
Robust energy-efficient power loading for MIMO system under imperfect CSI

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Abstract: In this paper, we will investigate the energy efficient power loading problem in multiple-input multiple-output singular value decomposition (MIMO-SVD) architecture. Most existing power loading schemes are developed on the assumption that a scheduler possesses perfect channel state

information (CSI). However, in practice, perfect CSI is not always available. In this paper, we propose an energy-efficient power loading scheme for MIMO system under imperfect CSI, taking into account the effects of channel estimation error (CEE). Specifically, we propose two algorithms to maximise the system energy efficiency. One is a global method which can achieve the global optimal solution with high computational complexity; the other is a suboptimal method which can obtain the approximate optimal solution with lower computational complexity. Simulation results validate the effectiveness and robustness of our proposed power loading scheme.

Keywords: energy efficiency; power loading; MIMO; multiple-input multiple-output; imperfect CSI.

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1 Introduction

Multiple-input multiple-output (MIMO) technology has attracted a great attention due to its high spectral efficiency (Telatar, 1999). With the transmit channel state information (CSI), singular value decomposition (SVD) can be utilised for MIMO channel to effectively create parallel independent channels, which possess different signal-to-noise ratio (SNR). Thus, by carefully performing power allocation to each subchannel, the system performance can be optimised to choose a few of the best quality channels or to use all channels to achieve the high rate (Dighe et al., 1999). However, the application of multiple radio chains incurs a higher circuit

power consumption. Recently, considerable research effort has been made to focus on optimising the energy efficiency (EE) of MIMO systems, mostly considering power loading under the assumption of perfect CSI at the transmitter (Prabhu and Daneshrad, 2010). However, it is not realistic to assume that the transmitter can always get perfect CSI in a MIMO cellular system. In this paper, we will study the power loading scheme under channel estimation error (CEE) for MIMO systems. To the best of our knowledge, we are the first to study the impact of CEE on energy efficiency in MIMO systems.

Specifically, we also consider a MIMO-SVD architecture for the transceivers. By modeling CEE as an independent complex Gaussian random variable (Wang et al., 2006), we

derive the effective signal-to-interference-plus-noise (SINR) at the receiver under CEE, given the availability of an estimated channel. We then obtain the energy efficiency model, which is the ratio of transmission rates to power consumption. Different from results derived in Prabhu and Daneshrad (2010), the objective function after the parameterisation transformation is still non-convex. Thus, we further propose two methods to solve this problem. One is transforming to canonical difference of convex (DC) programming (Horst and Thoai, 1999), which is proved to have the only global solution. Furthermore, considering the complexity, an approximate method is proposed to relax the objective function of the convex problem, which leads to a closed-form solution. Simulation results show the effectiveness and robustness of the proposed two algorithms against CEE.

The rest of the paper is outlined as follows. The system model is described in Section 2. Then, we propose two algorithms to solve the power loading problem in Section 3. Simulation results are provided in Section 4, followed by the conclusion in Section 5.

2 System model

We consider an uncorrelated flat fading MIMO system with N_t transmit antennas and N_r receive antennas. The output signals can be modelled as

$$\mathbf{r} = \mathbf{H}\mathbf{s} + \mathbf{n}, \quad (1)$$

where $\mathbf{s} \in \mathbb{C}^{N_t \times 1}$ denotes transmitted signals, $\mathbf{H} \in \mathbb{C}^{N_r \times N_t}$ denotes the channel matrix, and $\mathbf{n} \in \mathbb{C}^{N_r \times 1}$ is modelled as zero-mean additive white Gaussian noise (AWGN) with variance σ_n^2 . When CEE occurs, we assume that the MIMO transceiver can only obtain the imperfect CSI, which is modelled as (Wang et al., 2006; Song and Zhang, 2006)

$$\mathbf{H} = \hat{\mathbf{H}} + \mathbf{E}, \quad (2)$$

where \mathbf{E} is the estimation error matrix, and each element is with zero mean and variance σ_e^2 . Then, by SVD decomposition of $\hat{\mathbf{H}}$, we can obtain

$$\hat{\mathbf{H}} = \hat{\mathbf{U}} \cdot \hat{\mathbf{D}} \cdot \hat{\mathbf{V}}^H = \hat{\mathbf{U}} \cdot \text{diag} \left(\sqrt{\hat{\lambda}_1}, \dots, \sqrt{\hat{\lambda}_{N_{ss}}} \right) \hat{\mathbf{V}}^H, \quad (3)$$

where $N_{ss} = \min\{N_t, N_r\}$ is the rank of $\hat{\mathbf{H}}$ and $\{\hat{\lambda}_i\}_{i=1}^{N_{ss}}$ is the eigenvalue of matrix $\hat{\mathbf{H}}\hat{\mathbf{H}}^H$.

