INVITED PAPER *Special Section on Microwave and Millimeter-Wave Technology*

Scattered Reflections on Scattering Parameters —Demystifying Complex-Referenced S Parameters—

Shuhei AMAKAWA[†a)], ***Member***

SUMMARY The most commonly used scattering parameters (S parameters) are normalized to a real reference resistance, typically 50 Ω. In some cases, the use of S parameters normalized to some complex reference impedance is essential or convenient. But there are different definitions of complex-referenced S parameters that are incompatible with each other and serve different purposes. To make matters worse, different simulators implement different ones and which ones are implemented is rarely properly documented. What are possible scenarios in which using the right one matters? This tutorial-style paper is meant as an informal and not overly technical exposition of some such confusing aspects of S parameters, for those who have a basic familiarity with the ordinary, real-referenced S parameters.

key words: *S parameters, reflection coefficient, transmission coefficient, traveling waves, pseudo waves, power waves, reference impedance, renormalization transformation*

1. Introduction

According to Carlin [1], the earliest article that dealt with the scattering parameters or S parameters was [2], published in 1920. The first book [3] that gave extensive coverage of the subject was published in 1948 [1]. The S parameters described in this book are essentially the same S parameters as those we most often (but not always!) use today. Those are the real-referenced S parameters.

To define S parameters, we must first define an effective voltage and an effective current from the electric and magnetic fields in a waveguide, respectively. A "waveguide" here may refer to a (quasi-)TEM (transverse electromagnetic) transmission line, a hollow metallic waveguide, or some other form of waveguide. In the case of ideal TEM transmission lines, the mapping of electromagnetic (EM) fields to voltages and currents is unique. But in general, there is some arbitrariness in the mapping. This arbitrariness implies that there is arbitrariness in the definition of characteristic impedance, too. We don't delve here into the difficult and controversial problem of how the mapping should be done [4]–[8], and simply assume that effective voltage and current have been defined appropriately. We will hereafter refer to them simply as "voltage" and "current," respectively. We will also assume that our waveguide is a quasi-TEM transmission line.

Now that we have voltages and currents in waveguides somehow defined, can we define S parameters uniquely? Not yet. We have a choice between defining S parameters, including reflection coefficients, based on voltages or currents. The choice may [3], [9]–[13] or may not [14] affect the values of S parameters, depending on how you define current scattering parameters. We opt for the voltage-based definition, as is commonly practiced.

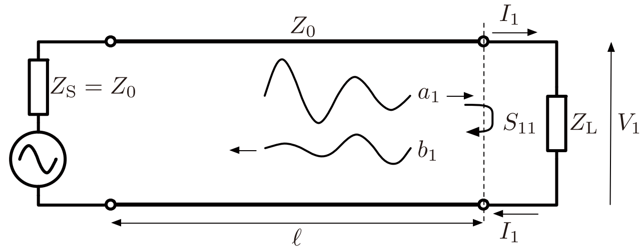

Fig. 1 A length, ℓ, of transmission line driven by a matched source (source impedance $Z_S = Z_0$) and terminated with a load impedance Z_L. Z_0 is the characteristic impedance of the line.

If a transmission line is terminated with an impedance Z_L as shown in Fig. 1, the *voltage* reflection coefficient at the terminating load is [11]–[13]

$$S_{11} = \frac{Z_L - Z_0}{Z_L + Z_0}, \tag{1}$$

where Z_0 is the characteristic impedance of the line. Z_0 appears in Eq. (1) because the reflection coefficient is a description of a 1-port in question (in this case, the load impedance Z_L) in terms of incident and reflected traveling wave amplitudes in a one-dimensional medium (i.e. transmission line) that feeds the 1-port. Z_0 is a physical property of the one-dimensional medium, not of the 1-port. The standard assumption, often made implicitly, is that the transmission line is lossless, and therefore Z_0 is real. Its standard value is 50 Ω [15], [16]. To emphasize the fact that the value of S_{11} depends on Z_0, and that its value is real, it is more appropriate to write instead

$$S_{11\,(R_{ref})} = \frac{Z_L - R_{ref}}{Z_L + R_{ref}}. \tag{2}$$

The above notation of explicitly showing the *reference resistance* R_{ref} of an S parameter in parentheses was introduced by Woods [17]. The notation is summarized in Table 1. The standard choice of R_{ref} is the *characteristic resistance* [18], [19] R_0 of the lossless transmission line, which the load Z_L terminates. Why don't we simply write $S_{11\,(R_0)}$?

Manuscript received February 15, 2016.
Manuscript revised June 27, 2016.
[†]The author is with the Graduate School of Advanced Sciences of Matter, Hiroshima University, Higashihiroshima-shi, 739–8530 Japan.
a) E-mail: amakawa@ieee.org
DOI: 10.1587/transele.E99.C.1100

Table 1 Notation for showing reference impedances.

Example	Description
$S_{ij\,(50\,\Omega)}$	S parameter with ports i and j referenced to 50 Ω
$\mathsf{S}_{(R_{\mathrm{ref}})}$	S matrix with all ports referenced to R_{ref}
$S_{21\,(R_{\mathrm{ref1}},R_{\mathrm{ref2}})}$	Port 1 referenced to R_{ref1}, port 2 referenced to R_{ref2}
$\mathsf{S}_{(Z_{\mathrm{ref1}},Z_{\mathrm{ref2}})}$	2-port S matrix referenced to Z_{ref1} and Z_{ref2}
$\mathsf{S}_{(\mathsf{Z}_{\mathrm{ref}})}$	S matrix referenced to reference impedance matrix $\mathsf{Z}_{\mathrm{ref}} = \mathrm{diag}(Z_{\mathrm{ref1}}, Z_{\mathrm{ref2}}, \cdots)$

Well, we could, but we might want to use an R_{ref} value different than R_0, which is a property of the transmission line. In some situations, we might want to assign to R_{ref} a value that is *not* a property of a physical object (transmission line) at hand. We will later see when such a need arises (§3.1). The characteristic resistance and the reference resistance should not be mixed up.

In the real world, all transmission lines are lossy, at least to a small degree. Then, the characteristic impedance Z_0 expressed in terms of the per-unit-length RLGC parameters,

$$Z_0 = \sqrt{\frac{R + \mathrm{j}\omega L}{G + \mathrm{j}\omega C}}, \tag{3}$$

assumes a *complex* value, unless the the distortionless condition [9], [11], [20],

$$\frac{R}{L} = \frac{G}{C}, \tag{4}$$

is met. What happens, then, to the reflection coefficient? Can we keep using Eq. (1) or (2) with a complex Z_{ref} (*reference impedance*) substituted for R_{ref} as follows,

$$S_{11\,(Z_{\mathrm{ref}})} = \frac{Z_{\mathrm{L}} - Z_{\mathrm{ref}}}{Z_{\mathrm{L}} + Z_{\mathrm{ref}}}, \tag{5}$$

or do we need something different? It is curious that most microwave textbooks refer to the complex characteristic impedance formula, Eq. (3), yet many of them are silent about how to define reflection coefficients and S parameters when Z_0 or Z_{ref} is complex. Even microwave metrologists don't appear to have looked very seriously into the issue [21], perhaps till mid 1980s.

Another question that springs to mind in this connection is this: how does the definition of S_{11} with a complex Z_0 relate to the well-known textbook problem (Fig. 2) of maximizing the power absorbed (and dissipated) by a load Z_{L} fed by a signal source with an impedance Z_{S}? We all know that, to maximize the power absorbed by the load in Fig. 2, the load impedance Z_{L} and the source impedance Z_{S} must be complex conjugate of each other ($Z_{\mathrm{L}} = Z_{\mathrm{S}}^*$). Does $S_{11} = 0$ (defined in what way?) imply that the power absorbed by the load in Fig. 1 is maximized? This question is not as trivial as it might appear (§2.5).

In this article, we look at *complex-referenced* S parameters. There are, at least, two distinct definitions of complex-referenced S parameters. They are incompatible with each other and serve different purposes. Depending on the system under consideration, the appropriate one to use differs. The most appropriate value to use as the reference impedance Z_{ref} might be the characteristic impedance Z_0 that feeds the network, the source impedance Z_{S} that directly feeds the network, or some other value. You might possibly think that complex-referenced S parameters are a matter of purely academic concern with little practical use. But that is not so. We work on millimeter-wave CMOS circuit design [22]–[29] and related measurements [30]–[42], and regularly use *both* types of complex-referenced S parameters out of necessity. Situations in which using the right one would matter include millimeter-wave and terahertz on-wafer measurements, where methods of vector network analyzer (VNA) calibration and de-embedding that work well at lower microwave frequencies fail. Also relevant would be power transfer systems, in which long transmission lines are deployed and minimizing losses is imperative. It is unfortunate that, in spite of the practical importance of the subject, resources are largely limited to research papers scattered about everywhere (a recent exception is [43]), at times with somewhat biased views.

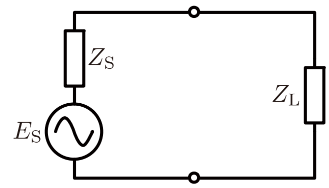

Fig. 2 A Thévenin signal source with an impedance Z_{S} feeds a load impedance Z_{L}. How can the power absorbed (dissipated) by the load be maximized?

To make matters worse, microwave engineers are left with microwave simulators, EM simulators and related programs that are strangely silent about which complex-referenced S parameters they implement (if they do), or whether they do. It is practically very important that you understand which S parameters you want to use and which ones your simulator implements. I hope this article helps develop practicing microwave engineers' awareness of the potential dangers of using wrong S parameters in the wrong context. This article was derived from an article that I presented at MWE 2015 [44], which, in turn, was based upon a tutorial that I gave in 2011 [45]. Articles that discuss related issues include [4], [17], [46]–[51].

2. Two Definitions of Reflection Coefficients

A reflection coefficient is the S parameter of a 1-port. We can learn a great deal about S parameters by looking at reflection coefficients.

2.1 Transmission Line and Reflection Coefficients

If the far end of a length of lossy transmission line is terminated with its complex characteristic impedance Z_0 as shown in Fig. 3, the line looks as if it were infinitely long ($Z_{\mathrm{in}} = Z_0$) as seen from the near end. It means that no

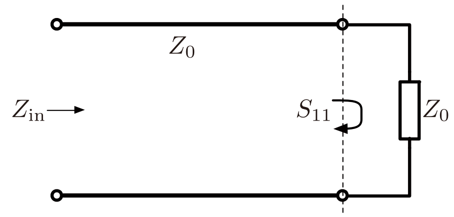

Fig. 3 A length of transmission line terminated with its characteristic impedance Z_0 at the far end looks as if the line were infinitely long as seen from the near end. $Z_{\rm in} = Z_0$ and $S_{11\,(Z_0)} = 0$.

waves reflect back when injected traveling waves reach the far end. The reflection coefficient S_{11} at the terminating load, $Z_{\rm L} = Z_0$, must be equal to 0. If $Z_{\rm L} \neq Z_0$, S_{11} will be nonzero. We, therefore, adopt Eq. (5) with $Z_{\rm ref} = Z_0$ to define the reflection coefficient of a 1-port $Z_{\rm L}$ that terminates the transmission line. If the terminating load has an impedance $Z_{\rm L} \neq Z_0$, reflected waves come back to the near end, and the line no longer appears infinitely long. The reflection coefficient, defined as above, is a representation of a 1-port in terms of traveling-wave amplitudes that appear in a transmission line, through which the 1-port is excited. That's why a property, Z_0, of the transmission line enters the expression, Eq. (5), through $Z_{\rm ref} = Z_0$. The characteristic impedance of the line physically connected to the load is the *natural reference impedance* (my preferred term) in this case. It is a property of the physical (as opposed to a virtual) environment in which the network in question is embedded.

