Utility Maximization for Uplink MU-MIMO: Combining Spectral-Energy Efficiency and Fairness

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Abstract—Driven by green communications, energy efficiency (EE) has become a new important criterion for designing wireless communication systems. However, high EE often leads to low spectral efficiency (SE), which spurs the research on EE-SE tradeoff. In this paper, we focus on how to maximize the utility in physical layer for an uplink multi-user multiple-input multiple-output (MU-MIMO) system, where we will not only consider EE-SE tradeoff in a unified way, but also ensure user fairness. We first formulate the utility maximization problem, but it turns out to be non-convex. By exploiting the structure of this problem, we find a convexization procedure to convert the original non-convex problem into an equivalent convex problem, which has the same global optima with the original problem. Following the convexization procedure, we present a centralized algorithm to solve the utility maximization problem, but it requires the global information of all users. Thus we propose a primal-dual distributed algorithm which does not need global information and just consumes a small amount of overhead. Furthermore, we have proved that the distributed algorithm can converge to the global optima. Finally, the numeric results show that our approach can both capture user diversity for EE-SE tradeoff and ensure user fairness, and they also validate the effectiveness of our primal-dual distributed algorithm.

Index Terms—MU-MIMO, Spectral Efficiency, Energy Efficiency, Fairness, Power Control, Primal-Dual

I. INTRODUCTION

Among the total worldwide energy consumption, communication networks have contributed increasingly from 1.3% in 2007 to 1.8% in 2012, and this proportion is anticipated to grow continuously in the coming years [1]. This stimulates the fast development of *green communications* recently [2]. Compared to spectral efficiency (SE), *energy efficiency* (EE), defined as the number of bits that can be transmitted with per energy consumption, becomes a new important criterion for designing green wireless systems. How to obtain optimal EE has become a hot research topic in different wireless communication systems [3].

On the other hand, multiple-input multiple-output (MIMO) has been a key technique in modern wireless communication systems, because it can significantly increase SE by exploiting transmit diversity and spatial multiplexing gains [5]. MIMO system is used for one single transmitter and one single receiver in a point-to-point way, so it is often referred to single-user MIMO (SU-MIMO). However, in some applications, especially in cellular networks, many devices such as smartphones and tablets, are often difficult to install many antennas due to size limitations. To increase the network-wide SE,

multi-user MIMO (MU-MIMO) technique has been proposed. Although users only have a small number of antennas or even just one, MU-MIMO allows distributed users to share the same time-frequency resource block [6]. In this paper, we are interested in the uplink MU-MIMO because users, such as smartphones and tablets, are often more energy-sensitive.

Recently there are some papers studying how to maximize EE for uplink MU-MIMO system. In [7], Miao investigates the uplink MU-MIMO system where each user deploys multi-antennas and he demonstrates that EE is maximized when some antennas are turned off if the corresponding spatial channel is not good or the corresponding circuit power consumption is large. In [8], Rui et al. study the uplink MU-MIMO system where each user deploys only one antenna and they maximize EE by jointly doing mode selection and optimal power allocation.

However, it is well-known that SE and EE are two conflicting objectives [4]. Often high EE leads to low SE and vice verse, which means it is more practical to consider SE and EE simultaneously. Thus how to study the EE-SE tradeoff has attracted a lot of attention [9]–[13] where, however, only a few articles study uplink MU-MIMO system. The authors in [12] consider how to get the EE-SE tradeoff for a large-scale uplink MU-MIMO system in a system level. They study EE-SE tradeoff in low and high SE regime asymptotically and do not involve user fairness explicitly which is important in multi-user system. Different from [12], our paper investigates uplink MU-MIMO system in the link level rather than the system level. More importantly, we study EE-SE tradeoff in a unified way and we also guarantee fairness among users. Specifically, our contributions are three-fold,

- We construct a utility function of all users which not only captures the user diversity for EE-SE tradeoff in a unified way, similar to [13], but also guarantees fairness among all users. And then we maximize the utility function. To the best of our knowledge, we are the first to study EE-SE tradeoff and user fairness together in uplink MU-MIMO system.
- Although the original utility maximization problem is not convex, we have proposed an approach to convert it into an equivalent convex programming problem which has the same optimal solution with the original problem. This convexization procedure also generates our optimal power allocation scheme in a centralized manner.

- Apart from the centralized algorithm, we further devise a primal-dual distributed algorithm which only consumes a small amount of overhead between users and Node-B. Moreover, we have proved that the distributed algorithm converges to the global optimal solution.

The rest of this paper is outlined as follows. We describe the system model and formulate the problem in Section II. In Section III, we analyze the optimal power allocation by converting the optimization problem into a convex programming problem. Next in Section IV, we propose a primal-dual distributed algorithm, which can achieve the global optima. The numeric results are shown in Section V, followed by conclusion in Section VI. Throughout this paper, we will use $[\cdot]_{ij}$ to denote the matrix's entry in i-th row and j-th column, $\mathbf{E}[\cdot]$ to denote expectation, $\mathbf{I}_n$ to denote the $n \times n$ identity matrix, and the superscript $\dagger$ to denote Hermitian transpose.

