

Dynamic Spectrum Allocation for Heterogeneous Cognitive Radio Network

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Abstract—One important issue associated with spectrum management in heterogeneous cognitive radio network is: how to appropriately allocate the spectrum to the secondary sender-destination (S-D) pair for sensing and utilization. In this work, the authors investigate the spectrum allocation problem under a more practical scenario, taking the heterogeneous characteristics of both secondary S-D and PU channels into consideration. With the objective to maximize the achievable throughput for secondary S-D, we formulate the spectrum allocation problem as a linear integer optimization problem under spectrum availability constraint, spectrum span constraint and interference free constraint. This problem is NP-complete, and a recent result in theoretical computer science called randomized rounding algorithm with polynomial computational complexity is developed to find the ρ -approximation solution. Evaluation results show that our proposed algorithm can achieve a close-to-optimal solution while keeping the complexity low.

I. INTRODUCTION

More and more spectrum resources are required to support the rapid development of wireless applications. However, a recent study by Federal Communications Commission (FCC) has shown that most of allocated frequency bands experience significant under-utilization [1]. Cognitive radio (CR) is therefore proposed as a potential technology to mitigate this spectrum scarcity problem. The basic idea of CR is to allow secondary users (SUs) to access licensed spectrum bands as far as they do not incur any harmful interference to the primary users (PUs) [2]. To achieve this goal, a well-designed spectrum management policy (which includes four aspects: spectrum sensing, spectrum allocation, spectrum sharing and spectrum handoff) is highly required [3].

As one of the most challenging problems in cognitive radio network, spectrum allocation received less attention compared with spectrum sensing. In [4], sensing and allocation strategies with one SU and multiple channels are proposed. This scheme may not be optimal when the channel characteristics are heterogeneous [4]. In [5], with the objective to minimize the difference between the expected channel available time and the expected service time, a heuristic matching algorithm is proposed to allocate spectrum to SU. In [6], a demand-matching spectrum sharing mechanism based on game theory for non-cooperative cognitive radio network is proposed. In [7], by jointly taking the heterogeneities of both PU channels and that of SUs into account, the authors formulate the spectrum allocation problem as a Maximum Weight Matching

problem. In [8], Hou. et al. consider the channel allocation problem with multiple primary channels and heterogeneous channel availability. However, the cognitive radio system considered in the above work only consider the one to one case (allocate one channel to one SU for utilization), which is a simple network scenario. Moreover, we observe that in the algorithms proposed above (except [7]), the heterogeneities of PU channels and those of SUs have not been fully considered jointly.

SU heterogeneity is a unique feature of cognitive radio network. The spectrum availability of SU is heterogeneous due to different geographical location among different SUs. Thus if we allocate different available channels to SU sender and destination, they cannot communicate with each other. Moreover, different SUs with different detection thresholds and received SNR will result in different detection performance. On the other hand, different PU channels may have different idle probability and channel capacity. Thus allocating different channels to different secondary S-D pair may result in different system performance. However in most of the above work, the important issue of interference has not been well investigated, and the spectrum temporal variation at the SU sender and destination is also not discussed. Thus, how to handle the spectrum allocation problem in heterogeneous cognitive radio network with the investigation into the interference and spectrum temporal problems has not drawn much attention before. In this paper, we mainly focus on the spectrum allocation problem, aiming at deciding how to appropriately allocate more than one channel to secondary S-D pair for utilization (many to one case), where the heterogeneities of both PU channels and that of secondary S-D pair are taken into consideration. Moreover, the interference between different S-D pairs is also directly studied. The contributions of this paper are listed as follows:

- 1) We optimize spectrum sensing and spectrum allocation for many to one case, while investigating the heterogeneous characteristics in both secondary S-D pairs and PU channels.
- 2) With the objective to maximize the achievable throughput for secondary S-D pairs, we formulate the spectrum allocation problem as a linear integer optimization problem. We leverage the randomized rounding algorithm to

