Robust Energy-Efficient Power Loading for MIMO System under Imperfect CSI

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Abstract. In this paper, we will analyze the energy efficient power loading in MIMO-SVD architecture. Existing power loading schemes are developed on assumption that a scheduler possesses perfect channel state information (CSI). But we take into account the effects of channel estimation error (CEE) and propose a robust energy-efficient power loading for MIMO system under imperfect CSI. We propose two algorithms to solve the optimization problem. The simulation results show the effectiveness of our proposed power loading scheme.

1 Introduction

Multiple-input multiple-output (MIMO) technology has attracted a great attention due to its high spectral efficiency [1]. However, the application of multiple radio chains incurs a higher circuit power consumption. On the other hand, with the transmit channel side information(CSIT), singular value decomposition(SVD) can be utilized for MIMO channel to effectively create parallel independent channels, which possess different signal-to-noise ratio(SNR). Thus, by carefully performing power allocation to each subchannel, the system performance can be optimized to choose a few of the best quality channels or to use all channels to achieve the high rate [2]. Recently, due to the higher circuit power consumption, considerable research effort has been made to focus on optimizing the energy efficiency of MIMO systems, mostly considering power loading under the assumption of perfect channel state information (CSI) at the transmitter [3]. However, it is not realistic to assume transmitter always with perfect CSI in a MIMO cellular system. This paper will study the power loading under channel estimation error (CEE) for MIMO systems, which focuses on energy efficiency of the wireless link. To the best of our knowledge, there have been no studies about the impact of CEE on energy efficiency for the MIMO systems.

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In this paper, we also consider MIMO-SVD architecture for the transceiver. By modeling CEE as an independent complex Gaussian random variable [5], we derive the effective signal-to-interference-plus-noise (SINR) at the receiver under CEE, given the availability of an estimated channel. Based on that, we can get the energy efficiency model, which is achievable rates to power consumption ratio. Different from the result in [4], the objective function after the transformation is still non-convex. Then, we further propose two method to solve this problem. One is transforming to canonical D.C.(difference of convex) programming [9], which is proved to have the only global solution. Considering the complexity, the other approximate method is proposed to relax the objective function to convex problem, which leads to the closed-form optimization solution. Simulation results show the effectiveness of the proposed two algorithms, which are fairly robust against CEE.

The rest of the paper is outlined as follows. The system model is described in Section 2. We propose two algorithms to solve the power loading problem in Section 3. Simulation results are provided in Section 4, followed by the conclusions drawn in Section 5.

2 System Model

We consider an uncorrelated flat fading MIMO system with N_t transmit and N_r receive antennas. The output signals can be modeled as

$$\mathbf{r} = \mathbf{H}\mathbf{s} + \mathbf{n}, \tag{1}$$

where $\mathbf{s} \in \mathbb{C}^{N_t \times 1}$ denotes transmitted signals, $\mathbf{H} \in \mathbb{C}^{N_r \times N_t}$ denotes the channel matrix, and $\mathbf{n} \in \mathbb{C}^{N_r \times 1}$ is modeled as zero-mean additive white Gaussian noise (AWGN) with variance σ_n^2. When channel estimation error occurs, we assume that the MIMO transceiver can only obtain the imperfect CSI, which is modeled as [5, 6]

$$\mathbf{H} = \hat{\mathbf{H}} + \mathbf{E}, \tag{2}$$

where $\mathbf{E}$ is the estimation error matrix, and the element is with zero mean and variance σ_e^2. Then, by SVD decomposition of $\hat{\mathbf{H}}$, we can obtain

$$\hat{\mathbf{H}} = \hat{\mathbf{U}} \cdot \hat{\mathbf{D}} \cdot \hat{\mathbf{V}}^H = \hat{\mathbf{U}} \cdot diag(\sqrt{\hat{\lambda}_1}, \cdots, \sqrt{\hat{\lambda}_{N_{ss}}})\hat{\mathbf{V}}^H, \tag{3}$$

where $N_{ss} = \min\{N_t, N_r\}$ is the rank of $\hat{\mathbf{H}}$ and $\{\hat{\lambda}_i\}_{i=1}^{N_{ss}}$ is the eigenvalue of matrix $\hat{\mathbf{H}}\hat{\mathbf{H}}^H$.