It is, however, not clear from Eq. (5) what the incident and reflected waves are. Equation (5) is a voltage reflection coefficient as noted in §1. Specifically,

$$S_{11\,(Z_{\rm ref})} \equiv \frac{V_1^-}{V_1^+} = \frac{b_1}{a_1}, \tag{6}$$

where V_1^+ is the complex amplitude of the rightward-traveling wave incident upon the load, V_1^- is that of the leftward-traveling, reflected wave. While V_1^+ and V_1^- are sufficient for defining a reflection coefficient, with a view to smooth extension to multiports†, normalized wave amplitudes, a_1 and b_1, are usually used [4].

$$a_1 \equiv \frac{\sqrt{\Re(Z_{\rm ref})}}{|Z_{\rm ref}|} V_1^+ = \frac{\sqrt{\Re(Z_{\rm ref})}}{|Z_{\rm ref}|} \frac{V_1 + Z_{\rm ref} I_1}{2}, \tag{7}$$

$$b_1 \equiv \frac{\sqrt{\Re(Z_{\rm ref})}}{|Z_{\rm ref}|} V_1^- = \frac{\sqrt{\Re(Z_{\rm ref})}}{|Z_{\rm ref}|} \frac{V_1 - Z_{\rm ref} I_1}{2}, \tag{8}$$

a_1 and b_1 are termed *pseudo waves* [4], because when $Z_{\rm ref}$ assumes a value different than the natural reference impedance (i.e. characteristic impedance Z_0 of the physical transmission line that feeds the network), a_1 and b_1 (and V_1^+ and V_1^-, too) are no longer directly related to the voltage- and current-traveling-wave amplitudes in the line. Only when $Z_{\rm ref}$ equals Z_0 do a_1 and b_1 correspond to actual traveling-wave amplitudes in the transmission line. But hereafter, we conveniently forget the fictitious nature of pseudo waves, and pretend that they are related to voltage- and current-traveling-wave amplitudes.

The port voltage V_1 and the port current I_1 at the load are related to the voltage- and current-traveling-wave amplitudes as

$$V_1 = V_1^+ + V_1^-, \tag{9}$$

$$I_1 = I_1^+ + I_1^-. \tag{10}$$

The characteristic impedance relates the voltage- and current-traveling-wave amplitudes traveling in the same direction:

$$\frac{V_1^+}{I_1^+} = -\frac{V_1^-}{I_1^-} = Z_0\,(= Z_{\rm ref}). \tag{11}$$

Z_0 being complex ($\arg Z_0 \neq 0$) means that there is a phase difference between the voltage and current traveling waves.

Although a_1 and b_1 have the dimensions of square root of power, they are just voltages multiplied by a real number, $\sqrt{\Re(Z_{\rm ref})}/|Z_{\rm ref}|$, as is clear from Eqs. (7) and (8). They are, therefore, essentially voltages, and the reflection coefficient, Eq. (6), should be understood as a *voltage* reflection coefficient. If $Z_{\rm ref}$ is real ($Z_{\rm ref} = \Re(Z_{\rm ref}) = R_{\rm ref}$), Eqs. (7) and (8) reduce to the widely known formulas:

$$a_1 = \frac{V_1^+}{\sqrt{R_{\rm ref}}} = \sqrt{R_{\rm ref}} I_1^+ = \frac{1}{\sqrt{R_{\rm ref}}} \frac{V_1 + Z_{\rm ref} I_1}{2}, \tag{12}$$

$$b_1 = \frac{V_1^-}{\sqrt{R_{\rm ref}}} = -\sqrt{R_{\rm ref}} I_1^- = \frac{1}{\sqrt{R_{\rm ref}}} \frac{V_1 - Z_{\rm ref} I_1}{2}. \tag{13}$$

a_1 and b_1 are often mixed up in the literature with *power waves* (Eqs. (19) and (20)) [54], which we will be discussing in §2.3. While it is not incorrect to regard Eqs. (12) and (13) as power waves, given the fact that Eqs. (19) and (20) reduce to Eqs. (12) and (13) for real $Z_{\rm ref}$, I would like to emphasize that a_1 and b_1 are voltage waves, expressed in square root of watts.

2.2 Current Reflection Coefficients

The real-referenced current reflection coefficient of a load $Z_{\rm L}$ (Fig. 1) is given usually [3], [9]–[13] by

$$S_{{\rm I}11\,(R_{\rm ref})} = -S_{11\,(R_{\rm ref})} = -\frac{Z_{\rm L} - R_{\rm ref}}{Z_{\rm L} + R_{\rm ref}}. \tag{14}$$

This follows from

$$S_{{\rm I}11\,(R_{\rm ref})} \equiv \frac{I_1^-}{I_1^+} = -\frac{b_1}{a_1}, \tag{15}$$

where we used Eqs. (12) and (13). Its complex-referenced

†If a scattering matrix is defined using voltage amplitudes V_i^+ and V_j^-, a reciprocal network's S matrix becomes symmetric only if reference resistances of all ports are equal [12], [52], [53]. If a_i and b_j are used instead to define an S matrix, a reciprocal network's S matrix becomes symmetric even when reference resistances are not all equal. But if reference impedances are complex, a reciprocal network's S matrix may be asymmetric.

extension is $S_{\mathrm{I}11\,(Z_{\mathrm{ref}})} = -S_{11\,(Z_{\mathrm{ref}})}$.

Less common but another valid definition of current reflection coefficient is [14]

$$S_{\mathrm{I}'11\,(R_{\mathrm{ref}})} \equiv \frac{I_1^{-\prime}}{I_1^{+}} = \frac{b_1}{a_1} = S_{11\,(R_{\mathrm{ref}})} = \frac{Z_{\mathrm{L}} - R_{\mathrm{ref}}}{Z_{\mathrm{L}} + R_{\mathrm{ref}}}, \tag{16}$$

where, $I_1^{-\prime} = -I_1^{-}$, and the port current is given by $I_1 = I_1^{+} - I_1^{-\prime}$, instead of Eq. (10). This amounts to accounting for the direction of reflected current "outside" $I_1^{-\prime}$. This not so popular definition is not completely worthless, because Eq. (16) is actually consistent with the power-wave reflection coefficient, Eq. (21), which is a current reflection coefficient (§2.7, §2.8).

2.3 Reflection Coefficient for Power Maximization

Let's get back to Fig. 2 and think about how a reflection coefficient should be defined if we want it to be zero when the power absorbed by the load is maximized. Since $Z_{\mathrm{L}} = Z_{\mathrm{S}}^*$ is the condition for power maximization, an appropriate definition of the reflection coefficient would be

$$S_{\mathrm{P}11\,(Z_{\mathrm{ref}})} = \frac{Z_{\mathrm{L}} - Z_{\mathrm{ref}}^*}{Z_{\mathrm{L}} + Z_{\mathrm{ref}}} \tag{17}$$

with $Z_{\mathrm{ref}} = Z_{\mathrm{S}}$. A subscript 'P' is added to the left-hand side of Eq. (17) to make it distinguishable from Eq. (5). Reflection coefficients of this type can be traced back to [55]. Equation (17) reduces to Eq. (2) when Z_{ref} is real. When $S_{\mathrm{P}11\,(Z_{\mathrm{S}})} = 0$, the power absorbed by the load Z_{L} is maximized, and the absorbed power equals the available power, P_{avs}, of the signal source.

$$P_{\mathrm{avs}} \equiv \frac{|E_{\mathrm{S,rms}}/2|^2}{\Re(Z_{\mathrm{S}})} = \frac{|E_{\mathrm{S}}|^2}{8\Re(Z_{\mathrm{S}})}. \tag{18}$$

E_{S} is the amplitude of the voltage source in Fig. 2, and $E_{\mathrm{S,rms}}$ is its root-mean-square (rms) value. The "waves" incident upon the load and reflected back in Eq. (17) are [54], [56]–[58]

$$a_{\mathrm{p}1} = \frac{1}{\sqrt{\Re(Z_{\mathrm{ref}})}}\frac{V_1 + Z_{\mathrm{ref}} I_1}{2}, \tag{19}$$

$$b_{\mathrm{p}1} = \frac{1}{\sqrt{\Re(Z_{\mathrm{ref}})}}\frac{V_1 - Z_{\mathrm{ref}}^* I_1}{2}, \tag{20}$$

$$S_{\mathrm{P}11\,(Z_{\mathrm{ref}})} = \frac{b_{\mathrm{p}1}}{a_{\mathrm{p}1}}. \tag{21}$$

$a_{\mathrm{p}1}$ and $b_{\mathrm{p}1}$ are usually referred to as the *power waves* [54], although they have the dimensions of square root of power. As mentioned earlier, Eqs. (7) and (8) are not power waves.

2.4 Power Absorbed by a Load

What about the power, P_{L}, absorbed by the load in Fig. 1? From Eqs. (6) through (10), we get

$$P_{\mathrm{L}} = \frac{\Re(V_1 I_1^*)}{2} = \frac{\Re[(V_1^+ + V_1^-)(I_1^+ + I_1^-)^*]}{2} \tag{22}$$

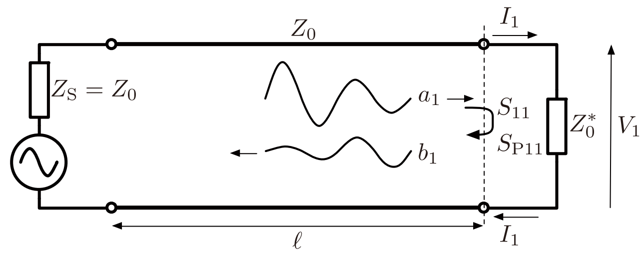

Fig. 4 A length of transmission line terminated with the complex conjugate, Z_0^*, of the characteristic impedance Z_0.

$$= \frac{1}{2}\left[|a_1|^2 - |b_1|^2 - 2\Im(a_1^* b_1)\frac{\Im(Z_{\mathrm{ref}})}{\Re(Z_{\mathrm{ref}})}\right] \tag{23}$$

$$= \frac{|a_1|^2}{2}\left[1 - \left|S_{11\,(Z_{\mathrm{ref}})}\right|^2 - 2\Im\left(S_{11\,(Z_{\mathrm{ref}})}\right)\frac{\Im(Z_{\mathrm{ref}})}{\Re(Z_{\mathrm{ref}})}\right]. \tag{24}$$

Note that in this article V_1 and I_1 are amplitudes, not rms values. If Z_{ref} is real, the last terms in Eqs. (23) and (24) disappear, and we obtain the well-known result:

$$P_{\mathrm{L}} = \frac{1}{2}\left(|a_1|^2 - |b_1|^2\right) = \frac{1}{2}|a_1|^2\left(1 - \left|S_{11\,(R_{\mathrm{ref}})}\right|^2\right), \tag{25}$$

where a_1 and b_1 are given by Eqs. (12) and (13). In Eq. (25), $|a_1|^2/2$ and $|b_1|^2/2$ can be interpreted as incident and reflected powers, respectively, and $|S_{11}|^2$ can be understood as the reflection coefficient for power. In contrast, when Z_{ref} is complex, the last term in Eq. (24) kicks in, and $|a_1|^2/2$ and $|b_1|^2/2$ can no longer be interpreted as powers [10], [52], [59]. This might appear undesirable properties of a_1 and b_1 as defined by Eqs. (7) and (8).

