Foundations of Graphics

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Sheet 1 - Transformations

Please upload your solutions to
`https://uni-bonn.sciebo.de/s/XfRkWhpWDEeR8Y7`
by
Sun, 20.10.2019.
Make sure to **list all group members** on all pages / source files.
Theoretical solutions must be texed or scanned, **photos will not be accepted**.

Practical Part

Assignment 1) Transformations *(1+1+2=4Pts)*

a) Download the framework and run `Transformations.py`. It will display a monkey head. Set the limits of all three axes to $[-2, ..., 2]$ so that the mesh is properly centered.
b) Scale up the model so that it appears 2 times larger and translate the mesh by $t = (1, 0, 0)^T$ and display the result.
c) Rotate the model from task a) by $\pi/2$ around the z-axis by using matrix/vector multiplication. In addition rotate the model from task b) by $\pi/4$ around an axis parallel to the z-axis passing the point $p = (1, 0, 0)^T$ and display the results of both rotations in a single coordinate frame.

Theoretical Part

Assignment 2) Cross and Dot Product *(1+1+2+1+1=6Pts)*

Let $\mathbf{a}, \mathbf{b}, \mathbf{c} \in \mathbb{R}^3$.

a) Show that $\mathbf{a} \times (\mathbf{b} + \mathbf{c}) = \mathbf{a} \times \mathbf{b} + \mathbf{a} \times \mathbf{c}$.
b) Show that $\mathbf{a} \cdot (\mathbf{a} \times \mathbf{b}) = 0$.
c) A vector $\mathbf{a}$ is defined by $a = a_1 e_x + a_2 e_y + a_3 e_z$ and $\mathbf{b} = b_1 e_x + b_2 e_y + b_3 e_z$. Show that

$$|\mathbf{a} \times \mathbf{b}| = \left| \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \cdot e_x - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \cdot e_y + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \cdot e_z \right|$$

Explain why the following definition is correct for the formal case.

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} e_x & e_y & e_z \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

d) Show that $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = \mathbf{b} \cdot (\mathbf{c} \times \mathbf{a}) = \mathbf{c} \cdot (\mathbf{a} \times \mathbf{b})$. (It is sufficient to show only one of the equations).
e) Show that $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$ represents the volume V of the parallelepiped (Fig. 1) defined by the three vector $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$.

Good luck!

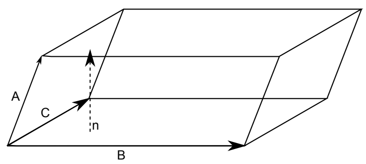

Figure 1: Parallelepiped with Volume V