Energy Efficiency Optimization in Uplink Virtual MIMO Systems

Yun Rui*, Q.T. Zhang†, *Fellow, IEEE*, Mingqi Li*, Lei Deng‡, and Peng Cheng‡
*Shanghai Advanced Research Institute, Chinese Academic of Science, China
†Department of Electronic Engineering, City University of Hong Kong, Hong Kong
‡Department of Electronic Engineering, Shanghai Jiao Tong University, China
Email: ruiy@sari.ac.cn, eekzhang@cityu.edu.hk, limq@sari.ac.cn, {dl0729, cp2001cp}@sjtu.edu.cn

***Abstract*—Energy-efficient transmission is increasing in importance for wireless system design because of limited battery power in mobile devices. In this paper, we consider energy efficiency optimization in uplink virtual MIMO systems. First, accurate closed-form expressions are derived for the achievable energy efficiency for both multi-user (MU) transmission mode and single-user (SU) transmission mode, which are based on the ergodic throughput, transmit power and circuit power. Then, we demonstrate the existence of unique globally optimal energy efficiency for SU and MU mode, respectively. Since users have data rates requirement and peak power limits, we further consider power constraint, and develop joint mode switching with power loading schemes to optimize energy efficiency for both homogeneous and heterogeneous networks. Finally, our simulation results show that the proposed adaptive transmission strategies significantly improve energy efficiency.**

***Index Terms*—Virtual MIMO, Energy Efficiency, Mode Switching, Power Allocation**

I. INTRODUCTION

Multiple-input multiple-output (MIMO) technology has attracted attention due to its high spectral efficiency (SE). However, the application of MIMO uplink is limited by practical implementation issues of multiple power amplifiers at users, especially in small hand-sets. To overcome this, virtual MIMO (V-MIMO) transmission is proposed for the uplink [1], which allows several users, each with one transmit antenna, to transmit independently on the same resource blocks.

Indeed, for most SE-oriented scenarios, systems are designed to maximize SE without further caring about power consumption as long as certain peak or average power constraints are satisfied. This may lead to transmitting with the full power for a long period and is deviated from energy-efficient design. Meanwhile, as green radio becomes a major trend [2], energy efficiency (EE), defined as the number of information bits that can be reliably conveyed over the channel per-unit energy consumption, is becoming increasingly important for mobile communications because of the slow progress of battery technology and growing requirements of multimedia applications [3].

Recently, energy-efficient system design has received much attention in academic fields. For the general MIMO case, the authors in [4] show that the MIMO system may not outperform single-input single-output (SISO) in terms of energy efficiency if the circuit power is also taken into account. However, by adapting modulation order to balance transmit energy and circuit energy consumption, the MIMO system can outperform SISO systems. In [5], adaptive switching between MIMO and single-input multiple-output (SIMO) is addressed to save energy at mobile terminal. For sensor networks, the single antenna terminals can cooperate among each other to form a V-MIMO system, and space time block code (STBC) cooperative transmission and adaptive clustering framework can be also exploited to improve EE [6]. In [7], the authors also derive an adaptive MIMO approach where the transmitter adapts its modulation and rate and chooses either space division multiplexing, space-time coding or single antenna transmission. It is shown that this adaptive technique can improve the EE up to 30% compared to the non-adaptive systems.

However, these adaptive switching schemes are mostly based on the instantaneous channel state information (CSI), which should be achieved at base station (BS) to schedule. Thus, the feedback overhead would be large due to the huge instantaneous CSI from many users. In this paper, we investigate the energy efficient communication from a different perspective, which aims to optimize EE based on the statistics of channels in uplink V-MIMO scenarios. Adaptive transmission strategies including joint mode switching and power optimization are proposed to improve the energy efficiency and reduce the instantaneous feedback amount for scheduling. First, we get the achievable EE expressions for multi-user (MU[1]) and single user (SU) transmission modes based on the ergodic rates as well as transmission and circuit power. For each mode, we demonstrate the existence of a globally optimal power solution to achieve maximum EE. Furthermore, considering the rate requirements and peak power limits, we study the problem for both mode switching and power loading under the given power constraint. In homogeneous network, it is shown that equal power can be performed among users and the reduced complexity threshold-based transmission strategy is designed. In the heterogeneous network where users experience different path losses, the power allocation could be first performed for MU transmission, then the optimal mode can be selected to maximize the system achievable EE.