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. System Model

Consider a MU-MIMO system with N users indexed from 1 to N, and one Node-B in a single cell. In this paper, we assume that each user is only equipped with one transmitting antenna, and the Node-B is equipped with $M(M \geq N)$ receiving antennas, as shown in Fig. 1. In uplink, all N users share the same time-frequency resource to transmit data to the Node-B. Denote P_i as the transmit power for user i and G_i as the path loss from user i to Node-B. Then the received signal vector $\mathbf{y} \in \mathbb{C}^{M \times 1}$ is,

$$\mathbf{y} = \mathbf{H}\mathbf{G}\mathbf{P}^{\frac{1}{2}}\mathbf{s} + \mathbf{n}, \tag{1}$$

where $\mathbf{s} \in \mathbb{C}^{N \times 1}$ denotes the transmit symbol vector with $\mathbf{E}[\mathbf{s}\mathbf{s}^{\dagger}] = \mathbf{I}_N$, $\mathbf{G} = \text{diag}\{G_1, G_2, \cdots, G_N\}$ denotes the path-loss matrix, $\mathbf{P}^{\frac{1}{2}} = \text{diag}\{\sqrt{P_1}, \cdots, \sqrt{P_N}\}$ denotes the power loading matrix, $\mathbf{H} \in \mathbb{C}^{M \times N}$ denotes the channel matrix where all entries are independent and identically distributed (i.i.d.) complex Gaussian distributed with zero mean and unit variance which however is fixed once it is chosen randomly, and $\mathbf{n} \in \mathbb{C}^{M \times 1}$ denotes the additive white Gaussian noise (AWGN) with zero mean and covariance matrix $\mathbf{E}[\mathbf{n}\mathbf{n}^{\dagger}] = \sigma_n^2 \mathbf{I}_M$.

In this paper, we assume that Node-B has perfect channel state information (CSI) for all users and the receiver at Node-B uses zero forcing (ZF) detection method. Thus, the signal-to-interference-plus-noise ratio (SINR) at the Node-B's receiver for user i is,

$$\gamma_i = \frac{P_i G_i}{\sigma_n^2 [(\mathbf{H}^{\dagger}\mathbf{H})^{-1}]_{ii}} = P_i \underbrace{\frac{G_i}{\sigma_n^2 [(\mathbf{H}^{\dagger}\mathbf{H})^{-1}]_{ii}}}_{\delta_i}. \tag{2}$$

Then we can obtain SE and EE for user i as

$$SE_i = \log(1 + \gamma_i) = \log(1 + \delta_i P_i), \tag{3}$$

$$EE_i = \frac{SE_i}{P_i + P_i^c} = \frac{\log(1 + \delta_i P_i)}{P_i + P_i^c}, \tag{4}$$

where P_i^c is a positive constant circuit power consumed by the relevant electronic devices for user i.
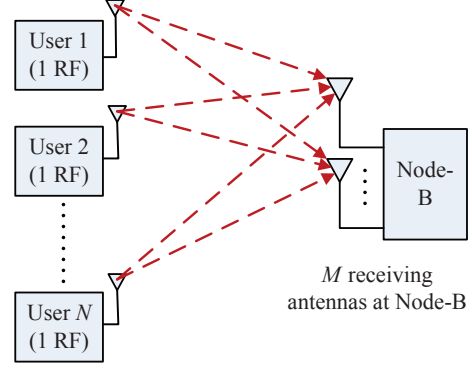

Fig. 1. System Model

B. Problem Formulation

Next, we will construct the utility function in two steps. First, we consider the EE-SE tradeoff. Inspired by the widely-used Cobb-Douglas production function in economics [14], we adopt this model *empirically* to get the "production" of SE and EE for user i,

$$u_i = (SE_i)^{w_i}(EE_i)^{1-w_i}, \tag{5}$$

where $w_i \in [0, 1]$. More specifically, we can regard $(w_i, 1-w_i)$ as a priori articulation of preferences for SE and EE, which captures EE-SE tradeoff in a unified way [13].

Second, we consider the fairness among all N users. If we apply the proportional fairness metric, we can define the final utility function for user i as

$$
\begin{aligned}
U_i(\mathbf{P}) &= \log(u_i) = \log[(SE_i)^{w_i}(EE_i)^{1-w_i}] \\
&= \log[\log(1 + \delta_i P_i)] - (1 - w_i)\log(P_i + P_i^c) \\
&= U_i(P_i),
\end{aligned} \tag{6}
$$

where $\mathbf{P} = (P_1, P_2, \cdots, P_N)$ and the last step shows that the utility for user i is not related to the transmit power of other users.

Based on the utility function in (6), we then formulate our utility maximization problem subject to a power constraint for each user and a power sum constraint for all users,

$$\text{maximize} \quad \sum_{i=1}^{N} U_i(P_i) \tag{7}$$

$$(\mathbf{P1}) \quad \text{subject to} \quad 0 \leq P_i \leq P_i^{\max}, \quad \forall i \tag{8}$$

$$\sum_{i=1}^{N} P_i \leq P_{\max}. \tag{9}$$

In (7), we aim at maximizing the sum of the utility for all users, i.e., the network-wide utility. (8) is the individual power constraints where $P_i^{\max}$ is the maximal transmit power for user i. (9) is the power sum constraint for the total MU-MIMO system where $P_{\max}$ is the maximal transmit power for all users. The reason why we set $P_{\max}$ is as follows. From the perspective of the whole uplink MU-MIMO system or Node-B, the system performance is more important than

individual utility. Thus the authors in [18] try to minimize the total power consumption of all users while meeting some pre-defined SINR targets. However, we capture the same idea in a reversed way, namely, we aim at achieving the best system performance given a maximal power sum constraint $P_{\max}$ for all users. In addition, by setting $P_{\max} = \infty$, our formula in (9) also includes the scenario that power sum constraint is not considered.

