Convergence proof for the steepest descent.

Proof.

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \alpha \mathbb{E}[(y - \mathbf{w}(n)^T \mathbf{x})\mathbf{x}] \quad \cdots (1)$$

$$\mathbb{E}[(y - \mathbf{w}(n)^T \mathbf{x})\mathbf{x}] = \mathbb{E}[\mathbf{x}y] - \mathbb{E}[\mathbf{x}\mathbf{x}^T]\mathbf{w}(n)$$

$$= \mathbf{r}_{xy} - R_x \mathbf{w}(n) \quad \cdots (2)$$

$$\mathbf{w}(n+1) = \alpha(\mathbf{r}_{xy} - R_x \mathbf{w}(n)) + \mathbf{w}(n) \quad (\because (1), (2))$$

$$= \alpha \mathbf{r}_{xy} - \alpha R_x \mathbf{w}(n) + \mathbf{w}(n)$$

$$= (I - \alpha R_x)\mathbf{w}(n) + \alpha \mathbf{r}_{xy}$$

$$\mathbf{w}(n+1) - \mathbf{w}^* = (I - \alpha R_x)\mathbf{w}(n) + \alpha \mathbf{r}_{xy} - \mathbf{w}^*$$

$$= (I - \alpha R_x)(\mathbf{w}(n) - \mathbf{w}^*)$$

where $\mathbf{w}^* = R_x^{-1} \mathbf{r}_{xy}$

Let $\mathbf{c}(n) = \mathbf{w}(n) - \mathbf{w}^*$

$$\mathbf{c}(n+1) = (I - \alpha R_x)\mathbf{c}(n)$$

$$= (I - \alpha U \Lambda U^T) c(n)$$

$$= (UU^T - \alpha U \Lambda U^T) c(n)$$

$$= U(I - \alpha \Lambda) U^T c(n)$$

Let $v(n) = U^T c(n)$

$$v(n+1) = (I - \alpha \Lambda) v(n)$$

Therefore,

$$v(n+1) = (I - \alpha \Lambda)^n v(0)$$

$$\lim_{n \to \infty} v(n+1) = 0$$

$$\left(\because 0 < \alpha < \frac{2}{\lambda_{max}}, \text{ theorem. } 3.2.10. \right.$$

$$\left. \rightarrow \forall i, \ |1 - \alpha \lambda_i| < 1 \right)$$

Therefore,

$$\lim_{n \to \infty} c(n) = 0.$$

and $\lim w(n) = w^* = w_{WF}$ ■

[illegible]