Comparing Delay-Constrained ALOHA and CSMA: A Learning-Based Low-Complexity Approximate Approach

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***Abstract*— Supporting delay-constrained traffic becomes more and more critical in multimedia communication systems, tactile Internet, networked control systems, and cyber-physical systems, etc. In delay-constrained traffic, each packet has a hard deadline—when it is not delivered before the hard deadline, it becomes useless and will be removed from the system. This feature is completely different from that of traditional delay-unconstrained traffic and brings new challenge to network protocol design. We focus on delay-constrained wireless communication in this paper. Many works design centralized scheduling policies in the downlink while a few works investigate distributed wireless access protocols in the uplink for delay-constrained traffic. In this work, we study the widely-used (slotted) ALOHA and CSMA wireless access protocols but under the new delay-constrained setting. Our goal is to compare delay-constrained ALOHA and CSMA for different system settings and thus give network operators guidelines on protocol selection. We use two Markov chains to analyze delay-constrained ALOHA and CSMA, respectively. However, the number of states of Markov chains increases exponentially with respect to the number of users in the network. Therefore, we can only compare the exact performance of delay-constrained ALOHA and CSMA for small-scale networks. To address the curse of dimensionality, we design a single-user parameterized ALOHA (resp. CSMA) system, where the parameters are to be learned to approximate the original multi-user ALOHA (resp. CSMA) system. We use small-scale networks to learn the best parameters and then apply the low-complexity approach to compare the approximate performance of delay-constrained ALOHA and CSMA in large-scale networks. We finally reveal the conditions under which ALOHA (resp. CSMA) outperforms CSMA (resp. ALOHA) in the delay-constrained setting.**

I. Introduction

Delay-constrained applications become widespread nowadays. Typical examples include multimedia communication systems such as real-time streaming and video conferencing [1], tactile Internet [2], [3], networked control systems (NCSs) such as remote control of unmanned aerial vehicles (UAVs) [4], [5], and cyber-physical systems (CPSs) such as medical tele-operations, X-by-wire vehicles/avionics, factory automation, and robotic collaboration [6]–[8]. In such applications, each packet has a hard deadline: if it is not delivered before the deadline, it becomes useless and will be removed from the system. On the other hand, wireless communication is ubiquitous because it can be easily deployed with low cost and low complexity. We focus on delay-constrained wireless communication in this paper. Many works design centralized scheduling policies in the downlink [1], [9]–[11], while a few works investigate distributed wireless access protocols in the uplink for delay-constrained traffics [12]–[14].

ALOHA and carrier sense multiple access (CSMA) are two widely used random access protocols in traditional delay-unconstrained wireless communication. The advantage of ALOHA is that it is extremely simple. Since Abramson invented pure ALOHA in 1970 [15], a variety of other ALOHA-type protocols have been designed. Among them, one popular type is *slotted ALOHA*, where users are synchronized and can only transmit data at the beginning of a slot [16]. We focus on slotted ALOHA in the rest of this paper. For simplicity, we will call it ALOHA if there is no ambiguity through the context. There are also many types of extension for slotted ALOHA protocol, including multi-packet reception [17], [18], framed slotted ALOHA [19]–[21], coded slotted ALOHA [22], slotted ALOHA with successive interference cancellation (SIC) [23], and stability analysis of the slotted ALOHA systems [24]–[26], etc. CSMA is a more sophisticated wireless access protocol than ALOHA, which is divided into non-persistent CSMA, p-persistent CSMA and 1-persistent CSMA [27]. Previous studies have shown that the access mechanism and the backoff-time mechanism are important design spaces to improve the performance of CSMA [28], [29]. There are also some works to compare ALOHA and CSMA under the delay-unconstrained setting [30], [31].

Due to the great success of ALOHA and CSMA, it deserves to investigate how they work in the delay-constrained setting. There are some existing works on delay-constrained ALOHA [14], [32] and delay-constrained CSMA [33]. Deng *et al.* in [14] analyze the asymptotic performance of ALOHA system for frame-synchronized delay-constrained traffic pattern. Zhang *et al.* in [32] study the system throughput and optimal retransmission probability of ALOHA under the saturated delay-constrained traffic. Campolo *et al.* in [33] analyze the p-persistent CSMA for broadcasting delay-constrained traffic. However, to the best of our knowledge,

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currently no work compares ALOHA and CSMA under the delay-constrained setting.

In this paper, we aim at theoretically providing a comprehensive comparison for delay-constrained ALOHA and delay-constrained CSMA protocols. We remark that previous studies [14], [32] assume that the packet size $L = 1$, i.e., a packet can be delivered in one slot. However, in many applications, the packet size can be large such that it cannot be delivered in one slot. To capture this case, in this paper we generalize L to be an arbitrary positive integer, and thus the delivery of a packet needs L slots. Partial delivery does not contribute to the throughput. Embedded with this new feature, we make the following contributions.