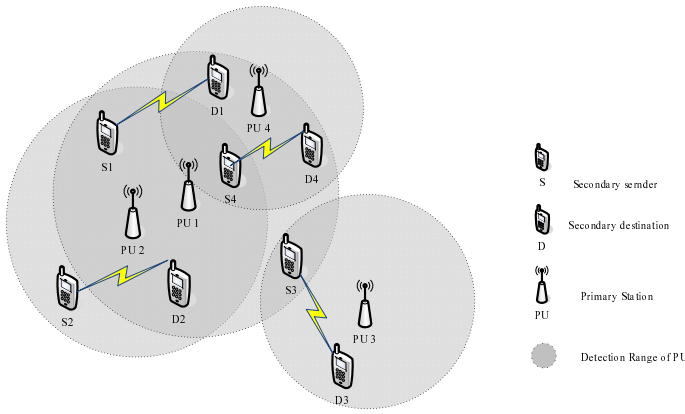


Fig. 1. The cognitive radio network architecture.

obtain a ρ -approximation solution.

- 3) In our work, more than one channel can be allocated to each S-D, such that if one channel is reclaimed by PU, the S-D can use the rest of channels to continue its transmission. Furthermore, we extend the work in [7] [8] [9] by adding the interference constraint in the problem formulation, which increases the complexity of this problem.

The rest of this paper is organized as follows. The overview of the system model is introduced in Section II. The problem analysis and spectrum allocation problem for heterogeneous cognitive radio network are described in Section III. The randomized rounding algorithm is proposed in Section IV. Simulation results and evaluations are given in Section V. Finally, Section VI concludes the paper.

II. SYSTEM MODEL

We consider a CRN with N secondary S-D pairs and M PU channels. Each channel is allocated to one PU. However the PU may not be active all the time and the SU sender and destination can opportunistically utilize the channel when it is not used by the PU. Let \mathcal{M} be the set of such PU channels and \mathcal{N} denote the set of secondary S-D pairs. In heterogeneous cognitive radio networks, different SUs may have different energy detection threshold, received SNR and geographical location, which results in different detection performance. Moreover some small-scale signal, e.g., wireless microphone uses a weak transmit power around 10-50mW, and its transmission range is limited to only 150-200m [10]. Thus the PU signal may not cover the whole network but only a part of the system. In this case the detection range of this kind of signal is relatively small. Some SUs located far from the PU cannot detect the PU signal and will report only the noise power. A channel j is said to be opportunistically accessible by SU i only if the SU is within the detection range of channel j , thus it can detect the PU activity. Otherwise, if the SU is outside the detection range of the PU channel, the detection probability is zero [11]. Therefore, different SUs may have different sets of available channels due to their het-

erogeneous geographical locations and environments. On the other hand, different PU channels may have different channel idle probability and channel capacity. Thus allocating different PU channels to different secondary S-D pair may result in different system performance. The CRN model is illustrated in Fig. 1. It demonstrates that the channel heterogeneity-spectrum availability varies across the SU S-D pairs. For simplicity, there are many PUs and secondary S-D pairs not marked in Fig. 1.

Spectrum sensing is a fundamental functionality in cognitive radio communications, it is required to be performed firstly before data transmission. For each channel j , the detection is basically a binary hypothesis between H_j^1 and H_j^0 , which denote the presence and absence of PU respectively. The sensing performance can be measured by two parameters: detection probability and false alarm probability, which are given by

$$P_{f,(i,j)} = Q\left(\left(\frac{\varepsilon_i}{\sigma_{u,i,j}^2} - 1\right)\sqrt{f_s\tau}\right) \quad (1)$$

$$P_{d,(i,j)} = Q\left(\left(\frac{\varepsilon_i}{\sigma_{u,i,j}^2} - 1 - \gamma_{i,j}\right)\sqrt{\frac{f_s\tau}{2\gamma_{i,j} + 1}}\right) \quad (2)$$

where the received primary signal is complex PSK with zero mean and variance $\sigma_{s,i,j}^2$, and the noise is the independent circular symmetric complex Gaussian with zero mean and variance $\sigma_{u,i,j}^2$. The energy detection threshold at SU i is ε_i , and $\gamma_{i,j} = \frac{\sigma_{s,i,j}^2}{\sigma_{u,i,j}^2}$ is the average SNR of the PU in channel j received by SU i , and $Q(x)$ is the tail probability of the standard normal distribution.

