η = efficiency ρ = ~~corr~~ pearson correlation coefficient

kladpapier/rough-work paper

$$\overline{a'b'} = \frac{1}{\eta_{ab}}\left[\sigma (a_s-\bar{a})(b_s-\bar{b}) + (1-\sigma)(a_e-\bar{a})(b_e-\bar{b})\right]$$

$$\bar{a} = \sigma a_s + (1-\sigma) a_e$$

$$\Rightarrow \overline{a'b'} = \frac{1}{\eta_{ab}}\left[\sigma(a_s-\bar{a})(b_s-\bar{b}) + \frac{\sigma^2}{(1-\sigma)}(a_s-\bar{a})(b_s-\bar{b})\right]$$

$$= \frac{1}{\eta_{ab}}\left[\frac{\sigma}{1-\sigma}\right]\left[(a_s-\bar{a})(b_s-\bar{b})\right]$$

$$\overline{a'a'} = \frac{1}{\eta_{aa}}\left[\frac{\sigma}{1-\sigma}\right]\left[(a_s-\bar{a})(a_s-\bar{a})\right] = \frac{1}{\eta_{aa}}\left(\frac{\sigma}{1-\sigma}\right)(a_s-\bar{a})^2$$

$$\overline{b'b'} = \frac{1}{\eta_{bb}}\left[\frac{\sigma}{1-\sigma}\right]\left[(b_s-\bar{b})(b_s-\bar{b})\right] = \frac{1}{\eta_{bb}}\left(\frac{\sigma}{1-\sigma}\right)(b_s-\bar{b})^2$$

Also:

$$\overline{a'b'} = \rho_{ab}\sqrt{\overline{a'a'}}\sqrt{\overline{b'b'}}$$

Hence:

(sign check!)

$$\boxed{\frac{1}{\eta_{ab}} = \rho_{ab}\frac{1}{\sqrt{\eta_{aa}}}\frac{1}{\sqrt{\eta_{bb}}}}$$

simple but powerful!

Maybe good   both poor.

Peters
Wyngaard
Moeng