Timely Wireless Flows with Arbitrary Traffic Patterns: Capacity Region and Scheduling Algorithms

Lei Deng [1] Chih-Chun Wang [2] Minghua Chen [1] Shizhen Zhao [2]

[1]Department of Information Engineering
The Chinese University of Hong Kong, Hong Kong

[2]School of Electrical and Computer Engineering
Purdue University, West Lafayette, IN, USA

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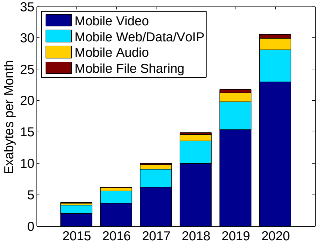
Delay-Constrained Wireless Communications

(source: Apple)



(source: Cisco)

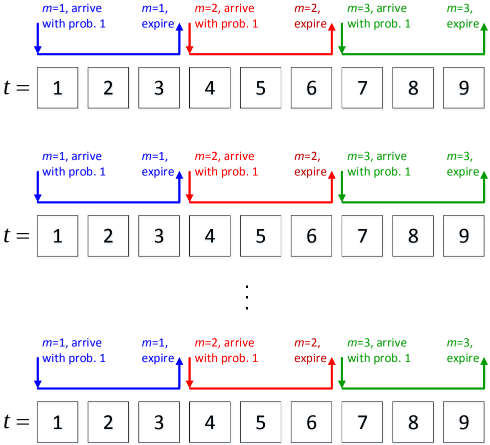
Arrive

Expire

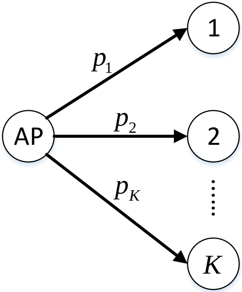
Packet Lifetime

Packet has a hard delay

A Commonly Studied Single-Hop Scenario

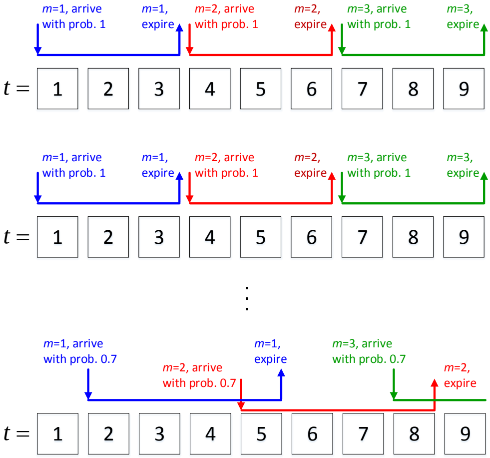


Frame-synchronized traffic pattern

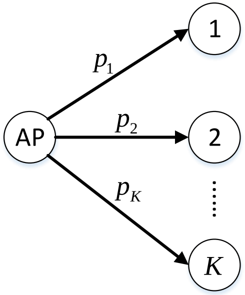


- Time is slotted
- One user per slot
- p_k is user-k packet's successful delivery prob.

How about More General Traffic Pattern?



Arbitrary traffic pattern



This traffic pattern captures more practical scenarios!

Our Contributions

Traffic Pattern	Capacity Region	Feasibility-Optimal Scheduling Policy	Utility-Optimal Scheduling Policy
Frame-Synchronized	Hou2009[1]	Hou2009[1]	Hou2010[2]
Arbitrary	This Work	This Work (Heuristic)	This Work (Heuristic)

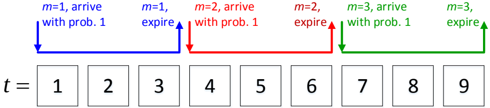
Three Fundamental Problems

- Characterize the capacity region in terms of timely throughput
  - Benchmark for any scheduling algorithms
  - Foundation for utility maximization
- Design efficient scheduling algorithms to fulfil feasibility
- Design efficient scheduling algorithms to maximize network utility

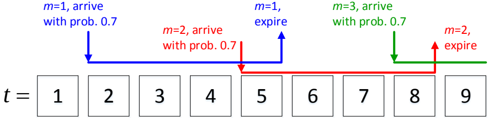
Reference

[1] I. Hou, V. Borkar, and P.R. Kumar, "A Theory of QoS for Wireless," INFOCOM, 2009.
[2] I. Hou and P.R. Kumar, "Utility Maximization for Delay Constrained QoS in Wireless," INFOCOM, 2010.