On the other hand, $a_{\mathrm{p}1}$ and $b_{\mathrm{p}1}$ are defined so that the same form as Eq. (25) results even when Z_{ref} is complex:

$$P_{\mathrm{L}} = \frac{1}{2}\left(|a_{\mathrm{p}1}|^2 - |b_{\mathrm{p}1}|^2\right) = \frac{1}{2}|a_{\mathrm{p}1}|^2\left(1 - \left|S_{\mathrm{P}11\,(Z_{\mathrm{ref}})}\right|^2\right). \tag{26}$$

This is highly pleasing compared to the seemingly awkward Eq. (24), and thereafter, power-wave S parameters became network theorists' favorite definition of complex-referenced S parameters [52], [60], [61]. Power-wave S parameters also saw widespread adoption by microwave engineers, too [12], [14], [62]–[65]. But as we will see, the pleasing property comes at a price. At this point, I only point out the fact that the last term in Eq. (24) can't be nulled out; it still lurks in Eq. (26). Otherwise, the conservation of energy would be violated. In this sense, nothing is fundamentally wrong with Eq. (24). Also note that in Eq. (26) the reflection coefficient for power is the scalar quantity $|S_{\mathrm{P}11}|^2$, not the complex $S_{\mathrm{P}11}$. The physical meaning of its phase, $\arg S_{\mathrm{P}11}$, is not as clear as $\arg S_{11}$ [48], [65].

2.5 Transmission Line Terminated with Z_0^*

What if the terminating load impedance in Fig. 1 is $Z_{\mathrm{L}} = Z_0^*$, as shown in Fig. 4? Looking leftward into the line from the load, the input impedance is Z_0. The natural reference impedance there, therefore, is $Z_{\mathrm{ref}} = Z_0$ for both S_{11} and

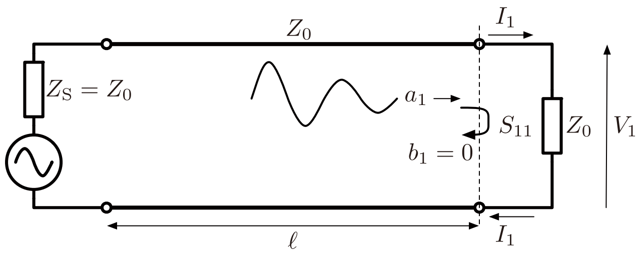

Fig. 5 A length of transmission line terminated with its characteristic impedance Z_0. Traveling waves emanating from the signal source are absorbed by the load Z_0 without any reflection.

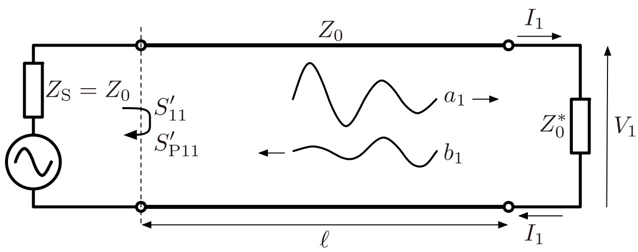

Fig. 6 Input reflection coefficients S'_{11} and S'_{P11} at the left end of the line.

S_{P11}. Since Fig. 5 corresponds to the case where $S_{11\,(Z_0)} = 0$, $S_{11\,(Z_0)} \neq 0$ in the case of Fig. 4. To be more specific, the voltage reflection coefficient of the load Z_0^* is, from Eq. (5),

$$S_{11\,(Z_0)} = \frac{Z_0^* - Z_0}{Z_0^* + Z_0} = -\mathrm{j}\frac{X_0}{R_0} \quad \text{(Fig. 4)}, \tag{27}$$

where $Z_0 = R_0 + \mathrm{j}X_0$. This means that the line wouldn't appear infinitely long as seen from the signal source. However, since the leftward and rightward input impedances at the load are, respectively, Z_0 and Z_0^*, the power-wave reflection coefficient of the load is, from Eq. (17),

$$S_{P11\,(Z_0)} = \frac{Z_0^* - Z_0^*}{Z_0^* + Z_0} = 0 \quad \text{(Fig. 4)}. \tag{28}$$

This means that the power P_L (Eq. (26)) absorbed by the load is maximized. But wait. In Fig. 5 (not Fig. 4), all traveling waves are absorbed by the load as suggested by $S_{11\,(Z_0)} = 0$. Equation (24) with $S_{11} = 0$ seems to suggest that P_L is maximized in the case of Fig. 5, too. What's going on here?

The short answer: $|a_{p1}|^2/2 = P_{avs} > |a_1|^2/2$; see the first terms of Eqs. (24) and (26). The power flowing out from the signal source in Fig. 5 is less than its available power, Eq. (18). In this rough sketch, we pretended that $\ell \to 0$ and ignored the power dissipated by the lossy transmission line itself. That must, of course, be taken into consideration in practice in power transfer problems. What is the actual power available at the right end of the transmission line in Fig. 4? It must be less than P_{avs}. Even at the left end of the line in Fig. 4, the power that flows into the line should, in general, be less than P_{avs} due to the mismatch there ($S'_{P11\,(Z_S)} \neq 0$ in Fig. 6). How can that power be made equal to P_{avs}? Perhaps by changing the load from Z_0^*? What happens then to S_{11} and S_{P11} at the right end of the line?

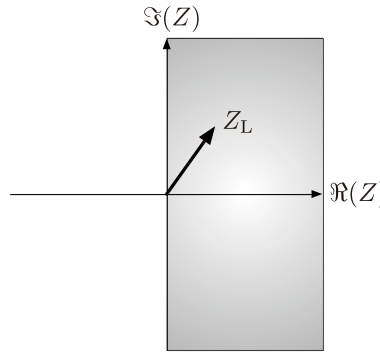

Fig. 7 A passive Z_L's range of $\arg Z_L$ on an impedance plane. $|\arg Z_L| \leq \pi/2$.

2.6 Moduli of Reflection Coefficients

Most textbooks state that the modulus of a passive load's (Z_L with $\Re(Z_L) \geq 0$) reflection coefficient is at most unity. This statement is valid for reflection coefficients defined by Eq. (2) or (17), but not for Eq. (5). If Z_{ref} is complex, $|S_{11\,(Z_{ref})}|$ might become greater than unity ($|S_{11\,(Z_{ref})}| > 1$) even if Z_L is passive. In contrast, Eq. (17) always satisfies $|S_{P11\,(Z_{ref})}| \leq 1$ for passive Z_L, which, again, might give the impression that power waves, Eqs. (19) and (20), are superior to pseudo waves, Eqs. (7) and (8).

Let's look more closely at what this is about [59]. Let

$$z_L \equiv \frac{Z_L}{Z_{ref}}. \tag{29}$$

Then, from Eq. (5),

$$S_{11\,(Z_{ref})} = \frac{z_L - 1}{z_L + 1}. \tag{30}$$

Since $\Re(Z_L) \geq 0$ by passivity assumption, $|\arg Z_L| \leq \pi/2$, as shown in Fig. 7. Let $Z_{ref} = Z_0$, where Z_0 is given by Eq. (3). Assuming that our transmission line is an ordinary right-handed line [66] with $R, L, G, C > 0$, we have $\Re(Z_{ref}) > 0$. Since the complex square root function is given by

$$z^{1/2} = \pm\sqrt{|z|}\exp\left(\mathrm{j}\frac{\arg z}{2}\right), \tag{31}$$

$|\arg Z_{ref}| < \pi/4$ as shown in Fig. 8. From Eq. (29) and Figs. 7 and 8, we get

$$|\arg z_L| < \frac{3\pi}{4}, \tag{32}$$

as shown in Fig. 9. Let us now take a look at the numerator and the denominator of Eq. (30) on a complex plane (Fig. 10). It is geometrically clear from Fig. 10 that $|z_L - 1|$ can be greater than $|z_L + 1|$, and hence $|S_{11}| > 1$ is possible. Analytically,

$$|S_{11\,(Z_{ref})}|^2 = \frac{z_L - 1}{z_L + 1}\cdot\frac{z_L^* - 1}{z_L^* + 1} = \frac{|z_L|^2 + 1 - 2\Re(z_L)}{|z_L|^2 + 1 + 2\Re(z_L)}. \tag{33}$$

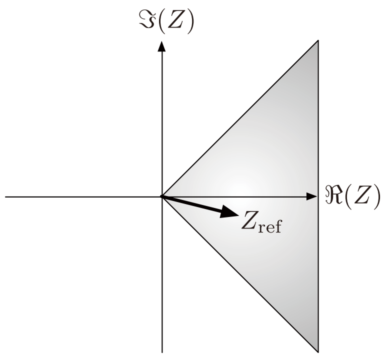

Fig. 8 The range of $\arg Z_{\mathrm{ref}}$ on an impedance plane when $Z_{\mathrm{ref}} = Z_0$. $|\arg Z_{\mathrm{ref}}| < \pi/4$.

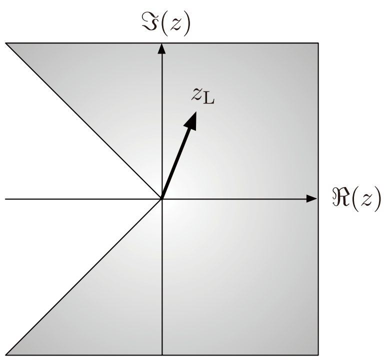

Fig. 9 The range of $\arg z_{\mathrm{L}}$, defined by Eq. (29), on a complex plane. $|\arg z_{\mathrm{L}}| < 3\pi/4$.

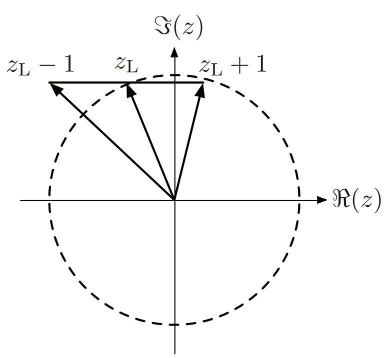

Fig. 10 Eq. (30)'s numerator $z_{\mathrm{L}} - 1$ and the denominator $z_{\mathrm{L}} + 1$ on a complex plane.