Moreover, the signals sent over transmit antennas $\mathbf{s}$ are obtained by performing a transformation $\mathbf{s} = \hat{\mathbf{V}}\mathbf{P}\mathbf{x}$, and $\mathbf{P}$ is power allocation diagonal matrix, $\mathbf{x}$ is the information symbol vector from unit-energy constellation set. Thus, the output signals can be rewritten as

$$\begin{aligned}\mathbf{r} &= (\hat{\mathbf{U}} \cdot \hat{\mathbf{D}} \cdot \hat{\mathbf{V}}^H + \mathbf{E})\mathbf{s} + \mathbf{n} = \hat{\mathbf{U}}\hat{\mathbf{D}}\hat{\mathbf{V}}^H\hat{\mathbf{V}}\mathbf{P}\mathbf{x} + \mathbf{E}\hat{\mathbf{V}}\mathbf{P}\mathbf{x} + \mathbf{n} \\ &= \hat{\mathbf{U}}\hat{\mathbf{D}}\mathbf{P}\mathbf{x} + \mathbf{E}\hat{\mathbf{V}}\mathbf{P}\mathbf{x} + \mathbf{n}.\end{aligned} \tag{4}$$

Then, at the receiver, after the linearly processed, yield

$$\begin{aligned}\mathbf{y} = \hat{\mathbf{U}}^H\mathbf{r} &= \hat{\mathbf{U}}^H\hat{\mathbf{U}}\hat{\mathbf{D}}\mathbf{P}\mathbf{x} + \hat{\mathbf{U}}^H\mathbf{E}\hat{\mathbf{V}}\mathbf{P}\mathbf{x} + \hat{\mathbf{U}}^H\mathbf{n}\\ &= \hat{\mathbf{D}}\mathbf{P}\mathbf{x} + \hat{\mathbf{U}}^H\mathbf{E}\hat{\mathbf{V}}\mathbf{P}\mathbf{x} + \hat{\mathbf{U}}^H\mathbf{n}\\ &= \hat{\mathbf{D}}\mathbf{P}\mathbf{x} + \hat{\mathbf{E}}\mathbf{P}\mathbf{x} + \hat{\mathbf{U}}^H\mathbf{n},\end{aligned} \tag{5}$$

and the received signal on the i-th sub-channel can be expressed as

$$\begin{aligned}y_i &= [\hat{\mathbf{D}}\mathbf{P}\mathbf{x} + \hat{\mathbf{E}}\mathbf{P}\mathbf{x} + \hat{\mathbf{U}}^H\mathbf{n}]_i\\ &= (\sqrt{\hat{\lambda}_i} + \hat{e}_{ii})P_i x_i + \sum_{j=1, j\neq i}^{N_{ss}} \hat{e}_{ij} P_j x_j + \hat{n}_i\\ &= \sqrt{\hat{\lambda}_i} P_i x_i + \sum_{j=1}^{N_{ss}} \hat{e}_{ij} P_j x_j + \hat{n}_i.\end{aligned} \tag{6}$$

Then, the SNR on the i-th subchannel can be approximated as [7]

$$SNR_i = \frac{\hat{\lambda}_i P_i}{\sigma_e^2 \sum_{j=1}^{N_{ss}} P_j + \sigma_n^2}, \tag{7}$$

where $i = 1, 2, \cdots, N_{ss},\ N_{ss} = \min(N_r, N_t)$.