- For given number of users N, hard deadline D, packet size L, we construct two Markov chains to analyze delay-constrained ALOHA and CSMA. By analyzing the state distributions of the Markov chains, we obtain the exact theoretical system timely throughput for delay-constrained ALOHA and CSMA.
- The number of states of Markov chains in the previous exact characterization increases exponentially with respect to the number of users. Therefore, we can only compare the exact performance of delay-constrained ALOHA and CSMA for small-scale networks. We thus design a parameterized ALOHA (resp. CSMA) system in view of only one user where there are two parameters to be learned to approximate the original multi-user ALOHA (resp. CSMA) system. We use the small-scale networks to learn the best parameters and then apply the low-complexity approach to compare the approximate performance of delay-constrained ALOHA and CSMA in large-scale networks. The numerical results show the effectiveness of this approximate approach.
- Using our proposed low-complexity approximate approach, we compare delay-constrained ALOHA and CSMA for different system settings, and then summarize the conditions under which delay-constrained ALOHA (resp. CSMA) outperforms delay-constrained CSMA (resp. ALOHA).

The rest of this paper is outlined as follows. Sec. II describe the system model. In Sec. III, we construct two Markov chains to analyze the exact performance of delay-constrained ALOHA and CSMA, respectively. In Sec. IV, we propose a learning-based low-complexity approximate approach. Sec. V shows our numerical results and summarizes the condition under which delay-constrained ALOHA (resp. CSMA) outperforms delay-constrained CSMA (resp. ALOHA). Sec. VI concludes this paper.

II. System Model

We consider a wireless network with N users who need to independently deliver delay-constrained traffics to a common receiver by competing for a shared wireless channel. We consider a slotted system indexed from 1. Similar to previous studies [1], [9], [10], [14], we consider the frame-synchronized traffic pattern. But we characterize the traffic pattern by parameters $L \in \mathbb{Z}^+$ and $D \in \mathbb{Z}^+$. More precisely, starting from slot 1, each user has a new packet arrival every D slots. All packets are of size L and have a hard delay/deadline of D slots. A packet will be useless and removed from the system if it cannot be delivered within D slots after its arrival. We also call the duration from slot $(k-1)T+1$ to slot kT *frame* k or *period* k where $k = 1, 2, \cdots$.

Note that existing studies [1], [9], [10], [14] assume that all packets are of unit size, i.e., $L = 1$, meaning that only a packet can be delivered successfully in a slot. In many practical applications, the packet size could be large so that it cannot be delivered in one slot. In this paper, to capture such applications, we generalize packet size to a positive integer L, meaning that a user needs L slots to deliver a packet. In addition, a packet can be divided into L different units for transmission but we do not allow partial delivery. Thus, a packet can contribute to the system performance in terms of timely throughput, which will be described in (1) shortly, only when the whole packet of size L has been completely delivered within D slots after its arrival. Since sending a packet needs at least L slots and the deadline is D, without loss of generality, we assume that $L \leq D$.

It is straightforward to see that any user has at most one *non-expired* packet for delivery in any slot. For any user, if the packet arrived at the beginning of a period has been completely delivered successfully to the receiver, this user remains idle until the end of this period. Therefore, for our wireless access problem, we do not need to handle the packet queueing problem. In addition, since each packet is of size L and the transmission capacity of each slot is only one unit, we divide each packet into L units and call them unit 1, unit 2, $\cdots$, and unit L. In our wireless access problem, we sequentially transmit these L units. Namely, we keep transmitting unit i until it has been successfully delivered to the receiver. After that, we keep transmitting unit $i+1$. If the final unit L has been delivered successfully to the receiver before the end of this period, this packet is successfully delivered and can contribute to this user's timely throughput.

In this paper, we adopt conventional wireless access setting—if two or more users send data in the same slot, a collision happens and no data can be delivered to the receiver; if only one user transmits data, it can be successfully delivered to the receiver. Same as previous studies on delay-constrained communications, we use *timely throughput* to characterize the system performance. The timely throughput of user i is defined as

$$R_i \triangleq \lim_{k \to \infty} \frac{L \cdot \mathbb{E}\left[\begin{smallmatrix}\text{number of packets of user } i \text{ delivered} \\ \text{before expiration from slot 1 to slot } kD\end{smallmatrix}\right]}{kD}. \quad (1)$$

Our goal is to try to maximize the system timely throughput $R = \sum_{i=1}^{N} R_i$.

In the following, we will consider two widely used wireless access protocols, slotted ALOHA and CSMA.

III. Theoretical Analysis

A. Slotted ALOHA

Mechanism. In slotted ALOHA protocol, in each slot, if a user has a packet that has not yet delivered successfully before its deadline, this user transmits the current unit of this packet to the receiver with probability $p \in [0,1]$; otherwise, if unit L of the packet has been delivered successfully, this user remains idle. Note that the transmission probability p needs to be optimized to maximize the system timely throughput. We assume that the transmission events of all users in any slot are independent as same as the traditional delay-unconstrained ALOHA protocol [16], [27].