Moreover, the signals sent over transmit antennas \mathbf{s} are obtained by performing a transformation $\mathbf{s} = \hat{\mathbf{V}}\mathbf{P}\mathbf{x}$, and \mathbf{P} is power allocation diagonal matrix, \mathbf{x} is the information symbol vector from unit-energy constellation set. Thus, the output signals can be rewritten as

$$\begin{aligned} \mathbf{r} &= (\hat{\mathbf{U}} \cdot \hat{\mathbf{D}} \cdot \hat{\mathbf{V}}^H + \mathbf{E})\mathbf{s} + \mathbf{n} \\ &= \hat{\mathbf{U}}\hat{\mathbf{D}}\hat{\mathbf{V}}^H\hat{\mathbf{V}}\mathbf{P}\mathbf{x} + \mathbf{E}\hat{\mathbf{V}}\mathbf{P}\mathbf{x} + \mathbf{n} \\ &= \hat{\mathbf{U}}\hat{\mathbf{D}}\mathbf{P}\mathbf{x} + \mathbf{E}\hat{\mathbf{V}}\mathbf{P}\mathbf{x} + \mathbf{n}. \end{aligned} \quad (4)$$

At the receiver, after linear transformation, it yields

$$\begin{aligned} \mathbf{y} &= \hat{\mathbf{U}}^H \mathbf{r} = \hat{\mathbf{U}}^H \hat{\mathbf{U}} \hat{\mathbf{D}} \mathbf{P} \mathbf{x} + \hat{\mathbf{U}}^H \mathbf{E} \hat{\mathbf{V}} \mathbf{P} \mathbf{x} + \hat{\mathbf{U}}^H \mathbf{n} \\ &= \hat{\mathbf{D}} \mathbf{P} \mathbf{x} + \hat{\mathbf{U}}^H \mathbf{E} \hat{\mathbf{V}} \mathbf{P} \mathbf{x} + \hat{\mathbf{U}}^H \mathbf{n} \\ &= \hat{\mathbf{D}} \mathbf{P} \mathbf{x} + \hat{\mathbf{E}} \mathbf{P} \mathbf{x} + \hat{\mathbf{U}}^H \mathbf{n}, \end{aligned} \quad (5)$$

and the received signal on the i th sub-channel can be expressed as

$$\begin{aligned} y_i &= [\hat{\mathbf{D}} \mathbf{P} \mathbf{x} + \hat{\mathbf{E}} \mathbf{P} \mathbf{x} + \hat{\mathbf{U}}^H \mathbf{n}]_i \\ &= \left(\sqrt{\hat{\lambda}_i} + \hat{e}_{ii} \right) P_i x_i + \sum_{j=1, j \neq i}^{N_{ss}} \hat{e}_{ij} P_j x_j + \hat{n}_i \\ &= \sqrt{\hat{\lambda}_i} P_i x_i + \sum_{j=1}^{N_{ss}} \hat{e}_{ij} P_j x_j + \hat{n}_i. \end{aligned} \quad (6)$$

Then, the SNR on the i th subchannel can be approximated as (Schaible, 1976)

$$SNR_i = \frac{\hat{\lambda}_i P_i}{\sigma_e^2 \sum_{j=1}^{N_{ss}} P_j + \sigma_n^2}, \quad (7)$$

where $i = 1, 2, \dots, N_{ss}$, $N_{ss} = \min(N_r, N_t)$.

Since energy efficiency is defined as the ratio of the transmitted bit energy consumption to the total energy consumption, we can obtain the energy efficiency under imperfect CSI as

$$\max EE(\mathbf{P}) \left\{ = \frac{\sum_{i=1}^{N_{ss}} \log(1 + SNR_i)}{\sum_{i=1}^{N_{ss}} P_i + P'_c} \right\} \quad (8)$$

$$\text{s.t.} \quad \sum_{i=1}^{N_{ss}} P_i \leq P_T \quad (9)$$

$$0 \leq P_i \leq P_{\max} \quad (10)$$

$$\sum_{i=1}^{N_{ss}} \log(1 + SNR_i) \geq R_{\min}, \quad (11)$$

where $P'_c = N_{ss}P_c$, P_c is the average circuit power consumption in a single transmitter or receiver chain.

3 The proposed algorithms

Since the optimisation problem in equation (8) is a fractional programming problem and the objective function, i.e., $EE(\mathbf{P})$, is non-convex and non-concave, we cannot apply convex optimisation methods to solve this problem. However, according to Schaible (1976), we can transform such a fractional programming problem into a two-layer optimisation problem. The following is the transformation process. First, we let

$$g(\mathbf{P}, q) = \sum_{i=1}^{N_{ss}} \log(1 + SNR_i) - q \left(\sum_{i=1}^{N_{ss}} P_i + P'_c \right), \quad (12)$$

$$f(q) = \left\{ \max_{\mathbf{P} \in \mathbb{D}} g(\mathbf{P}, q) \right\}, \quad (13)$$

where \mathbb{D} is the power constraint region consisting of equations (9)–(11), which is a convex set. Note that \mathbb{D} is non-empty and compact, and both the numerator part $\sum_{i=1}^{N_{ss}} \log(1 + SNR_i)$ and the denominator part $(\sum_{i=1}^{N_{ss}} P_i + P'_c)$ are continuous on \mathbb{D} , and the denominator part is positive. Then from Schaible (1976), we can obtain the optimal energy efficiency $EE^*(\mathbf{P}) = q^*$ when $f(q^*) = 0$. Therefore, the fraction programming problem in (8) can be solved by a two-layer algorithm as two layers:

- *inner layer*: for a given q , find the maximum g^* which is also $f(q)$, i.e., $f(q) = g^* = \{\max_{\mathbf{P} \in \mathbb{D}} g(\mathbf{P}, q)\}$
- *outer layer*: find the zero point of $f(q)$, i.e., $q^* = \{q | f(q) = 0\}$.

