Sheet 9 - Parametric Curves

Jan Scheffczyk - 3242317
Leif Van Holland - 2563657
Oliver Leuschner - 3205025

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Assignment 2) Regular Curves

a)

$$p(t) = R \begin{pmatrix} \cos(t) \\ \sin(t) \end{pmatrix}$$

where $t \in [0, 2\pi], R \in \mathbb{R} \geq 0$

b)

$$T(t) = R \begin{pmatrix} -\sin(t) \\ \cos(t) \end{pmatrix}$$

for $R = 2, t = \frac{\pi}{4}$ we get $T(\pi/4) = \begin{pmatrix} -\sqrt{2} \\ \sqrt{2} \end{pmatrix}$

c) Show regularity We have already seen in b) that the curve is differentiable. The magnitude of $T$is independent of the paramter t, $\left| \begin{pmatrix} -\sin(t) \\ \cos(t) \end{pmatrix} \right| = 1$. As such in order to ensure $T(t) \neq 0$ we simply need to restrict $R \in \mathbb{R} > 0$ in order for $p(t)$to be regular.

d) Arc length The circumfrence of two circles with different radii also differes. However in our function the parametrisation is independent of the radius and as such is only a arc length paramtrisation for the special case $R = 1$.

Assignment 3) Bezier Curves

a) The i-th out of nBernstein Polynomials is defined as:

$$B_i^n(t) = \left(\begin{array}{c} n \\ i \end{array} \right) t^i (1-t)^{n-i}, t \in [0,1]$$

With the properties:

$$\begin{array}{cc} \sum_{i=0}^n B_i^n(t) = 1 & \text{Partition of unity} \\ B_i^n(t) \geq 0, t \in [0,1] & \text{Nonnegativity} \\ B_i^n(t) = B_{n-i}^n(1-t) & \text{Symmetry} \\ B_i^n(t) = B_{i-1}^{n-1}(t) + (1-t)B_i^{n-1}(t) & \text{Recursive definition} \end{array}$$

b)

$$p(t) = \sum_{i=0}^n b_i B_i^n(t), \;\; t \in [0,1], \;\; b_i \in \mathbb{R}^d$$

where $b_i$are the control points that are approximated, execpt for the first and last point of the polygon. Since every polygon is strictly positive the approximated curve is completly contained in the convex hull of the control points, see .

c) In order to extend beyond the convex hull of the control points at least one Bernstein polynom would have to be less then 0 or more than 1 somewhere within the range of t. If we look at the individual factors $\left(\begin{array}{c} n \\ i \end{array} \right)$, t^i, $(1-t)^{n-i}$ it is apperent that they are all strictly greater than 0, thus their product is positive as well. The binomial therom states that $\sum_i^n B_i^n(t) = 1, \forall t \in [0,1]$ as such each individual polynom has to be less than 1. As such all Bernstein polynoms are contained in the range $[0,1]$ and as such the resulting curve is contained in the convex hull of its control points.

Assignment 4) Splines

a)

- Adjust control points only has a local effect in a certain range around the point. The range depends on the specific type of polynom used for each piece of the spline.
- The degree of the used polynomials is constant and usually small.
- Controls are more intuitive to the user as he can also adjust the differential properties(tagents) at the control points.

b) Splines are a list of polynomial curves that fullfill a application specific set of continuity constraints with respect to each other to achieve a smooth curve. The first set of constrains enforces smooth connection by setting the n-th-derivative of the two curve equal at the connection. The second constraint enforces a reparametrisation such that a single parameter can be used to traverse the whole curve.