III. SPECTRUM ALLOCATION PROBLEM STATEMENT

In cognitive radio networks, in order to conduct successful data transmission, it is a must that both the SU sender and destination should work on the same radio frequency channel. However, as been discussed before, due to the heterogeneous characteristics of SUs, SU sender and destination may have different sets of available channels. Besides, the available channels at each SU sender and destination vary from time slot to time slot due to the activity of PUs. In each time slot, each sender and destination should select one or more common idle channel as their working channel based on the available channel information. Therefore how to select working frequency band for each S-D pair becomes a key part of spectrum management in cognitive radio network.

To represent the spectrum availability at all S-D pairs, we define NM binary variables $c_{i,j}^s$ and $c_{i,j}^d$, $\forall i, j$, as follows:

$$c_{i,j}^s = \begin{cases} 1 & \text{if channel } j \text{ is available at SU sender } i \\ 0 & \text{Otherwise} \end{cases}$$

$$c_{i,j}^d = \begin{cases} 1 & \text{if channel } j \text{ is available at SU destination } i \\ 0 & \text{Otherwise} \end{cases}$$

In order to mitigate co-channel interference, we define a matrix $\mathbb{A}_{N \times N \times M}$ to represent the interference graph of any

pair of S-D, as follows:

$$\mathcal{A}_{i_1, i_2, j} = \begin{cases} 1 & \text{if S-D } i_1 \text{ and S-D } i_2 \text{ conflict on channel } j \\ 0 & \text{Otherwise} \end{cases}$$

Let Δ_i^s and Δ_i^d be the sets of channels that are available at SU sender and destination of pair i , respectively, that is

$$\Delta_i^s = \{j | c_{i,j}^s = 1, \forall j \in \mathcal{M}\}$$

$$\Delta_i^d = \{j | c_{i,j}^d = 1, \forall j \in \mathcal{M}\}$$

For each S-D pair, spectrum allocation is done by deciding on the following two vectors:

1) Φ_s represents the set of channels allocated to SU sender

$$\Phi_s = \{s_{i,j}, \forall i \in \mathcal{N}, j \in \mathcal{M}\}$$

2) Φ_d represents the set of channels allocated to secondary destination

$$\Phi_d = \{d_{i,j}, \forall i \in \mathcal{N}, j \in \mathcal{M}\}$$

where $s_{i,j}$ and $d_{i,j}$ are the decision variables, which are defined as

$$s_{i,j} = \begin{cases} 1 & \text{if channel } j \text{ is allocated to SU sender } i \\ 0 & \text{Otherwise} \end{cases}$$

$$d_{i,j} = \begin{cases} 1 & \text{if channel } j \text{ is allocated to SU destination } i \\ 0 & \text{Otherwise} \end{cases}$$

In other words, the spectrum allocation can be viewed as deciding on the two vectors Φ_s and Φ_d from the current feasible region Δ_i^s and Δ_i^d , for $i \in \mathcal{N}$ and $j \in \mathcal{M}$.

A. Analysis of System Throughput

The objective is to maximize the sum of achievable throughput for all secondary S-D pairs over all the PU channels. Let T denote the length of a time slot, τ be the total sensing time allocated to sense each PU channel. Then the available throughput of S-D pair i transmission over channel j can be expressed

$$R_{i,j} = \frac{T - \tau}{T} P(\mathcal{H}_j) C_{ij} (1 - P_{f,ij}^s P_{f,ij}^d) \quad (3)$$

where $P(\mathcal{H}_j)$ denotes the idle probability for channel j , and $C_{i,j}$ is the transmission capacity for S-D pair i on channel j . $P_{f,ij}^s$ is the false alarm probability which is defined as the probability of the SU sender i falsely declaring the presence of PU in channel j under H_j^0 . And $P_{f,ij}^d$ is the false alarm probability at the SU destination i .