III. OPTIMAL POWER ALLOCATION

In the previous section, we have formulated the problem to maximize the network-wide utility in (7), which however is not a concave function mainly since EE in (4) is neither convex nor concave [13]. Therefore, in this section, we will exploit the inner structure of (**P1**) and find that we can narrow down the feasible region without changing the global optima. Furthermore, we will prove that the objective function in (7) is concave in the new feasible region, which converts the original problem into a convex programming problem. After some analysis, we will get the optimal power allocation scheme for (**P1**) with a centralized algorithm.

A. Convexization Procedure

To narrow down the feasible region in (**P1**), we first consider individual power constraints in (8). Since the optimization problem can be changed as,

$$\max_{0 \leq P_i \leq P_i^{\max}, \forall i} \sum_{i=1}^{N} U_i(P_i) = \sum_{i=1}^{N} \max_{0 \leq P_i \leq P_i^{\max}} U_i(P_i), \quad (10)$$

we just need to find the maximal individual utility, i.e., $U_i(P_i)$ for any user $i \in \{1, 2, \cdots, N\}$. For the individual utility function $U_i(P_i)$ in (6), we have the following proposition.

Proposition 1: For any user i under individual power constraint in (8), there exists one and only one point $P_i^u \in (0, P_i^{\max}]$ that maximizes $U_i(P_i)$. The function $U_i(P_i)$ is strictly increasing and strictly concave over the interval $[0, P_i^u]$ while strictly decreasing over the interval $(P_i^u, P_i^{\max}]$. In addition, P_i^u can be derived as follows,

$$P_i^u = \begin{cases} P_i^{\max} & \text{if} \quad w_i > 1 - \beta(P_i^{\max}) \\ P_i^0 & \text{if} \quad w_i \leq 1 - \beta(P_i^{\max}) \end{cases} \quad (11)$$

where

$$\beta(P_i) = \frac{\delta_i(P_i + P_i^c)}{(1 + \delta_i P_i) \log(1 + \delta_i P_i)}, \quad (12)$$

and P_i^0 is the unique solution for the following equation when $w_i \leq 1 - \beta(P_i^{\max})$,

$$\beta(P_i) = 1 - w_i. \quad (13)$$

Proof: See Appendix A. ∎

Let us denote optimal solution under individual power constraints as $\mathbf{P}^u = \{P_1^u, P_2^u, \cdots, P_N^u\}$.

Now we consider the power sum constraint in (9). In (**P1**), since the feasible region is a compact set and the objective function is continuous, a global optimal solution can be attained. Let us denote the global optimal solution

as $\mathbf{P}^* = \{P_1^*, P_2^*, \cdots, P_N^*\}$. Then we have the following proposition.

Proposition 2: $\mathbf{P}^* \leq \mathbf{P}^u$, i.e., $P_i^* \leq P_i^u$, $\forall i \in \{1, 2, \cdots, N\}$.

Proof: $\forall i \in \{1, 2, \cdots, N\}$, suppose $P_i^* > P_i^u$. Then $\sum_{k \neq i} P_k^* + P_i^u < \sum_{k=1}^{N} P_k^* \leq P_{\max}$, which means $\mathbf{P}' = \{P_1^*, \cdots, P_{i-1}^*, P_i^u, P_{i+1}^*, \cdots, P_N^*\}$ is a feasible solution to (**P1**). And according to the *Proposition 1*, we have $U_i(P_i^*) < U_i(P_i^u)$. So $\sum_{k=1}^{N} U_k(P_k^*) < \sum_{k \neq i} U_k(P_k^*) + U_i(P_i^u)$, which is a contradiction with the fact that $\mathbf{P}^*$ is the optimal solution to (**P1**). This completes the proof. ∎

Proposition 2 shows that for any user i, the optimal transmit power P_i^* cannot be greater than P_i^u. Therefore we have the following main result of this section.

Theorem 1: (**P1**) is equivalent to the following problem,

$$\begin{aligned} \text{maximize} \quad & \sum_{i=1}^{N} U_i(P_i) \\ (\mathbf{P2}) \quad \text{subject to} \quad & 0 \leq P_i \leq P_i^u, \quad \forall i \quad (14) \\ & \sum_{i=1}^{N} P_i \leq P_{\max}. \quad (15) \end{aligned}$$

In addition, (**P2**) is a convex programming problem.

Proof: Following from *Proposition 2*, we immediately conclude that (**P1**) is equivalent to (**P2**). In addition, from *Proposition 1*, we know that $U_i(P_i)$ is strictly concave at $P_i \in [0, P_i^u]$. Thus, (**P2**) is a problem to maximize a strictly concave function in a convex region, which means it is a convex problem now. This completes the proof. ∎

B. Some Analysis

Next we will give some analysis for the optimal solution $\mathbf{P}^*$ in the following two cases.

Case 1: $\sum_{i=1}^{N} P_i^u \leq P_{\max}$
In this case, $\mathbf{P}^u$ is feasible for (**P2**), so it is also the optimal solution to (**P2**), i.e., $\mathbf{P}^* = \mathbf{P}^u$.

Case 2: $\sum_{i=1}^{N} P_i^u > P_{\max}$
In this case, we can further narrow down the feasible region for (**P2**) and achieve the following proposition.