Markov Chain Analysis. For given number of users N, packet size L, and hard deadline D, we construct an Markov chain to analyze the theoretical performance of slotted ALOHA. In particular, the system state in any slot is denoted by

$$s = [(l_1,t),(l_2,t),\cdots,(l_N,t)],$$

where $l_i \in \{0,1,\cdots,L\}$ is the number of units of the current (non-expired) packet that has been delivered successfully to the receiver before the current slot and $t \in \{1,2,\cdots,D,D+1\}$ is the index of the current slot relative to the beginning of this period. Note that we construct a *virtual slot*, i.e., $t = D+1$, to indicate the end of the period, which is used for calculating the system timely throughput from the state distribution. Let us consider an example with $N=2, L=2$ and $D=3$. State $s = [(l_1,t),(l_2,t)] = [(0,1),(0,1)]$ means that this slot is the beginning (the first slot) of the period and no unit has been delivered successfully to the receiver before this slot, since a new packet just arrives at the system in the beginning of this slot. State $s = [(l_1,t),(l_2,t)] = [(0,2),(1,2)]$ means that this slot is the second slot of the period and no unit of user 1 has been delivered successfully to the receiver before this slot while unit 1 of user 2 has been delivered successfully to the receiver before this slot. State $s = [(l_1,t),(l_2,t)] = [(1,4),(2,4)]$ means that this slot is the virtual slot to indicate the end of the period and user 1 only transmits one unit while user 2 has delivered the whole packet to the receiver. Therefore, the packet of user 2 contributes to the timely throughput of user 2 while the packet of user 1 is discarded and does not contribute to the timely throughput of user 1. Note that the state space, denoted by $\mathcal{S}_{\mathsf{ALOHA}}$, is of size

$$[(L+1)]^N \cdot (D+1). \tag{2}$$

Now we construct the transition probabilities of the constructed Markov chain for slotted ALOHA protocol. For state $s = [(l_1,t),(l_2,t),\cdots,(l_N,t)] \in \mathcal{S}_{\mathsf{ALOHA}}$, where $t \in \{1,2,\cdots,D\}$, we divide all N users into two set $\mathcal{N}_1$ and $\mathcal{N}_2$, where $\mathcal{N}_1$ is the set of users who have not yet completely delivered the packet and $\mathcal{N}_2$ is the set of users who have already completely delivered the packet before this slot. That is,

$$\mathcal{N}_1 = \{i : l_i < L, i = 1,2,\cdots,N\},$$

and

$$\mathcal{N}_2 = \{i : l_i = L, i = 1,2,\cdots,N\}.$$

Thus, in this slot, users in $\mathcal{N}_2$ remain idle and users in $\mathcal{N}_1$ transmit one unit with probability p. We can compute the transition probabilities as follows:

$$\begin{aligned}
&P\left\{[(l_1,t+1),\cdots,(l_i+1,t+1),\cdots,(l_N,t+1)]\,|s\right\}\\
&= p(1-p)^{|\mathcal{N}_1|-1}, \forall i \in \mathcal{N}_1,\\
&P\left\{[(l_1,t+1),(l_2,t+1),\cdots,(l_N,t+1)]\,|s\right\}\\
&= 1 - |\mathcal{N}_1|p(1-p)^{|\mathcal{N}_1|-1},
\end{aligned} \tag{3}$$

and the transition probabilities from state s to all other states are zero. Note that (3) follows from the fact that there exists at most one successful transmission in any slot.

Now we have a transition matrix $\boldsymbol{P} = [P(s'|s) : s, s' \in \mathcal{S}_{\mathsf{ALOHA}}]$. Next we give an initial state distribution (in the beginning of slot 1) $\boldsymbol{\pi}^1 = \left(\pi_s^1 : s \in \mathcal{S}_{\mathsf{ALOHA}}\right)$ as

$$\pi_s^1 = \begin{cases} 1, & \text{If } s = [(0,1),(0,1),\cdots,(0,1)];\\ 0, & \text{Otherwise.}\end{cases}$$

Then the state distribution in the beginning of slot 2 is

$$\boldsymbol{\pi}^2 = \boldsymbol{\pi}^1 \boldsymbol{P}.$$

Similarly, we can obtain the the state distribution in the beginning of the virtual slot $D+1$,

$$\boldsymbol{\pi}^{D+1} = \boldsymbol{\pi}^1 \boldsymbol{P}^D. \tag{4}$$

Based on the state distribution $\boldsymbol{\pi}^{D+1}$, we can compute the timely throughput of user i by

$$R_i = \frac{L}{D} \cdot \left\{ \sum_{\substack{s=[(l_1,D+1),\cdots(l_i,D+1),\cdots,(l_N,D+1)]:\\ l_i=L,l_j\in\{1,2,\cdots,L\},\forall j\neq i}} \pi_s^{D+1} \right\}.$$

Then we can obtain the exact theoretical system timely throughput $R = \sum_{i=1}^N R_i$.