$\Re(z_{\mathrm{L}})$ in Eq. (33) can be positive or negative because of Eq. (32). $|S_{11\,(Z_{\mathrm{ref}})}| > 1$ results if $\Re(z_{\mathrm{L}}) < 0$.

The fact that $|S_{11\,(Z_{\mathrm{ref}})}|$ can exceed unity even for a passive load has long been known [10], [11], [59], [67]–[70], but unfortunately, only to not so many of those who have known it. It is also well understood that no laws of physics are violated even when $|S_{11\,(Z_{\mathrm{ref}})}| > 1$, however uncomfortable you might feel with it. The theoretical maximum value of $|S_{11\,(Z_{\mathrm{ref}})}|$ is stupendous $1 + \sqrt{2} \simeq 2.41$ [59]! In reality, $|\Im(Z_0)|$ is usually a small fraction of $\Re(Z_0)\,(> 0)$, and the values of $|S_{11\,(Z_0)}|\,(> 1)$ we encounter in real life (measurements especially) will be fairly close to unity *except at very low frequencies* [67]. A $|S_{11\,(Z_{\mathrm{ref}})}|$ value significantly greater than unity might, therefore, be an artifact of unrealistic simulation, for example. But any discomfort associated with even the slightest deviation from $|S_{11\,(Z_{\mathrm{ref}})}| \leq 1$ might as well be an "artifact" of human mind, because no laws of physics demand that $|S_{11\,(Z_{\mathrm{ref}})}| \leq 1$.

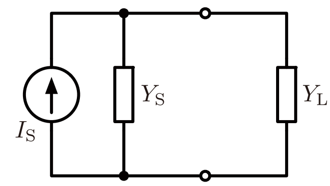

Fig. 11 A Norton signal source with an admittance Y_{S} feeds a load impedance Z_{L}.

2.7 Smith Chart and Reflection Coefficients

The result of §2.6 immediately leads to a disturbing conclusion: If Z_{ref} is complex, a locus of a passive load's $S_{11\,(Z_{\mathrm{ref}})}$ can stray out of the unit circle on a Z_{ref}-centered Smith chart [69]. Note also that $Z_{\mathrm{ref}} = Z_0$ is usually frequency-dependent, which might be another nuisance.

Given the fact that $|S_{\mathrm{P}11\,(Z_{\mathrm{ref}})}| \leq 1$ for passive loads, you might expect $S_{\mathrm{P}11\,(Z_{\mathrm{ref}})}$ helps here too. But it's not that simple. Recall that the Smith chart is derived from Eq. (5). Since Eq. (17) is different from Eq. (5), we cannot plot $S_{\mathrm{P}11\,(Z_{\mathrm{ref}})}$ on a Smith chart in the usual way we know. Let's consider an ideal "short" ($Z_{\mathrm{L}} = 0$). We all know that short's reflection coefficient is $S_{11} = -1$. This ($S_{11\,(Z_{\mathrm{ref}})} = -1$) is valid whether or not Z_{ref} is real in Eq. (5). But we get from Eq. (17) and $Z_{\mathrm{L}} = 0$

$$S_{\mathrm{P}11\,(Z_{\mathrm{ref}})} = -\frac{Z_{\mathrm{ref}}^{*}}{Z_{\mathrm{ref}}} \qquad \text{(short)}. \tag{34}$$

Evidently, $S_{\mathrm{P}11\,(Z_{\mathrm{ref}})} \neq -1$ unless Z_{ref} is real. This appears to go against microwave engineer's common sense. Now, how can we plot $S_{\mathrm{P}11\,(Z_{\mathrm{ref}})}$ on a Smith chart? Note that [54], [64]

$$S_{\mathrm{P}11\,(Z_{\mathrm{ref}})} = \frac{[R_{\mathrm{L}} + \mathrm{j}(X_{\mathrm{L}} + X_{\mathrm{ref}})] - R_{\mathrm{ref}}}{[R_{\mathrm{L}} + \mathrm{j}(X_{\mathrm{L}} + X_{\mathrm{ref}})] + R_{\mathrm{ref}}} = \frac{Z'_{\mathrm{L}} - R_{\mathrm{ref}}}{Z'_{\mathrm{L}} + R_{\mathrm{ref}}}, \tag{35}$$

$$Z_{\mathrm{ref}} = R_{\mathrm{ref}} + \mathrm{j}X_{\mathrm{ref}}, \tag{36}$$

$$Z'_{\mathrm{L}} \equiv R_{\mathrm{L}} + \mathrm{j}(X_{\mathrm{L}} + X_{\mathrm{ref}}). \tag{37}$$

Equation (35) has the same form as Eq. (5). By plotting Z'_{L} instead of Z_{L} on an R_{ref}-centered Smith chart, we can find the absolute value and the argument of $S_{\mathrm{P}11\,(Z_{\mathrm{ref}})}$. Note that R_{ref} will often be frequency-dependent.

But this is not the whole story. The above is valid if the signal source is a Thévenin equivalent as in Fig. 2. But if you start the theoretical development from a Norton-type signal source (Fig. 11), you can get a different conclusion! It can be shown that (see [48] and Appendix F of [65]) another possible and perfectly valid definition of the power-wave reflection coefficient is

$$S_{\mathrm{PV11}\,(Z_{\mathrm{ref}})} = \frac{Z_{\mathrm{ref}}}{Z^*_{\mathrm{ref}}} \cdot \frac{Z_{\mathrm{L}} - Z^*_{\mathrm{ref}}}{Z_{\mathrm{L}} + Z_{\mathrm{ref}}} = \frac{Z_{\mathrm{ref}}}{Z^*_{\mathrm{ref}}} S_{\mathrm{P11}\,(Z_{\mathrm{ref}})}. \tag{38}$$

The 'V' in the subscript indicates that this reflection coefficient is a voltage reflection coefficient. Although I didn't explain this, contrary to popular belief, $S_{\mathrm{P11}\,(Z_{\mathrm{ref}})}$ should be understood as a *current* reflection coefficient [48] (§2.8). Equation (17) reduces to Eq. (16) when Z_{ref} is real. Equation (38) follows from a somewhat different definition of power waves than Eqs. (19) and (20):

$$a'_{\mathrm{p1}} = \frac{1}{\sqrt{\Re(Y_{\mathrm{ref}})}} \frac{I_1 + Y_{\mathrm{ref}} V_1}{2}, \tag{39}$$

$$b'_{\mathrm{p1}} = \frac{1}{\sqrt{\Re(Y_{\mathrm{ref}})}} \frac{I_1 - Y^*_{\mathrm{ref}} V_1}{2}, \tag{40}$$

$$S_{\mathrm{PV11}\,(Y_{\mathrm{ref}})} \equiv \frac{b'_{\mathrm{p1}}}{a'_{\mathrm{p1}}}. \tag{41}$$

$Y_{\mathrm{ref}} = 1/Z_{\mathrm{ref}}$ is the reference admittance. From Eq. (38), short's reflection coefficient is $S_{\mathrm{PV11}} = -1$! This might appear more desirable than Eq. (34). However, the definition Eq. (41) is rarely adopted in practice.

In any case, why does such arbitrariness in the definition of power waves and associated $\mathrm{S_P}$ parameters arise? Thévenin and Norton signal sources are just two equivalent representations of the same physical source. Note that $|S_{\mathrm{P11}\,(Z_{\mathrm{ref}})}| = |S_{\mathrm{PV11}\,(Y_{\mathrm{ref}})}|$ and that the arbitrariness is in the phase. This is related to the remark made at the end of §2.4. Other definitions than Eqs. (21) and (41) are also possible. For example, yet another valid current-based definition of power-wave reflection coefficient is

$$S_{\mathrm{PI11}\,(Z_{\mathrm{ref}})} = -S_{\mathrm{P11}\,(Z_{\mathrm{ref}})} = -\frac{Z_{\mathrm{L}} - Z^*_{\mathrm{ref}}}{Z_{\mathrm{L}} + Z_{\mathrm{ref}}}. \tag{42}$$

Equation (42) reduces to Eq. (14) when Z_{ref} is real. It can further be shown that [51] the choice of phases of power waves is arbitrary. What does this physically mean?

2.8 Measurable Waves and S Parameters

Measurement is the act of sensing some physical quantities. As noted in §2.1, Eqs. (7) and (8) with $Z_{\mathrm{ref}} = Z_0$ are physical voltage traveling waves (multiplied by a real constant), and therefore these are measurable complex (phasor) quantities. Receivers in a calibrated VNA sense mathematically transformed version of these waves. Modern VNAs adopt analog-to-digital converters for voltage measurements [71].

What about the power waves, Eqs. (19) and (20) (or Eqs. (39) and (40))? Power, too, is a physical and measurable quantity, but it's a scalar quantity. Power measurements, therefore, reveal only $|a_{\mathrm{p1}}|$ and $|b_{\mathrm{p1}}|$. But $\mathrm{S_P}$ parameters given by Eq. (21) are complex quantities. How can we measure $\arg a_{\mathrm{p1}}$ and $\arg b_{\mathrm{p1}}$? Well, perhaps you can't [17], [46], [47]. If the arguments of power waves are not measurable quantities, that explains their arbitrariness (§2.7).

Pseudo waves, Eqs. (7) and (8), are fictitious in that they represent traveling waves that would be present in the transmission line if its characteristic impedance were Z_{ref} ($\neq Z_0$). Power waves are still more fictitious in that their phases are not measurable and can be dictated arbitrarily, usually following the convention started in the early days [56]–[58]. Is it possible that the phases of power waves will one day turn out physically significant somehow, just as the phase of the wave function† turned out to have observable significance in quantum mechanics [72]? I prefer to doubt that. The difference between Eqs. (17) and (38) arises only for a mathematical reason:

$$\Re\left(z^{-1}\right) \neq [\Re(z)]^{-1}, \tag{43}$$

where z is a complex number. For example, if

$$Z_{\mathrm{S}} = R_{\mathrm{S}} + \mathrm{j}X_{\mathrm{S}} = \frac{1}{Y_{\mathrm{S}}}, \tag{44}$$

then

$$Y_{\mathrm{S}} = \frac{1}{Z_{\mathrm{S}}} = \frac{R_{\mathrm{S}}}{R_{\mathrm{S}}^2 + X_{\mathrm{S}}^2} - \mathrm{j}\frac{X_{\mathrm{S}}}{R_{\mathrm{S}}^2 + X_{\mathrm{S}}^2}. \tag{45}$$

The available power of a signal source can be written as

$$P_{\mathrm{avs}} = \frac{1}{2}\Re(V_{\mathrm{i}} I^*_{\mathrm{i}}) \tag{46}$$

$$= \frac{1}{2}\Re(Z_{\mathrm{S}}) I_{\mathrm{i}} I^*_{\mathrm{i}} = \frac{1}{2} a_{\mathrm{p1}} a^*_{\mathrm{p1}} \quad \text{(Fig. 2)} \tag{47}$$

$$= \frac{1}{2}\Re(Y_{\mathrm{S}}) V_{\mathrm{i}} V^*_{\mathrm{i}} = \frac{1}{2} a'_{\mathrm{p1}} a'^*_{\mathrm{p1}} \quad \text{(Fig. 11)}, \tag{48}$$

where

$$I_{\mathrm{i}} \equiv \frac{E_{\mathrm{S}}}{2\Re(Z_{\mathrm{S}})} \quad \text{(incident current)}, \tag{49}$$

$$V_{\mathrm{i}} \equiv \frac{I_{\mathrm{S}}}{2\Re(Y_{\mathrm{S}})} \quad \text{(incident voltage)}, \tag{50}$$

$$a_{\mathrm{p1}} \equiv \sqrt{\Re(Z_{\mathrm{S}})} I_{\mathrm{i}} \quad \text{(Eq. (19))}, \tag{51}$$

$$a'_{\mathrm{p1}} \equiv \sqrt{\Re(Y_{\mathrm{S}})} V_{\mathrm{i}} \quad \text{(Eq. (39))}. \tag{52}$$

3. Scattering Matrices (S Matrices)

We have already learned significantly about 1-port S parameters (reflection coefficients). It is straightforward to extend them to multiports.

