Foundations of Graphics

Universität Bonn
Institut für Informatik II
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Sheet 5 - Model Fitting

Please upload your solutions to
`https://uni-bonn.sciebo.de/s/XfRkWhpWDEeR8Y7`
by **Thu, Nov 21, 2019, 23:59 CET**.
Make sure to **list all group members** on all pages / source files.
Theoretical solutions must be texed or scanned, **photos will not be accepted**.

Theoretical Part

Assignment 1) Fitting Circles *(1+1+2=4Pts)*

a) What is the minimum number of points to fit 1) a line and 2) a circle? Which constraints have to be fulfilled by the points? Explain your answers!

b) Write down the total least squares minimization problem formula to fit a circle (just $E = \ldots$ without the derivatives, compare fog-04 slide 7).

c) Consider a total least squares minimization of the form

$$\min_{\mathbf{p}} \mathbf{p}^T \mathbf{A}^T \mathbf{A} \mathbf{p} \quad \text{s.t. } \|\mathbf{p}\| = 1.$$

Can this be formulated without the constraint $\|\mathbf{p}\| = 1$? Explain your answer!

Practical Part

Assignment 2) Detecting Circles Using RANSAC *(1+2+1+2=6Pts)*

Finding certain shapes in images is important for several applications such as traffic sign detection in driving assistance systems. Here we want to locate coins in an image. Fill the missing parts in the given framework with the RANSAC steps:

a) In each RANSAC iteration, select s random edge points to fit a circle. s should be chosen to be the smallest required to completely define a circle.

b) Determine all inliers for the circle fit by the above samples. A edge point is an inlier when its distance to the circle is below a `distance_threshold`.

c) Compute the number N of iterations needed to select s points that belong to the same circle with a certain certainty at least once. You can find the formula in the slides.

d) Remove the edgepoints from the previous RANSAC run and re-run RANSAC to find the remaining circles.

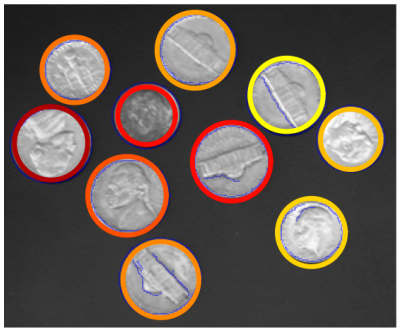

Figure 1: Examplary image with round object and the circles detected with Circular Hough Transform.

Assignment 3) Detecting Circles Using the Hough Transform *(4Pts)*

Now we want to detect the coins using the Hough transform. For taking into account that the circles might have different radii, we need to search over a whole range of possible radii (in comparison to RANSAC which does not need prior information about the circle radius).

Since the parameter space of the Circle Hough Transform is three-dimensional (a circle is defined by its center coordinates and radius, $(x,\ y,\ r) \in \mathbb{R}^3$), we also have a 3D array for the accumulation. To inspect the Hough space, we display some 2D-slices along the x- and y-axes of the three-dimensional array `acc`.

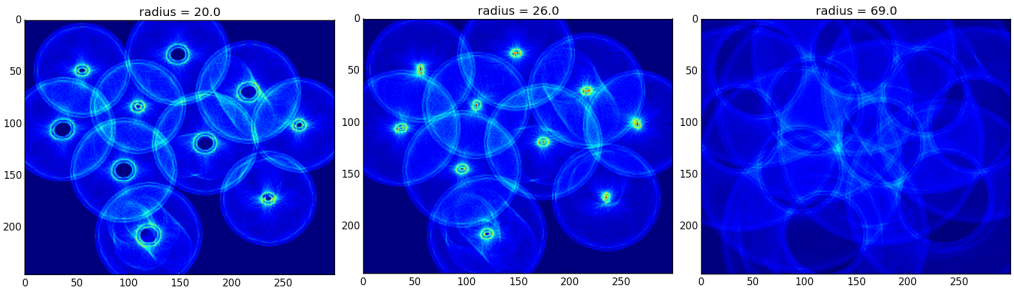

Figure 2: Examplary 2D slices of the 3D Hough volume for different radii.

Implement the function `hough_circles` which computes the Hough transformation for circles. **Hint:** You can compare it to the reference implementation to check if your solution is correct!

Good luck!