Also from Schaible (1976), $f(q)$ is continuous, convex and strictly decreasing on \mathbb{R} , so the outer layer problem can be easily solved by bisection search algorithm (Burden and Faires, 2000). Then in the rest of this paper, we will focus on the inner layer. Note that $g(\mathbf{P}, q)$ is still non-convex in equation (12), we cannot apply convex optimisation solutions directly. Here we will adopt two methods to solve optimisation problem in inner layer. The first one is a global method using D.C. programming, and the second one is a suboptimal method which can reduce the complexity and maintain reasonable performance.

3.1 Global method

In this subsection, we will transform the inner layer optimisation problem into a canonical D.C. programming problem. First, we rewrite equation (12) as

$$\begin{aligned} g(\mathbf{P}, q) &= \sum_{i=1}^{N_{ss}} \log(1 + SNR_i) - q \left(\sum_{i=1}^{N_{ss}} P_i + P'_c \right) \\ &= \sum_{i=1}^{N_{ss}} \log \left(1 + \frac{\hat{\lambda}_i P_i}{\sigma_e^2 \sum_{j=1}^{N_{ss}} P_j + \sigma_n^2} \right) \\ &\quad - q \left(\sum_{i=1}^{N_{ss}} P_i + P'_c \right) \\ &= \sum_{i=1}^{N_{ss}} \log \left(\frac{\hat{\lambda}_i P_i + \sigma_e^2 \sum_{j=1}^{N_{ss}} P_j + \sigma_n^2}{\sigma_e^2 \sum_{j=1}^{N_{ss}} P_j + \sigma_n^2} \right) \\ &\quad - q \left(\sum_{i=1}^{N_{ss}} P_i + P'_c \right) \\ &= \sum_{i=1}^{N_{ss}} \log(\hat{\lambda}_i P_i + \sigma_e^2 \sum_{j=1}^{N_{ss}} P_j + \sigma_n^2) \\ &\quad - \left[\sum_{i=1}^{N_{ss}} \log \left(\sigma_e^2 \sum_{j=1}^{N_{ss}} P_j + \sigma_n^2 \right) \right. \\ &\quad \left. + q \left(\sum_{i=1}^{N_{ss}} P_i + P'_c \right) \right]. \end{aligned} \quad (14)$$

Then, we let

$$m(\mathbf{P}) = - \sum_{i=1}^{N_{ss}} \log(\hat{\lambda}_i P_i + \sigma_e^2 \sum_{j=1}^{N_{ss}} P_j + \sigma_n^2), \quad (15)$$

$$\begin{aligned} n(\mathbf{P}, q) &= - \left[\sum_{i=1}^{N_{ss}} \log(\sigma_e^2 \sum_{j=1}^{N_{ss}} P_j + \sigma_n^2) \right. \\ &\quad \left. + q \left(\sum_{i=1}^{N_{ss}} P_i + P'_c \right) \right], \end{aligned} \quad (16)$$

$$f_0(\mathbf{P}, q) = -g(\mathbf{P}, q) = m(\mathbf{P}) - n(\mathbf{P}, q). \quad (17)$$

Thus, the inner layer optimisation problem can be rewritten as

$$\begin{aligned} f(q) &= \left\{ \max_{\mathbf{P} \in \mathbb{D}} g(\mathbf{P}, q) \right\} = - \left\{ \min_{\mathbf{P} \in \mathbb{D}} f_0(\mathbf{P}, q) \right\} \\ &= - \left\{ \min_{\mathbf{P} \in \mathbb{D}} [m(\mathbf{P}) - n(\mathbf{P}, q)] \right\}. \end{aligned} \quad (18)$$

We should solve the optimisation problem

$$\left\{ \min_{\mathbf{P} \in \mathbb{D}} f_0(\mathbf{P}, q) \right\} = \left\{ \min_{\mathbf{P} \in \mathbb{D}} [m(\mathbf{P}) - n(\mathbf{P}, q)] \right\}. \quad (19)$$

Since $m(\mathbf{P})$ and $n(\mathbf{P}, q)$ are all convex with \mathbf{P} , (19) is a D.C. programming problem. Next we show this problem can be further transformed into a canonical D.C. programming problem, which can be solved by some well-known algorithms (Horst and Thoai, 1999).

We first change the constraint region \mathbb{D} consisting of equations (9)–(11) in sequence as

$$f_1(\mathbf{P}) = g_1(\mathbf{P}) - h_1(\mathbf{P}) = \sum_{i=1}^{N_{ss}} P_i - P_T \leq 0, \quad (20)$$

where $g_1(\mathbf{P}) = \sum_{i=1}^{N_{ss}} P_i$ and $h_1(\mathbf{P}) = P_T$ are convex, and $f_1(\mathbf{P})$ is a D.C. function;

$$\mathbb{D}_0 = \{\mathbf{P} \in \mathbb{R}^{N_{ss}} | 0 \leq P_i \leq P_{\max}\}, \quad (21)$$

where \mathbb{D}_0 is a N_{ss} -dimensional rectangle in $\mathbb{R}^{N_{ss}}$;

