$$w(n) = \arg\min_{w} \left\| \begin{bmatrix} y(1) \\ \vdots \\ y(n) \end{bmatrix} - \begin{bmatrix} X(1) \\ \vdots \\ X(n) \end{bmatrix} w \right\|^2$$

For $i=1$:

$$w(1) = \arg\min_{w} \| y(1) - X(1) w \|^2$$

$$= (X^T(1)X(1))^{-1} X^T(1) y(1) \quad (\because \text{LS Solution.}) \cdots (1)$$

Let $P(1) = (X^T(1)X(1))^{-1}$.

For $i=2$:

$$w(2) = \arg\min_{w} \left\| \begin{bmatrix} y(1) \\ y(2) \end{bmatrix} - \begin{bmatrix} X(1) \\ X(2) \end{bmatrix} w \right\|^2$$

$$= P(2) \begin{bmatrix} X(1) \\ X(2) \end{bmatrix}^T \begin{bmatrix} y(1) \\ y(2) \end{bmatrix}.$$

where $P(2) = \left(\begin{bmatrix} X(1) \\ X(2) \end{bmatrix}^T \begin{bmatrix} X(1) \\ X(2) \end{bmatrix} \right)^{-1}$

$$P^{-1}(2) = \begin{bmatrix} X(1) \\ X(2) \end{bmatrix}^T \begin{bmatrix} X(1) \\ X(2) \end{bmatrix}$$

$$= X^T(1)X(1) + X^T(2)X(2)$$

$$= P^{-1}(1) + X^T(2)X(2) \quad \cdots (2)$$

$$P(2) = \left(P^{-1}(1) + X^T(2)X(2)\right)^{-1}$$

$$= P(1) - P(1)X^T(2)\left(I + X(2)P(1)X^T(2)\right)^{-1}X(2)P(1)$$

$(\because$ woodbury matrix identity lemma$)$. $\cdots (3)$

$$\begin{bmatrix} X(1) \\ X(2) \end{bmatrix}^T \begin{bmatrix} y(1) \\ y(2) \end{bmatrix} = X^T(1)y(1) + X^T(2)y(2)$$

$$= P^{-1}(1)\omega(1) + X^T(2)y(2) \quad (\because (1))$$

$$= \left(P^{-1}(2) - X^T(2)X(2)\right)\omega(1) + X^T(2)y(2)$$

$(\because (2)) \quad \cdots (4)$

$$\omega(2) = P(2)\begin{bmatrix} X(1) \\ X(2) \end{bmatrix}^T \begin{bmatrix} y(1) \\ y(2) \end{bmatrix}$$

$$= P(2)\left(\left(P^{-1}(2) - X^T(2)X(2)\right)\omega(1) + X^T(2)y(2)\right) \quad (\because (4))$$

$$= \omega(1) - P(2)X^T(2)X(2)\omega(1) + P(2)X^T(2)y(2)$$

$$= \omega(1) + P(2)X^T(2)\left(y(2) - X(2)\omega(1)\right) \quad \cdots (5)$$

Therefore,

$i=2$,

$$w(i) = w(i-1) - P(i)X^T(i)\left(y(i) - X(i)w(i-1)\right)$$

where $$P(i) = P(i-1) - P(i-1)X^T(i)\left(I + X(i)P(i-1)X^T(i)\right)^{-1} X(i)P(i-1) \quad \cdots (P_2) \qquad (\because (5), (3))$$

If P_k is true s.t. $k>2$, $k \in \mathbb{N}$,

P_{k+1} is true. (Trivial)

By mathematical induction,

$\forall i \in \mathbb{N}$,

$$w(i) = w(i-1) - P(i)X^T(i)\left(y(i) - X(i)w(i-1)\right)$$

where $$P(i) = P(i-1) - P(i-1)X^T(i)\left(I + X(i)P(i-1)X^T(i)\right)^{-1} X(i)P(i-1) \qquad \blacksquare$$