Since energy efficiency is defined as the ratio of the transmitted bit to the total energy consumptions, we can obtain the energy efficiency under imperfect CSI as

$$\max EE(\mathbf{P})\{ = \frac{\sum_{i=1}^{N_{ss}} \log(1 + SNR_i)}{\sum_{i=1}^{N_{ss}} GP_i \ + N_{ss}P_c} \overset{(a)}{=} \frac{\sum_{i=1}^{N_{ss}} \log(1 + SNR_i)}{\sum_{i=1}^{N_{ss}} P_i \ + P_c'}\} \tag{8}$$

$$s.t. \quad \sum_{i=1}^{N_{ss}} P_i \ \leq P_T \tag{9}$$

$$0 \leq P_i \leq P_{\max} \tag{10}$$

$$\sum_{i=1}^{N_{ss}} \log(1 + SNR_i) \geq R_{\min} \tag{11}$$

where $P_c' = \frac{N_{ss}P_c}{G}$, P_c is the average circuit power consumption in a single transmit or receiver chain, and G is defined as constant transmit power that is needed to overcome the path loss. Since it is positive constant, we can scale the objective by G, as shown in (a).

3 Proposed Algorithms

Since the optimization problem in (8) is fractional programming and the objective function, i.e., $EE(\mathbf{P})$, is non-convex and non-concave, we cannot apply convex optimization methods to solve this problem. However, according to [7], we can transform such fractional programming problem into a two-layer optimization problem. The following is the transformation process. First, we let

$$g(\mathbf{P}, q) = \sum_{i=1}^{N_{ss}} \log(1 + SNR_i) - q(\sum_{i=1}^{N_{ss}} P_i \ + P_c'), \tag{12}$$

$$f(q) = \{\max_{\mathbf{P} \in \mathbb{D}} g(\mathbf{P}, q)\}, \tag{13}$$

where $\mathbb{D}$ is the power constraint region consisting of (9)-(11), which is a convex set. Then from [7], we can obtain the optimal energy efficiency $EE^*(\mathbf{P}) = q^*$ when $f(q^*) = 0$. Therefore, the fraction programming problem in (8) can be solved by a two-layer algorithm as,

- *Inner Layer*: For a given q, find the maximum g^* which is also $f(q)$, i.e., $f(q) = g^* = \{\max_{\mathbf{P} \in \mathbb{D}} g(\mathbf{P}, q)\}$;
- *Outer Layer*: Find the zero point of $f(q)$, i.e., $q^* = \{q | f(q) = 0\}$.

Since the outer layer can be easily solved by bisection search algorithm [8], we focus on inner layer. Note that $g(\mathbf{P}, q)$ is still non-convex in (12), we cannot apply convex optimization solutions directly. Here we will adopt two methods to solve optimization problem in inner layer. The first is a global method using D.C. programming, and the second is a suboptimal method which can reduce the complexity.

3.1 Global Method

In this subsection, we will transform the inner layer optimization problem into a canonical D.C. programming problem. First, we rewrite (12) as

$$\begin{aligned} g(\mathbf{P}, q) &= \sum_{i=1}^{N_{ss}} \log(1 + SNR_i) - q(\sum_{i=1}^{N_{ss}} P_i \ + P_c') \\ &= \sum_{i=1}^{N_{ss}} \log(1 + \frac{\hat{\lambda}_i P_i}{\sigma_e^2 \sum_{j=1}^{N_{ss}} P_j + \sigma_n^2}) - q(\sum_{i=1}^{N_{ss}} P_i \ + P_c') \\ &= \sum_{i=1}^{N_{ss}} \log(\frac{\hat{\lambda}_i P_i + \sigma_e^2 \sum_{j=1}^{N_{ss}} P_j + \sigma_n^2}{\sigma_e^2 \sum_{j=1}^{N_{ss}} P_j + \sigma_n^2}) - q(\sum_{i=1}^{N_{ss}} P_i \ + P_c') \\ &= \sum_{i=1}^{N_{ss}} \log(\hat{\lambda}_i P_i + \sigma_e^2 \sum\nolimits_{j=1}^{N_{ss}} P_j + \sigma_n^2) \\ &\quad -[\sum_{i=1}^{N_{ss}} \log(\sigma_e^2 \sum\nolimits_{j=1}^{N_{ss}} P_j + \sigma_n^2) + q(\sum_{i=1}^{N_{ss}} P_i \ + P_c')]. \end{aligned} \tag{14}$$