$$\begin{aligned} f_2(\mathbf{P}) &= g_2(\mathbf{P}) - h_2(\mathbf{P}) = R_{\min} - \sum_{i=1}^{N_{ss}} \log(1 + SNR_i) \\ &= R_{\min} - \left[\sum_{i=1}^{N_{ss}} \log(\hat{\lambda}_i P_i + \sigma_e^2 \sum_{j=1}^{N_{ss}} P_j + \sigma_n^2) \right. \\ &\quad \left. - \sum_{i=1}^{N_{ss}} \log(\sigma_e^2 \sum_{j=1}^{N_{ss}} P_j + \sigma_n^2) \right] \\ &= \left[R_{\min} - \sum_{i=1}^{N_{ss}} \log(\hat{\lambda}_i P_i + \sigma_e^2 \sum_{j=1}^{N_{ss}} P_j + \sigma_n^2) \right] \\ &\quad - \left[- \sum_{i=1}^{N_{ss}} \log(\sigma_e^2 \sum_{j=1}^{N_{ss}} P_j + \sigma_n^2) \right] \leq 0, \end{aligned} \quad (22)$$

where $g_2(\mathbf{P}) = R_{\min} - \sum_{i=1}^{N_{ss}} \log(\hat{\lambda}_i P_i + \sigma_e^2 \sum_{j=1}^{N_{ss}} P_j + \sigma_n^2)$ and $h_2(\mathbf{P}) = -\sum_{i=1}^{N_{ss}} \log(\sigma_e^2 \sum_{j=1}^{N_{ss}} P_j + \sigma_n^2)$ are convex, and $f_2(\mathbf{P})$ is a D.C. function.

Therefore, the optimisation problem (19) can be transformed to:

$$\{\min f_0(\mathbf{P}, q) | \mathbf{P} \in \mathbb{D}_0, f_1(\mathbf{P}) \leq 0, f_2(\mathbf{P}) \leq 0\}. \quad (23)$$

Next we show how to obtain the canonical D.C. structure. By introducing another variable $P_{N_{ss}+1}$, the optimisation problem (23) can be transformed to,

$$\{\min P_{N_{ss}+1} | \mathbf{P} \in \mathbb{D}_0, f_0(\mathbf{P}, q) - P_{N_{ss}+1} \leq 0, f_1(\mathbf{P}) \leq 0, f_2(\mathbf{P}) \leq 0\}. \quad (24)$$

Denote

$$\begin{aligned} K(\mathbf{P}, P_{N_{ss}+1}, q) &= \max\{f_0(\mathbf{P}, q) - P_{N_{ss}+1}, f_1(\mathbf{P}), f_2(\mathbf{P})\} \\ &= \max\{m(\mathbf{P}) - [n(\mathbf{P}, q) + P_{N_{ss}+1}], g_1(\mathbf{P}) - h_1(\mathbf{P}), \\ &\quad g_2(\mathbf{P}) - h_2(\mathbf{P})\} \\ &\stackrel{(a)}{=} \max\{g_0(\mathbf{P}) - h_0(\mathbf{P}, P_{N_{ss}+1}, q), g_1(\mathbf{P}) - h_1(\mathbf{P}), \\ &\quad g_2(\mathbf{P}) - h_2(\mathbf{P})\} \\ &= M(\mathbf{P}, P_{N_{ss}+1}, q) - N(\mathbf{P}, P_{N_{ss}+1}, q), \end{aligned} \quad (25)$$

where $g_0(\mathbf{P}) = m(\mathbf{P})$ and $h_0(\mathbf{P}, P_{N_{ss}+1}, q) = n(\mathbf{P}, q) + P_{N_{ss}+1}$ as shown in (a). In addition, $M(\mathbf{P}, P_{N_{ss}+1}, q)$ and $N(\mathbf{P}, P_{N_{ss}+1}, q)$ in equation (25) can be achieved as

$$\begin{aligned} M(\mathbf{P}, P_{N_{ss}+1}, q) &= \max\{g_0(\mathbf{P}) + h_1(\mathbf{P}) + h_2(\mathbf{P}), \\ &\quad g_1(\mathbf{P}) + h_0(\mathbf{P}, P_{N_{ss}+1}, q) + h_2(\mathbf{P}), \\ &\quad g_2(\mathbf{P}) + h_0(\mathbf{P}, P_{N_{ss}+1}, q) + h_1(\mathbf{P})\}, \end{aligned} \quad (26)$$

and

$$\begin{aligned} N(\mathbf{P}, P_{N_{ss}+1}, q) &= h_0(\mathbf{P}, P_{N_{ss}+1}, q) \\ &\quad + h_1(\mathbf{P}) + h_2(\mathbf{P}). \end{aligned} \quad (27)$$

It is clear that both $M(\mathbf{P}, P_{N_{ss}+1}, q)$ and $N(\mathbf{P}, P_{N_{ss}+1}, q)$ are convex due to the convexity of $g_i, h_i, \forall i = 0, 1, 2$. Thus, $K(\mathbf{P}, P_{N_{ss}+1}, q)$ is a D.C. function. The optimisation problem in equation (24) is equivalent to

$$\{\min P_{N_{ss}+1} | \mathbf{P} \in \mathbb{D}_0, K(\mathbf{P}, P_{N_{ss}+1}, q) \leq 0\}. \quad (28)$$