Proposition 3: If $\sum_{i=1}^{N} P_i^u > P_{\max}$, (**P2**) is equivalent to the following convex optimization problem,

$$\begin{aligned} \text{maximize} \quad & \sum_{i=1}^{N} U_i(P_i) \\ (\mathbf{P3}) \quad \text{subject to} \quad & 0 \leq P_i \leq P_i^u, \quad \forall i \quad (16) \\ & \sum_{i=1}^{N} P_i = P_{\max}. \quad (17) \end{aligned}$$

Proof: Suppose $\sum_{i=1}^{N} P_i^* < P_{\max}$. Since $\sum_{i=1}^{N} P_i^u > P_{\max}$, there exists at least one $i \in \{1, 2, \cdots, N\}$ such that $P_i^* < P_i^u$ (Otherwise, $\sum_{i=1}^{N} P_i^* = \sum_{i=1}^{N} P_i^u > P_{\max}$, which is a contradiction). Therefore, there exists a $\epsilon > 0$ such that $P_i^* + \epsilon \leq P_i^u$ and $\sum_{k \neq i} P_k^* + (P_i^* + \epsilon) \leq P_{\max}$. So $\mathbf{P}' = \{P_1^*, \cdots, P_{i-1}^*, P_i^* + \epsilon, P_{i+1}^*, \cdots, P_N^*\}$ is a feasible solution for (**P2**). According to the *Proposition 1*, we have $U_i(P_i^*) <$

$U_i(P_i^* + \epsilon)$. Then $\sum_{k=1}^N U_k(P_k^*) < \sum_{k \neq i} U_k(P_k^*) + U_i(P_i^* + \epsilon)$, which is a contradiction with the fact that $\mathbf{P}^*$ is the optimal solution to (**P1**). Therefore, we must have $\sum_{i=1}^N P_i^* = P_{\max}$, which completes the proof. ∎

C. Centralized Algorithm

Based on the above analysis, we can readily get the optimal power allocation $\mathbf{P}^*$ for the optimization problem (**P1**) with a centralized algorithm, as shown in Algorithm 1. In practice, we can implement such centralized algorithm as follows. First, each user i transmits its parameters, including $P_i^{\max}, P_i^c$ and w_i to Node-B. After collecting all information of all users, Node-B runs Algorithm 1 to get the optimal power allocation $\mathbf{P}^*$, and then updates the optimal transmit power P_i^* to each user i. Finally, each user transmits data with the optimal transmit power.

Algorithm 1 Centralized Algorithm for (**P1**)

1: **for** $1 \leq i \leq N$ **do**
2:    **if** $w_i > 1 - \beta(P_i^{\max})$ **then**
3:      $P_i^u = P_i^{\max}$;
4:    **else**
5:      Get P_i^0 with Newton-Raphson iteration method for the equation (13);
6:      $P_i^u = P_i^0$;
7:    **end if**
8: **end for**
9: **if** $\sum_{i=1}^N P_i^u \leq P_{\max}$ **then**
10:    $\mathbf{P}^* = \mathbf{P}^u$;
11: **else**
12:    Get $\mathbf{P}^*$ with gradient projection method for (**P3**);
13: **end if**

IV. Distributed Primal-Dual Implementation

In the previous section, we provide Algorithm 1 in a centralized manner where Node-B needs to know all global information of all users. Since the centralized algorithm still involves some computational complexity and is not robust against time-varying system, we are more interested to implement the algorithm in a distributed manner. Inspired by the distributed algorithm in network flow optimization problem [15], we design the following primal-dual distributed algorithm to achieve the optimal power allocation $\mathbf{P}^*$,

$$\begin{cases} \dot{P}_i = k_i [U_i'(P_i) - \lambda]_{P_i}^{P_i^u +}, \forall i \in \{1, 2, \cdots, N\} \\ \dot{\lambda} = g[\sum_{i=1}^N P_i - P_{\max}]_\lambda^+, \end{cases} \quad (18)$$

where

$$[f]_z^+ = \begin{cases} \max(f, 0), & z \leq 0 \\ f, & z > 0 \end{cases} \quad (19)$$

and

$$[f]_z^{a+} = \begin{cases} \max(f, 0), & z \leq 0 \\ \min(f, 0), & z \geq a \\ f, & 0 < z < a \end{cases} \quad (20)$$

and $k_i > 0, g > 0$.

From (18), each user does not need the information of others but just the penalty λ. The only overhead is that Node-B broadcasts λ to all users and each user i updates P_i to Node-B until convergence. Therefore, such implementation only consumes a small amount of overhead between users and Node-B. In addition, our proposed distributed algorithm reduces computational complexity compared to Algorithm 1. Moreover, we also prove that this primal-dual distributed algorithm in (18) can converge to the global optimal, as shown in the following theorem.

Theorem 2: The distributed algorithm in (18) is globally asymptotically stable and the only equilibrium is $\mathbf{P}^*$.

Proof: See Appendix B. ∎

V. Numeric Results

In this section, simulation results are provided to validate our theoretic analysis, which show that our approach not only captures user diversity for EE-SE tradeoff, but also ensures user fairness. In addition, we also verify that our primal-dual distributed algorithm can converge to the global optimal solution. Throughout this section, we will set the maximal transmit power to be 1W (30dBm) for all users, by adopting the transmitter's power level 1 for 1800/1900 MHz mobile phones [16]. The circuit power is set to be 0.1W for all users.

