Dynamic Spectrum Allocation for Heterogeneous Cognitive Radio Networks With Multiple Channels

Wenjie Zhang ⬤, Yingjuan Sun, Lei Deng ⬤, Chai Kiat Yeo ⬤, and Liwei Yang

Abstract—**The rapid growth of wireless communication technology has resulted in the increasing demand on spectrum resources. However, according to a recent study, most of the allocated frequency experiences significant underutilization. One important issue associated with spectrum management in heterogeneous cognitive radio networks is: How to appropriately allocate the spectrum to secondary sender–destination (S–D) pair for sensing and utilization. In this paper, the authors investigate the spectrum allocation problem under a more practical scenario where the heterogeneous characteristics of both the secondary S–D and primary channels are taken into consideration. With the objective to maximize the achievable throughput for secondary S–D, we formulate the spectrum allocation problem as a linear integer optimization problem under spectrum availability constraint, spectrum span constraint, and interference free constraint. This problem is proven to be Non-deterministic Polynomial (NP)-complete, and a recent result in theoretical computer science called randomized rounding algorithm with polynomial computational complexity is developed to find the ρ-approximation solution. Evaluation results show that our proposed algorithm can achieve a close-to-optimal solution at a low level of computation complexity.**

Index Terms—**Cognitive radio (CR) networks, NP-complete, randomized rounding algorithm, spectrum allocation.**

I. INTRODUCTION

MORE and more spectrum resources are required to support the rapid development of wireless applications. However, a recent study by Federal Communications Commission (FCC) has shown that most of the allocated frequency bands experience significant underutilization. The current utilization of a licensed spectrum band varies from 15 to 85% [1]. Cognitive radio (CR) is, therefore, proposed as a potential technology to mitigate this spectrum scarcity problem. The basic idea of an CR is to allow the secondary users (SUs) to access licensed spectrum bands, so long as they do not inflict any harmful interference to the primary users (PUs) [2], [3]. To achieve this goal, the SU must monitor each channel's usage by means of spectrum sensing to identify spectrum holes [4], [5]. Whenever, the SU finds a channel that is not occupied by the PU, it can utilize this channel to transmit its own data. Due to high priority, once the return of PU on a channel is detected, the SU is required to promptly vacate the occupied channel in order to avoid interference to the PU, and then determine a new idle channel to resume its unfinished transmission. This process is referred to as spectrum handoff, which may consume a lot of system resources.

One of the most challenging problems in CR networks, spectrum allocation has been extensively investigated recently [6]–[11]. However, most prior works on spectrum allocation have mainly focused on the one to one case (allocate one channel to one SU for sensing and utilization), which is a simple network scenario. Moreover, as observed in [12], the operations of PUs are highly unpredictable, they can become active at any time without any notification. Thus, due to this temporal variation of the PU channels, the SU needs to promptly vacate the occupied channel and transfer its connection to an unused channel, if available. On the other hand, the spectrum availability at the SU is different due to different geographical locations. The measurement of available channels at the Harvard University shows significant variation in channel availability at different locations [12]. Therefore, different SUs may have different available channel sets, and one SU may have more than one available channel to exploit in its location. Thus, in order to reduce the number of spectrum handoff, more than one channel can be allocated to each SU for utilization simultaneously depending on their availability at that time (many-to-one case). These channels can be treated as if it were a single channel whose capacity is equal to the sum of all the other allocated channels [13]. In this way, when the PU becomes active, the SU should exclude the channel from usage. In a special case, if the channel to be vacated is the only one used by the SU, there will be no more channels to utilize, then spectrum handoff is required. Otherwise, the SU can use the rest of channels to continue its unfinished communication. Moreover, as discussed above, the spectrum availability of the SU is heterogeneous, thus, if we allocate different available channels to the secondary sender and

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destination, they cannot conduct communication among each other. Therefore, another advantage of many-to-one case is the ability to increase the probability for the secondary sender–destination (S–D) pair to find common idle channels to conduct communication.

On one hand, different SUs with different detection thresholds and received SNR will result in different detection performance. On the other hand, different PU channels may have different idle probability and channel capacity. Thus, allocating different channels to different secondary S–D pair may result in different system performance. Zhang *et al.* focused on how to appropriately assign the SUs to sense the PU channels under a practical scenario by taking the heterogeneous characteristics of both SUs and PU channels into consideration. However, in [14], the important issue of interference has not been well investigated, and the spectrum temporal variation at secondary sender and destination is also not discussed. Thus, how to handle the spectrum allocation problem in heterogeneous CR networks in the presence of interference and spectrum temporal variation has not drawn much attention before. In this paper, we mainly focus on the spectrum allocation problem, aiming at deciding how to appropriately allocate more than one channel to the secondary S–D pair for sensing and utilization, where the heterogeneities of both PU channels (in terms of channel idle probability and channel capacity) and secondary S–D pair (in terms of energy detection threshold, received SNR, and geographical location) are taken into consideration. Moreover, the interference among different S–D pairs is also studied directly. The contributions of this paper are as follows.

1) In Section IV, we optimize spectrum sensing and spectrum allocation for many-to-one case while investigating the heterogeneous characteristics of both secondary S–D pairs and PU channels. This paper completes the analysis of the spectrum allocation problem where the initial part of this paper has been done in [15].

2) With the objective to maximize the achievable throughput for secondary S–D pairs, we formulate the spectrum allocation problem as a linear integer optimization problem. We show that our formulated spectrum allocation problem is NP-complete. This observation reveals the inherent challenge of determining optimal spectrum allocation results for heterogeneous cognitive radio networks (CRNs). We leverage the randomized rounding algorithm to obtain a ρ-approximation solution.

3) This paper extends to one-to-one case in [11] and [14] to many-to-one case, and furthermore it adds the interference constraint and span constraint in the problem formulation, which increases the complexity of this problem.

The rest of this paper is organized as follows. Some related works are briefly reviewed in Section II. The system model and spectrum sensing are introduced in Section III. The problem analysis and spectrum allocation problem for heterogeneous CRNs are described in Section IV. The randomized rounding algorithm is proposed in Section V. Simulation results and evaluations are given in Section VI. Finally, Section VII concludes the paper.

II. RELATED WORK

Research on spectrum allocation has attracted a lot of attention. In general, prior works on spectrum allocation mainly focused on allocating one channel to one SU. Zhao *et al.* proposed a sensing and allocation strategy with one SU and multiple channels, and the optimal policy is obtained via linear programming. However, this scheme may not be optimal when the channel characteristics are heterogeneous [6]. In [7], by considering the traffic pattern of each channel, a stochastic multiple-channel sensing scheme is proposed. Noh *et al.* derive the optimal channel allocation probability by formulating and solving a linear programming problem. In [8], the spectrum allocation problem is formulated as an oligopoly market with the assumption that there are several service providers and one consumer, where multiple service providers compete with one another to offer the spectrum access opportunities to the consumer. In [9], with the objective to minimize the difference between the expected channel available time and the expected service time, a heuristic matching algorithm is proposed to allocate spectrum to the SU. In [10], based on game theory, a demand-matching spectrum sharing for noncooperative CRNs is proposed. Hou and Huang consider the channel allocation problem with multiple primary channels. With the objective to maximize the total channel utilization, the channel allocation problem is formulated as a binary integer nonlinear programming. Yi and Cai considered a framework of spectrum auction by integrating advanced features, such as local trading markets and spectrum recall. In [17], time-dependent buyer valuation information is taken into consideration in auction mechanism design. By joint consideration of flexible spectrum demands and the satisfaction of SUs' QoS expectations, a multiunit spectrum auction for CR networks with power-constrained is further studied in [18]. Resource allocation problem has been further investigated in [19] and [20]. In [19], energy-efficient (EE) downlink resource allocation in heterogeneous orthogonal frequency division multiple access (OFDMA) networks is studied, where the EE maximization problem is formulated as a mixed-integer nonlinear fractional programing. In [20], a weighted semimatching algorithm is proposed to allocate resources, i.e., allocating the SUs to base station, where the distance between the SU and a base station is considered as the weight. More resource allocation techniques for efficient spectrum access have been investigated by a recent survey paper [21]. Our work differs from [6]–[11] in the following three aspects.