Moreover, if we introduce another variable $P_{N_{ss}+2}$, the optimisation problem in (28) can be transformed to

$$\{\min P_{N_{ss}+1} | z \in \mathbb{Z}, \psi(z, q) \leq 0\}, \quad (29)$$

where $z = \{\mathbf{P}, P_{N_{ss}+1}, P_{N_{ss}+2}\} \in \mathbb{R}^{N_{ss}+2}$, $\mathbb{Z} = \{z \in \mathbb{R}^{N_{ss}+2} | \mathbf{P} \in \mathbb{D}_0, M(\mathbf{P}, P_{N_{ss}+1}, q) - P_{N_{ss}+2} \leq 0\}$ and $\psi(z, q) = P_{N_{ss}+2} - N(\mathbf{P}, P_{N_{ss}+1}, q)$. Note that \mathbb{Z} is a convex set and $\psi(z, q)$ is a concave function, indicating (29) is a canonical D.C. structure (Horst and Thoai, 1999). We can solve the canonical D.C. programming problem with two types of algorithms, branch-and-bound type and outer-approximation type (Horst and Thoai, 1999). By applying either of these two algorithms, we can obtain the global solution to the energy efficiency optimisation problem in equation (8).

3.2 Suboptimal method

Though the global optimal solution can be achieved after transforming equation (8) into a canonical D.C. programming problem as shown above, the computational complexity is still large for the solving process with branch-and-bound type and outer-approximation algorithms. In addition, we cannot get a closed-form optimal solution. To overcome these drawbacks, in this subsection, we will propose a suboptimal method to solve the inner layer optimisation problem, which can reduce complexity and obtain a closed-form suboptimal solution.

First, we use the following lower bound obtained in Papadriopoulos and Evans (2006)

$$\begin{aligned} &\alpha \log z + \beta \\ &\leq \log(1+z) \begin{cases} \alpha = \frac{z_0}{1+z_0} \\ \beta = \log(1+z_0) - \frac{z_0}{1+z_0} \log(z_0) \end{cases} \end{aligned} \quad (30)$$

This bound is tight with equality at a chosen value z_0 when the constants are chosen as specified above. As a result, the inner layer optimisation problem can be relaxed as

$$\begin{aligned} &\max g(\mathbf{P}, q) \\ &= \sum_{i=1}^{N_{ss}} [\alpha_i \log(SNR_i) + \beta_i] - q \left(\sum_{i=1}^{N_{ss}} P_i + P'_c \right). \end{aligned} \quad (31)$$

By doing the following transformation, $\tilde{P}_i = \log(P_i)$ and $P_i = e^{\tilde{P}_i}$, then we have

$$\begin{aligned} g(\tilde{\mathbf{P}}, q) &= \sum_{i=1}^{N_{ss}} [\alpha_i \log(SNR_i) + \beta_i] \\ &\quad - q \left(\sum_{i=1}^{N_{ss}} P_i + P'_c \right) \\ &= \sum_{i=1}^{N_{ss}} \left[\alpha_i \log \left(\frac{\hat{\lambda}_i e^{\tilde{P}_i}}{\sigma_e^2 \sum_{j=1}^{N_{ss}} e^{\tilde{P}_j} + \sigma_n^2} \right) + \beta_i \right] \\ &\quad - q \left(\sum_{i=1}^{N_{ss}} e^{\tilde{P}_i} + P'_c \right) \\ &= \sum_{i=1}^{N_{ss}} \left[\alpha_i \log(\hat{\lambda}_i) + \alpha_i \tilde{P}_i \right. \\ &\quad \left. - \alpha_i \log(\sigma_e^2 \sum_{j=1}^{N_{ss}} e^{\tilde{P}_j} + \sigma_n^2) + \beta_i \right] \\ &\quad - q \sum_{i=1}^{N_{ss}} e^{\tilde{P}_i} - q P'_c \\ &= \sum_{i=1}^{N_{ss}} [\alpha_i \log(\hat{\lambda}_i) + \alpha_i \tilde{P}_i + \beta_i] \\ &\quad - \sum_{i=1}^{N_{ss}} [\alpha_i \log(\sigma_e^2 \sum_{j=1}^{N_{ss}} e^{\tilde{P}_j} + \sigma_n^2) + q e^{\tilde{P}_i}] - q P'_c \end{aligned}$$

$$\begin{aligned}
&= \sum_{i=1}^{N_{ss}} [\alpha_i \log(\hat{\lambda}_i) + \alpha_i \tilde{P}_i + \beta_i] \\
&\quad - \underbrace{\left[\left(\sum_{i=1}^{N_{ss}} \alpha_i \right) \log \left(\sigma_e^2 \sum_{j=1}^{N_{ss}} e^{\tilde{P}_j} + \sigma_n^2 \right) \right]}_{\text{Convex}} \\
&\quad + \underbrace{\left[\sum_{i=1}^{N_{ss}} q e^{\tilde{P}_i} \right]}_{\text{Convex}} - q P'_c, \tag{32}
\end{aligned}$$

where $\tilde{\mathbf{P}} = \{\tilde{P}_1, \tilde{P}_2, \dots, \tilde{P}_{N_{ss}}\}$. Because the second item is the sum of convex function, the objective function is concave with $\tilde{\mathbf{P}}$. Next we show the constraint region is convex with $\tilde{\mathbf{P}}$. The power sum constraint in equation (9) can be transformed as

$$\tilde{f}_1(\tilde{\mathbf{P}}) = \sum_{i=1}^{N_{ss}} e^{\tilde{P}_i} - P_T \leq 0. \tag{33}$$