3.1 Definitions and Uses

Multiport extension of Eqs. (6) through (8) are

$$a_{i\,(Z_{\mathrm{ref}i})} = \mathrm{e}^{\mathrm{j}\phi_i} \frac{\sqrt{\Re(Z_{\mathrm{ref}i})}}{|Z_{\mathrm{ref}i}|} \frac{V_i + Z_{\mathrm{ref}i} I_i}{2} \quad \text{(port } i\text{)}, \tag{53}$$

†In quantum mechanics, probability is given by $|\psi|^2$, where ψ is the complex wave function. Obviously, $\arg\psi$ doesn't affect $|\psi|^2$, at least in most elementary problems.

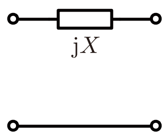

Fig. 12 Series reactance as a 2-port.

$$b_{j(Z_{\text{ref}j})} = e^{j\phi_j} \frac{\sqrt{\Re(Z_{\text{ref}j})}}{|Z_{\text{ref}j}|} \frac{V_j - Z_{\text{ref}j} I_j}{2} \quad \text{(port } j\text{)}, \tag{54}$$

$$S_{ji(Z_{\text{ref}i}, Z_{\text{ref}j})} = \frac{b_{j(Z_{\text{ref}j})}}{a_{i(Z_{\text{ref}i})}} \quad \text{(S parameter)}, \tag{55}$$

$$\mathsf{S}_{(\mathsf{Z}_{\text{ref}})} = \begin{bmatrix} & \cdots & & \\ \vdots & \ddots & S_{ij(Z_{\text{ref}i}, Z_{\text{ref}j})} & \\ & S_{ji(Z_{\text{ref}i}, Z_{\text{ref}j})} & \ddots & \vdots \\ & & \cdots & \end{bmatrix}. \tag{56}$$

$e^{j\phi_i}$ and $e^{j\phi_j}$ in Eqs. (53) and (54) are phase factors that may be needed depending on the mapping mentioned in §1 [4]. They can often be made to disappear in Eq. (55), which always does in Eq. (6). Z_{ref} in Eq. (56) is the reference impedance matrix:

$$\mathsf{Z}_{\text{ref}} = \text{diag}(Z_{\text{ref}1}, Z_{\text{ref}2}, \cdots). \tag{57}$$

The moduli of reflection coefficients of a passive network may exceed unity if Z_{ref} is complex (§2.6). Likewise, the moduli of transmission coefficients of a passive network may exceed unity if Z_{ref} is complex. For example, the S matrix of a series reactance (Fig. 12) is given by [12]

$$\mathsf{S}_{(Z_{\text{ref}})} = \frac{1}{jX + 2Z_{\text{ref}}} \begin{bmatrix} jX & 2Z_{\text{ref}} \\ 2Z_{\text{ref}} & jX \end{bmatrix}. \tag{58}$$

It follows that

$$\left|S_{21(Z_{\text{ref}})}\right|^2 = \frac{4|Z_{\text{ref}}|^2}{X^2 + 4X\Im(Z_{\text{ref}}) + 4|Z_{\text{ref}}|^2}. \tag{59}$$

If $X = 1$ and $Z_{\text{ref}} = e^{-j(\pi/4)}$ (Fig. 8),

$$\left|S_{21(Z_{\text{ref}})}\right|^2 = \frac{4}{1^2 - 4 \cdot \frac{1}{\sqrt{2}} + 4 \cdot |1|^2} \simeq 1.84 > 1. \tag{60}$$

This, of course, doesn't violate any laws of physics, because Eq. (60) isn't power gain. Recall that in Eq. (24), $|S_{11(Z_{\text{ref}})}|^2$ isn't a power reflection coefficient, either.

Similarly, Eqs. (19) through (21) can be extended to multiports.

$$a_{\text{p}i(Z_{\text{ref}i})} = \frac{1}{\sqrt{\Re(Z_{\text{ref}i})}} \frac{V_i + Z_{\text{ref}i} I_i}{2} \quad \text{(incident on port } i\text{)}, \tag{61}$$

$$b_{\text{p}j(Z_{\text{ref}j})} = \frac{1}{\sqrt{\Re(Z_{\text{ref}j})}} \frac{V_j - Z^*_{\text{ref}j} I_j}{2} \quad \text{(out of port } j\text{)}, \tag{62}$$

$$S_{\text{P}ji(Z_{\text{ref}i}, Z_{\text{ref}j})} = \frac{b_{\text{p}j(Z_{\text{ref}j})}}{a_{\text{p}i(Z_{\text{ref}i})}} \quad (\text{S}_{\text{P}} \text{ parameter}), \tag{63}$$

$$\mathsf{S}_{\text{P}(\mathsf{Z}_{\text{ref}})} = \begin{bmatrix} & \cdots & & \\ \vdots & \ddots & S_{\text{P}ij(Z_{\text{ref}i}, Z_{\text{ref}j})} & \\ & S_{\text{P}ji(Z_{\text{ref}i}, Z_{\text{ref}j})} & \ddots & \vdots \\ & & \cdots & \end{bmatrix}. \tag{64}$$

Equations (63) and (64) are also known as the generalized S parameter and the generalized S matrix, respectively [12], [14], [63].

For example, the S_P parameters of a series reactance (Fig. 12) with $Z_{\text{ref}1} = Z_\text{S}$ and $Z_{\text{ref}2} = Z_\text{L}$ are given by [14]

$$S_{\text{P}11(Z_\text{S}, Z_\text{L})} = \frac{jX + Z_\text{S} + Z_\text{L} - 2\Re(Z_\text{S})}{jX + Z_\text{S} + Z_\text{L}}, \tag{65}$$

$$S_{\text{P}21(Z_\text{S}, Z_\text{L})} = S_{\text{P}12(Z_\text{S}, Z_\text{L})} = \frac{2\sqrt{\Re(Z_\text{S})}\sqrt{\Re(Z_\text{L})}}{jX + Z_\text{S} + Z_\text{L}}, \tag{66}$$

$$S_{\text{P}22(Z_\text{S}, Z_\text{L})} = \frac{jX + Z_\text{S} + Z_\text{L} - 2\Re(Z_\text{L})}{jX + Z_\text{S} + Z_\text{L}}. \tag{67}$$

Since

$$\sqrt{\Re(Z_\text{S})\Re(Z_\text{L})} \le \frac{\Re(Z_\text{S}) + \Re(Z_\text{L})}{2}, \tag{68}$$

we can conclude from Eq. (66) that

$$\left|S_{\text{P}21(Z_\text{S}, Z_\text{L})}\right| \le 1 \tag{69}$$

as anticipated.

In majority of textbooks that cover complex-referenced S matrices, the S_P matrix is *the* complex-referenced S matrix. But that doesn't mean the pseudo-wave S matrices are unimportant. On the contrary, they are indispensable for microwave and millimeter-wave metrology, because some fundamental VNA calibration algorithms, such as thru-reflect-line (TRL) [4], [21], [73], are formulated using complex-referenced S matrices (not S_P matrices) [4], [74], [75]. Like it or not, you get complex-referenced S parameters (referenced to the natural reference impedance) from measurements, at least in some situations, and you have no choice but to work with them. When you do get them, you will most likely want to convert them to 50-Ω-referenced S parameters for further manipulation and saving into files, because results like Eq. (60) are, at best, confusing. Some simulators and file formats don't support $\mathsf{S}_{(\mathsf{Z}_{\text{ref}})}$. The mathematical operation of changing reference impedances is called the *renormalization transformation* [17], [47], [76], [77]. It can be done either by using a direct $\mathsf{S}_{(\mathsf{Z}_{\text{ref}})} \leftrightarrow \mathsf{S}'_{(\mathsf{Z}'_{\text{ref}})}$ conversion formula [47], [76], [77], or by cascading appropriate conversion networks [4].

On the other hand, the use of complex-referenced S_P matrices is not mandatory. They can be quite useful, for example, for amplifier design especially when lengths of interconnecting transmission lines (not including intentional stubs and delay lines) can be ignored. However, everything (including S_P parameters) can be expressed in terms of real-referenced S parameters [63], possibly derived from measured, complex-referenced pseudo-wave S parameters;

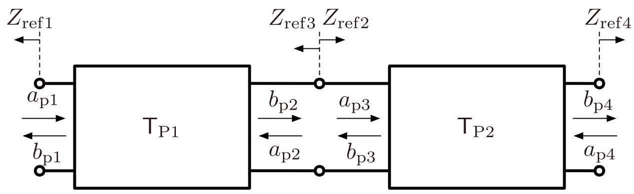

Fig. 13 Cascading is possible only if $b_{\mathrm{p2}} = a_{\mathrm{p3}}$ and $a_{\mathrm{p2}} = b_{\mathrm{p3}}$ are satisfied.

so you don't have to use S_{P} parameters if you don't want to. You use them if you find them useful and/or less disturbing because moduli of passive S_{P} parameters are guaranteed to be less than or equal to unity. It is advisable in this case too that you perform $\mathsf{S}_{\mathrm{P}\,(Z_{\mathrm{ref}})} \to \mathsf{S}'_{(50\,\Omega)}$ before saving your data. Make sure to use the right formula [54] (different from $\mathsf{S}_{(Z_{\mathrm{ref}})} \to \mathsf{S}'_{(50\,\Omega)}$) for the conversion.

In any case, you must be clear about which type of S parameter you are dealing with. Conversion formulas for $\mathsf{Z} \leftrightarrow \mathsf{S}_{(Z_{\mathrm{ref}})}$ and $\mathsf{Z} \leftrightarrow \mathsf{S}_{\mathrm{P}(Z_{\mathrm{ref}})}$, for example, are different. Note also that S_{P} parameters have further unusual properties.