1) First, in this paper, we mainly focus on the spectrum allocation problem where more than one channel can be allocated to each SU for utilization simultaneously depending on their availability at that instant (many-to-one case) while [6]–[11] only considers the one-to-one case (allocate one channel to one SU for sensing and utilization).

2) Second, we attempt to consider the spectrum allocation problem under a more practical scenario, where the heterogenous characteristics in both PU channels and SUs are investigated. The PU channel is characterized by channel idle probability and channel capacity, while the SU is de-

picted by the energy detection threshold, received SNR, and geographical location.

3) Finally, to avoid the cochannel interference, we use the interference graph to model the cochannel interference, which increases the inherent challenge of this spectrum allocation problem.

III. SYSTEM MODEL

In this paper, we attempt to consider the spectrum allocation problem under a more practical scenario, where the heterogeneous characteristics of both PU channels and SUs are investigated. In this case, different SUs may have different available channels. If we allocate different available channels to secondary sender and destination, they cannot communicate with each other. Thus, how to allocate channel to the secondary S–D pair based on current channel availability is one of the most important problems in the CRNs.

A. System Model

We consider a CRN with N secondary S–D pairs and M PU channels. Each channel is allocated to one PU. However, the PU may not be active all the time and the secondary S–D can opportunistically utilize the channel when it is not used by the PU. Let $\mathcal{M}$ be the set of PU channels and $\mathcal{N}$ denote the set of secondary S–D pairs.

In heterogeneous CRNs, different SUs may have different energy detection thresholds, received SNR, and geographical locations, which results in different detection performances. Moreover, small-scale signal, such as a wireless microphone always transmits a weak power at around 10–50 mW [22], where the transmission range is limited to only 150–200 m [23]. Thus, the PU signal may only cover a part of the network rather than the whole system. In this case, the detection range of this kind of signal is relatively small. Some SUs located far from the PU cannot detect the PU signals. A channel j is said to be opportunistically accessible by the SU i only if this SU is within the detection range of channel j, then it can detect the PU's activity. Otherwise, if the SU i is located outside the detection range of channel j, then the detection probability is set to 0 [24], [25]. Therefore, different SUs may have different set of available channels due to their different geographical locations and environments. On the other hand, different PU channels may have different channel idle probability and channel capacity. Thus, allocating different PU channels to different secondary S–D pairs may result in different system performance. The CRN model is illustrated in Fig. 1. It shows that the channel availability varies across the different secondary senders and destinations.

B. Spectrum Sensing

Spectrum sensing is one of the fundamental functionalities in CR communications as it has to be performed first before data transmission. Suppose that the received signal is sampled with sampling frequency f_s, and the sensing time is denoted by τ, then the sensing performance can be measured by two parameters: detection probability and false alarm probability,
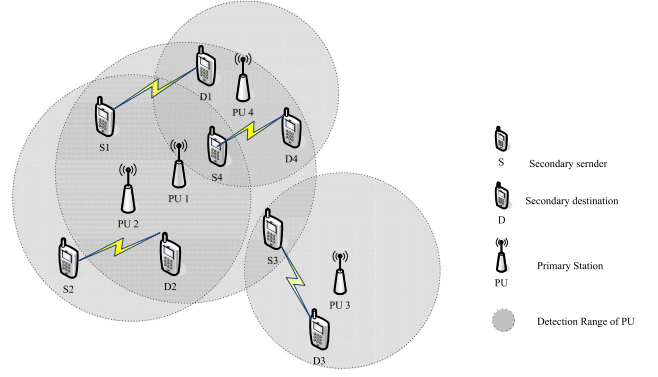

Fig. 1. CR networks architecture.

which are given by [26]

$$P_{f,(i,j)} = Q\left(\left(\frac{\varepsilon_i}{\sigma_{u_{i,j}}^2} - 1\right)\sqrt{f_s\tau}\right) \tag{1}$$

$$P_{d,(i,j)} = Q\left(\left(\frac{\varepsilon_i}{\sigma_{u_{i,j}}^2} - 1 - \gamma_{i,j}\right)\sqrt{\frac{f_s\tau}{2\gamma_{i,j}+1}}\right) \tag{2}$$

where the received primary signal is complex phase-shift keying (PSK) with zero mean and variance $\sigma_{s_{i,j}}^2$, and the noise is the independent circular symmetric complex Gaussian with zero mean and variance $\sigma_{u_{i,j}}^2$. The energy detection threshold at SU i is ε_i, and $\gamma_{i,j} = \frac{\sigma_{s_{i,j}}^2}{\sigma_{u_{i,j}}^2}$ is the average SNR in channel j received by SU i, and $Q(x)$ is the tail probability of the standard normal distribution.

Due to the heterogeneous characteristics of SUs, they may have different sensing outcomes for the same channel. Thus, the secondary sender and destination may have different available channel sets. On the other hand, the PUs channels are also generally heterogeneous. Thus, allocating different channels to different secondary S–D pairs will result in significant different performance. One of our main contributions is to take all these heterogeneous characteristics in both PU channels and SUs into consideration when studying this spectrum allocation problem.

IV. SPECTRUM ALLOCATION PROBLEM STATEMENT

In order to conduct successful data transmission, it is a must that both the secondary sender and destination should work on the same radio frequency channel. However, as discussed before, secondary sender and destination may have different sets of available channels. Besides, the available channels at each secondary sender and destination vary from time slot to time slot due to the activity of PU. At each time slot, each sender and destination should select one or more than one common idle channel as their working channels based on the available channel information. Therefore, how to select working frequency bands for each S–D pair becomes a key part of spectrum management in CRN. The key notations that will be used are listed in Table I.

To represent the spectrum availability at all S–D pairs, we define $N \times M$ binary variables $c_{i,j}^s$ and $c_{i,j}^d$, $\forall\ i,\ j$, as

TABLE I
SUMMARY OF KEY NOTATIONS

Notation	Definition
N	Number of SU S-D pairs
M	Number of PU channels
T	Frame duration
τ	Sensing time
f_s	Sampling frequency
Δ_i^s	Set of available channels at SU sender i
Δ_i^d	Set of available channels at SU destination i
d_0	Maximum number of channels allocated to S-D
$P(\mathcal{H})$	Channel idle probability
$\mathbb{C}_{N \times M}$	Channel capacity
$\mathbb{A}_{N \times N \times M}$	Interference map
Φ_s	Set of channels allocated to SU sender
Φ_d	Set of channels allocated to SU destination

follows:

$$c_{i,j}^s = \begin{cases} 1 & \text{if channel } j \text{ is available at SU sender } i \\ 0 & \text{Otherwise} \end{cases}$$

$$c_{i,j}^d = \begin{cases} 1 & \text{if channel } j \text{ is available at SU destination } i \\ 0 & \text{Otherwise.} \end{cases}$$

In order to mitigate cochannel interference, we define a matrix $\mathbb{A}_{N \times N \times M}$ to represent the interference, as follows:

$$\mathcal{A}_{i_1,i_2,j} = \begin{cases} 1 & \text{if S–D } i_1 \text{ and S–D } i_2 \text{ conflict on channel } j \\ 0 & \text{Otherwise.} \end{cases}$$

The meaning of $\mathcal{A}_{i_1,i_2,j} = 1$ is that S–D pair i_1 and S–D pair i_2 interfere with each other on channel j, thus channel j cannot be allocated to S–D pair i_1 and S–D pair i_2 simultaneously for communication.

