Sheet 3 - Raytracing

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November 3, 2019

1 Practical part

Please find the solution in the accompanying .py file.

2 The rendering complexity

2.1 a)

$640 \cdot 480 \cdot 4 \cdot 0.0001s = 122.88s$

2.2 b)

$640 \cdot 480 \cdot (100^3 + 1) \cdot 0.0001s \approx 30720030s \approx 355.5days \approx 1year$

2.3 c)

$640 \cdot 480 \cdot (100^{(n-1)} + 1) \cdot 0.0001s$

2.4 d)

- Using a proper acceleration structure will allow us to reject most reflection rays early thus drastically reducing the average time for an intersection test.
- Instead of choosing a random direction for our rays, we can sample the reflection in a directional cone around the perfect perfection direction. This will reduce total number of rays needed to get a similar result.

3 Plane reflection

3.1 a)

Law of reflection is given in the lecture:

$$\mathbf{R} = \mathbf{d} - 2\left(\mathbf{N} \cdot \mathbf{d}\right)\mathbf{N}$$

In our example $\mathbf{d} = \mathbf{L} - \mathbf{C} = \begin{pmatrix} x \\ -8 \end{pmatrix}$, $\mathbf{N} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ thus:

$$\mathbf{R} = \begin{pmatrix} x \\ -8 \end{pmatrix} + 2\left(\begin{pmatrix} 0 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ -8 \end{pmatrix}\right) \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ 8 \end{pmatrix}$$

By solving

$$\begin{aligned} & \mathbf{R} &&= \mathbf{P} - \mathbf{L} \\ \iff & \begin{pmatrix} x \\ 8 \end{pmatrix} \cdot k &&= \begin{pmatrix} 375 - x \\ 192 \end{pmatrix} \\ \iff & \begin{pmatrix} 24 \cdot x \\ 192 \end{pmatrix} &&= \begin{pmatrix} 375 - x \\ 192 \end{pmatrix} \\ \iff & x &&= 15 \end{aligned}$$

we know that the reflection of the plan will be seen at $\mathbf{L} = \begin{pmatrix} 15 \\ 2 \end{pmatrix}$.

3.2 b)

Using Snell's law as given in the lecture, we can derive

$$\begin{aligned} & \tfrac{\sin\theta_i}{\sin\theta_t} &&= \eta \\ \iff & \tfrac{\sqrt{1-\cos^2\theta_i}}{\sqrt{1-\cos^2\theta_t}} &&= \eta \\ \iff & \tfrac{\sqrt{1-\left(\frac{\mathbf{d}\cdot\mathbf{N}}{\|\mathbf{d}\|\cdot\|\mathbf{N}\|}\right)^2}}{\sqrt{1-\left(\frac{\mathbf{T}\cdot\mathbf{N}}{\|\mathbf{T}\|\cdot\|\mathbf{N}\|}\right)^2}} &&= \eta \end{aligned}$$

In our case we have $\mathbf{d} = \mathbf{L} - \mathbf{C} = \begin{pmatrix} 10 \\ -8 \end{pmatrix}$ and $\mathbf{T} = \mathbf{B} - \mathbf{L} = \begin{pmatrix} x - 10 \\ -10 \end{pmatrix}$. Then we get

$$\left(\frac{\mathbf{d}\cdot\mathbf{N}}{\|\mathbf{d}\|\cdot\|\mathbf{N}\|}\right)^2 = \left(\frac{\begin{pmatrix} 10 \\ -8 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix}}{\sqrt{10^2 + 8^2}}\right)^2 = \left(\frac{-8}{\sqrt{10^2+8^2}}\right)^2 = \frac{8^2}{10^2+8^2} = \frac{8^2}{10^2} + 1$$

$$\left(\frac{\mathbf{T}\cdot\mathbf{N}}{\|\mathbf{T}\|\cdot\|\mathbf{N}\|}\right)^2 = \left(\frac{\begin{pmatrix} x-10 \\ -10 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix}}{\sqrt{(x-10)^2 + 10^2}}\right)^2 = \left(\frac{-10}{\sqrt{(x-10)^2+10^2}}\right)^2 = \frac{10^2}{(x-10)^2+10^2} = \frac{10^2}{(x-10)^2} + 1$$

and when solving for x this amounts to

$$
\begin{aligned}
&& \frac{\sqrt{-\frac{8^2}{10^2}}}{\sqrt{-\frac{10^2}{(x-10)^2}}} &= \eta \\
\iff && \frac{\frac{8}{10}}{\frac{10}{x-10}} &= \eta \\
\iff && \frac{2(x-10)}{25} &= \eta \\
\iff && x &= \frac{25\eta}{2} + 10
\end{aligned}
$$

Together with $\eta = \frac{n_2}{n_1} = \frac{1.33}{1.000277}$ we can conclude that the boat has to be at

$$
\mathbf{B} = \begin{pmatrix} \frac{25\eta}{2} + 10 \\ -8 \end{pmatrix} \approx \begin{pmatrix} 26.62 \\ -8 \end{pmatrix}.
$$

3.3 c)

Given the equation for x in b), we can easily calculate the positions for other refractive indices:

$$
\begin{aligned}
\text{Ice: } \eta = \frac{1.31}{1.000277} &\implies \mathbf{B} \approx \begin{pmatrix} 26.37 \\ -8 \end{pmatrix} \\
\text{Diamond: } \eta = \frac{2.417}{1.000277} &\implies \mathbf{B} \approx \begin{pmatrix} 40.2 \\ -8 \end{pmatrix}
\end{aligned}
$$