$$\min_{\mathbf{w}} \left\{ J(\mathbf{w}) = \frac{1}{2} \mathbb{E}\left[(y - \mathbf{w}^T \mathbf{x})^2 \right] \right\}$$

$$\mathbf{w}_{WF} = \arg\min_{\mathbf{w}} \mathbb{E}\left[(y - \mathbf{w}^T \mathbf{x})^2 \right]$$

Proof.

$$\frac{\partial}{\partial \mathbf{w}} \mathbb{E}\left[(y - \mathbf{w}^T \mathbf{x})^2 \right]$$

$$= \frac{\partial}{\partial \mathbf{w}} \left(\frac{1}{N} \sum_{i=1}^{N} (y_i - \mathbf{w}^T \mathbf{x}_i)^2 \right)$$

$$= \frac{2}{N} \sum_{i=1}^{N} (y_i - \mathbf{w}^T \mathbf{x}_i) \mathbf{x}_i = \mathbf{0}$$

$$\frac{2}{N} \sum_{i=1}^{N} (y_i - \mathbf{w}^T \mathbf{x}_i) \mathbf{x}_i = \frac{2}{N} \sum_{i=1}^{N} y_i \mathbf{x}_i - \frac{2}{N} \sum_{i=1}^{N} (\mathbf{w}^T \mathbf{x}_i) \mathbf{x}_i$$

Therefore,

$$\sum_{i=1}^{N} (\mathbf{w}^T \mathbf{x}_i) \mathbf{x}_i = \sum_{i=1}^{N} y_i \mathbf{x}_i$$

$$\sum_{i=1}^{N} \left((\mathbf{w}^T \mathbf{x}_i) \mathbf{x}_i^T \right)^T$$

$$= \sum_{i=1}^{N} (\mathbf{x}_i \mathbf{x}_i^T)^T \mathbf{w} = N\mathbb{E}[\mathbf{x}\mathbf{x}^T]\mathbf{w} = N\mathbb{E}[\mathbf{x}y]$$

$$\therefore \mathbf{w}_{WF} = (\mathbb{E}[\mathbf{x}\mathbf{x}^T])^{-1}\mathbb{E}[\mathbf{x}y]$$

$$= R_x^{-1} r_{xy} \qquad \blacksquare$$