d) 4 Euler steps with $\frac{\pi}{4}$ $\quad x(t_0) = \begin{matrix}1\\0\end{matrix}$ $\quad a = \begin{matrix}\cos t\\ \sin t\end{matrix}$

$$\vec{v}_{t+1} = \vec{v}_t + \Delta t \vec{a} \qquad \vec{x}_{t+1} = \vec{x}_t + \Delta t \cdot \vec{v}_{t+1}$$

$$\vec{x}_{t+1} = \vec{x}_t + \Delta t \cdot \vec{v}_t + \Delta t^2 \cdot \vec{a}_t$$

$t = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi \qquad \Delta t = \frac{\pi}{4}$

$$\vec{v}_1 = \begin{pmatrix}0\\0\end{pmatrix} + \frac{\pi}{4} \cdot \begin{pmatrix}\cos \pi/4\\ \sin \pi/4\end{pmatrix}$$

$$\vec{x}_1 = \begin{pmatrix}0\\0\end{pmatrix} + \frac{\pi}{4} \cdot \frac{\pi}{4}\begin{pmatrix}\cos \pi/4\\ \sin \pi/4\end{pmatrix} + \frac{\pi^2}{16} \cdot \begin{pmatrix}\cos \pi/4\\ \sin \pi/4\end{pmatrix}$$

$$= \frac{\pi^2}{16} \cdot \begin{pmatrix}\sqrt{2}\\ \sqrt{2}\end{pmatrix}$$

$$\vec{v}_2 = \pi/4 \cdot \begin{pmatrix}\cos \pi/4\\ \sin \pi/4\end{pmatrix} + \frac{\pi}{4} \cdot \begin{pmatrix}\cos \pi/2\\ \sin \pi/2\end{pmatrix}$$

$$= \frac{\pi}{4} \cdot \begin{pmatrix}\cos \pi/4\\ \sin(\pi/4) + 1\end{pmatrix}$$

$$\vec{x}_2 = \frac{\pi^2}{16} \cdot \begin{pmatrix}\sqrt{2}\\ \sqrt{2}\end{pmatrix} + \frac{\pi}{4} \cdot \frac{\pi}{4} \cdot \begin{pmatrix}\cos(\pi/4)\\ 1 + \sin \pi/4\end{pmatrix}$$

$$= \frac{\pi^2}{16} \cdot \begin{pmatrix}\sqrt{2} + \cos \pi/4\\ \sqrt{2} + 1 + \sin \pi/4\end{pmatrix}$$

$$\vec{v}_3 = \frac{\pi}{4}\begin{pmatrix}\cos \pi/4\\ 1 + \sin \pi/4\end{pmatrix} + \frac{\pi}{4} \cdot \begin{pmatrix}\cos 3/4\pi\\ \sin 3/4\pi\end{pmatrix}$$

$$= \frac{\pi}{4}\begin{pmatrix}0\\ 1 + \sqrt{2}\end{pmatrix}$$

$$\vec{x}_3 = \frac{\pi^2}{16} \cdot \begin{pmatrix}\sqrt{2} + \cos \pi/4\\ \sqrt{2} + 1 + \sin \pi/4\end{pmatrix} + \frac{\pi}{4} \cdot \frac{\pi}{4}\begin{pmatrix}0\\ 1 + \sqrt{2}\end{pmatrix}$$

$$= \frac{\pi^2}{16} \cdot \begin{pmatrix}\sqrt{2} + \cos \pi/4\\ 2\sqrt{2} + 2 + \sin \pi/4\end{pmatrix}$$

$$\vec{v}_4 = \frac{\pi}{4}\begin{pmatrix} 0 \\ 1+\sqrt{2} \end{pmatrix} + \frac{\pi}{4}\cdot\begin{pmatrix} -1 \\ 0 \end{pmatrix} \quad \leftarrow \text{eventuell minus einen } \Delta t \text{ davon nehmen ?}$$

$$= \frac{\pi}{4}\begin{pmatrix} -1 \\ 1+\sqrt{2} \end{pmatrix}$$

$$\vec{x}_4 = \frac{\pi^2}{16}\cdot\begin{pmatrix} \sqrt{2} + \cos \pi/4 \\ 2\sqrt{2}+2+\sin \pi/4 \end{pmatrix} + \frac{\pi}{4}\cdot\frac{\pi}{4}\begin{pmatrix} -1 \\ 1+\sqrt{2} \end{pmatrix}$$

$$= \frac{\pi^2}{16}\begin{pmatrix} \sqrt{2}-1+\cos \pi/4 \\ 3\sqrt{2}+3+\sin \pi/4 \end{pmatrix}$$