The rest of the paper is outlined as follows. The system model is described in Section II. The achievable EE is

[1]It is noted here that MU mode means V-MIMO transmission in uplink.

analyzed for different transmission modes in Section III. In Section IV, we perform the joint mode switching and power optimization method to maximum EE. Simulation results are provided in Section V, followed by the conclusions given in Section VI.

II. System Model

Let us consider an uplink V-MIMO system with U mobile users within a cell communicating with the Node-B. Each user can use only one transmit antenna, i.e., $N_t = 1$, while the Node-B is equipped with N_r receive antennas. The scheduler in the Node-B chooses $N_u \leq N_r$ users to share the same time-frequency resource blocks. Also, it is assumed that the channel suffers quasi-static Rayleigh flat-fading. If the Node-B randomly selects N_u users among a total of U users to construct a V-MIMO, then at the receiver, the received signal vector $\mathbf{y}$ (whose dimension is $N_r \times 1$) can be expressed as

$$\mathbf{y} = \mathbf{HGPs} + \mathbf{n}, \tag{1}$$

where $\mathbf{s}$ is a $N_u \times 1$ vector representing the transmitted signals from N_u different users and $\mathbf{E}[\mathbf{ss}^H] = \mathbf{I}_{N_u}$. $(\cdot)^H$ represents the complex conjugate transpose, $\mathbf{E}[\cdot]$ denotes expectation operation and $\mathbf{I}_{N_u}$ is the identity matrix of size $N_u \times N_u$. $\mathbf{P}$ and $\mathbf{G}$ are the power loading matrix and path loss gain matrix for the transmitted signals, which are $\mathrm{diag}\{P_1, P_2, \cdots, P_u\}$ and $\mathrm{diag}\{G_1, G_2, \cdots, G_u\}$ respectively. Moreover, $\mathbf{H}$ is a $N_r \times N_u$ matrix with zero mean i.i.d complex Gaussian entries with unit variance. The vector $\mathbf{n}$ is modeled as zero-mean additive white Gaussian noise (AWGN) with variance $\mathbf{E}[\mathbf{nn}^H] = \sigma^2 \mathbf{I}_{N_r}$.

For defining the notion of a good V-MIMO channel which is constructed by the selected users pair, the energy efficiency is used as the metric, and the instantaneous energy efficiency of the MIMO channel is given by

$$EE_i = \frac{\log_2(1+\gamma_i)}{P_i + P_c}, \tag{2}$$

which defines the number of bits transmitted per Joule of energy. In fact, the numerator is the instantaneous capacity for i-th user, and the denominator is the sum of transmit power P_i and circuit power P_c. Therein, P_i is used for reliable data transmission, and P_c represents average energy consumption of device electronics. γ_i is the post-processing signal to noise ratio (SNR) for the i-th user. If perfect CSI is available at the Node-B, assuming that zero-forcing (ZF) detection method is employed at the receiver [8], γ_i can be denoted as

$$\gamma_i = \frac{P_i G_i}{\sigma^2 [(\mathbf{H}^\dagger (\mathbf{H}^\dagger)^H)]_{ii}}, \tag{3}$$

where $\mathbf{H}^\dagger = (\mathbf{H}^H \mathbf{H})^{-1} \mathbf{H}^H$, with $\dagger$ denoting the pseudo-inverse operation. $[\mathbf{A}]_{ii}$ denotes the i-th diagonal element of the matrix $\mathbf{A}$. Moreover, we can define the achievable energy efficiency for i-th user as

$$\bar{EE}_i = \mathbf{E_H}[\frac{\log_2(1+\gamma_i)}{P_i + P_c}]. \tag{4}$$

Actually, the achievable energy efficiency is the ratio of ergodic rates and power consume since $\mathbf{E_H}$ is the expectation on the random variable $\mathbf{H}$.

In particular, if single user transmission mode is activated, the channel $\mathbf{H}$ in (1) will convert into a $N_r \times 1$ vector, which is equivalent to SIMO situation with maximal ratio combing (MRC) receive. Thus, the achievable EE can be denoted as

$$EE_{\mathrm{SU}} = \mathbf{E_H}[\frac{\log_2(1+\gamma_{\mathrm{SU}})}{P + P_c}] = \mathbf{E_H}[\frac{\log_2(1+\|\mathbf{H}\|^2 PG/\sigma^2)}{P + P_c}], \tag{5}$$

where P is single user transmit power, and G is the corresponding path loss.