B. Analysis of Valid Allocation

The constraints that spectrum allocation imposes are as follows:

Availability Constraint: spectrum allocated to any S-D pair should be limited to the set of channels that are detected to be idle at SUs, that is

$$s_{i,j} = 1 \Rightarrow c_{i,j}^s = 1, \forall i \in \mathcal{N}, j \in \mathcal{M} \quad (4)$$

$$d_{i,j} = 1 \Rightarrow c_{i,j}^d = 1, \forall i \in \mathcal{N}, j \in \mathcal{M} \quad (5)$$

Spectrum Span Constraint: In order to guarantee a fairness among the secondary S-D pairs, each one should be allocated with at least one channel for data transmission (It is possible that no common channel is available for a S-D pair, because they might not be covered by one common PU. In this case the throughput achieved by this S-D pair is zero, we can just exclude this S-D pair from being considered); on the other hand the total number of channels allocated to each S-D pair should not exceed the maximum value d_0 due to some hardware limitations, that is

$$1 \leq \sum_{j=1}^M s_{i,j} d_{i,j} \leq d_0, \forall i \in \mathcal{N} \quad (6)$$

Interference Free Constraint: mutually interfering secondary S-D pairs should not be allocated with the same channels. So that the Interference Free Constraint can be represented as:

$$\mathcal{A}_{i_1, i_2, j} = 1 \Rightarrow s_{i_1, j} d_{i_1, j} s_{i_2, j} d_{i_2, j} = 0, \forall i_1, i_2, j \quad (7)$$

C. Problem Formulation

Finally, with the objective of maximizing the achievable throughput, the dynamic spectrum allocation problem can be formulated as the following optimization problem:

$$\max_{\Phi_s, \Phi_d} \sum_i \sum_j s_{i,j} d_{i,j} R_{ij} \quad (8)$$

$$s.t. \quad (5) - (8)$$

$$s_{i,j}, d_{i,j} \in \{0, 1\}, \quad \forall i, j \quad (9)$$

Due to the nonlinear constraints (5)-(8) and factor $s_{i,j} d_{i,j}$ in the objective function, the formulated problem above is nonlinear optimization problem. Let $m_{i,j} = s_{i,j} d_{i,j}$, we can transform the Dynamic sPectrum Allocation (DPA) problem into the following linear 0-1 integer optimization problem.

$$\max_{\Phi_s, \Phi_d} \sum_i \sum_j m_{i,j} R_{ij} \quad (10)$$

$$s.t. \quad s_{i,j} \leq c_{i,j}^s, \quad \forall i, j \quad (11)$$

$$d_{i,j} \leq c_{i,j}^d, \quad \forall i, j \quad (12)$$

$$1 \leq \sum_{j=1}^M m_{i,j} \leq d_0, \quad \forall i \quad (13)$$

$$s_{i_1, j} + d_{i_1, j} + s_{i_2, j} + d_{i_2, j} \leq 3, \quad (14)$$

$$s_{i,j}, d_{i,j}, m_{i,j} \in \{0, 1\}, \quad \forall i, j \quad (15)$$

It is obviously that the two formulated problems are equivalent. Solving the DPA Problem is NP-complete, the proof is omitted here due to page limitation. When the number of SU S-D pairs and PU channels increases, the complexity to find the optimal solution will grow exponentially.

IV. THE RANDOMIZED ROUNDING ALGORITHM

Since the DPA problem is NP-complete, it seems impossible to solve this problem in polynomial time. We resort to the randomized rounding algorithm as illustrated in Algorithm 1.

