c - 14a) = 0,$$
find the largest possible value of $\frac{b-14c}{c-14a}$.
13. (SMO) Let $m \neq n$ be two real numbers such that $m^2 = n + 2$ and $n^2 = m + 2$. Find the value of $4mn - m^3 - n^3$.
14. Given that $6(a + b + c) = 3(a^2 + b^2 + c^2) = 2(a^3 + b^3 + c^3) = 6$, find the value of abc.

4 Trivial Inequality

Not discussed in class: 3

1. Find all triples of reals (x, y, z) such that

$$22(x-22)^{22} + 24(y-24)^{24} + 26(y-26)^{26} = 0.$$

2. Find the minimum value of $x^2 + 2x + 4$ by completing the square.

3. Find all ordered pairs of real numbers (x, y) such that

$$(4x^2 + 4x + 3)(y^2 - 6y + 13) = 8.$$

4. Find all real solutions x, y, z of the equation

$$x^2 + 5y^2 + 10z^2 = 4xy + 6yx + 2z - 1.$$