III. Energy Efficiency Analysis for Different Transmission Modes

In this section, we investigate the achievable energy efficiency for both MU and SU transmission modes, and the perfect CSI is assumed. The analysis results developed under this assumption can be applied to joint mode switching and power loading design in Section IV.

A. Multi-user Transmission Mode

Based on the received SNR γ_i as expressed in (3), and from the analysis in [9], we can find that the effective channel gain of each parallel channel is a chi-square random variable with $2(N_r - N_u + 1)$ degrees of freedom, i.e., $\frac{1}{[(\mathbf{H}^\dagger(\mathbf{H}^\dagger)^H)]_{ii}} \sim \chi^2(2(N_r - N_u + 1))$. Besides, assuming $N_r = N_u$, then we can get $\frac{1}{[(\mathbf{H}^\dagger(\mathbf{H}^\dagger)^H)]_{ii}} \sim \chi^2(2)$. Therefore, the achievable energy efficiency for i-th user can be further derived as [10]

$$\begin{aligned} \bar{EE}_i &= \frac{\mathrm{E}_{\mathbf{H}}[\log_2(1+\gamma_i)]}{P_i + P_c} \\ &= \frac{\log_2(e)\exp(\sigma^2/P_i G_i) E_1(\sigma^2/P_i G_i)}{P_i + P_c}, \end{aligned} \tag{6}$$

where $E_1(\cdot)$ is the first order exponential function (see Appendix A). To facilitate reading, we define $\delta_i = P_i G_i / \sigma^2$ in the following formulas.

Then, the sum energy efficiency of all the users can be defined as

$$EE_{\mathrm{MU}} = \sum_{i=0}^{N_u - 1} \bar{EE}_i = \sum_{i=0}^{N_u - 1} \frac{\log_2(e)\exp(1/\delta_i) E_1(1/\delta_i)}{P_i + P_c}. \tag{7}$$

From the above equation, we can find that there is no inter-user interference and each user gets an equivalent interference-free channel under the ZF receiver. Hence, the problem is decoupled and the sum network EE is maximized when each user selects power to maximize their own EE. Thus, we will investigate the independent item for the individual user in the following.

First, we redefine the EE for i-th user as

$$\bar{EE}_i(P_i) = \frac{\log_2(e)\exp(1/\delta_i) E_1(1/\delta_i)}{P_i + P_c} = \frac{\log_2(e) f(\delta_i)}{P_i + P_c}, \tag{8}$$

where $f(x) = \exp(1/x) E_1(1/x)$.

From Appendix B, we have Lemma 1.

Lemma 1: EE for i-th user have the following two properties:

1) There is one and only one point P_i^* to maximize $\bar{EE}_i(P_i)$, i.e., $P_i^* = \arg\max_{P_i \in (0,+\infty)} \bar{EE}_i(P_i)$.

2) $\bar{EE}_i(P_i)$ is concave at the interval $P_i \in (0, P_i^*]$, while only quasiconcave at the interval $P_i \in (P_i^*, +\infty)$.

Moreover, according to Appendix B, the maximal value exists when $\frac{\partial \bar{EE}_i}{\partial P_i^*} = 0$, i.e.,

$$g(P_i^*) = G_i/\sigma^2 f'(P_i^* G_i/\sigma^2)(P_i^* + P_c) - f(P_i^* G_i/\sigma^2) = 0. \tag{9}$$

Since the closed-form solution for P_i^* can not be achieved on the basis of the above equation, we can use the Newton-Raphson method for the determination of P_i^* in a recursive manner, and the $(k+1)$-th iteration is related to the k-th one by

$$P_i^*(k+1) = P_i^*(k) - \frac{g(P_i^*(k))}{g'(P_i^*(k))}. \tag{10}$$

B. Single-user Transmission Mode

For SU transmission, it is a maximal ratio combining diversity system under perfect CSI, and the achievable EE in (5) can be further formulated as [11]

$$\begin{aligned} EE_{\mathrm{SU}} &= \mathrm{E}_{\mathbf{H}}[\frac{\log_2(1+\gamma_{\mathrm{SU}})}{P+P_c}] \\ &= \frac{\log_2(e)\exp(1/\delta)\sum\limits_{k=0}^{N_r-1}\frac{\Gamma(-k,1/\delta)}{\delta^k}}{P+P_c}, \end{aligned} \tag{11}$$

where $\delta = PG/\sigma^2$. Similar as the analysis for the MU transmission, we can get the derivative of EE_{SU} with P as

$$\frac{\partial EE_{\mathrm{SU}}}{\partial P} = \frac{G/\sigma^2 h'(\delta)(P+P_c) - h(\delta)}{\ln(2)(P+P_c)^2}, \tag{12}$$

where $h(x) = \exp(1/x)\sum\limits_{k=0}^{Nr-1}\frac{\Gamma(-k, 1/x)}{x^k}$, and Γ is the incomplete gamma function.