Note the the achieved system timely throughput R depends on number of users N, hard deadline D, packet size L, and the transmission probability p. To show this dependance clearly, we denote $R_{\mathsf{ALOHA}}(N,D,L,p)$ as the the exact theoretical system timely throughput of slotted ALOHA protocol for given N, D, L and p.

To maximize the system performance, we need to find the best transmission probability p, i.e.,

$$p^*(N,D,L) = \arg\max_{p\in[0,1]} R_{\mathsf{ALOHA}}(N,D,L,p).$$

It is difficult to find closed-form $p^*(N,D,L)$. In this paper, we numerically search it with an adjustable stepsize to control the precision. We denote the maximum system timely throughput of ALOHA by $R^*_{\mathsf{ALOHA}}(N,D,L)$.

B. CSMA

Mechanism. In CSMA protocol, each node has the capability of carrier sensing to check whether the wireless channel is idle or not. If a user has a new packet arrival (i.e., in the beginning of a period), it randomly selects an integer value b from $[0, D-1]$ as the backoff time. Then, in each slot, the user performs carrier sensing to check whether the channel is idle or not. If the channel is busy, the backoff-time value b is frozen; otherwise the backoff-time value b decreases by 1. If $b = 0$, there are two cases. If the packet has been delivered or the remaining number of units is larger than the remaining number of slots before expiration, the user remains idle in this slot; otherwise, the user transmits the current unit to the receiver in this slot. If the transmitted unit is delivered successfully, the user keeps transmitting the next unit in the next slot; otherwise, if the transmitted unit is not delivered successfully (i.e., a collision happens), the user restarts the backoff process by randomly selecting an integer value b from $[0, D-1]$ as the backoff time. The detailed behavior of each user is shown in Algorithm 1.

Markov Chain Analysis. For given number of users N, packet size L, and hard deadline D, we present an Markov chain to analyze the exact theoretical performance of CSMA. In particular, the system state in any slot is denoted by

$$s = [(b_1, l_1, t), (b_2, l_2, t), \cdots, (b_N, l_N, t)],$$

where $b_i \in \{0, 1, \cdots, D-1\}$ is the backoff-time value of user i at the current slot, $l_i \in \{0, 1, \cdots, L\}$ is the number of units that has been delivered successfully to the receiver before the current slot and $t \in \{0, 1, 2, \cdots, D, D+1\}$ is the index of the current slot relative to the beginning of this period. Similar to ALOHA, we construct a virtual slot $t = D+1$ to indicate the end of a period. In addition, we construct a virtual state

$$s_0 = [(b_1, l_1, t), \cdots, (b_N, l_N, t)] = [(0,0,0), \cdots, (0,0,0)]$$

to indicate the start of a period. This means that we construct another virtual slot $t = 0$ for each period. The state space of CSMA, denoted by $\mathcal{S}_{\mathsf{CSMA}}$, is of size

$$1 + [D(L+1)]^N \cdot (D+1). \tag{5}$$

For the initial state s_0, the transition probability to state

$$s = [(b_1, 0, 1), \cdots, (b_N, 0, 1)], \forall b_i \in \{0, 1, \cdots, D-1\}$$

is $\left[\frac{1}{D}\right]^N$.

For any state $s = [(b_1, l_1, t), (b_2, l_2, t), \cdots, (b_N, l_N, t)]$ where $t \in \{1, 2, \cdots, D\}$, we divide the user set into three sets,

$$\begin{aligned}\mathcal{N}_1 = \{&i : l_i = L, i = 1, \cdots, N\} \cup \\ &\{i : L - l_i > D - [(t-1) \mod D], i = 1, \cdots, N\},\\ \mathcal{N}_2 = \{&i : b_i = 0, i \in \{1, 2, \cdots, N\} \setminus \mathcal{N}_1\},\\ \mathcal{N}_3 = \{&i : b_i > 0, i \in \{1, 2, \cdots, N\} \setminus \mathcal{N}_1\}.\end{aligned}$$

Any user $i \in \mathcal{N}_1$ either has already successfully delivered the packet or cannot complete the delivery even it transmits in all the rest of slots before the end of the period. Thus, any user $i \in \mathcal{N}_1$ remains idle in this slot. The backoff-time value of any user $i \in \mathcal{N}_2$ is 0, i.e., $b_i = 0$, and thus it transmits the current unit in this slot. The backoff-time value of any user $i \in \mathcal{N}_3$ is larger than 0, i.e., $b_i > 0$, and thus it performs carrier sensing.