Then, we let

$$m(\mathbf{P}) = -\sum_{i=1}^{N_{ss}} \log(\hat{\lambda}_i P_i + \sigma_e^2 \sum\nolimits_{j=1}^{N_{ss}} P_j + \sigma_n^2), \tag{15}$$

$$n(\mathbf{P},q) = -[\sum_{i=1}^{N_{ss}} \log(\sigma_e^2 \sum\nolimits_{j=1}^{N_{ss}} P_j + \sigma_n^2) + q(\sum_{i=1}^{N_{ss}} P_i \ + P_c')], \tag{16}$$

$$f_0(\mathbf{P},q) = -g(\mathbf{P},q) = m(\mathbf{P}) - n(\mathbf{P},q). \tag{17}$$

Thus, the inner layer optimization problem can be rewritten as

$$f(q) = \{\max_{\mathbf{P}\in\mathbb{D}} g(\mathbf{P},q)\} = -\{\min_{\mathbf{P}\in\mathbb{D}} f_0(\mathbf{P},q)\} = -\{\min_{\mathbf{P}\in\mathbb{D}}[m(\mathbf{P}) - n(\mathbf{P},q)]\}. \tag{18}$$

Then we should solve the optimization problem

$$\{\min_{\mathbf{P}\in\mathbb{D}} f_0(\mathbf{P},q)\} = \{\min_{\mathbf{P}\in\mathbb{D}}[m(\mathbf{P}) - n(\mathbf{P},q)]\}. \tag{19}$$

Since $m(\mathbf{P})$ and $n(\mathbf{P},q)$ are all convex with $\mathbf{P}$, (19) is a D.C. programming problem. Next we show it can be further transformed into a canonical D.C. programming problem, which can be solved by some famous algorithms [9].

We first change the constraint region $\mathbb{D}$ consisting of (9-11) in sequence as

$$f_1(\mathbf{P}) = g_1(\mathbf{P}) - h_1(\mathbf{P}) = \sum_{i=1}^{N_{ss}} P_i \ - P_T \leq 0, \tag{20}$$

where $g_1(\mathbf{P}) = \sum\limits_{i=1}^{N_{ss}} P_i$ and $h_1(\mathbf{P}) = P_T$ are convex, and $f_1(\mathbf{P})$ is a D.C. function;

$$\mathbb{D}_0 = \{\mathbf{P} \in \mathbb{R}^{N_{ss}} | 0 \leq P_i \leq P_{\max}\}, \tag{21}$$

where $\mathbb{D}_0$ is a N_{ss}-dimensional rectangle in $\mathbb{R}^{N_{ss}}$;

$$\begin{aligned} f_2(\mathbf{P}) &= g_2(\mathbf{P}) - h_2(\mathbf{P}) = R_{\min} - \sum_{i=1}^{N_{ss}} \log(1 + SNR_i) \\ &= R_{\min} - [\sum_{i=1}^{N_{ss}} \log(\hat{\lambda}_i P_i + \sigma_e^2 \sum\nolimits_{j=1}^{N_{ss}} P_j + \sigma_n^2) - \sum_{i=1}^{N_{ss}} \log(\sigma_e^2 \sum\nolimits_{j=1}^{N_{ss}} P_j + \sigma_n^2)] \\ &= [R_{\min} - \sum_{i=1}^{N_{ss}} \log(\hat{\lambda}_i P_i + \sigma_e^2 \sum\nolimits_{j=1}^{N_{ss}} P_j + \sigma_n^2)] \\ &\quad -[-\sum_{i=1}^{N_{ss}} \log(\sigma_e^2 \sum\nolimits_{j=1}^{N_{ss}} P_j + \sigma_n^2)] \leq 0, \end{aligned} \tag{22}$$

where $g_2(\mathbf{P}) = R_{\min} - \sum\limits_{i=1}^{N_{ss}} \log(\hat{\lambda}_i P_i + \sigma_e^2 \sum_{j=1}^{N_{ss}} P_j + \sigma_n^2)$ and $h_2(\mathbf{P}) = -\sum\limits_{i=1}^{N_{ss}} \log(\sigma_e^2 \sum_{j=1}^{N_{ss}} P_j + \sigma_n^2)$ are convex, and $f_2(\mathbf{P})$ is a D.C. function.

