$$f(x) = f^*(x) = \operatorname{sgn}(\cos(x)) = \begin{cases} 1 & , 0 < x \le \pi/2 \\ -1 & , \pi/2 < x \le 3/2\,\pi \\ 1 & , 3/2\,\pi < x \le 2\pi \end{cases}$$

f is real

$$g(x) = \begin{cases} 2/\pi\, x & , 0 < x \le \pi/2 & \text{I} \\ -2/\pi\, x + 2 & , \frac{\pi}{2} < x \le \frac{3}{2}\pi & \text{II} \\ 2/\pi\, x - 4 & , 3/2\,\pi < x \le 2\pi & \text{III} \end{cases}$$

$$\int g(x)\, f(x)\, dx = g(x) \cdot F(x) - \int g'(x) \cdot F(x)\, dx$$

Integrate each region separatly:

I: $\forall x : 0 < x \le \pi/2$

$$= 2/\pi\, x \cdot (x) - \int 2/\pi \cdot x \, dx = 2/\pi\, x^2 - \frac{x^2}{\pi} = \frac{x^2}{\pi}(2-1) = \frac{x^2}{\pi}$$

II: $\forall x : \frac{\pi}{2} < x \le \frac{3}{2}\pi$

$$= (-\tfrac{2}{\pi}x + 2)(-x) - \int (-2/\pi)(-x)\, dx$$

$$= 2/\pi\, x^2 - 2x - \frac{x^2}{\pi} = \frac{x^2}{\pi}(2-1) - 2x = \frac{x^2}{\pi} - 2x$$

III: $\forall x : 3/2\,\pi < x \le 2\pi$

$$= (\tfrac{2}{\pi}x - 4)(x) - \int (2/\pi)(x)\, dx = \tfrac{2}{\pi}x^2 - 4x - \frac{x^2}{\pi}$$

$$= \frac{x^2}{\pi} - 4x$$

Plug in the integral boundaries

I: $\frac{1}{\pi}\left(\frac{\pi}{2}\right)^2 = \pi/4$

II: $\frac{1}{\pi}\left(\frac{3}{2}\pi\right) - 2\left(\frac{3}{2}\pi\right) - \left(\frac{1}{\pi}(\pi/2)^2 - 2(\pi/2)\right)$

$$= 9/4\,\pi - 3\pi - \pi/4 + \pi = 0$$

III: $\frac{1}{\pi}(2\pi)^2 - 4(2\pi) - \left(\frac{1}{\pi}\left(\frac{3}{2}\pi\right)^2 - 4(3/2\,\pi)\right)$

$$= 4\pi - 8\pi - 9/4\,\pi + 6\pi = -1/4\,\pi$$

$\Rightarrow$ I + II + III $= 0$ The integral is 0 over $[0, 2\pi]$

Assignment 2)

$$\int_0^1 |f(t)|^2 \, dt = \sum_{n=-\infty}^{\infty} |\hat{f}(n)|^2$$

$$\int_0^1 |f(t)|^2 \, dt = \int_0^1 \overline{f(t)} \, f(t) \, dt$$

with: $f(t) = \sum_{n=-\infty}^{\infty} \hat{f}(n) \cdot e^{2\pi i n t}$

$$= \int_0^1 \sum_{n=-\infty}^{\infty} \overline{\hat{f}(n)} \cdot e^{-2\pi i n t} \quad f(t) \quad dt$$

$$= \sum_{n=-\infty}^{\infty} \left(\overline{\hat{f}(n)} \cdot e^{-2\pi i n t} \right) \cdot \sum_{n=-\infty}^{\infty} \left(\hat{f}(n) \, e^{2\pi i n t} \right)$$

$$= \sum_{n=-\infty}^{\infty} \left(\overline{\hat{f}(n)} \; \hat{f}(n) \cdot 1 \right) = \sum_{n=-\infty}^{\infty} |\hat{f}(n)|$$

$\square$