With the lower bound in equation (30), the minimal throughput constraint in equation (11) can be transformed as

$$\begin{aligned}
\tilde{f}_2(\tilde{\mathbf{P}}) &= R_{\min} - \sum_{i=1}^{N_{ss}} \log(1 + \text{SNR}_i) \\
&= R_{\min} - \sum_{i=1}^{N_{ss}} \log \left(1 + \frac{\hat{\lambda}_i e^{\tilde{P}_i}}{\sigma_e^2 \sum_{j=1}^{N_{ss}} e^{\tilde{P}_j} + \sigma_n^2} \right) \\
&= R_{\min} - \sum_{i=1}^{N_{ss}} \left(\alpha_i \log \frac{\hat{\lambda}_i e^{\tilde{P}_i}}{\sigma_e^2 \sum_{j=1}^{N_{ss}} e^{\tilde{P}_j} + \sigma_n^2} + \beta_i \right) \\
&= R_{\min} - \sum_{i=1}^{N_{ss}} \left[\alpha_i \log \hat{\lambda}_i + \alpha_i \tilde{P}_i \right. \\
&\quad \left. - \alpha_i \log \left(\sigma_e^2 \sum_{j=1}^{N_{ss}} e^{\tilde{P}_j} + \sigma_n^2 \right) + \beta_i \right] \\
&= R_{\min} - \sum_{i=1}^{N_{ss}} \left[\alpha_i \log \hat{\lambda}_i + \beta_i \right] - \sum_{i=1}^{N_{ss}} \alpha_i \tilde{P}_i \\
&\quad + \sum_{i=1}^{N_{ss}} \alpha_i \log \left(\sigma_e^2 \sum_{j=1}^{N_{ss}} e^{\tilde{P}_j} + \sigma_n^2 \right) \leq 0 \tag{34}
\end{aligned}$$

Because $\tilde{f}_1(\tilde{\mathbf{P}})$ and $\tilde{f}_2(\tilde{\mathbf{P}})$ are convex function, the constraint region is a convex set. Therefore, we can use Karush-Kuhn-Tucker (KKT) conditions to solve the optimisation problem. Denote $\lambda^* \geq 0$, $v^* \geq 0$ as KKT multipliers, then $\forall i = 1, 2, \dots, N_{ss}$, the global optima can be obtained by

$$\frac{\partial g(\tilde{\mathbf{P}}, q)}{\partial \tilde{P}_i^*} + \lambda^* \frac{\partial \tilde{f}_1(\tilde{\mathbf{P}})}{\partial \tilde{P}_i^*} + v^* \frac{\partial \tilde{f}_2(\tilde{\mathbf{P}})}{\partial \tilde{P}_i^*} = 0, \tag{35}$$

where

$$\frac{\partial g(\tilde{\mathbf{P}}, q)}{\partial \tilde{P}_i^*} = \alpha_i - \left(q e^{\tilde{P}_i^*} + \sum_{i=1}^{N_{ss}} \alpha_i \cdot \frac{\sigma_e^2 e^{\tilde{P}_i^*}}{\sigma_e^2 \sum_{j=1}^{N_{ss}} e^{\tilde{P}_j^*} + \sigma_n^2} \right), \tag{36}$$

$$\frac{\partial \tilde{f}_1(\tilde{\mathbf{P}})}{\partial \tilde{P}_i^*} = e^{\tilde{P}_i}, \tag{37}$$

and

$$\frac{\partial \tilde{f}_2(\tilde{\mathbf{P}})}{\partial \tilde{P}_i^*} = -\alpha_i + \sum_{i=1}^{N_{ss}} \alpha_i \frac{\sigma_e^2 e^{\tilde{P}_i^*}}{\sigma_e^2 \sum_{j=1}^{N_{ss}} e^{\tilde{P}_j^*} + \sigma_n^2}. \tag{38}$$

Then we can sum equation (35) with respect to i , yielding

$$\bar{\alpha} - (qx + \bar{\alpha} \cdot \frac{x}{x + \theta}) + \lambda^* x - v^* \bar{\alpha} + v^* \bar{\alpha} \cdot \frac{x}{x + \theta} = 0, \tag{39}$$

where $\bar{\alpha} = \sum_{i=1}^{N_{ss}} \alpha_i$, $x = \sum_{i=1}^{N_{ss}} e^{\tilde{P}_i^*}$, and $\theta = \frac{\sigma_n^2}{\sigma_e^2}$. After some derivations for equation (39), we can get

$$x^2 + \theta x + \frac{\bar{\alpha} \theta (1 - v^*)}{(\lambda^* - q)} = 0, \tag{40}$$

with the solution

$$x = \pm \sqrt{\frac{\bar{\alpha} \theta (v^* - 1)}{(\lambda^* - q)} + \frac{\theta^2}{4}} - \frac{\theta}{2}. \tag{41}$$