Let Δ_i^s and Δ_i^d be the sets of channels that are available at secondary sender and destination of S–D pair i, respectively, that is

$$\Delta_i^s = \{j | c_{i,j}^s = 1, \forall j \in \mathcal{M}\}$$
$$\Delta_i^d = \{j | c_{i,j}^d = 1, \forall j \in \mathcal{M}\}.$$

For each S–D pair, spectrum allocation is done by deciding the following two vectors:

1) Φ_s represents the set of channels allocated to secondary sender

$$\Phi_s = \{s_{i,j}, \forall i \in \mathcal{N}, j \in \mathcal{M}\};$$

2) Φ_d represents the set of channels allocated to secondary destination

$$\Phi_d = \{d_{i,j}, \forall i \in \mathcal{N}, j \in \mathcal{M}\}$$

where $s_{i,j}$ and $d_{i,j}$ are the decision variables, which are defined as

$$s_{i,j} = \begin{cases} 1 & \text{if channel } j \text{ is allocated to SU sender } i \\ 0 & \text{Otherwise} \end{cases}$$

$$d_{i,j} = \begin{cases} 1 & \text{if channel } j \text{ is allocated to SU destination } i \\ 0 & \text{Otherwise.} \end{cases}$$

In other words, the spectrum allocation problem can be viewed as deciding the two vectors Φ_s and Φ_d from the current feasible region Δ_i^s and Δ_i^d, for $i \in \mathcal{N}$ and $j \in \mathcal{M}$.

A. Analysis of System Throughput

The objective is to maximize the sum of achievable throughput for all secondary S–D pairs over all the PU channels. Let T denote the length of a time slot and τ be the total sensing time allocated to sense each PU channel. Then, the achievable throughput of S–D pair i transmitted over channel j can be expressed as

$$R_{i,j} = \frac{T - \tau}{T} P(\mathcal{H}_j) C_{ij} (1 - P_{f,ij}^s P_{f,ij}^d) \qquad (3)$$

where $P(\mathcal{H}_j)$ denotes the idle probability of channel j, $C_{i,j}$ is the transmission capacity for S–D pair i on channel j, $P_{f,ij}^s$ is the false alarm probability at SU sender i, and $P_{f,ij}^d$ is the false alarm probability at SU destination i, respectively.

B. Analysis of Valid Allocation

The constraints that spectrum allocation imposes are as follows.

1) *Availability Constraint:* Spectrum allocated to any S–D pair should be limited to the set of channels that are detected to be idle, that is

$$s_{i,j} = 1 \Rightarrow c_{i,j}^s = 1, \forall \quad i \in \mathcal{N}, \quad j \in \mathcal{M} \qquad (4)$$
$$d_{i,j} = 1 \Rightarrow c_{i,j}^d = 1, \forall \quad i \in \mathcal{N}, \quad j \in \mathcal{M}. \qquad (5)$$

2) *Spectrum Span Constraint:* In order to guarantee a fairness among the secondary S–D pairs, each one should be allocated with at least one channel for data transmission (It is possible that no common channel is available for a S–D pair, because they might not be covered by one common PU detection range. In this case, the throughput achieved by this S–D pair is zero and we can just exclude this S–D pair from being considered.). On the other hand, the total number of channels allocated to each S–D pair should not exceed the maximum value d_0 due to some hardware limitations, that is

$$1 \le \sum_{j=1}^{M} s_{i,j} d_{i,j} \le d_0, \forall i \in \mathcal{N}. \qquad (6)$$

3) *Interference Free Constraint:* Mutually interfering secondary S–D pairs should not be allocated with the same channels. Thus, the interference free constraint can be

represented as

$$\mathcal{A}_{i_1,i_2,j} = 1 \Rightarrow s_{i_1,j} d_{i_1,j} s_{i_2,j} d_{i_2,j} = 0,$$
$$\forall i_1, \quad i_2 \in \mathcal{N}, \quad j \in \mathcal{M}. \quad (7)$$

C. Problem Formulation

Finally, with the objective of maximizing the achievable throughput, the dynamic spectrum allocation problem can be formulated as the following optimization problem:

$$\max_{\Phi_s,\Phi_d} \quad \sum_i \sum_j s_{i,j} d_{i,j} R_{ij} \quad (8)$$

$$\text{s.t.} \quad (4)\text{--}(7)$$

$$s_{i,j}, d_{i,j} \in \{0,1\}, \quad \forall i \in \mathcal{N}, \quad j \in \mathcal{M}. \quad (9)$$

Due to the nonlinear constraints (4)–(7) and factor $s_{i,j} d_{i,j}$ in the objective function, the formulated problem above is a nonlinear optimization problem. Let $m_{i,j} = s_{i,j} d_{i,j}$, we can transfer the Dynamic sPectrum Allocation (DPA) problem into the following linear 0-1 integer optimization problem

$$\max_{\Phi_s,\Phi_d} \quad \sum_i \sum_j m_{i,j} R_{ij} \quad (10)$$

$$\text{s.t.} \quad s_{i,j} \leq c_{i,j}^s, \quad \forall i,j \quad (11)$$

$$d_{i,j} \leq c_{i,j}^d, \quad \forall i,j \quad (12)$$

$$1 \leq \sum_{j=1}^M m_{i,j} \leq d_0, \quad \forall i \quad (13)$$

$$s_{i_1,j} + d_{i_1,j} + s_{i_2,j} + d_{i_2,j} \leq 3$$
$$if \quad \mathcal{A}_{i_1,i_2,j} = 1, \forall \quad i_1, i_2, j \quad (14)$$

$$s_{i,j}, d_{i,j}, m_{i,j} \in \{0,1\}, \quad \forall i,j. \quad (15)$$

It is obviously that the two formulated problems are equivalent. This DPA problem is NP-complete. The complexity to find the optimal solution will grow exponentially as the number of S–D pairs and PU channels increases. In the next section, we prove the NP-complete of this problem.

D. Complexity Analysis of DPA Problem

To prove an optimization problem is NP-complete, it is equivalent to prove its corresponding decision problem is NP-complete [11], [27]. Therefore, we start with the definition of a decision problem corresponding to our formulated DPA problem as shown below.

Definition 1: DPA decision problem. Given the inputs: the secondary S–D pairs set $\mathcal{N}$, the PU channels set $\mathcal{M}$, the interference graph $\mathbb{A}$, the heterogeneous characteristics of both PU channels and secondary S–D pairs (e.g., the available channel sets Δ_i^s and Δ_i^d, the channel idle probability $P(\mathcal{H})$, the channel capacity $\mathbb{C}$ etc.), and a value of total achievable throughput α. Does there exist the allocation matrix (Φ_s and Φ_d) that satisfies all the constraints of the DPA problem, and the total achievable throughput is α?

To prove that the DPA decision problem is NP-complete, we have to prove the following.

1) The DPA decision problem is an NP problem.
2) The DPA decision problem is an NP-hard problem:
   a) select a well known NP-complete problem, in our paper, say circuit satisfiability (SAT) problem is used;
   b) find a mapping algorithm, such that the DPA decision problem can be transformed to the SAT problem in polynomial time.

Lemma 1: The DPA decision problem is an NP problem.