Let us denote the state in the next slot by

$$s' = [(b'_1, l'_1, t+1), (b'_2, l'_2, t+1), \cdots, (b'_N, l'_N, t+1)],$$

which depends on the value of $|\mathcal{N}_2|$. We initialize $b'_i = b_i, l'_i = l_i, \forall i = 1, 2, \cdots N$ in state s' and then discuss which of them will be changed (the changing parts of state s'). If $|\mathcal{N}_2| = 0$, no user transmits data in this slot. Thus, all users in $\mathcal{N}_3$ sense to know that the channel is idle and decrease

Algorithm 1 CSMA Protocol of A User

```
Require: Hard deadline $D$, packet size $L$
Set $b = 0$, $u = 1$
for $t = 1, 2, \cdots,$ do
    if $(t-1) \mod D = 0$ then
        Randomly select an integer $b$ from $[0, D-1]$
        Set $u = 1$
        if $b = 0$ then
            Transmit unit $u$ to the receiver in slot $t$
            if The unit is delivered successfully then
                Set $u = u + 1$
            else
                Randomly select an integer $b$ from $[0, D-1]$
            end if
        else
            Perform carrier sensing in slot $t$
            if The channel is idle then
                Set $b = b - 1$
            end if
        end if
    else
        if $u = L + 1$ or $L - u + 1 > D - [(t-1) \mod D]$ then
            Remain idle
        else
            if $b = 0$ then
                Transmit unit $u$ to the receiver in slot $t$
                if The unit is delivered successfully then
                    Set $u = u + 1$
                else
                    Randomly select an integer $b$ from $[0, D-1]$
                end if
            else
                Perform carrier sensing in slot $t$
                if The channel is idle then
                    Set $b = b - 1$
                end if
            end if
        end if
    end if
end for
```

the backoff-time value by 1. Thus, the updated portion of state s' is

$$b_i' = b_i - 1, \forall i \in \mathcal{N}_3.$$

The transition probability is $P(s'|s) = 1$. If $|\mathcal{N}_2| = 1$, only one user $i \in \mathcal{N}_2$ transmits unit $l_i + 1$ in this slot. Thus, no collision happens and the transmission is successful. In addition, since the channel is busy, the backoff-time values of all users in $\mathcal{N}_3$ are frozen. Thus, the updated portion of state s' is

$$l_i' = l_i + 1, i \in \mathcal{N}_2.$$

Again, the transition probability is $P(s'|s) = 1$. If $|\mathcal{N}_2| > 1$, all users (more than one) in $\mathcal{N}_2$ transmit data in this slot. Thus, a collision happens and all transmissions fail. Therefore, each of all users in $\mathcal{N}_2$ restarts the backoff process by randomly selecting a backoff-time value from $[0, D-1]$. In addition, since the channel is busy, the backoff-time values of users in $\mathcal{N}_3$ are frozen. Therefore, the updated portion of state s' is

$$b_i' \in \{0, 1, \cdots, D-1\}, \forall i \in \mathcal{N}_2.$$

There are in total $D^{|\mathcal{N}_2|}$ possibilities for state s' and the transition probability is $P(s'|s) = \frac{1}{D^{|\mathcal{N}_2|}}$ for each possible s'.

Now we have a transition matrix $\boldsymbol{P} = [P(s'|s) : s, s' \in \mathcal{S}_{\mathsf{CSMA}}]$. Next we give an initial state distribution (in the beginning of virtual slot 0) $\boldsymbol{\pi}^0 = \left(\pi_s^0 : s \in \mathcal{S}_{\mathsf{CSMA}}\right)$ as

$$\pi_s^0 = \begin{cases} 1, & \text{If } s = [(0,0),(0,0),\cdots,(0,0)]; \\ 0, & \text{Otherwise.} \end{cases}$$

Then the state distribution in the beginning of slot 1 is

$$\boldsymbol{\pi}^1 = \boldsymbol{\pi}^0 \boldsymbol{P}.$$

Similarly, we can obtain the state distribution in the beginning of the virtual slot $D+1$ as

$$\boldsymbol{\pi}^{D+1} = \boldsymbol{\pi}^0 \boldsymbol{P}^{D+1}. \tag{6}$$

Based on the state distribution $\boldsymbol{\pi}^{D+1}$, we can compute the timely throughput of user i by

$$R_i = \frac{L}{D} \cdot \sum_{\substack{s=[(b_1,l_1,D+1),\cdots(b_i,l_i,D+1),\cdots,(b_N,l_N,D+1)]:l_i=L, \\ l_j \in \{1,2,\cdots,L\}, \forall j \neq i, b_j \in \{0,1,\cdots,D-1\}, \forall j \in \{1,2\cdots,N\}}} \pi_s.$$

Then we can obtain the exact theoretical system timely throughput $R = \sum_{i=1}^N R_i$.

Note that the achieved system timely throughput R depends on number of users N, hard deadline D, and packet size L. To show this dependance clearly, we denote $R_{\mathsf{CSMA}}(N, D, L)$ as the exact theoretical system timely throughput of CSMA protocol for given N, D and L.

IV. A Learning-based Low-complexity Approximate Approach

In the previous section, we use Markov-chain analysis to obtain the exact system timely throughput for both ALOHA and CSMA protocols. However, when we solve the equations (4) and (6) to obtain the final-slot state distributions for ALOHA and CSMA, respectively, the computational complexity exponentially increases with respect to the total number of users, i.e., N. This is because the number of states for both ALOHA and CSMA exponentially increase with respect to N, as shown in (2) and (5). Therefore, we can only apply the exact approach in the previous section for small-scale networks. For example, under our computational resources, we can only obtain the exact system timely throughput for $N \leq 10$. To compare ALOHA and CSMA broadly, we need to figure out how to evaluate the system timely throughput for large-scale networks.