Therefore, the optimization problem (19) can be transformed to:

$$\{\min f_0(\mathbf{P},q) | \mathbf{P} \in \mathbb{D}_0, f_1(\mathbf{P}) \leq 0, f_2(\mathbf{P}) \leq 0\}. \tag{23}$$

Since $f_0(\mathbf{P},q)$, $f_1(\mathbf{P})$, $f_2(\mathbf{P})$ are D.C. functions and $\mathbb{D}_0$ is a N_{ss}-dimensional rectangle in $\mathbb{R}^{N_{ss}}$, we can transform (23) to into a canonical D.C. programming problem [9] and solve it with two types of algorithms, branch-and-bound type and outer-approximation type [9]. Therefore, we can get the global solution to optimization problem in (8).

3.2 Suboptimal Method

In this subsection, we will propose a suboptimal method to solve the inner layer optimization problem, which can reduce complexity compared to the global method in Section 3.1.

First, the following lower bound is used [10],

$$\alpha \log z + \beta \le \log(1+z) \begin{cases} \alpha = \frac{z_0}{1+z_0} \\ \beta = \log(1+z_0) - \frac{z_0}{1+z_0}\log(z_0) \end{cases} \tag{24}$$

That is tight with equality at a chosen value z_0 when the constants are chosen as specified above. As a result, the inner layer optimization problem can be relaxed as

$$\max g(\mathbf{P},q) = \sum_{i=1}^{N_{ss}} [\alpha_i \log(SNR_i) + \beta_i] - q(\sum_{i=1}^{N_{ss}} P_i \ + P_c'). \tag{25}$$

We do the following transformation, $\tilde{P}_i = \log(P_i)$ and $P_i = e^{\tilde{P}_i}$, then we have

$$\begin{aligned} g(\tilde{\mathbf{P}},q) &= \sum_{i=1}^{N_{ss}} [\alpha_i \log(SNR_i) + \beta_i] - q(\sum_{i=1}^{N_{ss}} P_i \ + P_c') \\ &= \sum_{i=1}^{N_{ss}} [\alpha_i \log(\frac{\hat{\lambda}_i e^{\tilde{P}_i}}{\sigma_e^2 \sum_{j=1}^{N_{ss}} e^{\tilde{P}_j} + \sigma_n^2}) + \beta_i] - q(\sum_{i=1}^{N_{ss}} e^{\tilde{P}_i} \ + P_c') \\ &= \sum_{i=1}^{N_{ss}} [\alpha_i \log(\hat{\lambda}_i) + \alpha_i \tilde{P}_i - \alpha_i \log(\sigma_e^2 \sum_{j=1}^{N_{ss}} e^{\tilde{P}_j} + \sigma_n^2) + \beta_i] - q\sum_{i=1}^{N_{ss}} e^{\tilde{P}_i} \ - qP_c' \\ &= \sum_{i=1}^{N_{ss}} [\alpha_i \log(\hat{\lambda}_i) + \alpha_i \tilde{P}_i + \beta_i] - \sum_{i=1}^{N_{ss}} [\ \alpha_i \log(\sigma_e^2 \sum_{j=1}^{N_{ss}} e^{\tilde{P}_j} + \sigma_n^2) + qe^{\tilde{P}_i}] - qP_c' \\ &= \sum_{i=1}^{N_{ss}} [\alpha_i \log(\hat{\lambda}_i) + \alpha_i \tilde{P}_i + \beta_i] - [\underbrace{(\sum_{i=1}^{N_{ss}} \alpha_i) \log(\sigma_e^2 \sum_{j=1}^{N_{ss}} e^{\tilde{P}_j} + \sigma_n^2)}_{Convex} + \underbrace{\sum_{i=1}^{N_{ss}} qe^{\tilde{P}_i}}_{Convex}] - qP_c' \end{aligned}, \tag{26}$$