Proof: To show the DPA decision problem is an NP problem, we have to prove that the instance of this decision problem for which the answer is "yes" can be verified in polynomial time. Suppose we are given the allocation matrix (Φ_s and Φ_d), so we can verify if it is a solution of the DPA decision problem by checking:

1) whether $\sum_i \sum_j m_{i,j} R_{ij} = \sum_i \sum_j s_{i,j} d_{i,j} R_{ij} = \alpha$;
2) whether the availability constraint is satisfied, that is $s_{i,j} \leq c_{i,j}^s$ and $d_{i,j} \leq c_{i,j}^d$, for $\forall i,j$;
3) whether the spectrum span constraint is satisfied, that is $1 \leq \sum_{j=1}^M m_{i,j} = \sum_{j=1}^M s_{i,j} d_{i,j} \leq d_0, \forall i$;
4) whether the interference free constraint is satisfied, that is if S–D pair i_1 conflicts with S–D pair i_2 on channel j, they cannot be allocated with channel j for communication, by checking if $\mathcal{A}_{i_1,i_2,j} = 1$ then $s_{i_1,j} + d_{i_1,j} + s_{i_2,j} + d_{i_2,j} \leq 3, \quad \forall i_1, i_2, j$.

Verifying (1)–(3) takes a running time of $O(NM)$. Furthermore, it takes a running time of $O(N^2 M)$ to verify (4). Thus, if the allocation matrix (Φ_s and Φ_d) is a solution of the DPA decision problem, it can be verified in polynomial time. The DPA decision problem is an NP problem. ∎

Lemma 2: The DPA decision problem is NP-hard.

To prove the DPA decision problem is an NP-hard problem, we use the approach proposed in [11] by restricting the DPA decision problem to an instance for small values of N, M, and d_0, and then transforming this restricted the DPA decision problem to a well known NP-hard SAT problem in polynomial time. The detailed proof of NP-hard is given in the appendix.

Theorem 1: The DPA problem is an NP-complete problem.

Proof: Combining Lemmas 1 and 2, we make a conclusion that the DPA problem is an NP-complete problem. ∎

V. RANDOMIZED ROUNDING ALGORITHM

Since the DPA problem is NP-complete, it is difficult to solve this problem in polynomial time. We resort to the randomized rounding algorithm as illustrated in Algorithm 1. Here, the linear programming relaxation (LPR) of the 0-1 integer programming (IP) is defined as follows.

Definition 2: The LPR of the 0-1 IP is obtained by relaxing the integrality constraint to $0 \leq x_i \leq 1$ for all the variables.

As stated by in [28, Th. 2.1] and [29], if we have an approximation heuristic algorithm to the max-IP DPA problem, let (Φ_s^*, Φ_d^*) be the optimal solution to the LPR of DPA problem, then $(\Phi_s^*, \Phi_d^*)/\rho$ dominates a convex combination of all feasible

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Algorithm 1: The Randomized Rounding Algorithm.

1: *Step 1:* **Relaxation of the DPA problem**
2: -Calculate the optimal spectrum allocation result (Φ_s^*, Φ_d^*) for the LPR of DPA problem.
3: *Step 2:* **Convex decomposition**
4: -Decompose the fractional solution (Φ_s^*, Φ_d^*) to a convex combination of mixed integer solutions, i.e., $\sum_{q \in \Psi} \lambda^q(\Phi_s^q, \Phi_d^q) \geq (\Phi_s^*, \Phi_d^*)/\rho$. This can be done by solving a pair of primal-dual LP in (19) and (20) using ellipsoid method.
5: *Step 3:* **Pick the integer solution (Φ_s^q, Φ_d^q) with λ^q**
6: -Select each feasible integer solution (Φ_s^q, Φ_d^q) of the DPA problem with probability λ^q.

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integer solutions of the DPA problem, that is, we have

$$\sum_{q \in \Psi} \lambda^q(\Phi_s^q, \Phi_d^q) \geq (\Phi_s^*, \Phi_d^*)/\rho$$

where $\lambda^q \geq 0$ for all q and $\sum_{q \in \Psi} \lambda^q = 1$. (Φ_s^q, Φ_d^q) is a feasible integer solution to the DPA problem and Ψ is the index set for all feasible integer solutions.

A. Detailed Analysis for the Randomized Rounding Algorithm

The randomized rounding algorithm which consists of three main steps.

Step 1: Relaxation of the DPA Problem: The first step is to solve the LPR of DPA problem by relaxing constraint (15) to ($s_{i,j} \leq 1, d_{i,j} \leq 1, m_{i,j} \leq 1 \quad \forall \quad i \in \mathcal{N}, \quad j \in \mathcal{M}$, are redundant and hence ignored)

$$s_{i,j} \geq 0, \quad \forall\, i \in \mathcal{N}, \quad j \in \mathcal{M}$$
$$d_{i,j} \geq 0, \quad \forall\, i \in \mathcal{N}, \quad j \in \mathcal{M}$$
$$m_{i,j} \geq 0, \quad \forall\, i \in \mathcal{N}, \quad j \in \mathcal{M}.$$

The LPR of DPA problem is linear programmable, obviously, it can be optimally solved in polynomial time. Let (Φ_s^*, Φ_d^*) denote the optimal solution to the LPR of DPA problem.

Step 2: Convex decomposition: Applying the recent convex decomposition technique [29], [30], we decompose the optimal fractional solution (Φ_s^*, Φ_d^*) into a convex combination of integral solutions each with a fractional weight that sums up to 1. This step requires an effective polynomial-time approximation algorithm to the DPA problem, that satisfies

$$\sum_i \sum_j m_{i,j} R_{ij} \geq \mathrm{OPT_{LPR}}/\rho. \tag{16}$$

The left side represents the achievable throughput obtained using the approximation algorithm, and $\mathrm{OPT_{LPR}}$ is the value of the objective function for the LPR of DPA problem when the optimal solution is (Φ_s^*, Φ_d^*).

Thus, the goal of the convex decomposition is to find combination weights $\lambda^q \geq 0$, for all q, such that

$$\sum_{q \in \Psi} \lambda^q = 1, \text{ and, } \sum_{q \in \Psi} \lambda^q(\Phi_s^q, \Phi_d^q) \geq (\Phi_s^*, \Phi_d^*)/\rho. \tag{17}$$

Next, we will compute each λ^q, which is the weight required in the convex decomposition for solution (Φ_s^q, Φ_d^q). In order to obtain λ^q that satisfies (17), we wish to solve the following LP problem:

$$\text{Primal} : \min \quad \sum_{q \in \Psi} \lambda^q \tag{18}$$

$$\text{s.t.} \quad \sum_{q \in \Psi} \lambda^q(s_{i,j}^q, d_{i,j}^q) \geq (s_{i,j}^*, d_{i,j}^*)/\rho$$

$$\sum_{q \in \Psi} \lambda^q \geq 1, \lambda^q \geq 0, \forall q \in \Psi.$$

Our goal is to solve this primal LP problem optimally with $\sum_{q \in \Psi} \lambda^q = 1$. However, we note that the problem described in (19) has an exponential number of variables, which is difficult to solve. We instead resort to its dual problem that has an exponential number of constraints. The dual problem of (19) is defined as follows:

$$\text{Dual: } \max \quad \left(\sum_{i,j} \omega_{i,j} s_{i,j}^* + \sum_{i,j} \gamma_{i,j} d_{i,j}^* \right) /\rho + \delta \tag{19}$$

$$\text{s.t.} \quad \sum_{i,j} \omega_{i,j} s_{i,j}^q + \sum_{i,j} \gamma_{i,j} d_{ij}^q + \delta \leq 1, \forall q \in \Psi$$

$$\omega_{i,j} \geq 0, \gamma_{i,j} \geq 0, \delta \geq 0, \forall i, j.$$

In the following, we will first demonstrate that this dual problem can be solved in polynomial time. Then, according to strong duality, we can solve the primal LP problem (19) optimally in polynomial time with $\sum_{q \in \Psi} \lambda^q = 1$.

Theorem 2: Both LP problems (19) and (20) can be solved in polynomial time and the optimal value of objective function is 1.

