
homework sheet 03

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1 Assignment: Basic Probability

Problem 1

$T \triangleq$ terrorist and $\neg T \triangleq$ not a terrorist

$S \triangleq$ positive scan as terrorist and $\neg S \triangleq$ negative scan as terrorist

The following information about probabilities are included in the text:

$$p(T) = 0.01, p(\neg T) = 0.99$$

$$p(S|T) = 0.95, p(\neg S|T) = 0.05$$

$$p(\neg S|\neg T) = 0.95, p(S|\neg T) = 0.05$$

To find: $p(T|S)$

$$p(T|S) = \frac{p(S|T)p(T)}{p(S)} \quad (1)$$

$$= \frac{p(S|T)p(T)}{p(S|T)p(T) + p(S|\neg T)p(\neg T)} \quad (2)$$

$$= \frac{0.95 \cdot 0.01}{0.95 \cdot 0.01 + 0.05 \cdot 0.99} \quad (3)$$

$$\approx 0.161 \quad (4)$$

Problem 2

$B \triangleq$ Box and $D \triangleq$ Drawn

$r \triangleq$ red and $w \triangleq$ white

To find: $p(B = 2red|D = 3red)$

$$p(B = 2r|D = 3r) = \frac{p(D = 3r|B = 2r)p(B = 2r)}{p(D = 3r)} \quad (5)$$

$$= \frac{p(D = 3r|B = 2r)p(B = 2r)}{p(D = 3r|B = 2r)p(B = 2r) + p(D = 3r|B = 2w)p(B = 2w) + p(D = 3r|B = 1r1w)p(B = 1r1w)} \quad (6)$$

$$= \frac{1 \cdot (0.5 \cdot 0.5)}{1 \cdot (0.5 \cdot 0.5) + 0 \cdot (0.5 \cdot 0.5) + (0.5 \cdot 0.5 \cdot 0.5) \cdot (2 \cdot 0.5 \cdot 0.5)} \quad (7)$$

$$= \frac{0.25}{0.25 + 0.125 \cdot 0.5} \quad (8)$$

$$= \frac{0.25}{0.3125} \quad (9)$$

$$= 0.8 \quad (10)$$

Problem 3

Mean:

$$E[X] = \int_{-\infty}^{\infty} x \cdot p(x) dx \quad (11)$$

$$= \int_0^1 x \cdot 1 \quad (12)$$

$$= \left[\frac{1}{2} x^2 \right]_0^1 \quad (13)$$

$$= \frac{1}{2} - 0 \quad (14)$$

$$= \frac{1}{2} \quad (15)$$

Variance:

$$Var[X] = E[X^2] - E[X]^2 \quad (16)$$

$$= \int_{-\infty}^{\infty} x^2 \cdot p(x) dx - \left(\frac{1}{2} \right)^2 \quad (17)$$

$$= \left[\frac{1}{3} x^3 \right]_0^1 - \frac{1}{4} \quad (18)$$

$$= \frac{1}{3} - 0 - \frac{1}{4} \quad (19)$$

$$= \frac{1}{12} \quad (20)$$

Problem 4

2 Assignment: Probability Inequalities

2.1 Markov Inequality

Problem 5

2.2 Chebyshev Inequality

Problem 6

2.3 Jensen's Inequality

Problem 7

3 Assignment: a shorter solution to Fermat's last theorem

Assignment 3 is solved, but takes too much space to write down here.