Since the second item is the sum of convex function, the objective is concave. Also, the constraint region is convex. Therefore, we can use Karush-Kuhn-Tucker(KKT) conditions to solve the optimization problem. Due to space limitation, we only give the final closed-form optimization solution, i.e.,

$$P_i^* = \left[\frac{\alpha_i(v^*-1)(\sqrt{\frac{\bar{\alpha}\theta(v^*-1)}{(\lambda^*-q)} + \frac{\theta^2}{4}} + \frac{\theta}{2})}{(\lambda^*-q)(\sqrt{\frac{\bar{\alpha}\theta(v^*-1)}{(\lambda^*-q)} + \frac{\theta^2}{4}} + \frac{\theta}{2}) + \bar{\alpha}(v^*-1)} \right]_0^{P_{\max}}, \tag{27}$$

where λ^*, v^* are KKT multipliers, $\bar{\alpha} = \sum_{i=1}^{N_{ss}} \alpha_i$, and $\theta = \frac{\sigma_n^2}{\sigma_e^2}$.

Therefore, we propose the suboptimal method as shown in Algorithm A.

A. Suboptimal Method

```
Initialize:
iteration counter t = 0.
α_i = 1, β_i = 0, for 1 ≤ i ≤ N_ss, (high SNR approximate).
Repeat:
    maximize: solve (25) to give solution P.
    set: P* = P.
    tighten: update α_i = z_i/(1+z_i), β_i = log(1 + z_i) − z_i/(1+z_i) log(z_i), with z_i = SNR_i(P).
    increment t.
Until convergence
```

4 Simulation Results

In this section, Monte Carlo simulations are used to illustrate proposed power loading algorithms to optimize EE in MIMO systems under imperfect CSI. The detailed simulation parameters are listed in Table 1. At the receiver, the ZF structure is adopted, where perfect synchronization is assumed.

Table 1. Simulation Parameters

Frame duration	0.5 ms
N_t (# of Tx antenna at user)	2
N_r (# of Rx antenna at Node-B)	2
Carrier frequency	2 GHz
Sampling frequency	7.68 MHz
Receiver	Zero forcing
Traffic model	Full buffer
σ_n^2 (Noise variance)	1
# of frames	5000

Fig. 1 shows that the energy efficiency varies on estimation error variance with both global method and suboptimal method. From this figure, we can see as the estimation error variance, i.e., σ_e^2, increases, the EE performance declines. In addition, we can compare the global method and suboptimal method in this figure. The global methods can always achieve better EE performance than suboptimal method, but the gap is very little. This substantiates the effectiveness of our proposed suboptimal method.

Fig. 2 substantiates the robustness of our proposed power loading scheme. In this figure, we use the global method to obtain the global maximal EE. We can find that when the estimation error variance $\sigma_e^2 < 10^{-2}$, the EE performance is almost flat with σ_e^2. So our proposed energy efficient power loading scheme is robust against CEE.