Proof: First, suppose that $\omega_{i,j} = 0, \gamma_{i,j} = 0$, for all i, j, and $\delta = 1$, we note that this is a feasible solution to the dual problem, because it satisfies the dual constraint and the value of objective function is 1. Hence, the optimal value is at least 1. Next, we will prove that the optimal value of objective function is 1 by way of contradiction. We assume that

$$\left(\sum_{i,j} \omega_{i,j} s_{i,j}^* + \sum_{i,j} \gamma_{i,j} d_{i,j}^* \right) /\rho + \delta > 1. \tag{20}$$

Then, we have

$$\left(\sum_{i,j} \omega_{i,j} s_{i,j}^* + \sum_{i,j} \gamma_{i,j} d_{i,j}^* \right) /\rho > 1 - \delta. \tag{21}$$

Since the integrality gap between LPR and DPA is at least $1/\rho$, as stated in first primal constraint, there exists a $q \in \Psi$,

such that

$$(s_{i,j}^q, d_{i,j}^q) \geq (s_{i,j}^*, d_{i,j}^*)/\rho. \tag{22}$$

Combining (21) and (22), resulting in

$$\sum_{i,j} \omega_{i,j} s_{i,j}^q + \sum_{i,j} \gamma_{i,j} d_{i,j}^q > 1 - \delta \tag{23}$$

which violates the first dual constraint, and a contradiction occurs. Hence, the optimal objective value of the dual LP is 1. Due to the strong duality, the optimal objective value of primal LP is 1 as well. ∎

We observe that the primal LP has an exponential number of variables, which may take exponential time to solve directly. We instead resort to the dual LP that has an exponential number of constraints. The ellipsoid method can be used to solve the problem in polynomial time despite an exponential number of constraints [31]. In order to make the dual LP solvable in polynomial time, the ellipsoid method requires an approximation algorithm to serve as a separation hyperplane [29]. Each hyperplane corresponds to a constraint in the dual problem, providing a feasible solution (Φ_s^q, Φ_d^q) corresponding to each primal variable λ^q. The primal LP can then be transformed to an optimization problem with a polynomial number of variables corresponding to these hyperplanes. We can, hence, solve the primal LP in polynomial time, obtaining the weights of the convex decomposition that sum to 1.

Step 3: Pick the Integer Solution With λ^q: Following the decomposition, each possible integer solution (Φ_s^q, Φ_d^q) is selected with a probability equal to its corresponding convex multiplier λ^q computed in the convex decomposition in the second step. Then, the expected throughput is

$$\sum_q \sum_i \sum_j \lambda^q s_{i,j}^q d_{i,j}^q R_{ij} \geq \sum_i \sum_j s_{i,j}^* d_{i,j}^* R_{ij}/\rho. \tag{24}$$

The above inequality implies that the decomposition algorithm can achieve an approximation ratio of ρ with respect to the aggregated gain.

B. Approximation Algorithm for DPA

Next, we will present a greedy heuristic algorithm to obtain the feasible integer solutions of the DPA problem, in which we relate the DPA problem to the multiple maximum bipartite matching problem. The proposed algorithm is described in Algorithm 2, which consists of the following four main steps.

Step 1: Select Available Channel Set for Each S–D Pair: In the initialization phase, we select the set of common channels that are available at both sender and destination for each S–D pair i, we use $\Delta_{sd,i}$ to represent this set, that is

$$\Delta_{sd,i} = \{j | c_{i,j}^s = c_{i,j}^d = 1, \forall j\}.$$

Step 2: Construct a Bipartite Graph: A bipartite graph $G(V_1 \cup V_2, \varepsilon)$ is a graph whose vertices are divided into two disjoint sets, such that every edge in ε connects a vertex in V_1 to one in V_2 [32]. In CRNs, the topology of S–D pairs and PU channels can be represented as a bipartite graph $G(V_1 \cup V_2, \varepsilon)$. Vertex set V_1 contains the S–D pairs, and set V_2 corresponds to the

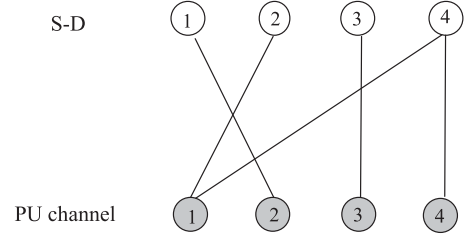
Fig. 2. Illustration of bipartite graph.

PU channels in the network. An edge exists between (i, j), $i \in V_1$ and $j \in V_2$, if and only if the channel j is the common available channel for sender and destination of S–D pair i, that is $j \in \Delta_{sd,i}$. For instance, Fig. 2 shows the bipartite graph corresponding to the Fig. 1, where the set of common channels for S–D pairs 1–4 are $\Delta_{sd,1} = \{2\}$, $\Delta_{sd,2} = \{1\}$, $\Delta_{sd,3} = \{3\}$, and $\Delta_{sd,4} = \{1, 4\}$, respectively.

Step 3: Channel Allocation Using Kuhn–Munkres Matching Algorithm: In graph theory, the maximum matching is a set of independent edges with the largest possible cardinality. Here, we use Kuhn–Munkres matching algorithm to match S–D pairs with their common available channels such that as many as S–D pairs can select different common channels to achieve a high channel utilization.

Step 4: Update the Bipartite Graph: More than one channel is available for each S–D pair, and we allow each S–D pair to transmit over more than one channel if possible. Therefore, we are required to update the bipartite graph. Let $Q(S \cup B, \eta)$ be the maximum matching from the bipartite graph $G(V_1 \cup V_2, \varepsilon)$, then we use the following steps to update the bipartite graph.

1) Remove all the edges in η from ε, that is $\varepsilon = \varepsilon/\eta$.
2) If S–D pair i_1 conflicts with S–D pair i_2, and if channel j has been allocated to S–D pair i_1, then remove edge $e_{i_2 j}$ from ε.

Then go back to step 3 until either one of the following termination conditions is satisfied: no more available channel can be allocated to the S–D pair, which means that all the nodes in V_2 have become isolated nodes; and all the S–D pairs have been allocated with the maximum allowable number of channels, that is $\sum_{j=1}^M s_{i,j} d_{i,j} \geq d_0, \forall i \in \mathcal{N}$. The algorithm is described in detailed in Algorithm 2.

VI. SIMULATION RESULTS

In this section, the simulation results are displayed to evaluate the proposed spectrum allocation method, the system parameters are taken similarly to [33]. We set $f_s = 6$ MHz and $T = 200$ ms. In order to model the heterogeneous characteristics of secondary S–D pairs and PU channels, the noise power and energy detection threshold are randomly generated with means 1 and 1.03, and the channel capacity and channel idle probability are randomly generated with means 0.9 and 0.7, respectively. To provide a better understanding on how our proposed spectrum allocation algorithm behaves, we compute the allocation results for the following two interference graph settings.

Algorithm 2: Heuristic Approximation Algorithm for DPA.