Algorithm 1 The Randomized Rounding Algorithm

- 1: **Step 1. Relaxation of the DPA problem**
 - 2: - Calculate the optimal spectrum allocation result (Φ_s^*, Φ_d^*) for the Linear Programming Relaxation (LPR) of DPA problem.
 - 3: **Step 2. Convex decomposition**
 - 4: - Decompose the fractional solution (Φ_s^*, Φ_d^*) to a convex combination of mixed integer solutions, i.e., $\sum_{q \in \Psi} \lambda^q (\Phi_s^q, \Phi_d^q) \geq (\Phi_s^*, \Phi_d^*)/\rho$. This can be done by solving a pair of primal-dual LPs in (19) and (20) using ellipsoid method.
 - 5: **Step 3. Pick the integer solution (Φ_s^q, Φ_d^q) with λ^q**
 - 6: - Select each feasible integer solution (Φ_s^q, Φ_d^q) of DPA problem with probability λ^q .
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Here, the Linear Programming Relaxation (LPR) of the 0-1 Integer Programming (IP) is defined as follows:

Definition 4: The LPR of the 0-1 IP is obtained by relaxing the integrality constraint to $0 \leq x_i \leq 1$ for all the variables.

As stated by Theorem 2.1 of [12], if we have an approximation heuristic algorithm to the max-IP DPA problem, and let (Φ_s^*, Φ_d^*) be the optimal solution to the LPR of DPA, then $(\Phi_s^*, \Phi_d^*)/\rho$ dominates a convex combination of all feasible integer solutions of DPA, that is, we have

$$\sum_{q \in \Psi} \lambda^q (\Phi_s^q, \Phi_d^q) \geq (\Phi_s^*, \Phi_d^*)/\rho$$

where $\lambda^q \geq 0$ for all q and $\sum_{q \in \Psi} \lambda^q = 1$. (Φ_s^q, Φ_d^q) is a feasible integer solution to the DPA problem and Ψ is the index set for all feasible integer solutions.

A. Detailed Analysis for the Randomized Rounding Algorithm

In the following, we present a detailed description of the randomized rounding algorithm which consists of three main steps.

Step 1. Relaxation of the integer DPA problem. The first step is to solve the LPR of DPA problem by relaxing constraint (16) to $(s_{i,j} \leq 1, d_{i,j} \leq 1, m_{i,j} \leq 1 \quad \forall i \in \mathcal{N}, j \in \mathcal{M})$, is redundant and hence ignored):

$$s_{i,j}, d_{i,j}, m_{i,j} \geq 0, \quad \forall i \in \mathcal{N}, j \in \mathcal{M}$$

The LPR of DPA problem is linear programmable, obviously, it can be optimally solved in polynomial time. Let (Φ_s^*, Φ_d^*) denote the optimal solution to the LPR of DPA problem.

Step 2. Convex decomposition. Applying the recent convex decomposition technique [13], we decompose the optimal fractional solution (Φ_s^*, Φ_d^*) into a convex combination of integral solutions each with a fractional weight that sums up to 1. This step requires an effective polynomial-time approximation algorithm to DPA problem, that satisfies:

$$\sum_i \sum_j m_{i,j} R_{ij} \geq OPT_{LPR}/\rho \quad (16)$$

The left side represents the achievable throughput using the approximation algorithm, and OPT_{LPR} is the value of the

objective function for LPR of DPA problem when the optimal solution is (Φ_s^*, Φ_d^*) .

Thus, the goal of the convex decomposition is to find combination weights $\lambda^q \geq 0$, for all q , such that

$$\sum_{q \in \Psi} \lambda^q = 1, \text{ and, } \sum_{q \in \Psi} \lambda^q (\Phi_s^q, \Phi_d^q) \geq (\Phi_s^*, \Phi_d^*)/\rho \quad (17)$$