System Model

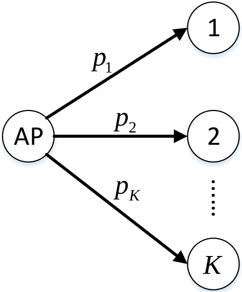


Flow 1: $(\text{offset}_1, \text{prd}_1, \text{D}_1, \text{B}_1) = (0, 3, 3, 1)$



Flow 2: $(\text{offset}_2, \text{prd}_2, \text{D}_2, \text{B}_2) = (1, 3, 4, 0.7)$

$\vdots$



The timely throughput of flow k is defined as

$$R_k \triangleq \liminf_{\mathsf{T}\to\infty} \frac{\mathsf{E}\left\{\#\text{ of flow-}k\text{ packets delivered before expiration in }[1,\mathsf{T}]\right\}}{\mathsf{T}}$$

Problem Formulation

$$(\mathbf{P1}) \qquad \max_{\text{All possible scheduling policies}} \sum_{k=1}^{K} U_k(R_k) = \sum_{k=1}^{K} w_k R_k$$

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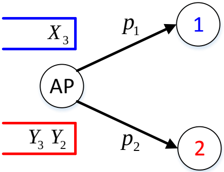
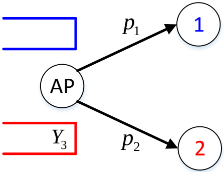
It is nature to formulate it as an Markov Decision Process (MDP)

The system looks Markovian and the long-term-wise timely throughput is similar to the average reward criteria in MDP

Formulate as An MDP



$$S_8 = (\{X_3\}, \{Y_2, Y_3\})$$
$$S_8 = (010, 1001)$$



$$S_9 = (\emptyset, \{Y_3\})$$
$$S_9 = (000, 0010)$$

- State: $S_t \triangleq (S_t^1, S_t^2, \cdots, S_t^K)$
- Action: schedule which flow (and which packet), $\mathcal{A} = \{1, 2, \cdots, K\}$
- Transition Probability:
  $P_8(S_9 = (000, 0010) | S_8 = (010, 1001), A_8 = 1) = p_1$
- Reward Function: $S_8 = (010, 1001), A_8 = 1$,
  $r(S_8, A_8) = w_1 r_1(S_8, A_8) + w_2 r_2(S_8, A_8) = w_1 \cdot p_1 + w_2 \cdot 0 = w_1 p_1$

Formulate as An MDP

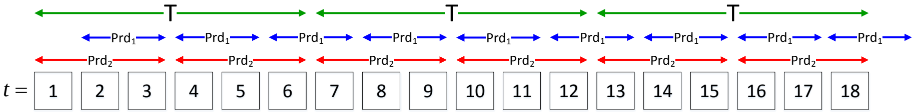
MDP Formulation with Average Reward Criteria

$$(\mathbf{P2}) \quad \max_{\text{solutions of the MDP}} \quad \liminf_{\mathsf{T}\to\infty} \frac{\sum_{t=1}^{\mathsf{T}} \mathsf{E}\{r(S_t, A_t)\}}{\mathsf{T}}$$

Benefit

MDP is a systematic approach and has rich literature

Transition Probabilities Are almost Cyclo-Stationary



$$T = \text{Least.Common.Multiple}(\text{prd}_1, \text{prd}_2, \cdots, \text{prd}_K)$$

Stationary MDP

- Randomized stationary polices are optimal
- One-slot LP

Cyclo-stationary MDP

- Randomized cyclo-stationary (RAC) polices are optimal
- T-slot LP

Main Result

Theorem 1

$(\mathbf{P2})$ is equivalent to an LP $(\mathbf{P3})$ where $\vec{R} = (R_1, R_2, \cdots, R_K)$ are variables and there are $O(|\mathcal{S}| \cdot K \cdot T)$ other variables

$$(\mathbf{P3}) \quad \max_{\vec{R}, \vec{x} \geq 0} \quad \sum_{k=1}^{K} w_k R_k$$

s.t.

$$\sum_{a \in \mathcal{A}} x_{t+1}(s', a) = \sum_{s \in \mathcal{S}} \sum_{a \in \mathcal{A}} P_t(s'|s,a) x_t(s,a), \forall s' \in \mathcal{S}, t \in [1, T-1]$$

$$\sum_{a \in \mathcal{A}} x_1(s', a) = \sum_{s \in \mathcal{S}} \sum_{a \in \mathcal{A}} P_T(s'|s,a) x_T(s,a), \forall s' \in \mathcal{S}$$

$$R_k \leq \sum_{t=1}^{T} \sum_{s \in \mathcal{S}} \sum_{a \in \mathcal{A}} \frac{r_k(s,a)}{T} x_t(s,a), \forall k \in [1, K]$$

$$\sum_{s \in \mathcal{S}} \sum_{a \in \mathcal{A}} x_t(s,a) = 1, \forall t \in [1, T]$$

