$\mathcal{H}_1, \mathcal{H}_2 \Rightarrow$ Linearly separable.

n_0 번 후 $\rightarrow$ $\omega(n_0) = \omega(n_0+1) = \cdots \Rightarrow$ Solution of n_0.

i : (n+1) data 분류 잘된 경우)

$\omega(n+1) = \omega(n)$ (In both cases)

$\begin{cases} X(n) \in C_1 \\ \quad \rightarrow \omega^T(n)X(n) > 0 \\ X(n) \in C_2 \\ \quad \rightarrow \omega^T(n)X(n) \le 0 \end{cases}$

ii : (n+1) data 분류 안된 경우)

$\omega(n+1) = \omega(n) - \eta X(n) \cdots (1)$

$\rightarrow \mathrm{sgn}(\omega^T(n)X(n)) = 1,$

근데 $X(n) \in C_2$.

$\omega(n+1) = \omega(n) + \eta X(n) \cdots (2)$

$\rightarrow \mathrm{sgn}(\omega^T(n)X(n)) = -1,$

but, $X(n) \in C_1$.

Assumptions (Sequence of worst cases)

$\eta = 1,\quad \omega(0) = 0,\quad \omega^T(n)X(n) < 0,\quad X(n) \in C_1,\ \forall n.$

(i) $\omega(n+1) = \omega(n) + X(n)$ (∵ Widrow hoff rule)

$= \sum_{i=1}^{n} X(i)$ (∵)

$\Rightarrow \exists\, \omega_0$ s.t. $\omega^T X(n) > 0$ (선형 분리 가정하에)

$\omega_0^T \omega(n+1) = \omega_0^T X(1) + \omega_0^T X(2) + \cdots + \omega_0^T X(n)$

$\ge n \cdot \min_{X(n)} \omega_0^T X(n)$ (이건 넘어야 됨.)

$$\therefore \|\omega(n+1)\|^2 \geq \frac{n^2\alpha^2}{\|\omega_0\|^2} \quad (\because \text{②}\lambda)$$

(ii) $\omega(k+1) = \omega(k) + X(k)$

$, \quad k=1,2,\cdots,n, \quad X(k) \in C_1.$

$$\|\omega(k+1)\|^2 = \|\omega(k)\|^2 + \|X(k)\|^2 + 2\omega^T(k)X(k)$$

$(\because$ misclassified $\omega^T(k)X(k) < 0)$

$$\leq \|\omega(k)\|^2 + \|X(k)\|^2$$

$$\Rightarrow \|\omega(k+1)\|^2 - \|\omega(k)\|^2 \leq \|X(k)\|^2, \quad k=1,2,\cdots,n.$$

$$\sum_{k=0}^{n}\|X(k)\|^2 \geq \cancel{\|\omega(1)\|^2} - \|\omega(0)\|^2$$

$$+\cancel{\|\omega(2)\|^2} - \cancel{\|\omega(1)\|^2}$$

$$+\cancel{\|\omega(3)\|^2} - \cancel{\|\omega(2)\|^2}$$

$$+ \cdots$$

$$\vdots$$

$$+\|\omega(n+1)\|^2 - \cancel{\|\omega(n)\|^2}$$

$$+\|\omega(n+1)\|^2 - \|\omega(0)\|^2 \geq \|\omega(k+1)\|^2$$

$$\therefore \|\omega(k+1)\|^2 \leq \sum_{k=1}^{n}\|X(k)\|^2 \leq n \cdot \max_{X(k)}\|X(k)\|^2$$

$$\Rightarrow \|\omega(k+1)\|^2 \le n\beta$$

$$\frac{n^2\alpha^2}{\|\omega_0\|^2} \le \|\omega(k+1)\|^2 \le n\beta.$$

$$\Rightarrow n_{max} = \max\left\{ n = 1, 2, \cdots \;\middle|\; \frac{n^2\alpha^2}{\|\omega_0\|^2} = n\beta \right\}$$

$$n \not\to \infty$$

$$\therefore n_{max} = \frac{\beta\|\omega_0\|^2}{\alpha^2} \quad \blacksquare$$