A. User Diversity for EE-SE Tradeoff

As shown in (5), different users can have different preferences for SE and EE, determined by parameter w_i. In this part, we will show how to capture user diversity with w_i. We consider two users, i.e., $N = 2$, and let the number of receiving antennas for Node-B to be 2, i.e., $M = 2$. In addition, we regard two users as homogeneous such that they have the same CSI. Specifically, let $\delta_1 = \delta_2 = 20$dB and $P_{\max} = 1.5$W.

Fig. 2 shows the impact of users' different preferences, i.e., w_1 and w_2. From Fig. 2(a) and 2(d), we can see the optimal transmit power for two users. Under the optimal transmit power for user 1, Fig. 2(b) demonstrates that SE increases as w_1 increases while Fig. 2(c) demonstrates that EE decreases as w_1 increases. Similarly, Fig. 2(e) and 2(f) show the same effect of w_2 for user 2. Such results comply with our intuition for the effect of w_i in (5), and therefore verify that our utility function can capture user diversity for EE-SE tradeoff very well with the preference w_i.

B. User Fairness

In this part, we will show that our utility function in (6) can ensure fairness among all users. We still consider two users, i.e., $N = 2$, and let $M = 2$. And we set $w_1 = w_2 = 0.5$ which means SE and EE are equally important for both users. $P_{\max}$ is set to be 1.5W. We will vary the channel conditions for two users. Specifically, we fix δ_1 to three different levels: -20dB (worst), 0dB (normal) and 20dB (best) respectively, while changing δ_2 from -20dB (worst) to 20dB (best).

From Fig. 3, we can see the utility for user 1, i.e., U_1, changes very slowly with respect to δ_2, while the utility
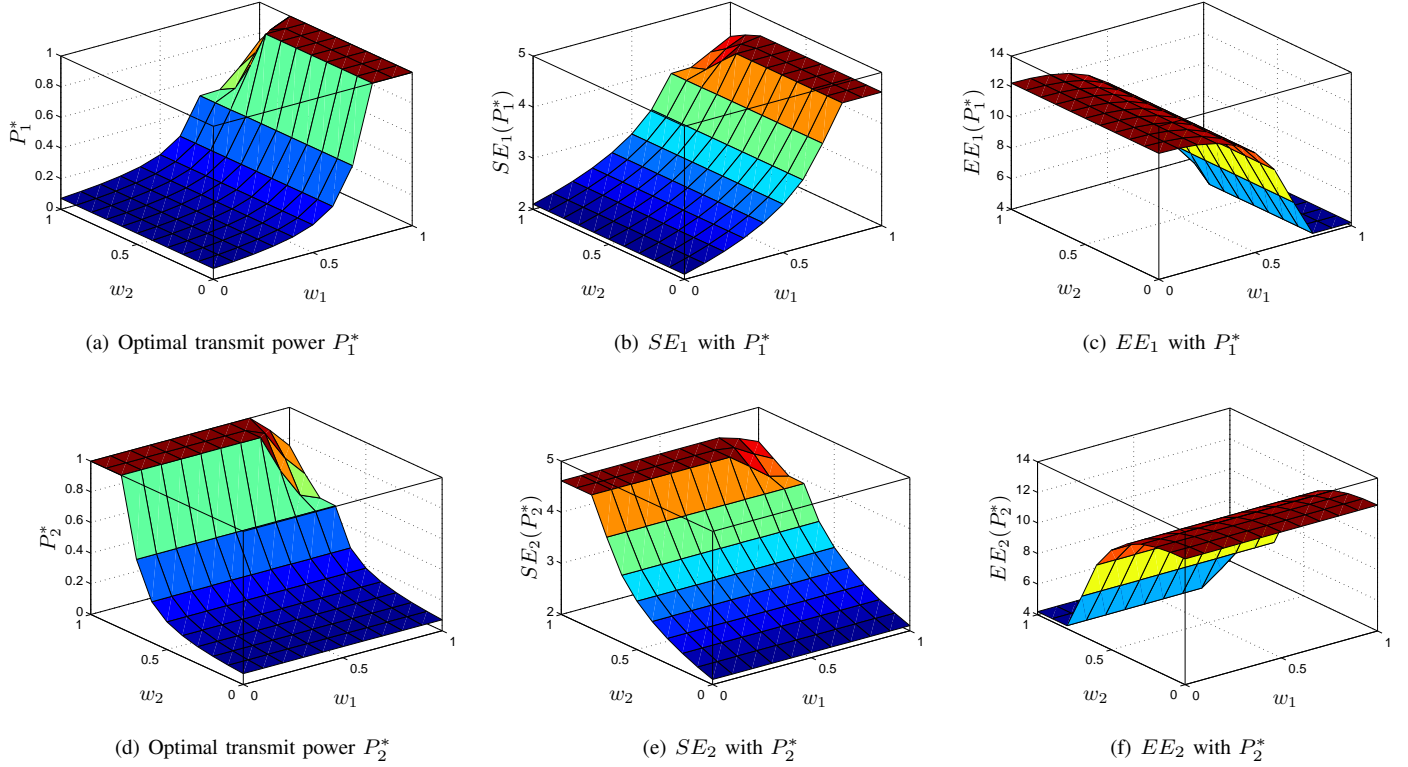
(a) Optimal transmit power P_1^*

(b) SE_1 with P_1^*

(c) EE_1 with P_1^*

(d) Optimal transmit power P_2^*

(e) SE_2 with P_2^*

(f) EE_2 with P_2^*

Fig. 2. User diversity for EE-SE tradeoff with $N = 2, M = 2, P_1^{\max} = P_2^{\max} = 1$W, $P_{\max} = 1.5$W, $P_1^c = P_2^c = 0.1$W, $\delta_1 = \delta_2 = 20$dB.

for user 2, i.e., U_2, increases as δ_2 increases (or as channel condition goes better). In addition, when $\delta_1 = \delta_2$, they have the same utility, so no bias exists. Furthermore, the utility for both users will not go to very low. For example, in Fig. 3(a) where the channel condition is worst for user 1 with $\delta_1 = -20$dB, even when the channel condition is best for user 2 with $\delta_2 = 20$dB, U_1 is still larger than -5, which means user 1 can still transmit data with a positive transmit power. Therefore, the user fairness can be guaranteed under our proposed utility function in (6).