Since $x \geq 0$, we have

$$x = \sqrt{\frac{\bar{\alpha} \theta (v^* - 1)}{(\lambda^* - q)} + \frac{\theta^2}{4}} - \frac{\theta}{2}. \tag{42}$$

Then, equation (35) is equivalent to

$$\begin{aligned}
&(\lambda^* - q) e^{\tilde{P}_i^*} + (v^* - 1) \sum_{i=1}^{N_{ss}} \alpha_i \frac{\sigma_e^2 e^{\tilde{P}_i^*}}{\sigma_e^2 \sum_{j=1}^{N_{ss}} e^{\tilde{P}_j^*} + \sigma_n^2} \\
&= (v^* - 1) \alpha_i. \tag{43}
\end{aligned}$$

Inserting equation (42) into equation (43) with $x = \sum_{i=1}^{N_{ss}} e^{\tilde{P}_i^*}$, and considering the power constraint (10), we obtain the final closed-form suboptimal solution, i.e.,

$$P_i^* = \left[\frac{\alpha_i (v^* - 1) \left(\sqrt{\frac{\bar{\alpha} \theta (v^* - 1)}{(\lambda^* - q)} + \frac{\theta^2}{4}} + \frac{\theta}{2} \right)}{(\lambda^* - q) \left(\sqrt{\frac{\bar{\alpha} \theta (v^* - 1)}{(\lambda^* - q)} + \frac{\theta^2}{4}} + \frac{\theta}{2} \right) + \bar{\alpha} (v^* - 1)} \right]_{0}^{P_{\max}} \tag{44}$$

Note that the KKT multipliers λ^* and v^* can be obtained by the water-filling procedure, the same as (Prabhu and Daneshrad, 2010), with the complementary slackness, i.e.,

$$\lambda^* \tilde{f}_1(\tilde{\mathbf{P}}) = 0, \tag{45}$$

$$v^* \tilde{f}_2(\tilde{\mathbf{P}}) = 0. \tag{46}$$

We summarise the suboptimal method as shown in Algorithm A.

A.	Suboptimal Method
1:	Initialize:
2:	iteration counter $t = 0$.
3:	$\alpha_i = 1, \beta_i = 0$, for $1 \leq i \leq N_{ss}$, (high SNR approximate).
4:	Repeat:
5:	maximize: solve (31) to give solution \mathbf{P} .
6:	set: $\mathbf{P}^* = \mathbf{P}$.
7:	tighten: update $\alpha_i = \frac{z_i}{1+z_i}$, $\beta_i = \log(1+z_i) - \frac{z_i}{1+z_i} \log(z_i)$, with $z_i = \text{SNR}_i(\mathbf{P})$.
8:	increment t .
9:	Until convergence

4 Simulation results

In this section, Monte Carlo simulations are used to illustrate proposed power loading algorithms to optimise EE in MIMO systems under imperfect CSI. The detailed simulation parameters are listed in Table 1. At the receiver, the ZF (zero-forcing) structure is adopted, where perfect synchronisation is assumed.

Table 1 Simulation parameters

Frame duration	0.5 ms
N_t (No. of Tx antenna at user)	2
N_r (No. of Rx antenna at Node-B)	2
Carrier frequency	2 GHz
Sampling frequency	7.68 MHz
Receiver	Zero forcing
Traffic model	Full buffer
σ_n^2 (Noise variance)	1
# of frames	5000

Figure 1 shows that the energy efficiency varies on estimation error variance with both the global method and the suboptimal method. From this figure, we can see as the estimation error variance, i.e., σ_e^2 , increases, the EE performance deteriorate. In addition, we compare the global method and suboptimal method in this figure. The global method can always achieve better EE performance than suboptimal method, but the gap is very little. This substantiates the effectiveness of our proposed suboptimal method.

Figure 1 EE vs. σ_e^2 (see online version for colours)

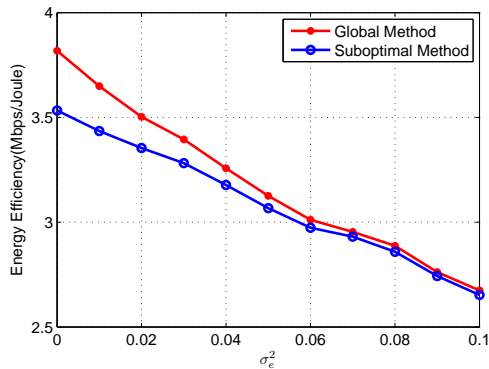


Figure 2 validates the robustness of our proposed power loading scheme. In this figure, we use the global method to obtain the global maximal EE. As shown, when the estimation error variance $\sigma_e^2 < 10^{-2}$, the EE performance is almost flat with σ_e^2 . That means our proposed energy efficient power loading scheme is robust against CEE.