Benefits and Problems

Traffic Pattern	Capacity Region	Feasibility-Optimal Scheduling Policy	Utility-Optimal Scheduling Policy
Arbitrary	Feasible Region of the LP	-	RAC (based on the optimal sol. of the LP)

RAC (Utility-Optimal) Scheduling Policy

Choose action $a \in \mathcal{A} = \{1, 2, \cdots, K\}$ at slot t with probability

$$\mathsf{Prob}_{A_t|S_t}(a|s) = \frac{\mathsf{Prob}_{S_t,A_t}(s,a)}{\mathsf{Prob}_{S_t}(s)} = \frac{x_t(s,a)}{\sum_{a' \in \mathcal{A}} x_t(s,a')}$$

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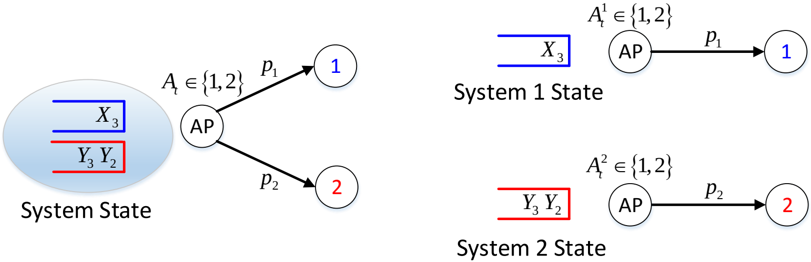
MDP Suffers Curse of Dimensionality

Exponential number of states, i.e., $|\mathcal{S}| = |\mathcal{S}^1| \times |\mathcal{S}^2| \times \cdots \times |\mathcal{S}^K| = O(2^K)$

Decouple the System and Relaxation



- Synchronized action: $A_t^1 = A_t^2$
- Relax to common-scheduling-frequency action:

$$\text{Prob}(A_t^1 = a) = \text{Prob}(A_t^2 = a), \forall a \in \mathcal{A} = \{1, 2\}$$

Theorem 2

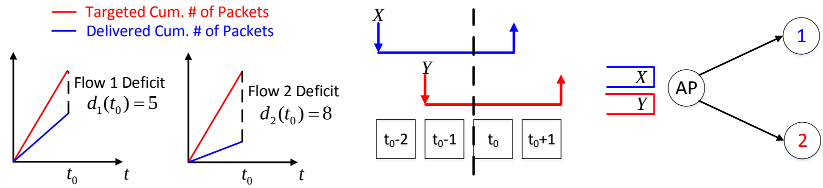
The relaxed problem is equivalent to an LP where $\vec{R} = (R_1, R_2, \cdots, R_K)$ *are variables and there are* $O((|\mathcal{S}_1| + \cdots + |\mathcal{S}_K|) \cdot K \cdot T)$ *other variables*

Complexity Reduction

From $|\mathcal{S}| = |\mathcal{S}_1| \times |\mathcal{S}_2| \times \cdots \times |\mathcal{S}_K| = O(2^K)$
to $|\mathcal{S}_1| + \cdots + |\mathcal{S}_K| = O(K)$

Traffic Pattern	Capacity Region	Feasibility-Optimal Scheduling Policy	Utility-Optimal Scheduling Policy
Arbitrary	An Outer Bound	-	RAC-Approx (heuristic, based on the optimal sol. of the LP)

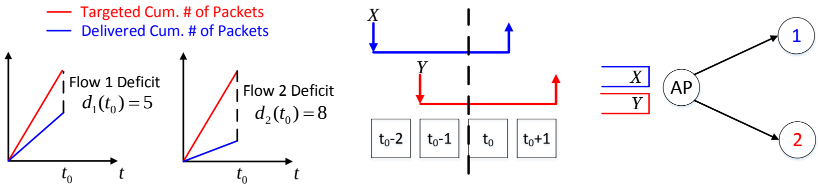
A Feasibility-Suboptimal Scheduling Policy



Largest-Deficit-First Policy (LDF) (Hou09[1]): $\max$ deficit

Since $d_2(t_0) = 8 > d_1(t_0) = 5$, schedule flow 2 and transmit packet Y

A Feasibility-Suboptimal Scheduling Policy



Largest-Deficit-First Policy (LDF) (Hou09[1]): $\max$ deficit

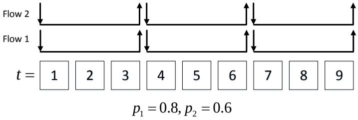
Since $d_2(t_0) = 8 > d_1(t_0) = 5$, schedule flow 2 and transmit packet Y