In this section, we propose a learning-based low-complexity approach to obtain an approximate value of the exact system timely throughput for both ALOHA and CSMA. Note that the exact system timely throughput for ALOHA (resp. CSMA), i.e., $R^*_{\mathsf{ALOHA}}(N, D, L)$ (resp. $R_{\mathsf{CSMA}}(N, D, L)$), depends on three parameters, the number of users N, the hard deadline D, and the packet size L. If we can learn functions $R^*_{\mathsf{ALOHA}}(N, D, L)$ and $R_{\mathsf{CSMA}}(N, D, L)$ based on small-scale networks, we can predict the system performance for large-scale networks. This is the basic idea of the proposed learning-based low-complexity approach. In particular, we use the proposed exact Markov-chain analysis to obtain a dataset for small-scale networks, and then apply machine learning approaches to learn the functions $R^*_{\mathsf{ALOHA}}(N, D, L)$ and $R_{\mathsf{CSMA}}(N, D, L)$. Note that the function types could be very arbitrary and we also have a large number of machine learning approaches. We next propose our learning approach by leveraging Markov-chain structures of ALOHA and CSMA. The basic idea is as follows. The exponential complexity of the exact approach comes from the number of users, i.e., N. We thus try to reduce the total number of users but find a way to maintain the structure of the multi-user Markov chains. Concretely, we will respectively construct two parameterized single-user Markov chains to mimic the original multi-user Markov chains for ALOHA and CSMA where the parameters need to be learned.

A. ALOHA

For slotted ALOHA, we construct a single-user Markov chain parameterized by a transmission probability $p_t \in [0, 1]$ and a success probability $p_s \in [0, 1]$. The single-user Markov chain behaves as follows. In each slot of a period, if the packet of the user in this period has not been successfully delivered to the receiver, the user transmits the current unit to the receiver with probability p_t. The transmitted unit is then delivered successfully with probability p_s. The transmission probability p_t mimics the transmission probability p in the original multi-user ALOHA system, while the success probability p_s mimics the potential collision from other users in the original multi-user ALOHA system, as shown in Sec. III-A. Clearly, we can construct an Markov chain for this particular user similar to Sec. III-A. We obtain the final-slot state distribution to calculate the timely throughput, denoted by $R^{\text{Approx}}_{\text{ALOHA}}(p_t, p_s)$. The total number of state is $(L+1)(D+1)$, which is linear with L and D, regardless of N. Thus, the computational complexity is exponentially reduced. Our goal is to try to estimate the achieved timely throughput of the original multi-user ALOHA system for given N, D, L, i.e., $R^{*}_{\text{ALOHA}}(N, D, L)$, from the achieved timely throughput of this single-user system. Namely, for given N, D, L, we aim at finding suitable p_t and p_s to minimize

$$\left| N \cdot R^{\text{Approx}}_{\text{ALOHA}}(p_t, p_s) - R^{*}_{\text{ALOHA}}(N, D, L) \right|.$$

The reason that we use two parameters p_t and p_s to construct the single-user Markov chain for ALOHA is because the combination of p_t and p_s reasonably mimics the original multi-user ALOHA system. However, it is easy to check that the transition matrix of the single-user Markov chain only depends on the product of p_t and p_s, but does not depend on the individual value of p_t or p_s. Therefore, the achieved timely throughput $R^{\text{Approx}}_{\text{ALOHA}}(p_t, p_s)$ also only depends on the product of p_t and p_s. Without loss of generality, next we assume that $p_t = 1$, i.e., the single user transmits the current unit in all slots for sure. We then denote the achieved timely throughput by $R^{\text{Approx}}_{\text{ALOHA}}(p_s)$.

Thus, for given N, D, L, we need to find suitable p_s to minimize

$$\left| N \cdot R^{\text{Approx}}_{\text{ALOHA}}(p_s) - R^{*}_{\text{ALOHA}}(N, D, L) \right|. \tag{7}$$

Namely, we need to learn function

$$p_s = f_1(N, D, L). \tag{8}$$

We construct the dataset as follows. For given D, L, and a small N, we obtain the exact system timely throughput, i.e., $R^{*}_{\text{ALOHA}}(N, D, L)$, by using the multi-user Markov-chain analysis in Sec. III-A. We find the best p_s to minimize (7) via the binary-search approach (since $R^{\text{Approx}}_{\text{ALOHA}}(p_t, p_s)$ strictly increases as p_s increases). Then we obtain a dataset

$$(N^i, D^i, L^i, p^i_s), \quad i = 1, 2, \cdots, K,$$

where K is the total number of data points and i is the index of a data point in the dataset. We apply machine learning approaches to predict function p_s as shown in (8).

Now given D, L, and a large N, we can use our learned model to predict p_s and then construct a single-user Makrov chain to obtain $N \cdot R^{\text{Approx}}_{\text{ALOHA}}(p_s)$, which serves as an approximate value of $R^{*}_{\text{ALOHA}}(N, D, L)$.