3.2 Cascading

Let us introduce the power-wave cascading matrix T_{P} (Fig. 13).

$$\begin{bmatrix} b_{\mathrm{p1}} \\ a_{\mathrm{p1}} \end{bmatrix} = \mathsf{T}_{\mathrm{P}} \begin{bmatrix} a_{\mathrm{p2}} \\ b_{\mathrm{p2}} \end{bmatrix} = \begin{bmatrix} T_{\mathrm{P11}} & T_{\mathrm{P12}} \\ T_{\mathrm{P21}} & T_{\mathrm{P22}} \end{bmatrix} \begin{bmatrix} a_{\mathrm{p2}} \\ b_{\mathrm{p2}} \end{bmatrix}. \tag{70}$$

In terms of the elements of S_{P},

$$\mathsf{T}_{\mathrm{P}} = \frac{1}{S_{\mathrm{P21}}} \begin{bmatrix} S_{\mathrm{P12}} S_{\mathrm{P21}} - S_{\mathrm{P11}} S_{\mathrm{P22}} & S_{\mathrm{P11}} \\ -S_{\mathrm{P22}} & 1 \end{bmatrix}. \tag{71}$$

We want

$$\begin{bmatrix} b_{\mathrm{p1}} \\ a_{\mathrm{p1}} \end{bmatrix} = \mathsf{T}_{\mathrm{P1}} \begin{bmatrix} a_{\mathrm{p2}} \\ b_{\mathrm{p2}} \end{bmatrix} = \mathsf{T}_{\mathrm{P1}} \mathsf{T}_{\mathrm{P2}} \begin{bmatrix} a_{\mathrm{p4}} \\ b_{\mathrm{p4}} \end{bmatrix} \tag{72}$$

to be valid. This requires that $b_{\mathrm{p2}} = a_{\mathrm{p3}}$ and $a_{\mathrm{p2}} = b_{\mathrm{p3}}$ be satisfied at the interconnecting plane (Fig. 13). Since $V_2 = V_3$ and $I_2 = -I_3$ hold there, $Z_{\mathrm{ref2}} = Z^*_{\mathrm{ref3}}$ follows from Eqs. (61) and (62) as a requirement [46]. This is in stark contrast with the ordinary T matrices, for which $Z_{\mathrm{ref2}} = Z_{\mathrm{ref3}}$ is required. So be extra careful when doing cascading operations with S_{P} parameters or T_{P} matrices.

3.3 S Matrices of a Length of Transmission Line

The complex-referenced S matrix of a length, ℓ, of transmission line is

$$\mathsf{S}_{(Z_{\mathrm{ref}})} = \frac{1}{Z_0^2 + Z_{\mathrm{ref}}^2 + 2Z_0 Z_{\mathrm{ref}} \coth(\gamma\ell)} \times \begin{bmatrix} Z_0^2 - Z_{\mathrm{ref}}^2 & 2Z_0 Z_{\mathrm{ref}} / \sinh(\gamma\ell) \\ 2Z_0 Z_{\mathrm{ref}} / \sinh(\gamma\ell) & Z_0^2 - Z_{\mathrm{ref}}^2 \end{bmatrix}. \tag{73}$$

With $Z_{\mathrm{ref}} = Z_0$, Eq. (73) reduces to

$$\mathsf{S}_{(Z_0)} = \begin{bmatrix} 0 & \mathrm{e}^{-\gamma\ell} \\ \mathrm{e}^{-\gamma\ell} & 0 \end{bmatrix}, \tag{74}$$

regardless of the value of ℓ. This property is used in the formulation of TRL [4], [21], [73]. It also is the reason for the line's (generally unknown) Z_0 becoming the reference impedance ($Z_{\mathrm{ref}} = Z_0$) of the new reference planes after performing TRL calibration.

More generally, a pair of reference impedances Z_{i1} and Z_{i2} that makes $S_{11} = S_{22} = 0$ are known as the *image impedances* [78] of the 2-port. If

$$\mathsf{S}_{(Z_{\mathrm{ref1}}, Z_{\mathrm{ref2}})} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}, \tag{75}$$

then

$$\mathsf{S}_{(Z_{\mathrm{i1}}, Z_{\mathrm{i2}})} = \begin{bmatrix} 0 & \mathrm{e}^{-\theta_{\mathrm{i12}}} \\ \mathrm{e}^{-\theta_{\mathrm{i21}}} & 0 \end{bmatrix}, \tag{76}$$

where θ_{i21} and θ_{i12} are the image propagation parameters. Obviously, $Z_{\mathrm{i1}} = Z_{\mathrm{i2}} = Z_0$ and $\theta_{\mathrm{i21}} = \theta_{\mathrm{i12}} = \gamma\ell$ in the case of transmission lines.

The matrix elements of the complex-referenced S_{P} matrix of a length of transmission line are

$$S_{\mathrm{P11}\,(Z_{\mathrm{ref1}}, Z_{\mathrm{ref2}})} = \frac{(Z_0^2 - Z^*_{\mathrm{ref1}} Z_{\mathrm{ref2}}) \tanh\gamma\ell + Z_0 (Z_{\mathrm{ref2}} - Z^*_{\mathrm{ref1}})}{(Z_0^2 + Z_{\mathrm{ref1}} Z_{\mathrm{ref2}}) \tanh\gamma\ell + Z_0 (Z_{\mathrm{ref1}} + Z_{\mathrm{ref2}})}, \tag{77}$$

$$S_{\mathrm{P22}\,(Z_{\mathrm{ref1}}, Z_{\mathrm{ref2}})} = \frac{(Z_0^2 - Z_{\mathrm{ref1}} Z^*_{\mathrm{ref2}}) \tanh\gamma\ell + Z_0 (Z_{\mathrm{ref1}} - Z^*_{\mathrm{ref2}})}{(Z_0^2 + Z_{\mathrm{ref1}} Z_{\mathrm{ref2}}) \tanh\gamma\ell + Z_0 (Z_{\mathrm{ref1}} + Z_{\mathrm{ref2}})}, \tag{78}$$

$$S_{\mathrm{P21}\,(Z_{\mathrm{ref1}}, Z_{\mathrm{ref2}})} = S_{\mathrm{P12}\,(Z_{\mathrm{ref1}}, Z_{\mathrm{ref2}})} = \frac{2Z_0 \sqrt{\Re(Z_{\mathrm{ref1}}) \Re(Z_{\mathrm{ref2}})} / \cosh\gamma\ell}{(Z_0^2 + Z_{\mathrm{ref1}} Z_{\mathrm{ref2}}) \tanh\gamma\ell + Z_0 (Z_{\mathrm{ref1}} + Z_{\mathrm{ref2}})}. \tag{79}$$

Note that $Z_{\mathrm{ref1}} = Z_{\mathrm{ref2}} = Z_0$ doesn't make $S_{\mathrm{P11}} = S_{\mathrm{P22}} = 0$. In this sense, S_{P} matrices of lossy transmission lines are not terribly useful.

A pair of reference impedances that makes $S_{\mathrm{P11}} = S_{\mathrm{P22}} = 0$ is called the *conjugate image impedances* [55]. $S_{\mathrm{P11}} = S_{\mathrm{P22}} = 0$ means that simultaneous conjugate matching is achieved at the input and output ports. Since a transmission line is a symmetric 2-port, the conjugate image impedances are the same for both ports. Unlike the image impedance Z_0, the conjugate image impedance depends on ℓ. This implies that it will be difficult to formulate VNA calibration algorithms based on power waves and to measure S_{P} parameters directly.

3.4 Amplifier Gains

The use of $S_{\mathrm{P11}\,(Z_{\mathrm{ref}})}$, instead of $S_{11\,(Z_{\mathrm{ref}})}$, can be beneficial for reasons explained in §2.6. What about the use of 2-port S_{P} parameters? Suppose you are designing a multi-stage amplifier. Consider a single stage within it (Fig. 14). Its 50-Ω-referenced power gain is $|S_{21\,(50\,\Omega)}|^2$. What is its power gain

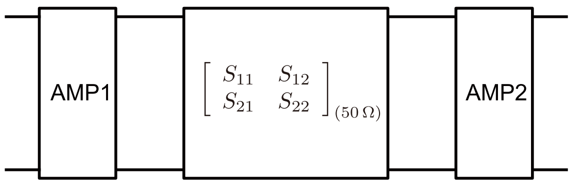

Fig. 14 A single amplifying stage within a multi-stage amplifier.

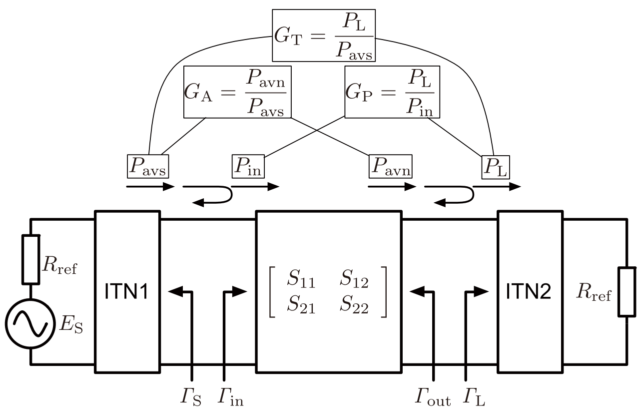

Fig. 15 Power gains of a 2-port. Γ_{in}: Input reflection coefficient. Γ_{out}: Output reflection coefficient. P_{avs}: Power available from the source. P_{in}: Power absorbed by the 2-port network. P_{avn}: Power available from the 2-port network. P_{in}: Power absorbed by the load. ITN: Impedance transforming network.

under the operating condition (mismatches with the preceding and following stages included)?

The transducer gain G_T is the gain that includes the mismatches with the "source" and the "load" (Fig. 15). G_T, therefore, depends both on the source reflection coefficient Γ_S and the load reflection coefficient Γ_L.

$$\Gamma_S = \frac{Z_S - 50}{Z_S + 50}, \tag{80}$$

$$\Gamma_L = \frac{Z_L - 50}{Z_L + 50}, \tag{81}$$

$$G_T(\Gamma_S, \Gamma_L) \equiv \frac{P_L}{P_{avs}} \tag{82}$$

$$= \frac{(1 - |\Gamma_L|^2)(1 - |\Gamma_S|^2)|S_{21}|^2}{|(1 - S_{22}\Gamma_L)(1 - S_{11}\Gamma_S) - S_{12}S_{21}\Gamma_S\Gamma_L|^2}. \tag{83}$$

While Eq. (83) already answers the question about the gain in terms of 50-Ω-referenced S parameters, it is worth mentioning that G_T can also be written concisely as follows:

$$G_T(\Gamma_S, \Gamma_L) = \left|S_{P21\,(Z_S,Z_L)}\right|^2. \tag{84}$$

Although Eqs. (83) and (84) are mathematically completely the same, the latter should be much easier to understand intuitively. When, for example, writing a program for design optimization, if a library of functions are available for manipulating S_P parameters, it is not only conceptually easier to use Eq. (84) but also less error-prone than writing Eq. (83) in the program.