From Appendix A, since $h'(\delta) > 0$, and $h''(\delta) < 0$, we can also find EE_{SU} is quasiconcave with P. Thus, there exists the maximal value when $\frac{\partial EE_{\mathrm{SU}}}{\partial P} = 0$, and the optimal solution $P_{\mathrm{SU-OPT}}$ also can be achieved with the Newton-Raphson iteration.

IV. Energy Efficiency Optimization with both Mode Switching and Power Loading Schemes

In this section, multi-mode transmission is proposed to adaptively select the active mode to maximize the energy efficiency. We both consider the joint mode selection and power loading for MU and SU transmission. Moreover, the homogeneous and heterogeneous networks are both exploited.

From [12], we know that there is a tradeoff for throughput between SU and MU transmission, and the diversity gain dominates the performance at low SNR while the spatial multiplexing gain dominates at high SNR. Moreover, there exists the power consume discrepancy for different transmission modes due to different circuit power consume. Hence, considering the total power constraint, the power loading and mode selection should be jointly decided under the power constraint for MU and SU transmission respectively, which can be denoted as

$$\begin{aligned} &\{\mathrm{Mode\_OPT}^*, \mathbf{P}^*\} = \arg\max EE_{\mathrm{Mode\_OPT}} \\ &\quad \text{s.t. } P_{\min} - P_c \le P_T^{\mathrm{SU}} \le P_{\max} - P_c \\ &\qquad P_{\min} - N_u P_c \le P_T^{\mathrm{MU}} \le P_{\max} - N_u P_c, \end{aligned} \tag{13}$$

where

$$EE_{\mathrm{Mode\_OPT}} = \begin{cases} EE_{\mathrm{MU}}, & \text{MU transmision mode applied.} \\ EE_{\mathrm{SU}}, & \text{SU transmision mode applied.} \end{cases} \tag{14}$$

Note that the transmit power constraint in (13) is different due to different circuit power consumption with each mode.

A. Homogeneous Networks

For Homogeneous Networks, all the users have the same path loss. Hence, under MU transmission mode, we can omit the path loss factor, then (6) can be simplified expressed as

$$EE_{\mathrm{MU}} = \sum_{i=0}^{N_u-1}\frac{\log_2(e)\exp(\sigma^2/P_i)E_1(\sigma^2/P_i)}{P_i+P_c} \tag{15}$$

From the above equation, we can find that the optimal equal power allocation will be performed among users, i.e., $P_i = P_T^{\mathrm{MU}}/N_u$, then the total energy efficiency can be denoted as

$$\begin{aligned} EE_{\mathrm{MU}} &= \frac{N_u^2\log_2(e)\exp(N_u\sigma^2/P_T^{\mathrm{MU}})E_1(N_u\sigma^2/P_T^{\mathrm{MU}})}{P_T^{\mathrm{MU}}+N_uP_c}, \\ &P_{\min} - N_uP_c \le P_T^{\mathrm{MU}} \le P_{\max} - N_uP_c. \end{aligned} \tag{16}$$

On the other hand, the energy efficiency for SU mode can be expressed as

$$\begin{aligned} EE_{\mathrm{SU}} &= \frac{\log_2(e)\exp(\sigma^2/P_T^{\mathrm{SU}}G)\sum\limits_{k=0}^{N_r-1}\frac{\Gamma(-k,\sigma^2/P_T^{\mathrm{SU}}G)}{(P_T^{\mathrm{SU}}G/\sigma^2)^k}}{P_T^{\mathrm{SU}}+P_c}, \\ &P_{\min} - P_c \le P_T^{\mathrm{SU}} \le P_{\max} - P_c. \end{aligned} \tag{17}$$

Since EE_{MU} and EE_{SU} is the function of only one variable respectively, and based on the analysis in Section III, we can first achieve the optimal solution for each mode. Then, the optimal EE need to be compared to decide the optimal mode.

Furthermore, with equal power allocation for MU mode, there exists only one mode switching point for achievable rates [12]. Thus, further considering the power consume difference for SU and MU mode, we propose the reduced complexity algorithm based on threshold comparison to jointly optimize the mode and power.