C. Primal-Dual Distributed Algorithm

In this part, we will validate the effectiveness of our primal-dual distributed algorithm in (18). We consider four users, i.e., $N = 4$ and let $M = 4$. In addition, we will let users have random CSI, i.e., δ_i, but they are fixed once chosen. Also, we set different EE-SE preferences for all users, which are $w_1 = 0, w_2 = 0.3, w_3 = 0.7$ and $w_4 = 1$. The maximal sum power is $P_{\max} = 3$W. In addition, the step sizes, k_i, $\forall i \in \{1, 2, 3, 4\}$ and g in (18) are set to be 0.001.

Fig. 4 shows the simulation results. From Fig. 4(a), we can see that the power allocation can be converged and it also shows that our approach can capture user diversity when different users have different preferences for EE-SE tradeoff. From Fig. 4(b), we can see that the distributed algorithm can converge to the global optima, which verifies Theorem 2.

VI. CONCLUSION

In this paper, we consider utility maximization for the uplink MU-MIMO system. We define the utility function combining both EE-SE tradeoff and user fairness. After formulating the utility maximization problem with individual power constraints and sum power constraint, we analyze the optimal power allocation scheme. Although the original optimization problem is not convex, we propose a convexization procedure to convert it into an equivalent convex programming problem, which is proved to have the same global optimal solution with the original problem. Moreover, we have proposed two algorithms to get the optimal solution: one is the centralized algorithm which requires all global information; the other is the primal-dual distributed algorithm which only needs a small amount of overhead between users and Node-B. Furthermore, we have proved that our proposed distributed algorithm can converge to the global optimal solution.
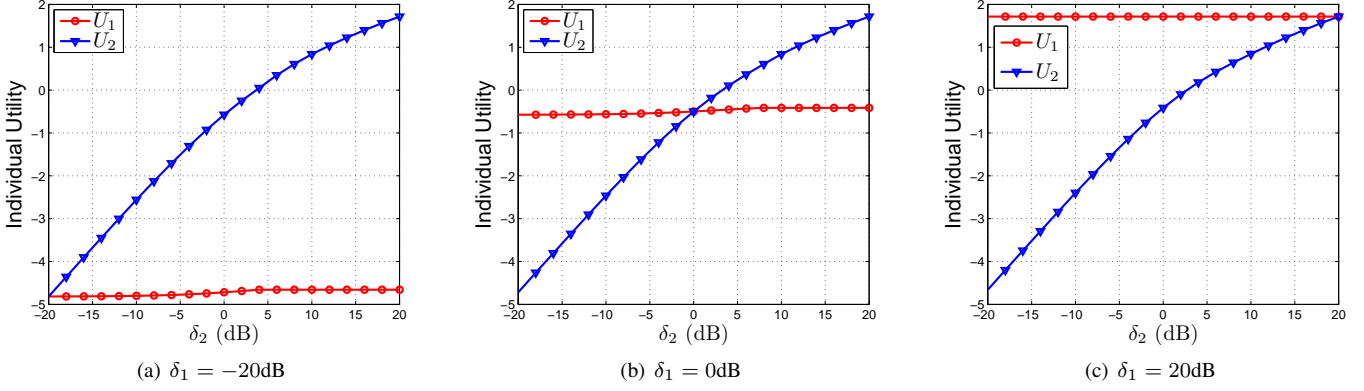
(a) $\delta_1 = -20$dB (b) $\delta_1 = 0$dB (c) $\delta_1 = 20$dB

Fig. 3. User fairness with $N = 2, M = 2, P_1^{\max} = P_2^{\max} = 1\text{W}, P_{\max} = 1.5\text{W}, P_1^c = P_2^c = 0.1\text{W}, w_1 = w_2 = 0.5$.

(a) Power Convergence

(b) Utility convergence

Fig. 4. Convergence of primal-dual distributed algorithm.