Figure 2 EE vs. σ_e^2 (see online version for colours)

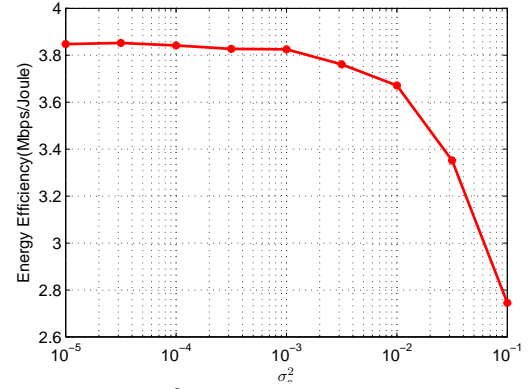


Figure 3 evaluates the effect of circuit power P_c . It can be seen that larger P_c leads to worse EE performance. This is because P_c does not affect the transmission rate, but only the energy consumption. If P_c is increased, we cannot obtain any improvement in the transmission rate. In contrast, the energy consumption will increase accordingly. So EE performance will deteriorate. We can also see the suboptimal methods can achieve close performance to the global method.

Figure 3 EE vs. P_c (see online version for colours)

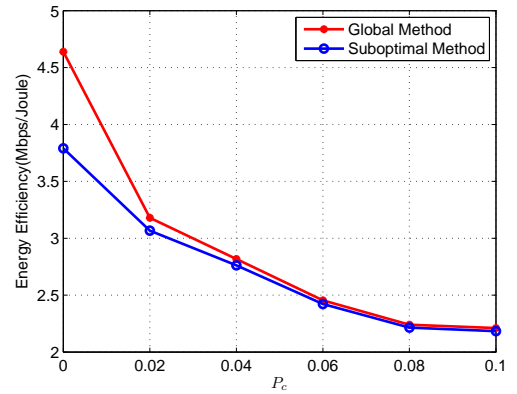
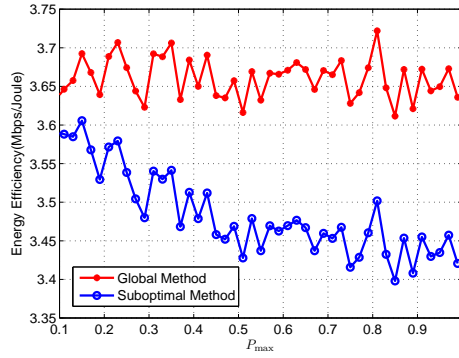


Figure 4 evaluates the effect of P_{\max} , which shows that the EE performance could be better or worse with larger P_{\max} . This is because larger transmit power can increase both the transmission rate and the power consumption, which is not necessary to increase the EE performance. In addition, from this figure, we also see the big similarity between the global method and suboptimal method, not only for the small gap of EE performance, but also for the nearly identical change direction varying with P_{\max} .

Figure 4 EE vs. P_{\max} (see online version for colours)

5 Conclusion

In this paper, we proposed an energy efficient power loading scheme in MIMO-SVD architecture under imperfect CSI. We first derived the closed-form expression of EE in MIMO-SVD system by taking into account the effects of CEE. We then proposed two algorithms to solve the optimisation problem: one is a global method and the other is a low-complexity suboptimal method. The simulation results validate the effectiveness and robustness of our proposed EE power loading scheme. In our future work, we will try to apply our energy efficient power loading scheme to large-scale networks (Wang et al., 2011).

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References

- Burden, R.L. and Faires, J.D. (2000) *Numerical Analysis*, 7th ed., Brooks/Core, Pacific Grove.
- Dighe, P.A., Mallik, R.K. and Jamuar, S.S. (2003) 'Analysis of transmit-receive diversity in Rayleigh fading', *IEEE Trans. Commun.*, Vol. 51, pp.694–703.
- Horst, R. and Thoai, N.V. (1999) 'DC programming: overview', *J.Optim. Theory Appl.*, Vol. 103, pp.1–43.
- Papandriopoulos, J. and Evans, J.S. (2006) 'Low-complexity distributed algorithms for spectrum balancing in multi-user DSL networks', *2006 IEEE International Conference on Communications*, Istanbul, Turkey, pp.3270–3275.
- Prabhu, R.S. and Daneshrad, B. (2010) 'An energy-efficient water-filling algorithm for OFDM systems', *2010 IEEE International Conference on Communications*, Cape Town, South Africa, pp.1–5.
- Prabhu, R.S. and Daneshrad, B. (2010) 'Energy-efficient powerloading for a MIMO-SVD system and its performance in flat fading', *2010 IEEE Global Telecommunications Conference*, Miami, FL, USA, pp.1–5.
- Schaible, S. (1976) 'Fractional programming, II: on Dinkelbach's algorithm', *J. Manage. Sci.*, Vol. 22, pp.868–873.
- Song, S.H. and Zhang, Q.T. (2009) 'Mutual information of multipath channels with imperfect channel information', *IEEE Trans. Commun.*, Vol. 57, pp.1523–1531.
- Telatar, E. (1999) 'Capacity of multi-antenna Gaussian channels', *Eur. Trans. Telecommun.*, Vol. 10, pp.585–595.
- Wang, X.B., Fu, L., Tian, X., Bei, Y., Peng, Q., Gan, X., Yu, H. and Liu, J. (2011) 'Converge-cast: on the capacity and delay tradeoffs', *IEEE Trans. Mobile. Computing*, Vol. 11, pp.970–982.
- Wang, Z.Y., He, C. and He, A. (2006) 'Robust AM-MIMO based on minimized transmission power', *IEEE Commun. Lett.*, Vol. 10, pp.432–434.