$$y = f(x) + \varepsilon$$

Since ε and $\hat{f}$ are independent,

$$\mathbb{E}[(y-\hat{f})^2] = \mathbb{E}[(f+\varepsilon-\hat{f})^2]$$

$$= \mathbb{E}[(\underline{f} + \underbrace{\varepsilon}_{②} \underbrace{- \hat{f} + \mathbb{E}[\hat{f}]}_{③} \underbrace{- \mathbb{E}[\hat{f}]}_{①})^2]$$

$$= \mathbb{E}[(f-\mathbb{E}[\hat{f}])^2] + \mathbb{E}[\varepsilon^2] + \mathbb{E}[(\mathbb{E}[\hat{f}]-\hat{f})^2]$$

$$+ 2\mathbb{E}[(f-\mathbb{E}[\hat{f}])\varepsilon] + 2\mathbb{E}[\varepsilon(\mathbb{E}[\hat{f}]-\hat{f})]$$

$$+ 2\mathbb{E}[(\mathbb{E}[\hat{f}]-\hat{f})(f-\mathbb{E}[\hat{f}])]$$

$$= (f-\mathbb{E}[\hat{f}])^2 + \mathbb{E}[\varepsilon^2] + \mathbb{E}[(\mathbb{E}[\hat{f}]-\hat{f})^2]$$

$$+ 2(f-\mathbb{E}[\hat{f}])\mathbb{E}[\varepsilon] + 2\mathbb{E}[\varepsilon]\,\mathbb{E}[\mathbb{E}[\hat{f}]-\hat{f}]$$

$$+ 2\mathbb{E}[(\mathbb{E}[\hat{f}]-\hat{f})](f-\mathbb{E}[\hat{f}])$$

$$= (f-\mathbb{E}[\hat{f}])^2 + \mathbb{E}[\varepsilon^2] + \mathbb{E}[(\mathbb{E}[\hat{f}]-\hat{f})^2]$$

$$= \mathrm{Bias}[\hat{f}]^2 + \sigma^2 + \mathrm{Var}[\hat{f}] \quad \blacksquare$$