APPENDIX A
PROOF OF PROPOSITION 1

The first derivative of $U_i(P_i)$ in (6) is

$$
\begin{aligned}
U_i'(P_i) &= \frac{1}{\log(1+\delta_i P_i)} \cdot \frac{\delta_i}{1+\delta_i P_i} - \frac{1-w_i}{P_i + P_i^c} \\
&= \frac{1}{P_i + P_i^c}\left[\frac{\delta_i(P_i + P_i^c)}{(1+\delta_i P_i)\log(1+\delta_i P_i)} - (1-w_i)\right] \\
&= \frac{1}{P_i + P_i^c}[\beta(P_i) - (1-w_i)], \quad (21)
\end{aligned}
$$

where

$$
\beta(P_i) = \frac{\delta_i(P_i + P_i^c)}{(1+\delta_i P_i)\log(1+\delta_i P_i)}. \quad (22)
$$

And the second derivative of $U_i(P_i)$ is

$$
U_i''(P_i) = \frac{\beta'(P_i)(P_i + P_i^c) - [\beta(P_i) - (1-w_i)]}{(P_i + P_i^c)^2}. \quad (23)
$$

In (22), we can get first derivative of $\beta(P_i)$ as

$$
\begin{aligned}
&\beta'(P_i) \\
&= \delta_i \cdot \frac{[\log(1+\delta_i P_i) - \delta_i P_i] - P_i^c \delta_i[\log(1+\delta_i P_i) + 1]}{[(1+\delta_i P_i)\log(1+\delta_i P_i)]^2} \\
&< 0, \quad (24)
\end{aligned}
$$

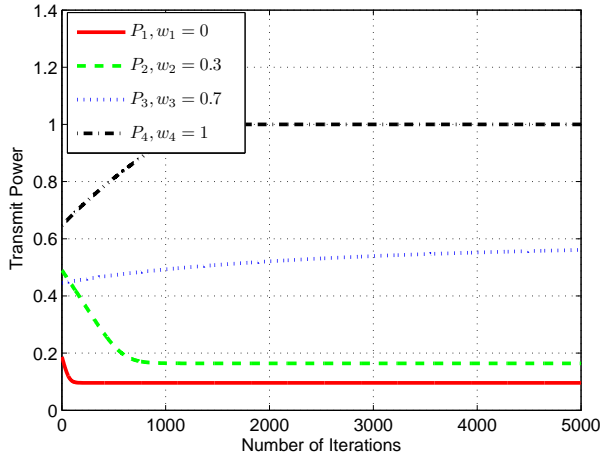which shows that $\beta(P_i)$ is strictly decreasing with P_i.

Case 1: If $w_i > 1 - \beta(P_i^{\max})$, then $1 - w_i < \beta(P_i^{\max}) \leq \beta(P)$, which yields $U_i'(P_i) > 0$. So $U_i(P_i)$ is strictly increasing at $[0, P_i^{\max}]$. The utility $U_i(P_i)$ is maximized when $P_i = P_i^{\max}$. In addition, $U_i''(P_i) < 0$, so $U_i(P_i)$ is strictly concave at $[0, P_i^{\max}]$.
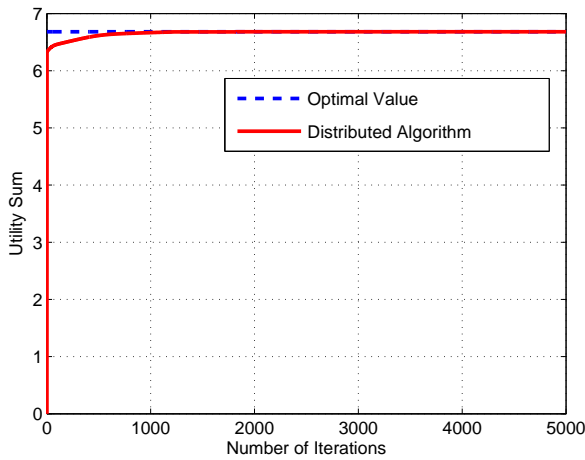
Case 2: If $w_i \leq 1 - \beta(P_i^{\max})$, then $U_i'(P_i) = 0$ has one and only one solution $P_i^0 \in (0, P_i^{\max}]$. This is because $\beta(P_i)$ is strictly decreasing and $\lim_{P_i \to 0} \beta(P_i) = +\infty$ and $\beta(P_i^{\max}) \leq 1 - w_i$. Note that P_i^0 is the unique solution of the following equation,

$$
\beta(P_i) = \frac{\delta_i(P_i + P_i^c)}{(1+\delta_i P_i)\log(1+\delta_i P_i)} = 1 - w_i. \quad (25)
$$

At the interval $[0, P_i^u)$, we have $\beta(P_i) - (1-w_i) > 0$, so $U_i'(P_i) > 0$ and $U_i''(P_i) < 0$. At the interval $[P_i^0, P_i^{\max}]$,

$\beta(P_i) - (1 - w_i) \leq 0$, so $U_i'(P_i) \leq 0$. Then we obtain that the utility $U_i(P_i)$ is maximized at $P = P_i^0$ and $U_i(P_i)$ is strictly increasing and concave at $[0, P_i^0]$ and strictly decreasing at $(P_i^0, P_i^{\max}]$.

The proof is completed.

APPENDIX B
PROOF OF THEOREM 2

Proof: The proof is similar to [17]. Readers can find more information about primal-dual algorithm for network flow optimization in [15] [17].