B. CSMA

For CSMA, we construct a single-user Markov chain parameterized by a channel-busy probability $p_b \in [0, 1]$ and a collision probability $p_c \in [0, 1]$. The single user follows the same steps in Algorithm 1 except the followings. First, when the user performs carrier sensing as shown in lines 14 and 31 in Algorithm 1, the channel is busy with probability p_b. Second, when the user transmits data as shown in lines 7 and 24, a collision happens with probability p_c and thus the transmitted unit can be delivered successfully with probability $1 - p_c$. The channel-busy probability p_b and the collision probability p_c mimic the potential transmissions from other users in the original multi-user CSMA system, as shown in Sec. III-B. Clearly, we can construct an Markov chain for this single user similar to Sec. III-B. We then obtain the final-slot state distribution to calculate the timely throughput, denoted by $R^{\text{Approx}}_{\text{CSMA}}(p_b, p_c)$. The total number of state is $1 + D(L+1)(D+1)$, which is linear with L and quadratic with D, regardless of N. Again, the computational complexity is exponentially reduced. Our goal is to try to estimate the achieved timely throughput of the original multi-user CSMA system for given N, D, L, i.e., $R_{\text{CSMA}}(N, D, L)$, from the achieved timely throughput of this single-user system.

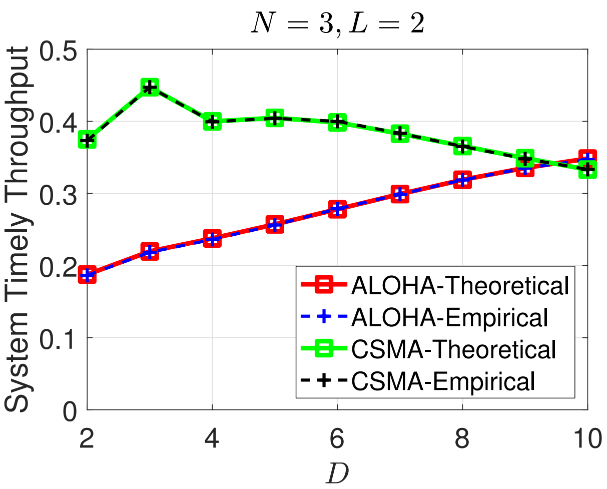


Fig. 1. The system timely throughput of $N = 3$ and $L = 2$.

Thus, we need to predict two parameters p_b and p_c for given parameters N, D and L, i.e.,

$$p_b = f_2(N, D, L),$$

and

$$p_c = f_3(N, D, L).$$

The learning process for p_b and p_c is similar to that for p_s in ALOHA as shown in Sec. IV-A.

Now given D, L , and a large N, we can use our learned model to predict p_b and p_c and then construct a single-user Makrov chain to obtain $N \cdot R^{\text{Approx}}_{\text{CSMA}}(p_b, p_c)$, which serves as an approximate value of $R^{*}_{\text{CSMA}}(N, D, L)$.

V. Numerical Results

A. Confirm Our Exact Theoretical Analysis

In Sec. III, we propose using Markov-chain analysis to obtain the theoretical system timely throughput for both ALOHA and CSMA. We need to confirm the correctness of this exact approach. We consider $N = 3, L = 2$ and vary D from 2 to 10. We first use the proposed exact

TABLE I

THE COST VALUE FOR PREDICTING PARAMETERS IN ALOHA AND CSMA UNDER DIFFERENT LEARNING APPROACHES.

Cost		Linear Regression		Neural Network		
		First-order	Second-order	One Layer	Two Layers	Three Layers
ALOHA	p_s	0.0665	0.2137	0.00256	0.00529	0.00649
CSMA	p_b	0.0930	0.2847	0.0640	0.06886	0.06986
	p_c	0.1686	0.2350	0.02695	0.03711	0.04185

approach to get the theoretical system timely throughput. We then simulate a multi-user ALOHA system and a multi-user CSMA system to get the empirical system timely throughput by running 100,000 periods. The results are shown in Fig. 1. As we can see, the theoretical values match well with the empirical values for both ALOHA and CSMA. This confirms the correctness of our exact theoretical analysis in Sec. III.

B. Learning Results of the Approximate Approach

In our approximate approach, we need to learn parameter p_s for ALOHA and learn parameters p_b and p_c for CSMA with respect to N, D, and L. As we can only obtain the theoretical system timely throughput for small-scale networks, we have collected 900 data points for ALOHA and 2400 data points for CSMA. We randomly choose 70% of the dataset to be the training dataset and leave the rest 30% as the test dataset. We can use many possible machine learning approaches. In this paper, we consider two classic approaches: linear regression and neural network. The cost function is defined as the mean square error. Taking p_s as an example, the cost function is

$$J = \frac{1}{K}\sum_{i=1}^{K}[\bar{p}_s^i - p_s^i]^2, \tag{9}$$

where K is the total number of data points in the dataset, $\bar{p}_s^i$ is the predicted value, and p_s^i is the true value.