Next, we will compute each λ^q , which is the weight required in the convex decomposition for solution (Φ_s^q, Φ_d^q) . In order to obtain λ^q that satisfies (18), we wish to solve the following LP problem:

$$\begin{aligned} \text{Primal : min} \quad & \sum_{q \in \Psi} \lambda^q \\ \text{s.t.} \quad & \sum_{q \in \Psi} \lambda^q (s_{i,j}^q, d_{i,j}^q) \geq (s_{i,j}^*, d_{i,j}^*)/\rho \\ & \sum_{q \in \Psi} \lambda^q \geq 1, \lambda^q \geq 0, \forall q \in \Psi \end{aligned} \quad (18)$$

Our goal is to solve this primal LP problem optimally with $\sum_{q \in \Psi} \lambda^q = 1$. However, we can note that the problem described in (19) has an exponential number of variables, which is difficult to solve. We instead resort to its dual problem that has an exponential number of constraints. The dual problem of (19) is defined as follows:

$$\begin{aligned} \text{Dual: max} \quad & (\sum_{i,j} \omega_{i,j} s_{i,j}^* + \sum_{i,j} \gamma_{i,j} d_{i,j}^*)/\rho + \delta \\ \text{s.t.} \quad & \sum_{i,j} \omega_{i,j} s_{i,j}^q + \sum_{i,j} \gamma_{i,j} d_{i,j}^q + \delta \leq 1, \forall q \in \Psi \\ & \omega_{i,j} \geq 0, \gamma_{i,j} \geq 0, \delta \geq 0, \forall i, j \end{aligned} \quad (19)$$

The ellipsoid method can solve the problem within polynomial time despite an exponential number of constraints [?]. In order to make the dual LP solvable in polynomial time, the ellipsoid method requires an approximation algorithm to serve as a separation hyperplane. Each hyperplane corresponds to a constraint in the dual problem, providing a feasible solution. The primal LP (19) then can be transformed to an optimization problem with a polynomial number of variables corresponding to these hyperplanes. We hence can solve the primal LP in polynomial time, obtaining weights of the convex decomposition that sum to 1.

Step 3. Pick the integer solution with λ^q . Following the decomposition, each possible integer solution (Φ_s^q, Φ_d^q) is selected with a probability equal to its corresponding convex multiplier λ^q computed in the convex decomposition in the second step. Then the expected throughput is

$$\sum_q \sum_i \sum_j \lambda^q s_{i,j}^q d_{i,j}^q R_{ij} \geq \sum_i \sum_j s_{i,j}^* d_{i,j}^* R_{ij}/\rho \quad (20)$$

The above inequality implies that the decomposition algorithm can achieve an approximation ratio of ρ with respect to the aggregated gain.

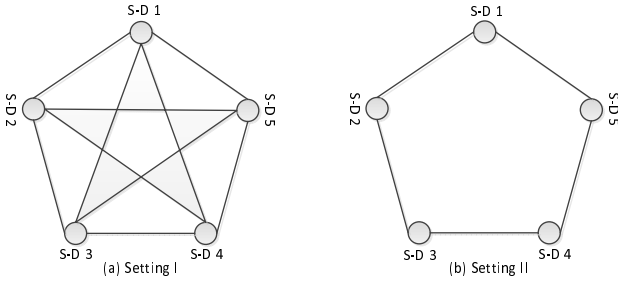


Fig. 2. Two interference graph setting for simulation.

B. The Approximation Algorithm for DPA

The randomized rounding algorithm in Algorithm 1 requires an efficient heuristic approximation algorithm for solving the DPA problem. Such approximation algorithm should compute a feasible integer solution efficiently, with a cost that is as close as possible to the LPR of DPA problem. The proposed algorithm is described in Algorithm 2, which consists of the following four main steps.

Step 1. Select Available Channel Set for Each S-D Pair:

We first select the set of common channels that are available at both SU sender and destination for each S-D pair i , that is

$$\Delta_{sd,i} = \{j | c_{i,j}^s = c_{i,j}^d = 1, \forall j\}$$

Step 2. Construct a Bipartite Graph: In cognitive radio networks, the topology of secondary S-D pairs and PU channels can be represented as a bipartite graph $G(V_1 \cup V_2, \varepsilon)$. Vertex set V_1 corresponds to the S-D pairs in the network, and set V_2 contains the PU channels. An edge exists between $(i, j) \in \varepsilon$, $i \in V_1$ and $j \in V_2$, if and only if $j \in \Delta_{sd,i}$.

