$$p(w \mid X, y) \propto p(y \mid X, w)\, p(w) \qquad (\because \text{Bayes Thm}).$$

$$\propto -\frac{1}{2\beta}(y - Xw)^T (y - Xw) - \frac{1}{2}(w-\mu_0)^T \Sigma_0^{-1} (w - \mu_0)$$

Inverse of covariance matrix. But, If it goes like this, we cannot derive correct result.

$$= -\frac{1}{2}\Big\{ \underline{w^T \Sigma_0^{-1} w + \frac{1}{\beta} w^T X^T X w}$$

$$\underline{- w^T \Sigma_0^{-1} \mu_0 - \frac{1}{\beta} w^T X^T y}$$

$$- \mu_0^T \Sigma_0^{-1} w - y^T X w + \frac{1}{\beta} y^T y + \mu_0^T \Sigma_0^{-1} \mu_0 \Big\}$$

$$\Sigma_N^{-1} = \Sigma_0^{-1} + \frac{1}{\beta} X^T X.$$

$$\Sigma_N^{-1} \mu_N = \Sigma_0^{-1} \mu_0 + \frac{1}{\beta} X^T y$$

$$\therefore \quad \mu_N = \Sigma_N \left(\Sigma_0^{-1} \mu_0 + \frac{1}{\beta} X^T y \right)$$

$$\Sigma_N = \left(\Sigma_0^{-1} + \frac{1}{\beta} X^T X \right)^{-1}$$

$$= \Sigma_0 - \Sigma_0 X^T (X \Sigma_0 X^T + \beta I)^{-1} X \Sigma_0 \quad (\because \text{Matrix Inversion Lemma.})$$