For linear regression, we consider the first-order linear regression and the second-order linear regression. For neural network, we consider the number of layers from one to three. We report the results of these different learning approaches in Table I. We remark that it is possible to apply high-order linear regressions and multi-layer neural networks (even deep learning with many layers). However, based on our investigation, such approaches do not have benefits but lead to over-fitting. From Table I, we can see that the one-layer neural network is the best learning approach for both predicting p_s for ALOHA and for predicting p_b and p_c for CSMA.

Although we have predicted parameters for ALOHA and CSMA, they are used to approximate the theoretical system timely throughput. Thus, we need to evaluate the accuracy of our proposed approximate approach. Again, we use the mean square error as the performance metric. Given N, D, L, for ALOHA (resp. CSMA) system, we use the single-user Markov chain parameterized by the best-learned success probability $\bar{p}_s$ (resp. the best-learned channel-busy probability $\bar{p}_b$ and the best-learned collision probability $\bar{p}_c$) to obtain the timely throughput, N times of which serves as the predicted (approximate) value for the theoretical system timely throughput of the multi-user ALOHA (resp. multi-user CSMA) system. The results are shown in Tabel II. As we can see, our prediction is relatively accurate with mean-square error in the order of 10^{-1} or 10^{-2}. This shows the effectiveness of our approach.

TABLE II

THE COST VALUE FOR PREDICTING THE THEORETICAL SYSTEM TIMELY THROUGHPUT UNDER OUR BEST APPROACH.

Cost	Our best Approach
ALOHA	0.0108
CSMA	0.1339

C. Compare ALOHA and CSMA

Our goal of this paper is to compare ALOHA and CSMA for a broad number of system settings. With the help of our learning-based low-complexity approximate approach, we can compare ALOHA and CSMA for different N, D, L. It is not easy to find patterns from a 3-D figure. Thus, we plot the 2-D projections for given N, given D or given L.

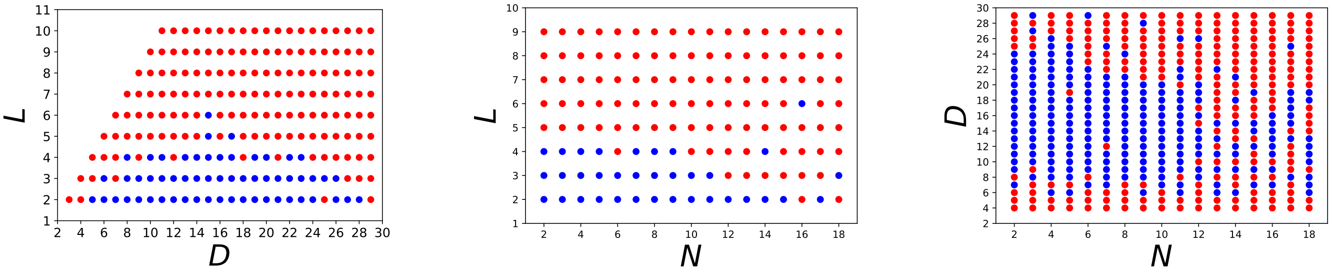
For $N = 4$, we vary L from 2 to 10 and D from L to 29. The result is shown in Fig. 2(a). As we can see, for this given N, In most cases, CSMA is better than ALOHA. But when L is small, ALOHA is better than CSMA for some D values.

For $D = 10$, we vary N from 2 to 18 and L from 2 to 9. The result is shown in Fig. 2(b). Again, CSMA is better than ALOHA when L is large. When L is small, ALOHA is superior to CSMA for small N values.

For $L = 3$, we vary N from 2 to 18 and D from 4 to 29. The result is shown in Fig. 2(c). Since L is relatively small, either ALOHA or CSMA could be better. However, the set $\{(N, D) : 2 \leq N \leq 18, 4 \leq D \leq 29, N \in \mathbb{Z}^+, D \in \mathbb{Z}^+\}$ is divided into several "closed" subsets, in each of which either ALOHA or CSMA is better.

VI. CONCLUSION

In this paper, we apply Makrov chains to analyze the theoretical system performance for delay constrained ALOHA and CSMA systems. Since the number of states of Markov chains grows exponentially, the exact approach can only be applied for small-scale networks. We thus exploit the structures of ALOHA and CSMA protocols and propose a learning-based low-complexity approximate approach. Our numerical results show the effectiveness of our proposed approximate approach. We further use this approximate approach to compare ALOHA and CSMA for a broad number of system settings and summarize the conditions under which each of them outperforms the other one. In the future, it is interesting and important to generalize the frame-synchronized traffic pattern.



(a) Fix $N = 4$

(b) Fix $D = 10$

(c) Fix $L = 3$

Fig. 2. Comparing ALOHA and CSMA for different settings where the red bullet (●) means that CSMA outperforms ALOHA and the blue bullet (●) means that ALOHA outperforms CSMA.

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