First, we can do some transformations for MU mode, substituting $\bar{P}_T^{\mathrm{MU}} = P_T^{\mathrm{MU}} + (N_u-1)P_c$ to let the power constraint for MU and SU transmission mode identical. Then, we can compare the power interval, the calculated maximal power of each mode and mode switching threshold to jointly decide the optimal mode and power. The detailed algorithm is omitted due to the space limitation.

B. Heterogeneous Networks

For heterogeneous networks, all the users have different average SNR due to different path loss. Therefore, it is necessary to perform power allocation among users to exploit multi-user diversity. Assuming round-robin scheduling performed here, we will first solve the power allocation problem to maximize EE for MU transmission mode under the power constraint, which can be denoted as follows

$$(\mathbf{P1}) \max_{\mathbf{P}} EE_{\text{sum}} = \sum_{i=0}^{N_u-1} \frac{\log_2(e)\exp(\sigma^2/P_iG_i)E_1(\sigma^2/P_iG_i)}{P_i+P_c}$$

$$s.t.\ P_{\min} - N_uP_c \le \sum_{i=0}^{N_u-1} P_i \le P_{\max} - N_uP_c$$

where $\mathbf{P} = (P_0, P_1, \cdots, P_{N_u-1})$.

First, we can achieve the optimal power for every user independently as follows

$$g(P_i^*) = G_i/\sigma^2 f'(P_i^*G_i/\sigma^2)(P_i^*+P_c) - f(P_i^*G_i/\sigma^2) = 0. \tag{18}$$

There are three possibilities as follows where $P'_{\min} = P_{\min} - N_uP_c$ and $P'_{\max} = P_{\max} - N_uP_c$.

If $\mathbf{P}^* = (P_0^*, P_1^*, \cdots, P_{N_u-1}^*)$ satisfies the constraint condition in Problem $(\mathbf{P1})$, i.e., $P'_{\min} \le \sum_{i=0}^{N_u-1} P_i^* \le P'_{\max}$, $\mathbf{P}^*$ is also the solution to Problem $(\mathbf{P1})$.

If $\sum_{i=0}^{N_u-1} P_i^* > P'_{\max}$, we will have Lemma 2 (see Appendix B).

Lemma 2: Let $\mathbf{P}^{**} = (P_0^{**}, P_1^{**}, \cdots, P_{N_u-1}^{**})$ be the solution to the Problem $(\mathbf{P1})$. Then $P_i^{**} \le P_i^*, \forall i \in \{0, 1, \cdots, N_u-1\}$.

According to Lemma 1, when $P_i^{**} \le P_i^*$, $\bar{EE}_i$ is actually concave with P_i and EE_{sum} is also concave with $\mathbf{P}$. Therefore, the local optimum is the global optimum for the problem $(\mathbf{P1})$. We can use the gradient Projection (GP) method to solve the problem $(\mathbf{P1})$ [13].

If $\sum_{i=0}^{U-1} P_i^* < P'_{min}$, we will have Lemma 3 (see Appendix B).

Lemma 3: Let $\mathbf{P}^{**} = (P_0^{**}, P_1^{**}, \cdots, P_{N_u-1}^{**})$ as the solution to the Problem $(\mathbf{P1})$. Then $P_i^{**} \ge P_i^*, \forall i \in \{0, 1, \cdots, N_u-1\}$.

However, according to Lemma 1, when $P_i^{**} \ge P_i^*$, $\bar{EE}_i$ is only quasiconcave. In this way, the Problem $(\mathbf{P1})$ is non-linear non-concave optimization problem. The global optimum of non-concave optimization problem generally is difficult to find by conventional optimization techniques. In this paper, since the number of pairing users is limited in uplink, we can use the simulated annealing (SA) algorithm. SA algorithm is a probabilistic method developed by Kirkpatrick [14] to find the global optimum of a cost function which might have several local optimum.