1: **Input:** Available channel sets Δ_i^s, Δ_i^d, d_0 and interference $\mathbb{A}$.
2: Construct $\Delta_{sd,i}$ for each S–D pair i, and bipartite graph $G(V_1 \cup V_2, \varepsilon)$.
3: **Initialization:** $k = 0$, $\Phi_s = [0]_{N \times M}$, $\Phi_d = [0]_{N \times M}$
4:     $G(V_1 \cup V_2, \varepsilon^{(0)}) = G(V_1 \cup V_2, \varepsilon)$
5: **while** $\sum_{j=1}^{M} s_{i,j} d_{i,j} < d_0$ and $|\varepsilon^{(k)}| > 0$ **do**
6:     Invoke Kuhn–Munkres algorithm to obtain the maximum matching $Q(S^{(k)} \cup B^{(k)}, \eta^{(k)})$
7:     **for** all S-D pair i **do**
8:       **if** $e_{ij} \in \eta^{(k)}$ **then**
9:         $s_{ij} = 1$ and $d_{ij} = 1$;
10:      **end if**
11:    **end for**
12:    **for** all S-D pair i_1 and i_2 **do**
13:      **if** $\mathcal{A}_{i_1 i_2 j} = 1$ **then**
14:        **if** $e_{i_1 j} \in \eta^{(k)}$ and $e_{i_2 j} \in \varepsilon^{(k)}$ **then**
15:          $\varepsilon^{(k)} = \varepsilon^{(k)}/e_{i_2 j}$;
16:        **end if**
17:      **end if**
18:    **end for**
19:    $\varepsilon^{(k+1)} = \varepsilon^{(k)}/\eta^{(k)}$;
20:    $k = k + 1$;
21: **end while**
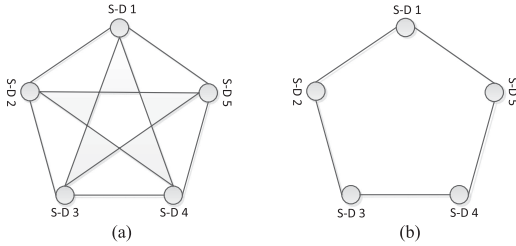22: **Output:** $\Phi_s = [s_{ij}]_{N \times M}$, $\Phi_d = [d_{ij}]_{N \times M}$.

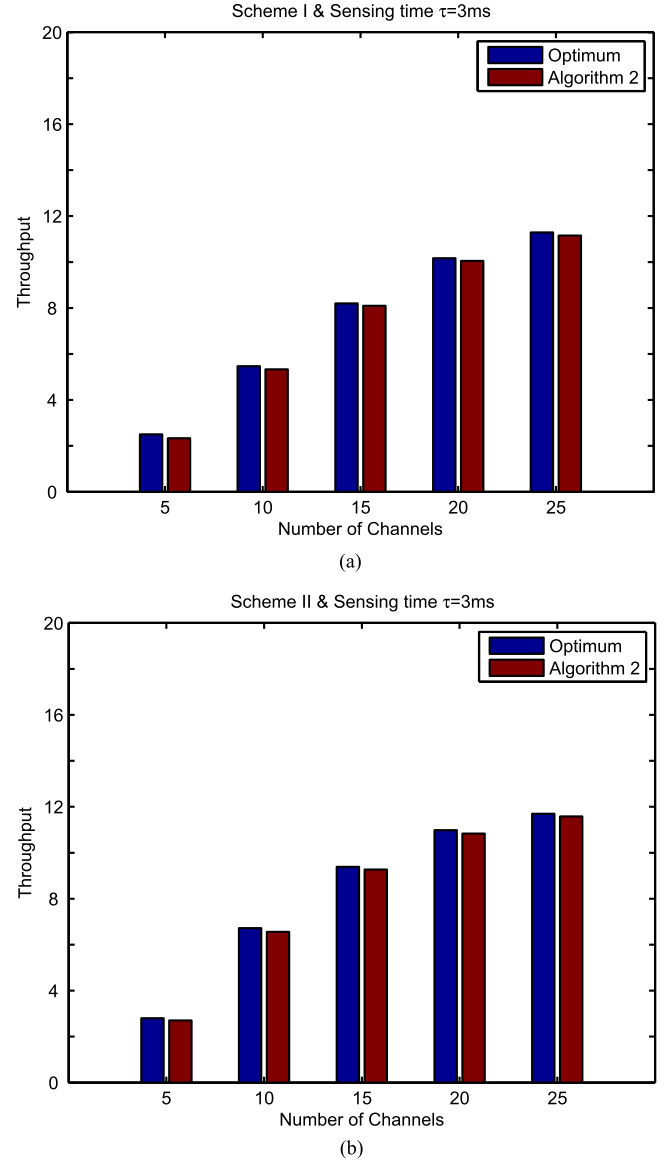Fig. 3. Two interference graph settings for simulation. (a) Setting I. (b) Setting II.

Fig. 4. Comparison between optimum and Algorithm 2 for setting I and II. (a) Achievable throughput for sensing time $\tau = 3$ ms under setting I. (b) Achievable throughput for sensing time $\tau = 3$ ms under setting II.

1) *Setting I:* As shown in Fig. 3(a), all the S–D pairs interfere with each other, which means that any two S–D pairs cannot be allocated with the same channel.
2) *Setting II:* As shown in Fig. 3(b), S–D pair 1 conflicts with S–D pairs 2 and 5, S–D pair 2 conflicts with S–D pairs 1 and 3, etc. In this case, if one channel is allocated to S–D pair 1, it cannot be allocated to S–D pairs 2 and 5 simultaneously. However, this channel is able to be utilized by S–D pairs 3 and 4.

A. Evaluation of Our Proposed Algorithm

To provide a better understanding of how our proposed algorithm performs, we first implement and evaluate Algorithm 2. Fig. 4(a) and (b) compare the spectrum allocation results for the proposed algorithm as well as the optimal solution obtained using exhaustive search for settings I and II. From Fig. 4(a) and (b), we observe that Algorithm 2 achieves an impressive performance, approaching the optimum rather closely in most cases with a maximum performance loss of 6.8% for setting I and 3.5% for setting II. This result shows that our spectrum allocation problem based on the proposed algorithm is reasonable and can achieve a close-to-optimal performance. It can also be observed that the achievable throughput goes up when the number of PU channels increases. This is because that as the number of PU channels increases, more transmission opportunities can be detected, therefore, more channels can be allocated to each S–D pair, and more throughput can be achieved.