Likewise, the available gain G_A, which depends only on Γ_S, and the operating gain (or power gain) G_P, which depends only on Γ_L, can be written concisely as S_{P21} with appropriate reference impedances (Fig. 15).

$$G_A(\Gamma_S) \equiv \frac{P_{avn}}{P_{avs}} = G_T(\Gamma_S, \Gamma_{out}^*) \tag{85}$$

$$= \frac{1 - |\Gamma_S|^2}{|1 - S_{11}\Gamma_S|^2} \cdot |S_{21}|^2 \cdot \frac{1}{1 - |\Gamma_{out}|^2} \tag{86}$$

$$= \frac{\left|S_{P21\,(Z_S,Z_L)}\right|^2}{1 - \left|S_{P22\,(Z_S,Z_L)}\right|^2} = \left|S_{P21\,(Z_S,Z_{out}^*)}\right|^2, \tag{87}$$

$$\Gamma_{out} = S_{22} + \frac{S_{21}\Gamma_S S_{12}}{1 - S_{22}\Gamma_S}, \tag{88}$$

$$G_P(\Gamma_L) \equiv \frac{P_L}{P_{in}} = G_T(\Gamma_{in}^*, \Gamma_L) \tag{89}$$

$$= \frac{1}{1 - |\Gamma_{in}|^2} \cdot |S_{21}|^2 \cdot \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2} \tag{90}$$

$$= \frac{\left|S_{P21\,(Z_S,Z_L)}\right|^2}{1 - \left|S_{P11\,(Z_S,Z_L)}\right|^2} = \left|S_{P21\,(Z_{in}^*,Z_L)}\right|^2, \tag{91}$$

$$\Gamma_{in} = S_{11} + \frac{S_{12}\Gamma_L S_{21}}{1 - S_{22}\Gamma_L}. \tag{92}$$

Equations (87) and (91) are easier to grasp than Eqs. (86) and (90). This conceptual gain is the power of using S_P parameters. Since G_A and G_P are the gains when conjugately matched on the load side and the source side, respectively, $G_T(\Gamma_S, \Gamma_L) \le G_A(\Gamma_S)$ and $G_T(\Gamma_S, \Gamma_L) \le G_P(\Gamma_L)$ hold.

When the 2-port in question is unconditionally stable, the maximum possible value of G_T equals the maximum available gain G_{MA}. Since G_{MA} is a property of a 2-port, it depends neither on Γ_S nor on Γ_L.

$$G_{MA} \equiv G_T(\Gamma_{ci1}, \Gamma_{ci2}) = \left|S_{P21\,(Z_{ci1},Z_{ci2})}\right|^2, \tag{93}$$

$$Z_{cij} = \frac{1 + \Gamma_{cij}}{1 - \Gamma_{cij}} \quad (j = 1, 2). \tag{94}$$

Z_{ci1} and Z_{ci2} are the conjugate image impedances [55] mentioned in §3.3. $\Gamma_{in} = \Gamma_{ci1}^*$ and $\Gamma_{out} = \Gamma_{ci2}^*$ hold.

4. Concluding Remarks

In this article we looked at two types of complex-referenced S parameters. Both show somewhat weird properties compared to the familiar 50-Ω-referenced S parameters and serve different purposes. Pseudo-wave S parameters (Eq. (55)) are indispensable for millimeter-wave metrology because your measurement reference planes might end up having complex reference impedances. In such a case, you have no choice but to deal with them. The most important thing to do about them is to convert them to real-referenced S parameters by renormalization transformation

$\mathsf{S}_{(Z_{\mathrm{ref}})} \to \mathsf{S}'_{(50\,\Omega)}$. Power-wave S parameters (Eq. (63)) can be useful for amplifier design and evaluation of antennas, but their use is not required because S_{P} parameters are derived quantities from measurable S parameters (§2.8). Either way, it is essential that you clarify which variant of complex-referenced S parameters you are using and what the values of your reference impedances are.

One very practically important point that I haven't been able to discuss is simulators and related programs. We need simulators written by somebody else because we can't write everything ourselves. But the confusion surrounding complex-referenced S parameters seen in the literature is, unfortunately and understandably, reflected in simulators and their documents, too. Very often, complex-referenced S parameters are poorly or not at all documented, even if they are implemented somehow. If you have a support contract with your simulator vendor, ask your support engineer. You are lucky if your support engineer doesn't share the said "misfortune." If you are not so lucky, you must somehow figure out which ones are implemented in your simulator. The outcome might not be as simple as "this simulator implements which variant of S parameters." Some inconsistency could possibly exist in integrated simulation environments. Some commercial microwave simulators implement $\mathsf{S}_{\mathrm{P}\,(Z_{\mathrm{ref}})}$ in their circuit simulation environment. But in EM simulation, more suitable one to use would be $\mathsf{S}_{(Z_{\mathrm{ref}})}$. How are different parts of an integrated environment interfaced with each other when reference impedances are complex?

What can we do with poorly documented simulators? We must at least be vigilant and be very clear about what we want to do using which kind of S parameter. Perhaps, we should also be putting more effort into application engineering, at least in the short term, till awareness of the importance of the issue grows in the field. It seems to me that the significance of application engineering in a situation like this is grossly underappreciated. See [79], an excellent application-engineering paper on a different but related subject.

Acknowledgments

I would like to thank Professor Kiyomichi Araki and Professor Atsushi Sanada for giving me the opportunity to present the earlier version of this article at MWE 2015 [44]. This work was supported in part by Fujikura Ltd. and JSPS.KAKENHI.

References

[1] H.J. Carlin, "The scattering matrix in network theory," IRE Trans. Circuit Theory, vol.3, no.2, pp.88–97, June 1956.
[2] G.A. Campbell and R.M. Foster, "Maximum output networks for telephone substation and repeater circuits," Trans. American Institute of Electrical Engineers, vol.39, no.1, pp.231–290, 1920.
[3] C.G. Montgomery, R.H. Dicke, and E.M. Purcell, editors, Principles of Microwave Circuits, Dover, 1965; Republication of McGraw-Hill, 1948.
[4] R.B. Marks and D.F. Williams, "A general waveguide circuit theory," J. Res. Natl. Inst. Stand. Technol., vol.97, no.5, pp.533–562, 1992.
[5] D.F. Williams, L.A. Hayden, and R.B. Marks, "A complete multimode equivalent-circuit theory for electrical design," J. Res. Natl. Inst. Stand. Technol., vol.102, no.4, pp.405–423, 1997.
[6] D.F. Williams and B.K. Alpert, "Causality and waveguide circuit theory," IEEE Trans. Microw. Theory Tech., vol.49, no.4, pp.615–623, April 2001.
[7] F. Olyslager, D. De Zutter, and A.T. de Hoop, "New reciprocal circuit model for lossy waveguide structures based on the orthogonality of the eigenmodes," IEEE Trans. Microw. Theory Tech., vol.42, no.12, pp.2261–2269, Dec. 1994.
[8] F. Olyslager, Electromagnetic Waveguides and Transmission Lines, Oxford University Press, 1999.
[9] W.C. Johnson, Transmission Lines and Networks, McGraw-Hill, 1950.
[10] N. Marcuvitz, editor, Waveguide Handbook, Dover, 1965; Republication of McGraw-Hill, 1951.
[11] R.W.P. King, Transmission-Line Theory, McGraw-Hill, 1955.
[12] R.E. Collin, Foundations for Microwave Engineering, 2nd ed., Wiley-Interscience, 2001.
[13] R. Collier, Transmission Lines: Equivalent Circuits, Electromagnetic Theory, and Photons, Cambridge University Press, 2013.
[14] R. Mavaddat, Network Scattering Parameters, World Scientific, 1996.
[15] T.H. Lee, Planar Microwave Engineering: A Practical Guide to Theory, Measurement, and Circuits, Cambridge University Press, 2004.
[16] E. Bogatin, Signal and Power Integrity—Simplified, 2nd ed., Prentice Hall, 2009.
[17] D. Woods, "Multiport-network analysis by matrix renormalisation employing voltage-wave *S*-parameters with complex normalisation," Proc. IEE, vol.124, no.3, pp.198–204, March 1977.
[18] L.N. Dworsky, Modern Transmission Line Theory and Applications, Wiley-Interscience, 1979.
[19] G. Miano and A. Maffucci, Transmission Lines and Lumped Circuits, Academic Press, 2001.
[20] J.C. Freeman, Fundamentals of Microwave Transmission Lines, Wiley-Interscience, 1996.
[21] G.F. Engen, Microwave Circuit Theory and Foundations of Microwave Metrology, Peter Peregrinus, 1992.
[22] M. Fujishima, S. Amakawa, K. Takano, K. Katayama, and T. Yoshida, "Tehrahertz CMOS design for low-power and high-speed wireless communication," IEICE Trans. Electron., vol.E98-C, no.12, pp.1091–1104, Dec. 2015.
[23] S. Hara, K. Katayama, K. Takano, I. Watanabe, N. Sekine, A. Kasamatsu, T. Yoshida, S. Amakawa, and M. Fujishima, "Compact 160-GHz amplifier with 15-dB peak gain and 41-GHz 3-dB bandwidth," Radio Freq. Integrated Circuits Symp., pp.7–10, May 2015.
[24] S. Hara, I. Watanabe, N. Sekine, A. Kasamatsu, K. Katayama, K. Takano, T. Yoshida, S. Amakawa, and M. Fujishima, "Compact 138-GHz amplifier with 18-dB peak gain and 27-GHz 3-dB bandwidth," IEEE Int. Symp. Radio-Freq. Integration Technol., pp.55–57, Aug. 2015.
[25] K. Takano, K. Katayama, S. Amakawa, T. Yoshida, and M. Fujishima, "Wireless digital data transmission from a 300-GHz CMOS transmitter," Electron. Lett., vol.52, no.15, pp.1353–1355, 2016.
[26] K. Katayama, K. Takano, S. Amakawa, S. Hara, A. Kasamatsu, K. Mizuno, K. Takahashi, T. Yoshida, and M. Fujishima, "A 300GHz 40nm CMOS transmitter with 32-QAM 17.5Gb/s/ch capability over 6 channels," Int. Solid-State Circuits Conf., pp.342–343, Feb. 2016.
[27] K. Takano, K. Katayama, T. Yoshida, S. Amakawa, and M. Fujishima, "124-GHz CMOS quadrature voltage-controlled oscillator with fundamental injection locking," Asian Solid-State Circuits Conf., pp.77–80, Nov. 2015.
[28] S. Mizukusa, K. Takano, K. Katayama, S. Amakawa, T. Yoshida, and M. Fujishima, "Analytical design of small-signal amplifier with maximum gain in conditionally stable region," Asia-Pacific Micro-