As a result, the mode switching and power allocation algorithms are presented as follows

i) For the SU mode, first we choose the best user with maximal path loss gain. Then we can easily obtain the optimal power loading

$$\bar{P}^*_{\text{SU-OPT}} = \begin{cases} P_{\max} - P_c & \text{if } \mathrm{P_{\max} - P_c <\ P_{SU-OPT}} \\ P_{\min} - P_c & \text{if } \mathrm{P_{\min} - P_c >\ P_{SU-OPT}} \\ P_{\text{SU-OPT}} & \text{otherwise} \end{cases}$$

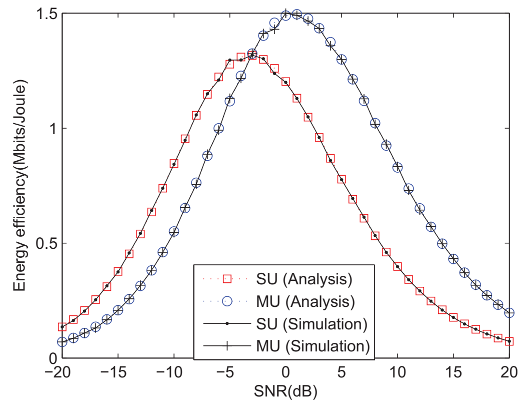

Fig. 1. Energy Efficiency performance comparison for derivations and simulations.

ii) For MU mode, we get the optimal power allocation for the Problem $(\mathbf{P1})$.

iii) Finally, the maximal EE for two modes can be compared to decide the optimal mode.

V. Simulation Results

In this section, Monte Carlo simulations are used to illustrate the analysis and proposed mode switching joint with power loading algorithms to optimize EE in uplink V-MIMO systems. For simplicity, we assume a single cell scenario. We let $N_u = 2$ and $U = 20$. At the receiver, the ZF structure is adopted, where perfect synchronization is assumed. Moreover, the scheduling and detection are assumed under perfect channel estimation at the BS side.

In Fig. 1, EE performance comparison is investigated for both SU and MU modes. The circuit power P_c is set to be 0.2. The homogeneous network is considered for MU mode, which means equal power allocation is performed. From the figure, we can see the theoretical analysis and simulation for EE are well matched, which shows the accuracy of the derived EE expressions for both MU and SU modes.

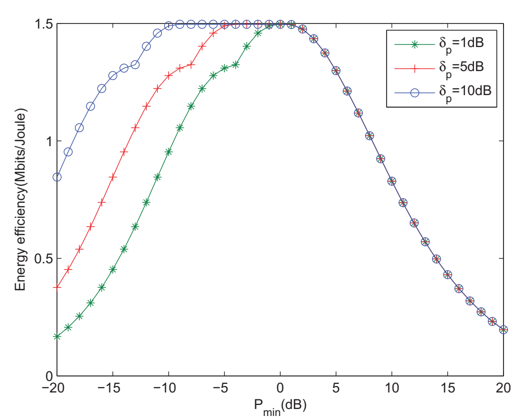

Fig. 2. Energy efficiency with different power constraints in the homogeneous network.

Fig. 2 compares energy efficiency of different power constraints with mode switching and power loading schemes. Note that $\delta_p = P_{\max} - P_{\min}$. When δ_p is small, it means at first, the maximum power is less than the handover power P_{THR}, SU mode is activated, and $P_{\max}$ is the power loading result. At the medium, MU mode is selected to achieve global optimality. By increasing δ_p from 1 dB to 10dB, the maximum achievable EE range will expand. While for larger $P_{\min}$, EE is decreasing and only determined by the minimum transmit power.

For the heterogeneous networks, we consider 2 users with different path loss for $P_c = 2$. The proposed power allocation algorithm is performed between these two users, which is shown in Fig. 3. Note that "Optimal PC" means the proposed power allocation rule, while "Equal PC" outputs the maximal EE with equal power allocation between users, searching for the total power constraint range. δ_p is the same as that in Fig. 2. It can be demonstrated the effectiveness of the proposed power allocation rule, especially in the larger power constraint region.

Fig. 4 shows the achievable energy efficiency joint with mode switching and power allocation for heterogeneous networks, with $P_c = 0.2$ and $\delta_p = 5$. Besides, EE under SU mode and MU mode are compared. This confirms that the achievable energy efficiency can be maximized with proper mode transmission under certain power constraints.

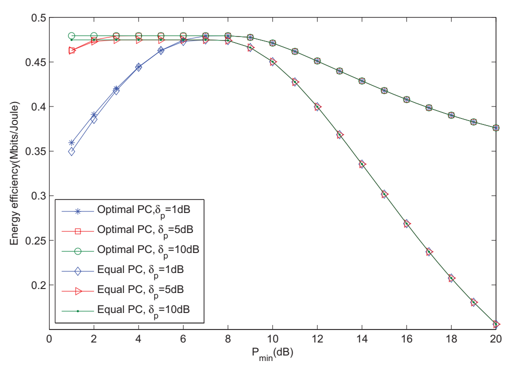

Fig. 3. Comparison of optimal power allocation and equal power allocation with different power constraints in the heterogeneous network.