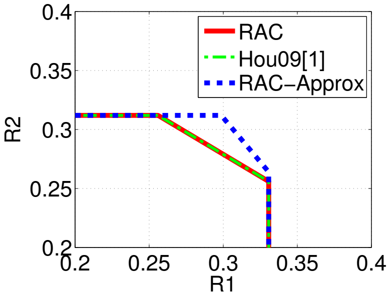
Lead-time-normalized-LDF Policy (L-LDF): $\max \frac{\text{deficit}}{\text{urgency}}$

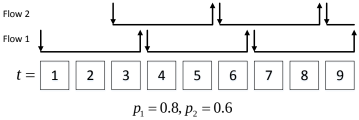
Since $\frac{d_1(t_0)}{1} = 5 > \frac{d_2(t_0)}{2} = 4$, schedule flow 1 and transmit packet X

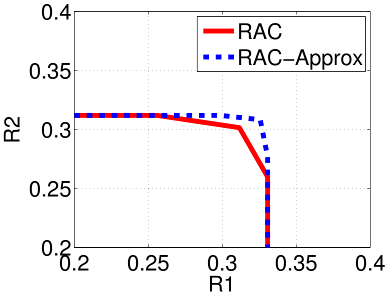
Traffic Pattern	Capacity Region	Feasibility-Optimal Scheduling Policy	Utility-Optimal Scheduling Policy
Arbitrary	-	L-LDF (Heuristic)	-

Simulation: Capacity Region

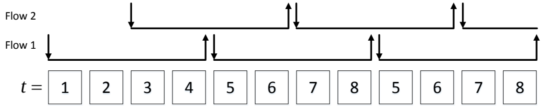




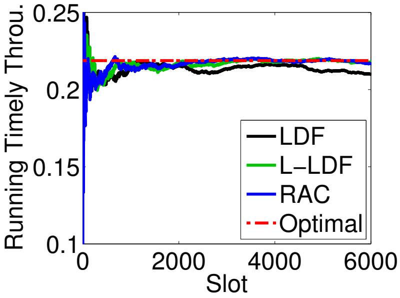




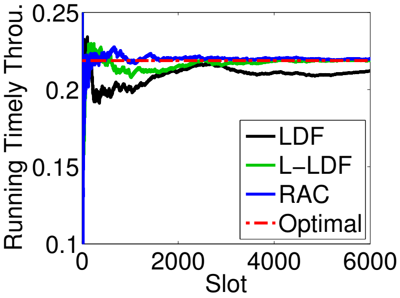
Simulation: LDF (Hou09[1]) Is Strictly Sub-optimal



$$p_1 = 0.5, p_2 = 0.5, U_1(R_1) = R_1, U_2(R_2) = R_2$$

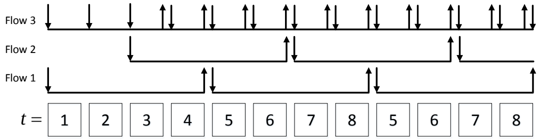


Flow 1

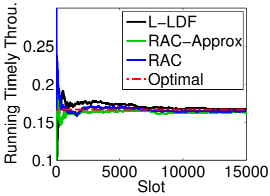


Flow 2

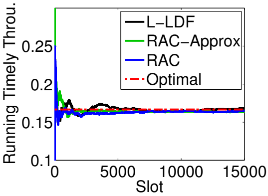
Simulation: Performance of Scheduling Policies



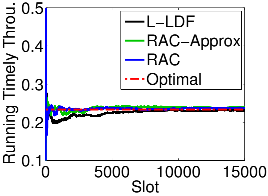
$$p_1 = 0.5, p_2 = 0.5, p_3 = 0.7, U_1(R_1) = \log R_1, U_2(R_2) = \log R_2, U_3(R_3) = \log R_3$$



Flow 1

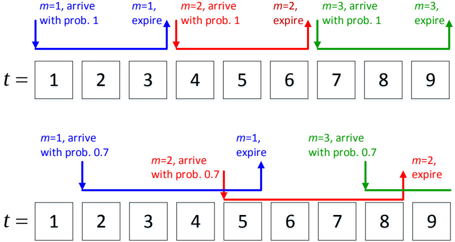


Flow 2



Flow 3

Conclusion and Future Work



Traffic Pattern	Capacity Region	Feasibility-Optimal Scheduling Policy	Utility-Optimal Scheduling Policy
Arbitrary	This Work	This Work, L-LDF (Heuristic)	This Work, RAC-Approx (Heuristic)

- How to handle the curse of dimensionality?
- How to extend results to multi-hop wireless networks?

Q&A

Thank You