First, we rewrite the problem (**P2**),

$$\text{maximize} \quad \sum_{i=1}^{N} U_i(P_i)$$
$$\text{subject to} \quad -P_i \leq 0, \quad \forall i \tag{26}$$
$$P_i - P_i^u \leq 0, \quad \forall i \tag{27}$$
$$\sum_{i=1}^{N} P_i - P_{\max} \leq 0. \tag{28}$$

Since $\mathbf{P}^* = \{P_1^*, P_2^*, \cdots, P_N^*\}$ is the optimal solution of this problem and the Slater's condition holds for this convex problem, there exists KKT multipliers μ_i^* for (26), ν_i^* for (27) and λ^* for (28) such that they satisfy the following KKT conditions,

$$U_i'(P_i^*) + \mu_i^* - \nu_i^* - \lambda^* = 0, \forall i \tag{29}$$
$$0 \leq P_i^* \leq P_i^u, \forall i \tag{30}$$
$$\sum_{i=1}^{N} P_i^* \leq P_{\max}, \tag{31}$$
$$\mu_i^* \geq 0, \nu_i^* \geq 0, \lambda^* \geq 0, \forall i \tag{32}$$
$$\mu_i^* P_i^* = 0, \forall i \tag{33}$$
$$\nu_i^*(P_i^* - P_i^u) = 0, \forall i \tag{34}$$
$$\lambda^*(\sum_{i=1}^{N} P_i^* - P_{\max}) = 0. \tag{35}$$

Now given such KKT conditions, we can prove this theorem by constructing the following Lyapunov function for the system in (18),

$$V(\mathbf{P}, \lambda) = \frac{1}{2} \sum_{i=1}^{N} \frac{(P_i - P_i^*)^2}{k_i} + \frac{(\lambda - \lambda^*)^2}{2g}. \tag{36}$$

It is easy to verify that $V(\mathbf{P}, \lambda)$ in (36) is positive definite. To show that $V(\mathbf{P}, \lambda)$ is a Lyapunov function, it suffices to verify its Lie derivative with respect to (18) is nonnegative,

i.e., $\dot{V} \leq 0$. This is true because,

$$\dot{V} = \sum_{i=1}^{N} \frac{P_i - P_i^*}{k_i} \dot{P}_i + \frac{\lambda - \lambda^*}{g} \dot{\lambda} \tag{37}$$
$$= \sum_{i=1}^{N}(P_i - P_i^*)[U_i'(P_i) - \lambda]_{P_i}^{P_i^u +}$$
$$\quad + (\lambda - \lambda^*)[\sum_{i=1}^{N} P_i - P_{\max}]_\lambda^+ \tag{38}$$
$$\leq \sum_{i=1}^{N}(P_i - P_i^*)[U_i'(P_i) - \lambda]$$
$$\quad + (\lambda - \lambda^*)[\sum_{i=1}^{N} P_i - P_{\max}] \tag{39}$$
$$= \sum_{i=1}^{N}(P_i - P_i^*)[U_i'(P_i) - U_i'(P_i^*)] \tag{40}$$
$$\quad + \sum_{i=1}^{N}(P_i - P_i^*)[U_i'(P_i^*) - \lambda^*] \tag{41}$$
$$\quad + (\lambda - \lambda^*)[\sum_{i=1}^{N} P_i^* - P_{\max}] \tag{42}$$
$$\leq 0, \tag{43}$$

where (40) is nonpositive since $U'(P_i)$ is strictly decrease over the interval $[0, P_i^u]$ according to *Proposition 1*, and (42) is nonpositive following from the KKT conditions as,

$$(\lambda - \lambda^*)[\sum_{i=1}^{N} P_i^* - P_{\max}]$$
$$= \lambda[\sum_{i=1}^{N} P_i^* - P_{\max}] - \lambda^*[\sum_{i=1}^{N} P_i^* - P_{\max}]$$
$$= \lambda[\sum_{i=1}^{N} P_i^* - P_{\max}], \quad //\text{from (35)} \tag{44}$$
$$\leq 0, \quad //\text{from (32)} \tag{45}$$

and (41) is nonpositive following from the KKT conditions

as,

$$\sum_{i=1}^{N}(P_i - P_i^*)[U_i'(P_i^*) - \lambda^*]$$

$$= \sum_{i=1}^{N}(P_i - P_i^*)(\nu_i^* - \mu_i^*) \quad //\text{from (29)} \qquad (46)$$

$$= \sum_{i=1}^{N}(P_i - P_i^*)\nu_i^* - \sum_{i=1}^{N}(P_i - P_i^*)\mu_i^* \qquad (47)$$

$$= \sum_{i=1}^{N}(P_i - P_i^*)\nu_i^* - \sum_{i=1}^{N} P_i\mu_i^* + \sum_{i=1}^{N} P_i^*\mu_i^* \qquad (48)$$

$$\leq \sum_{i=1}^{N}(P_i - P_i^*)\nu_i^* \quad //\text{from (30, 32, 33)} \qquad (49)$$

$$= \sum_{i=1}^{N}(P_i - P_i^u)\nu_i^* + \sum_{i=1}^{N}(P_i^u - P_i^*)\nu_i^* \qquad (50)$$

$$= \sum_{i=1}^{N}(P_i - P_i^u)\nu_i^* \quad //\text{from (34)} \qquad (51)$$

$$\leq 0. \quad //\text{from (30, 32)} \qquad (52)$$

Therefore, we have verified $V(\mathbf{P}, \lambda)$ in (36) is a Layponouv function. So the primal-dual distributed system in (18) is globally asymptotically stable and will converge to the equilibria set $\{\mathbf{P} : \dot{V} = 0\}$ [19]. On the other hand, $\dot{V} = 0$ only if (40) is equal to 0, i.e.,

$$\sum_{i=1}^{N}(P_i - P_i^*)[U_i'(P_i) - U_i'(P_i^*)] = 0, \qquad (53)$$

which holds only if $P_i = P_i^*$ for all $i \in \{1, 2, \cdots, N\}$. Therefore, the equilibria set $\{\mathbf{P} : \dot{V} = 0\}$ only contains one point, i.e., $\mathbf{P}^*$. This completes the proof. ∎

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