VI. Conclusion

In this paper, we propose a joint mode switching and power loading scheme that adaptively select active mode based on the achievable energy efficiency in uplink V-MIMO systems. Joint transmit and circuit power consumptions are taken into account to improve energy efficiency for both SU and MU transmission modes. We first demonstrate the existence of a unique globally optimal solution to maximize energy efficiency for SU and MU mode without power constraint, respectively. Considering the sum power constraint, we propose a threshold-based algorithm to jointly optimize mode and power in the homogeneous network. For the heterogeneous network, we further investigate power allocation for MU mode to exploit the multi-user diversity, as well as performing mode switching. From the simulation results, energy utilization in both scenarios can be significantly improved within the throughput and peak power requirement.

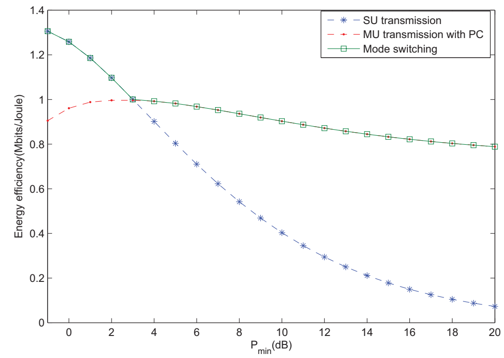

Fig. 4. Energy efficiency for joint mode switching and optimal power allocation.

Appendix A

For

$$h(x) = \exp(1/x) \sum_{k=0}^{M-1} \frac{\Gamma(-k, 1/x)}{x^k}, \tag{19}$$

we find that

$$E_{k+1}(1/x) = x^{-k}\Gamma(-k, 1/x), \tag{20}$$

where

$$E_k(x) = \int_1^{\infty} \frac{\exp(-tx)}{t^k} dt. \tag{21}$$

Therefore,

$$h(x) = \exp(1/x) \sum_{k=0}^{M-1} E_{k+1}(1/x). \tag{22}$$

Since $E'_n(z) = -E_{n-1}(z)$ for $n > 0$ and $|Arg(z)| < \pi$, for $x > 0$, then $E'_{k+1}(1/x) = x^{-2}E_k(1/x)$. As a result,

$$\begin{aligned} \frac{dh(x)}{dx} &= \exp(1/x)x^{-2}[\sum_{k=0}^{M-1} E_k(1/x) - \sum_{k=0}^{M-1} E_{k+1}(1/x)] \\ &= \exp(1/x)x^{-2}[E_0(1/x) - E_M(1/x)] > 0, \end{aligned} \tag{23}$$

as $E_k(1/x)$ is decreasing function with variable k.

For the second derivative of $h(x)$, we can get

$$\begin{aligned} &\frac{d^2h(x)}{dx^2} \\ &= \exp(1/x)\Gamma(1-M, 1/x)[\frac{1}{x^{M+3}} + \frac{(m+1)}{x^{M+2}}] - x^{-2} - x^{-3}. \end{aligned} \tag{24}$$

We further define

$$\ell(x) = \frac{(x^{-2}+x^{-3})\exp(-1/x)}{x^{-m-3}+(m+1)x^{-m-2}} - \Gamma(1-M, {}^1\!/_x)$$
$$= \frac{(x^{m+1}+x^{m})\exp(-1/x)}{(m+1)x+1} - \Gamma(1-M, {}^1\!/_x), \quad (25)$$

then

$$\ell'(x) = \frac{m(m+1)x^{m+1}\exp(-1/x)}{((m+1)x+1)^2} > 0, \quad (26)$$

and

$$\lim_{x\to 0} \ell(x) = 0, \quad (27)$$

hence, $\ell(x) > 0$, and $\frac{d^2\ell(x)}{dx^2} < 0$.

In addition, for $f(x) = \exp(1/x)E_1(1/x),\ x > 0$, it is actually a special case of $h(x)$ when $M = 1$. Therefore, we can also get $f'(x) > 0$ and $f''(x) < 0$.