Step 3. Channel Allocation Using Kuhn-Munkres(KM)

Algorithm: We use KM algorithm to match S-D pairs with their common channels such that as many S-D pairs as possible can select different common channels to achieve a high utilization.

Step 4. Update the Bipartite Graph: Let $Q(S \cup B, \eta)$ be the maximum matching from the bipartite graph $G(V_1 \cup V_2, \varepsilon)$, then we use the following steps to update the bipartite graph:

- 1) Remove all the edges in η from ε , that is $\varepsilon = \varepsilon / \eta$;
- 2) If S-D pair i_1 conflicts with S-D pair i_2 , and if channel j has been allocated to S-D pair i_1 , then remove edge $e_{i_2 j}$ from ε .

Then go back to step 3 until 1) no more available channel can be allocated to S-D pair; 2) all the S-D pairs have been allocated with maximum allowable number of channels.

V. SIMULATION RESULTS

In all the following simulations, we set the sampling frequency $f_s = 6\text{MHz}$ and the slot duration $T = 200\text{ms}$. To model the heterogeneous characteristics of PU channels and secondary S-D pairs, the channel idle probability and channel capacity are randomly generated with means 0.7 and 0.9, respectively. The energy detection threshold and noise power are generated randomly with means 1.03 and 1 respectively. To

provide a better understanding on how our proposed spectrum allocation algorithm behaves, we depict the allocation results for the following two interference graph settings:

- Setting I: As shown in Fig. 2(a), all the S-D pairs interfere with each other, which means that any two S-D pairs cannot be allocated with the same channel.
- Setting II: As shown in Fig. 2(b), S-D pair 1 conflicts with S-D pairs 2 and 5, and S-D pair 2 conflicts with S-D pairs 1 and 3, and so on. In this case, if one channel is allocated to S-D pair 1, it cannot be allocated to S-D pairs 2 and 5 simultaneously. However, this channel is able to be utilized by S-D pairs 3 and 4.

A. Evaluation of our proposed algorithm

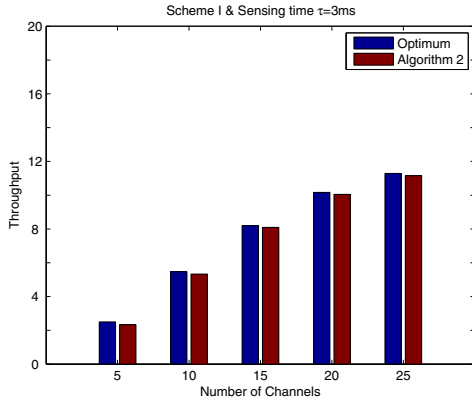
To provide a better understand of how our proposed algorithm performs, we first implement and evaluate Algorithm 2. Fig. 3(a) and Fig. 3(b) compare the spectrum allocation results for the proposed algorithm as well as the optimal solution obtained using exhaustive search for both setting I and setting II. From Fig. 3(a) and Fig. 3(b), we observe that Algorithm 2 achieves an impressive performance, approaching the optimum rather closely in most cases with a maximum performance loss of 6.8% for setting I and 3.5% for setting II. This result shows that our spectrum allocation problem based on the proposed algorithm is reasonable and can achieve a close-to optimal performance.