B. Evaluation of Maximum Number of Allocated Channels d_0

Fig. 5(a) and (b) depict the achievable throughput of S–D pairs as a function of the number of PU channels for different values of $d_0 \in \{1, 3, 5\}$ under settings I and II. It is easy
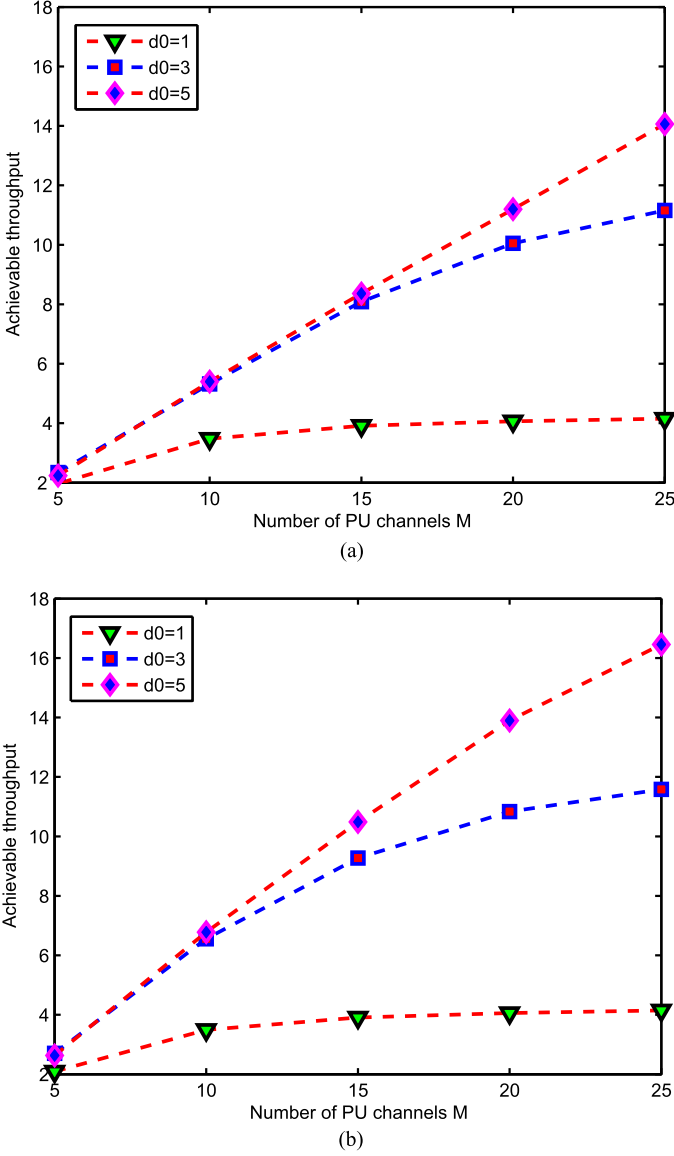
Fig. 5. Achievable throughput for different d_0 under settings I and II. (a) Achievable throughput for different d_0 under setting I. (b) Achievable throughput for different d_0 under setting II.
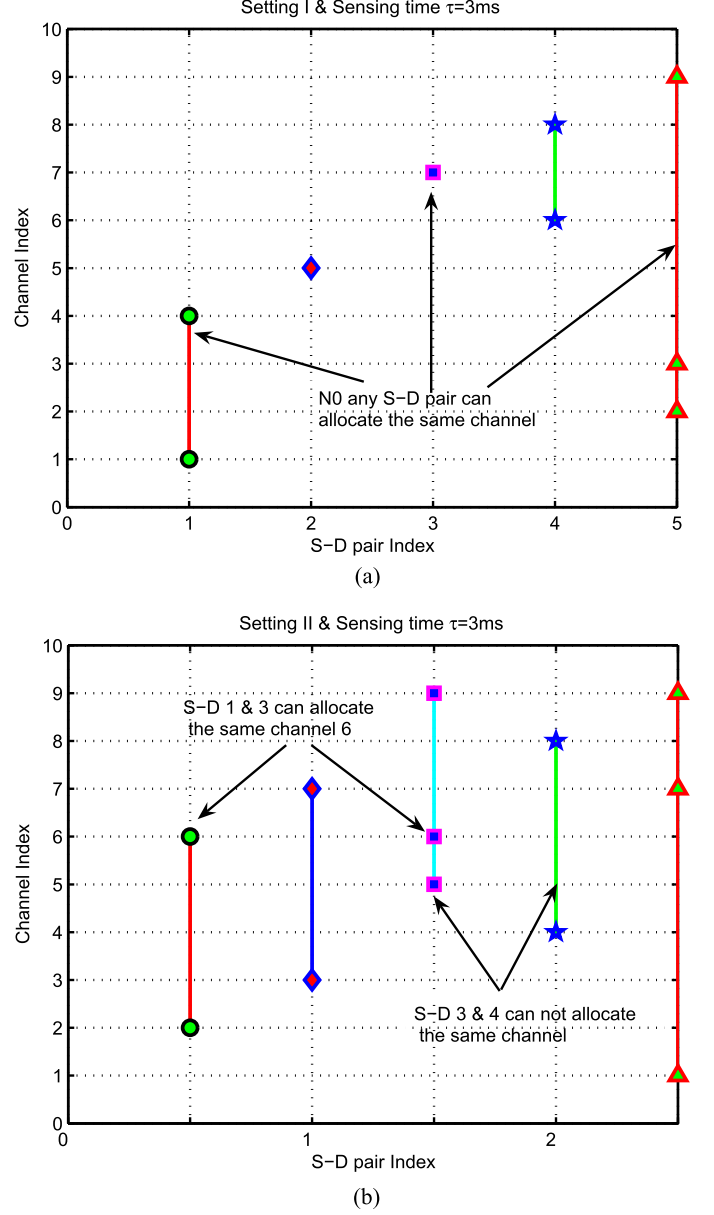

Fig. 6. Spectrum allocation results for sensing time $\tau = 3$ ms and $d_0 = 3$ under settings I and II. (a) Spectrum allocation for setting I. (b) Spectrum allocation for setting II.

to observe that the achievable throughput increases with d_0. Physically speaking, if we increase the maximum number of allocated channels, more than one channel can be allocated to each S–D pair, leading to an increase in the achievable throughput. However, Fig. 5(a) and (b) also show that when the number of PU channels grows larger and larger, the achievable throughput will increase slowly. Due to the spectrum span constraint, when the number of allocated channels reaches the maximum value d_0, no more channel can be allocated to each S–D pair, even though there still exist idle ones. Thus, for the case of $d_0 = 1$, only one channel can be allocated to each S–D pair, this is why when the number of channel grows continuously after 10, the achievable throughput will hardly change. Moreover, as seen in Figs. 4 and 5, setting II outperforms setting I in achievable throughput. The reason is obvious since for setting I, all the S–D pairs conflict with one another, thus no channel can be

reutilized by another S–D pair. While for setting II, the same channel can be allocated to different nonconflicting S–D pairs, which increases the achievable throughput.

C. Spectrum Allocation Results

In this section, we depict the spectrum allocation results for the two settings. In Fig. 6(a), the spectrum allocation result is shown for system setting I. As discussed before, we take the heterogeneous characteristics of both PU channels and S–D pairs into consideration so that more detailed result that accurately indicates which S–D pair should utilize which channel can be achieved. As shown in Fig. 6(a), channels 1 and 4 are allocated to S–D pair 1 for sensing and utilization; channels 5 and 7 are allocated to S–D pairs 2 and 3, respectively; channels 6 and 8 are
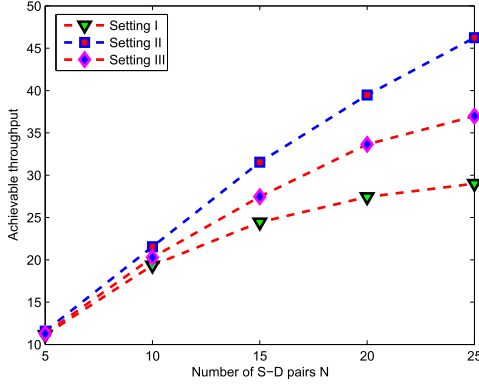
Fig. 7. Achievable throughput for different number of S–D pairs N.

Fig. 8. Average number of iterations versus the number of PU channels for different $d_0 = \{1, 3, 5\}$.

allocated to S–D pair 4; and channels 2, 3, and 9 are allocated to S–D pair 5. From Fig. 6(a), we can see that no channel can be allocated to two different S–D pairs, since in setting I, all the S–D pairs conflict with one another. Moreover, we note that channel 10 is not allocated to any S–D pair; this is because that no S–D pair is within the detection range of channel 10, thus it cannot be detected and utilized by any S–D pair. Furthermore, Fig. 6(b) illustrates the spectrum allocation result for setting II. Different from setting I, in setting II, some S–D pairs can be allocated with the same channel. For example, channel 6 is allocated to S–D 1 and S–D 3 simultaneously, since S–D 1 does not conflict with S–D 3; and channel 7 is also allocated to S–D 2 and S–D 5 simultaneously.

D. Evaluation of Number of S–D Pairs N

Next, we study the performance of the proposed algorithm for a relatively large-scaled network when the number of S–D pairs N varies from 5 to 25. Three interference graph settings are taken into consideration: setting I [any two S–D pairs conflict with each other, as shown in Fig. 3(a)], setting II [the interference graph of S–D pairs is depicted as Fig. 3(b)], and setting III [the interference graph of S–D pairs is randomly generated]. As shown in Fig. 7, the achievable throughput increases as the number of S–D pairs increases. In addition, it can be observed from Fig. 7 that setting II has better performance than settings I and III in achievable throughput. Furthermore, this improvement becomes larger and larger as the number of S–D pairs increases. This is because that as the number of S–D pairs increases there may not exist extra idle channel for two conflicted S–D pairs in settings I and III. While for setting II, one channel can be allocated to different S–D pairs simultaneously, if they do not interfere with each other, which can increase the achievable throughput.

