Machine Learning Worksheet 8

Gaussian Processes

1 Covariance

Problem 1: Given mean $\mu(x)$ and kernel $k(x, x')$ (symmetric definite positive), we have a Gaussian Process $f(x)$. There are two real values $a_1 \neq a_2$. Another Gaussian Process $t(x)$ with mean 0 and kernel of $p(t(a_1), t(a_2))$ is an identity matrix. Let L be a lower triangular matrix such that $K = LL^T$. we define $s(x) = \mu(x) + Lt(x)$. What is the shape of the distribution $p(s(a_1))$? What are the mean for $p(s(a_1))$, $p(s(a_2))$, and $p(s(a_1), s(a_2))$? What is the covariance of $p(s(a_1), s(a_2))$?

2 Regression

Problem 2: Given a training data set with input $\boldsymbol{x}$ and output $\boldsymbol{y}$:

$$\boldsymbol{x} = (-0.8372, -0.4558, 0.6902, 0.1114, -0.4678) \quad \boldsymbol{y} = (-1.1414, -1.5286, -1.1893, -1.9021, -1.5594)$$

Suppose it is zero mean and we have kernel:

$$k(x, x') = \sigma_f^2 \exp(-\frac{1}{2l^2}(x - x')^2) + \sigma_n^2 \delta_{pq}$$

where $\sigma_f = 1$ and $\sigma_n = 0.5$

(a) Given test data set with input $\boldsymbol{x}_*$:

$$\boldsymbol{x}_* = (-0.5, 0.5)$$

compute the mean value $\overline{\boldsymbol{y}_*}$ of the output of the testing data set with $l = 1$

(b) plot $\boldsymbol{y}_*$ with $\boldsymbol{x}_*$ in the range of $[-1, 1]$ using different hyperparameter l, and show the differences