Appendix B

A. Proof of Lemma 1

We can get the first derivative of $\bar{EE}_i(P_i)$ with P_i, that is

$$\frac{\partial \bar{EE}_i(P_i)}{\partial P_i} = \frac{G_i/\sigma^2 f'(\delta_i)(P_i+P_c) - f(\delta_i)}{\ln(2)(P_i+P_c)^2} = \frac{\alpha(P_i)}{(P_i+P_c)^2}. \quad (28)$$

For the numerator part, from Appendix A, we know that $f'(\delta_i) > 0$ and $f''(\delta_i) < 0$, thus it is easily demonstrated for $\alpha'(P_i) < 0$, $\lim\limits_{P_i\to 0} \alpha(P_i) > 0$, and $\lim\limits_{P_i\to\infty} \alpha(P_i) < 0$. As a result, we can deduce that $\alpha(P_i)$ is first larger than 0, then decreasing to less than zero. Since the denominate item in (28) is always larger than 0, there exists the unique P_i^* for $\frac{\partial \bar{EE}_i(P_i^*)}{\partial P_i^*} = 0$. Moreover, for $P_i < P_i^*$, $\bar{EE}_i(P_i)$ is increasing as $\frac{\partial \bar{EE}_i(P_i)}{\partial P_i} > 0$, and for $P_i > P_i^*$, $\bar{EE}_i(P_i)$ is decreasing as $\frac{\partial \bar{EE}_i(P_i)}{\partial P_i} < 0$. Thus, $\bar{EE}_i(P_i)$ is a quasiconcave function, and P_i^* will be the unique point which is a global maximizer of $\bar{EE}_i$, i.e., $P_i^* = \arg\max_{P_i\in(0,+\infty)} \bar{EE}_i(P_i)$.

Meanwhile, we can get the second derivative of $\bar{EE}_i(P_i)$ with P_i, i.e.,

$$\frac{\partial^2 \bar{EE}_i(P_i)}{\partial P_i^2} = \frac{\alpha'(P_i)(P_i+P_c)^3 - 2\alpha(P_i)(P_i+P_c)}{(P_i+P_c)^4}. \quad (29)$$

Since $\alpha'(P_i) < 0$ and $\alpha(P_i) > 0$ at the interval $P_i \in (0, P_i^*]$, we can obtain $\frac{\partial^2 \bar{EE}_i(P_i)}{\partial P_i^2} < 0$, which further means $\bar{EE}_i(P_i)$ is concave. While for the interval $P_i \in (P_i^*, +\infty)$, $\frac{\partial^2 \bar{EE}_i(P_i)}{\partial P_i^2}$ can be either positive or negative, thus $\bar{EE}_i(P_i)$ is only quasiconcave, and neither concave nor convex. In conclusion, $\bar{EE}_i(P_i)$ is concave at the interval $P_i \in (0, P_i^*)$, but only quasiconcave at the interval $P_i \in [P_i^*, +\infty)$.

B. Proof of Lemma 2 and Lemma 3

We will prove by contradiction. Assume $\exists i \in \{0, 1, \cdots, N_u - 1\}$, $P_i^{**} > P_i^*$ when $\sum_{i=0}^{N_u-1} P_i^* > P'_{\max}$. Since $\mathbf{P}^{**}$ is the solution to the Problem $(\mathbf{P1})$, $\mathbf{P}^{**}$ will satisfy the constraint of Problem $(\mathbf{P1})$, i.e., $P'_{\min} \le \sum_{j=0}^{N_u-1} P_j^{**} \le P'_{\max}$. Thus $\exists P'_i \in [P_i^*, P_i^{**}]$, which also satisfies $P'_{\min} \le \sum_{j=0, j\ne i}^{N_u-1} P_j^{**} + P'_i < \sum_{j=0}^{N_u-1} P_j^{**} \le P'_{\max}$. Meanwhile, according to the above analysis, $\bar{EE}_i$ is monotonically increasing in (0, P_i^*] and monotonically decreasing in [P_i^*, $P_{\max}$]. That means $\bar{EE}_i(P'_i) > \bar{EE}_i(P_i^{**})$. As a result, $\bar{EE}_{\text{sum}}(\mathbf{P_{-i}}^{**}, P'_i) = \sum_{j=0, j\ne i}^{N_u-1} \bar{EE}_j(P_j^{**}) + \bar{EE}_i(P'_i) > \sum_{j=0, j\ne i}^{N_u-1} \bar{EE}_j(P_j^{**}) + \bar{EE}_i(P_i^{**}) = \sum_{j=0}^{N_u-1} \bar{EE}_j(P_j^{**}) = \bar{EE}_{\text{sum}}(\mathbf{P}^{*})$, which contradicts with the assumption that $\mathbf{P}^{**}$ is the global optimum. Therefore, Lemma 2 holds. Similarly, Lemma 3 holds.

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