E. Complexity evaluation

Next, we illustrate the complexity of approximation algorithm for spectrum allocation problem, when the number of S–D pairs is 5. Since the time complexity of the optimal matching Kuhn–Munkres algorithm has been known as $O(M^3)$, thus the complexity highly depends on the number of iterations taken in the approximation algorithm, which has a maximum value of
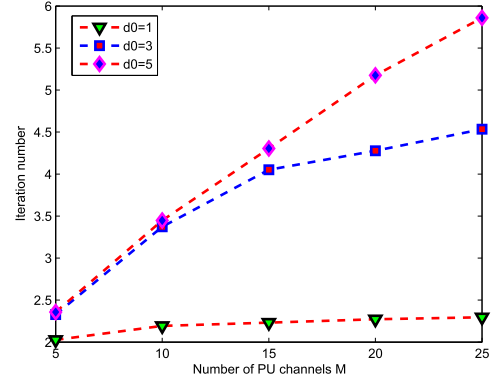
M. Obviously, this algorithm can be implemented in a polynomial time. More specifically, Fig. 8 shows the complexity when the number of channels increases from 5 to 25 for different values of $d_0 = \{1, 3, 5\}$.

VII. CONCLUSION

In this paper, we focus on the spectrum allocation problem: How to appropriately allocate the available PU channels to secondary S–D pairs? We take the heterogeneities of both PU channels and secondary S–D pairs into consideration, which has not been fully studied in most of the literatures. With the objective to maximize the achievable throughput for secondary S–D pairs, the spectrum allocation problem is formulated as a linear integer problem, where the availability constraint, spectrum span constraint, and interference free constraint are taken into consideration. This problem has been proved to be NP-complete. The proposed solution leverages a recent result in theoretical computer science that can decompose an optimal fractional solution to NP-hard problem into a convex combination of internal solutions. Evaluation results show that the proposed algorithm can achieve a close-to-optimal solution with far less complexity.

APPENDIX

Proof: To prove the DPA decision problem is an NP-hard problem, we adopt the approach used in [11] by restricting the DPA decision problem to an instance for small values of N, M, and then transform this restricted DPA decision problem to a well known NP-hard SAT problem in polynomial time. The restricted DPA decision problem is defined as follows.

Definition 2: The restricted DPA decision problem is a special instance of the DPA decision problem in *Definition 1* with the number of secondary S–D pair $N = 2$, the number of PU channels $M = 2$, the maximum number of allocated channels $d_0 = 1$. The sets of available channels at secondary senders and destinations are $\Delta_1^s = \Delta_2^s = \{1, 2\}$, and $\Delta_1^d = \Delta_2^d = \{1\}$. For further simplification, we assume that the achievable throughput for each PU channel is 1 regardless of which S–D pair utilizes this channel, that is $R_{ij} = 1, \forall i, j$. We further simplify such that these two S–D pairs conflict with each other and,

hence, they cannot transmit over the same channel. The total achievable throughput is $\alpha = 1$.

Definition 3: The SAT problem is a decision problem of determining whether a given boolean circuit has an assignment of its inputs that makes the output true, which has been proven to be NP-complete. The boolean formula of the SAT problem used in our work is given as

$$((x_{s,1}^1 \wedge x_{d,1}^1) \wedge (\overline{x_{s,2}^1} \wedge x_{s,2}^2 \wedge x_{d,2}^1))$$
$$\vee ((x_{s,2}^1 \wedge x_{d,2}^1) \wedge (\overline{x_{s,1}^1} \wedge x_{s,1}^2 \wedge x_{d,1}^1)) \tag{25}$$

where $x_{s,i}^j$ and $x_{d,i}^j$, $i, j \in \{1, 2\}$ denote boolean variables for the SAT problem, and $\overline{x_{s,i}^j}$ and $\overline{x_{d,i}^j}$, are the complements of $x_{s,i}^j$ and $x_{d,i}^j$, respectively. The output of the SAT problem is a boolean value (True or False). Given the boolean expression defined above, can we assign values to these variables $x_{s,i}^j$ and $x_{d,i}^j$, $i, j \in \{1, 2\}$ such that the expression is True?

To show that the restricted DPA decision problem can, in polynomial time, be transformed to the SAT problem, we need to verify that for a given set of inputs, the restricted DPA decision problem has "yes" answer if and only if there exists a set of assignments to each variable so that the SAT problem defined above can obtain the output of True.

First, given that the spectrum allocation vectors Φ_s and Φ_d are a "yes" answer instance for the restricted DPA decision problem. There are two possible selections for the S–D pairs: both sender and destination of S–D pair 1 select channel 1, and the selection of sender and destination of S–D pair 2 are channel 1 and channel 2, respectively. In this case S–D pair 1 can carry out the transmission, all the constraints are satisfied; and both sender and destination of S–D pair 2 select channel 1, and the selection of sender and destination of S–D pair 1 are channel 1 and channel 2, respectively. In this case, S–D pair 2 can carry out the transmission. Without loss of generality, we assume the first possible spectrum allocation solution where the elements of the allocation matrices are $s_{1,1} = d_{1,1} = 1, s_{2,1} = 0, d_{2,1} = s_{2,2} = 1$. That is $m_{1,1} = 1$ and $m_{i,j} = 0$, for $i \neq 1$ and $j \neq 1$. Therefore, the SAT problem can make the output true by setting the input variables $x_{s,i}^j = s_{i,j}$ and $x_{d,i}^j = d_{i,j}$ $i, j \in \{1, 2\}$. Obviously, this transformation takes polynomial time. For the other possible spectrum allocation solution, the transformation can be similarly made.

On the other hand, we also have to show that if there is a set of input variables that can make the SAT problem output True, we can get a Yes-instance for the restricted DPA decision problem. According to the SAT problem defined above, it can be concluded that if a set of assignments can make the SAT problem output True, the input variables satisfy $x_{s,1}^1 = x_{d,1}^1 = 1$, $x_{s,2}^1 = 0, x_{s,2}^2 = x_{d,2}^1 = 1$, or $x_{s,2}^1 = x_{d,2}^1 = 1, x_{s,1}^1 = 0, x_{s,1}^2 = x_{d,1}^1 = 1$. Without loss of generality, we consider the first setting. In this case, if we set $s_{i,j} = x_{s,i}^j$ and $d_{i,j} = x_{d,i}^j$ $i, j \in \{1, 2\}$, then we have $s_{1,1} = d_{1,1} = 1, s_{2,1} = 0, d_{2,1} = s_{2,2} = 1$, in this case, both the sender and destination of pair 1 are allocated with channel 1, and the sender and destination of pair 2 are allocated with channels 2 and 1, respectively. Thus, $m_{1,1} = 1$ and

$m_{i,j} = 0$, for $i \neq 1$ and $j \neq 1$, all the constraints of the DPA problem can be satisfied and the total achievable throughput is $\alpha = \sum_i \sum_j m_{i,j} R_{ij} = m_{1,1} R_{11} = 1$. Of course, this transformation takes polynomial time.

Therefore, we have proved that the restricted DPA problem can be transformed into the SAT problem in polynomial time. Thus, we conclude that the DPA decision problem is NP-hard. ∎

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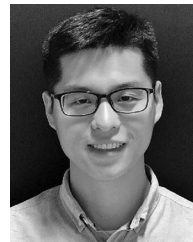
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