B. Spectrum Allocation Results

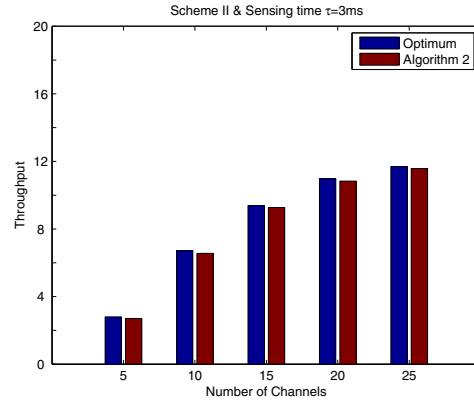
In this subsection, we depict the spectrum allocation results for the two settings: In Fig. 4(a), the spectrum allocation results are shown for system setting I. As discussed before, we take the heterogeneous characteristics of both PU channels and secondary S-D pairs into consideration so that a more detailed result that accurately indicates which S-D pair should utilize which channel can be achieved. As shown in Fig. 4(a), channels 1 and 4 are allocated to S-D pair 1 for sensing and utilization; channels 5 and 7 are allocated to S-D pairs 2 and 3, respectively; channels 6 and 8 are allocated to S-D pair 4; and channels 2, 3 and 9 are allocated to S-D pair 5. From Fig. 4(a), we can see that no channel can be allocated to two different S-D pairs, since in setting I all the S-D pairs conflict with each other. Moreover, we note that channel 10 is not allocated to any S-D pair, this is because no S-D pair is within the detection range of channel 10, thus it cannot be detected and utilized by secondary S-D pair. Furthermore, Fig. 4(b) illustrates the spectrum allocation results for setting II. Different from setting I, in setting II, some S-D pairs can allocate with the same channel. For example, channel 6 is allocated to S-D 1 and S-D 3 simultaneously, since S-D 1 does not conflict with S-D 3; and channel 7 is also allocated to S-D 2 and S-D 5 simultaneously.

VI. CONCLUSIONS

In this paper, we focus on the specific spectrum allocation problem: How to appropriately allocate the available PU channels to secondary S-D pairs? We take the heterogeneities

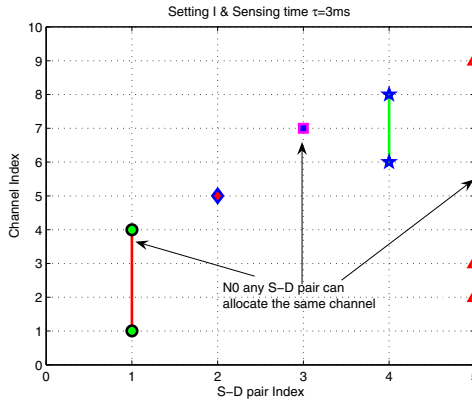


(a) Achieved throughput for Setting I.

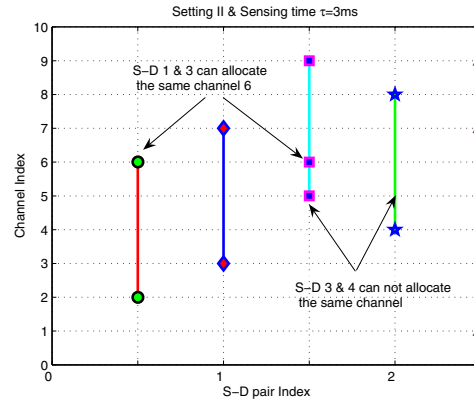


(b) Achieved throughput for Setting II.

Fig. 3. Comparison between optimum and Algorithm 2 for setting I and II.



(a) Spectrum allocation result for Setting I.



(b) Spectrum allocation result for Setting II.

Fig. 4. Spectrum allocation results for setting I and II.

of both the PU channels and secondary S-D pairs into consideration, which has not been fully studied in most of the literatures. With the objective to maximize the achievable throughput for secondary S-D pairs, the spectrum allocation problem is formulated as a linear integer problem, where the availability constraint, spectrum span constraint and interference free constraint are taken into consideration. The proposed solution leverages on a recent result in theoretical computer science that can decompose an optimal fractional solution to a NP-hard problem into a convex combination of internal solutions. Evaluation results show that the proposed algorithm can achieve a close-to-optimal solution with far less complexity.

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