wave Conf., pp.774–776, Nov. 2014.
[29] S. Amakawa, "Theory of gain and stability of small-signal amplifiers with lossless reciprocal feedback," Asia-Pacific Microwave Conf., pp.1184–1186, Nov. 2014.
[30] S. Amakawa, A. Orii, K. Katayama, K. Takano, M. Motoyoshi, T. Yoshida, and M. Fujishima, "Design of well-behaved low-loss millimetre-wave CMOS transmission lines," IEEE Workshop on Signal and Power Integrity, pp.1–4, May 2014.
[31] K. Takano, S. Amakawa, K. Katayama, M. Motoyoshi, and M. Fujishima, "Modeling of short-millimeter-wave CMOS transmission line with lossy dielectrics with specific absorption spectrum," IEICE Trans. Electron., vol.E96-C, no.10, pp.1311–1318, Oct. 2013.
[32] K. Takano, S. Amakawa, K. Katayama, M. Motoyoshi, and M. Fujishima, "Characteristic impedance determination technique for CMOS on-wafer transmission line with large substrate loss," 79th Automatic RF Techniques Group (ARFTG) Conf., pp.1–4, June 2012.
[33] S. Amakawa, K. Katayama, K. Takano, T. Yoshida, and M. Fujishima, "Comparative analysis of on-chip transmission line de-embedding techniques," IEEE Int. Symp. Radio-Freq. Integration Technol., pp.91–93, Aug. 2015.
[34] K. Katayama, S. Amakawa, K. Takano, and M. Fujishima, "300-GHz MOSFET model extracted by an accurate cold-bias de-embedding technique," IEEE MTT-S Int. Microw. Symp., pp.1–4, May 2015.
[35] R. Goda, S. Amakawa, K. Katayama, K. Takano, T. Yoshida, and M. Fujishima, "Modeling of wideband decoupling power line for millimeter-wave CMOS circuits," IEEE Int. Symp. Radio-Freq. Integration Technol., pp.151–153, Aug. 2015.
[36] S. Amakawa, R. Goda, K. Katayama, K. Takano, T. Yoshida, and M. Fujishima, "Wideband CMOS decoupling power line for millimeter-wave applications," IEEE MTT-S Int. Microw. Symp., pp.1–4, May 2015.
[37] R. Goda, S. Amakawa, K. Katayama, K. Takano, T. Yoshida, and M. Fujishima, "Characterization of wideband decoupling power line with extremely low characteristic impedance for millimeter-wave CMOS circuits," Int. Conf. Microelectronic Test Struct., pp.220–223, March 2015.
[38] A. Orii, M. Suizu, S. Amakawa, K. Katayama, K. Takano, M. Motoyoshi, T. Yoshida, and M. Fujishima, "On the length of thru standard for TRL de-embedding on Si substrate above 110 GHz," Int. Conf. Microelectronic Test Struct., pp.81–86, March 2013.
[39] K. Takano, M. Motoyoshi, T. Yoshida, K. Katayama, S. Amakawa, and M. Fujishima, "Evaluation of CMOS differential transmission lines as two-port networks with on-chip baluns in millimeter-wave band," 83rd Automatic RF Techniques Group (ARFTG) Conf., pp.1–5, June 2014.
[40] K. Takano, K. Katayama, T. Yoshida, S. Amakawa, M. Fujishima, S. Hara, and A. Kasamatsu, "Calibration of process parameters for electromagnetic field analysis of CMOS devices up to 330 GHz," IEEE Int. Symp. Radio-Freq. Integration Technol., pp.94–96, Aug. 2015.
[41] K. Takano, K. Katayama, S. Mizukusa, S. Amakawa, T. Yoshida, and M. Fujishima, "Systematic calibration procedure of process parameters for electromagnetic field analysis of millimeter-wave CMOS devices," Int. Conf. Microelectronic Test Struct., pp.1–5, March 2015.
[42] S. Amakawa, A. Orii, K. Katayama, K. Takano, M. Motoyoshi, T. Yoshida, and M. Fujishima, "Process parameter calibration for millimeter-wave CMOS back-end device design with electromagnetic field analysis," Int. Conf. Microelectronic Test Struct., pp.182–187, March 2014.
[43] J.A. Dobrowolski, Microwave Network Design Using the Scattering Matrix, Artech House, 2010.
[44] S. Amakawa, "Demystifying S parameters: confusion surrounding S-parameter definitions," Microwave Workshops & Exhibition, FR6A-1, pp.1–6, Nov. 2015 (in Japanese).
[45] S. Amakawa, "Yet another introduction to S parameters," 27th Workshop on Silicon Analog & RF Electronics, Tutorial, Nov. 2011 (in Japanese).
[46] R.B. Marks and D.F. Williams, Comments on "Conversions between *S*, *Z*, *Y*, *H*, *ABCD*, and *T* parameters which are valid for complex source and load impedances," IEEE Trans. Microw. Theory Tech., vol.43, no.4, p.914, April 1995.
[47] H. Dropkin, Comments on "A rigorous technique for measuring the scattering matrix of a multiport device with a two-port network analyzer," and reply by J.C. Tippet and R.A. Speciale, IEEE Trans. Microw. Theory Tech., vol.31, no.1, pp.79–81, Jan. 1983.
[48] J.C. Freeman, "On the interpretation of scattering parameters," NASA Technical Documents, nasa_techdoc_20000056853, NASA, 1999.
[49] D. Williams, "Traveling waves and power waves," IEEE Microw. Mag., vol.14, no.7, pp.38–45, 2013.
[50] S. Llorente-Romano, A. Garca-Lampérez, T.K. Sarkar, and M. Salazar-Palma, "An exposition on the choice of the proper S parameters in characterizing devices including transmission lines with complex reference impedances and a general methodology for computing them," IEEE Antennas Propag. Mag., vol.55, no.4, pp.94–112, Aug. 2013.
[51] T. Hirano, "Review and another derivation of the power wave," Microw. Opt. Technol. Lett., vol.57, no.1, pp.26–28, Jan. 2015.
[52] H.J. Carlin and A.B. Giordano, Network Theory: An Introduction to Reciprocal and Nonreciprocal Circuits, Prentice-Hall, 1964.
[53] T.K. Ishii, Microwave Engineering, 2nd ed., Harcourt Brace Jovanovich, 1989.
[54] K. Kurokawa, "Power waves and the scattering matrix," IEEE Trans. Microw. Theory Tech., vol.13, no.2, pp.194–202, March 1965.
[55] S. Roberts, "Conjugate-image impedances," Proc. IRE, vol.34, no.4, pp.198P–204P, April 1946.
[56] P. Penfield, Jr., "Noise in negative-resistance amplifiers," IRE Trans. Circuit Theory, vol.7, no.2, pp.166–170, June 1960.
[57] D.C. Youla, "On scattering matrices normalized to complex port numbers," Proc. IRE, vol.49, no.7, p.1221, July 1961.
[58] K. Kurokawa, "Actual noise measure of linear amplifiers," Proc. IRE, vol.9, no.9, pp.1391–1397, Sept. 1961.
[59] R.B. Adler, L.J. Chu, and R.M. Fano, Electromagnetic Energy Transmission and Radiation, Wiley, 1960.
[60] V. Belevitch, Classical Network Theory, Holden-Day, 1968.
[61] N. Balabanian and T.A. Bickart, Electrical Network Theory, Wiley, 1969.
[62] K. Kurokawa, An Introduction to the Theory of Microwave Circuits, Academic Press, 1969.
[63] G. Gonzalez, Microwave Transistor Amplifiers: Analysis and Design, 2nd ed., Prentice Hall, 1996.
[64] M.W. Medley, Jr., Microwave and RF Circuits: Analysis, Synthesis and Design, Artech House, 1992.
[65] R.J. Weber, Introduction to Microwave Circuits: Radio Frequency and Design Applications, IEEE Press, 2001.
[66] C. Caloz and T. Itoh, Electromagnetic Metamaterials: Transmission Line Theory and Microwave Applications, Wiley-Interscience, 2006.
[67] R.A. Chipman, Theory and Problems of Transmission Lines, McGraw-Hill, 1968.
[68] R.J. Vernon and S.R. Seshadri, "Reflection coefficient and reflected power on a lossy transmission line," Proc. IEEE, vol.57, no.1, pp.101–102, Jan. 1969.
[69] J. Kretzschmar and D. Schoonaert, "Smith chart for lossy transmission lines," Proc. IEEE, vol.57, no.9, pp.1658–1660, Sept. 1969.
[70] S.C. Dutta Roy, "Some little-known facts about transmission lines and some new results," IEEE Trans. Education, vol.53, no.4, pp.556–561, Nov. 2010.
[71] J.P. Dunsmore, Handbook of Microwave Component Measurements with Advanced VNA Techniques, Wiley, 2012.
[72] J.J. Sakurai, Modern Quantum Mechanics, revised edition, Addison-

Wesley, 1994.
[73] G.F. Engen and C.A. Hoer, “ ‘Through-reflect-line’: An improved technique for calibrating the dual six-port automatic network analyzer,” IEEE Trans. Microw. Theory Tech., vol.27, no.12, pp.987–993, Dec. 1979.
[74] H.-J. Eul and B. Schiek, “A generalized theory and new calibration procedures for network analyzer self-calibration,” IEEE Trans. Microw. Theory Tech., vol.39, no.4, pp.724–731, April 1991.
[75] A. Rumiantsev and N. Ridler, “VNA calibration,” IEEE Microw. Mag., vol.9, no.3, pp.86–99, June 2008.
[76] J.C. Tippet and R.A. Speciale, “A rigorous technique for measuring the scattering matrix of a multiport device with a 2-port network analyzer,” IEEE Trans. Microw. Theory Tech., vol.30, no.5, pp.661–666, May 1982.
[77] E. Van Lil, Comments on “A rigorous technique for measuring the scattering matrix of a multiport device with a two-port network analyzer,” IEEE Trans. Microw. Theory Tech., vol.33, no.3, pp.286–287, March 1985.
[78] G. Matthaei, L. Young, and E.M.T. Jones, Microwave Filters, Impedance-Matching Networks, and Coupling Structures, Artech House, Norwood, Massachusetts, 1980; Republication of McGraw-Hill, 1964.
[79] E. Episkopou, S. Papantonis, W.J. Otter, and S. Lucyszyn, “Defining material parameters in commercial EM solvers for arbitrary metal-based THz structures,” IEEE Trans. Terahertz Sci. Technol., vol.2, no.5, pp.513–524, Sept. 2012.

Shuhei Amakawa received B.S., M.S., and Ph.D. degrees in engineering from the University of Tokyo in 1995, 1997, and 2001, respectively. He also received an MPhil degree in physics from the University of Cambridge. He was a research fellow at the Cavendish Laboratory, University of Cambridge from 2001 to 2004. After working for a couple of electronic design automation (EDA) companies, he joined the Integrated Research Institute, Tokyo Institute of Technology in 2006. Since 2010, he has been with the Graduate School of Advanced Sciences of Matter, Hiroshima University, where he is currently an associate professor. He serves as Associate Editor of *Electronics Letters* since 2015. His research interests include modeling and simulation of nanoelectronic devices and systems, design of RF circuits and interconnects, and microwave theory and measurement.