$$\max_{\omega} \left\{ J(\omega) = \frac{\omega^T S_B \omega}{\omega^T S_W \omega} \right\}$$

where $S_B = (m_2 - m_1)(m_2 - m_1)^T$

$$, S_W = \sum_{n \in C_1} (x_n - m_1)(x_n - m_1)^T + \sum_{n \in C_2} (x_n - m_2)(x_n - m_2)^T$$

Let's prove by constraint optimization.

$$\max_{\omega} \{ \omega^T S_B \omega \} \quad \cdots (1)$$

Since we are only interested in direction, length is not important.

Therefore, $\omega^T S_W \omega = 1 \cdots (2)$

$$\max_{\omega} \{ \omega^T S_B \omega \} \quad s.t. \ \omega^T S_W \omega = 1 \quad (\because (1), (2))$$

$$\mathcal{L}(\omega ; \mu) = \omega^T S_B \omega - \mu (\omega^T S_W \omega - 1)$$

$$\frac{\partial}{\partial \omega} \mathcal{L}(\omega ; \mu) = 2 S_B \omega - 2 \mu S_W \omega$$

$\partial_{w} L$

$$\therefore S_W^{-1} S_B w^* = \mu w^* \quad \text{--- (3)}$$

Therefore,

μ and w^* are the eigenvalues and eigenvectors of $S_W^{-1} S_B$. ($\because$ (3))

$$\therefore S_W^{-1}(m_2 - m_1) \propto w \quad \blacksquare$$