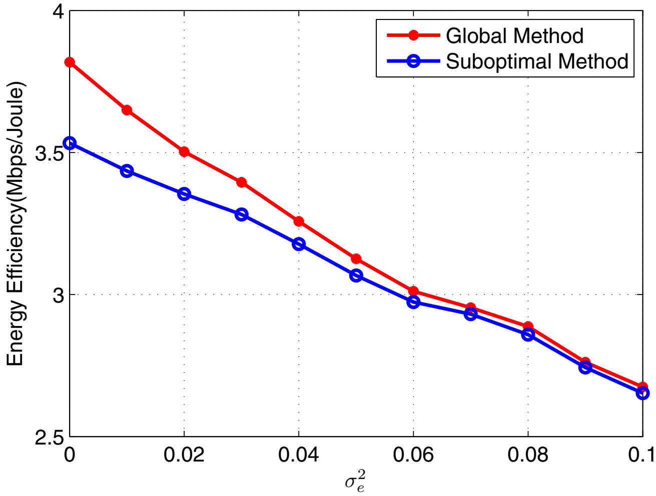

Fig. 1. EE v.s. σ_e^2

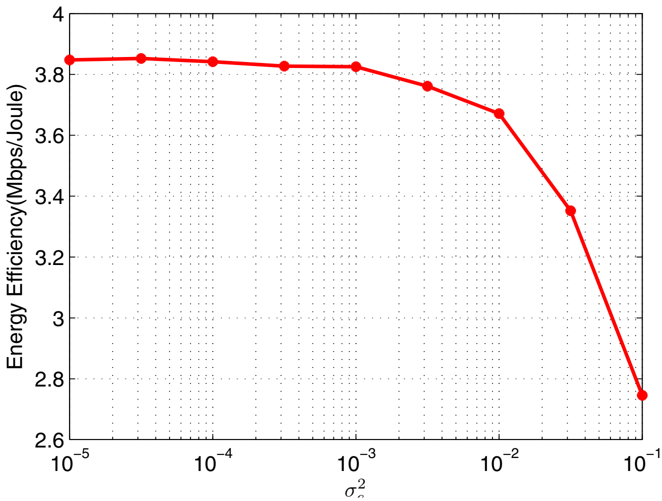

Fig. 2. EE v.s. σ_e^2

5 Conclusion

In this paper, we proposed an energy efficient power loading scheme in MIMO-SVD architecture under imperfect CSI. We first derive the close-form expression of EE in MIMO-SVD system taking into consideration of the effects of channel esti- mation error (CEE). Then we propose two algorithms to solve the optimization problem, one global method and one low-complexity suboptimal method.

The simulation results substantiate the effectiveness of our proposed EE power loading scheme. In our future work, we will try to apply our energy efficient power loading scheme to the large scale networks [11] [12].

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References

1. Telatar, E.: Capacity of Multi-antenna Gaussian Channels. Eur. Trans. Telecommun. 10, 585–595 (1999)
2. Dighe, P.A., Mallik, R.K., Jamuar, S.S.: Analysis of Transmit-receive Diversity in Rayleigh Fading. IEEE Trans. Commun. 51, 694–703 (2003)
3. Prabhu, R.S., Daneshrad, B.: An Energy-efficient Water-filling Algorithm for OFDM Systems. In: 2010 IEEE International Conference on Communications (2010)
4. Hassibi, B., Hochwald, B.M.: How Much Training Is Needed In Multiple-antenna Wireless Links? IEEE Trans. Inform. Theory 49, 951–963 (2003)
5. Wang, Z.Y., He, C., He, A.: Robust AM-MIMO Based on Minimized Transmission Power. IEEE Commun. Lett. 10, 432–434 (2006)
6. Song, S.H., Zhang, Q.T.: Mutual Information of Multipath Channels with Imperfect Channel Information. IEEE Trans. Commun. 57, 1523–1531 (2009)
7. Schaible, S.: Fractional Programming, II: on Dinkelbach's Algorithm. J. Manage. Sci. 22, 868–873 (1976)
8. Burden, R.L., Faires, J.D.: Numerical Analysis, 7th edn. Brooks/Core, Pacific Grove (2000)
9. Horst, R., Thoai, N.V.: DC Programming: Overview. J. Optim. Theory Appl. 103, 1–43 (1999)
10. Papandriopoulos, J., Evans, J.S.: Low-Complexity Distributed Algorithms for Spectrum Balancing in Multi-User DSL Networks. In: 2006 IEEE International Conference on Communications (2006)
11. Wang, X.B., Fu, L., Tian, X., Bei, Y., Peng, Q., Gan, X., Yu, H., Liu, J.: Converge-Cast: On the Capacity and Delay Tradeoffs. IEEE Trans. Mobile. Computing 11, 970–982 (2011)
12. Wang, X.B., Huang, W., Wang, S., Zhang, J., Hu, C.: Delay and Capacity Tradeoff Analysis for MotionCast. IEEE/ACM Trans. Networking 19, 1354–1367 (2011)