Sheet 2 - Physics

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1 Using Lyx

2 Practical part

Please find the solution in the accompanying .py file.

3 Particle systems

3.1 a)

$$a(t) = \begin{bmatrix} r\,cos\,(t) \\ r\,sin\,(t) \end{bmatrix}, \forall t \in \mathbb{R} | t \geq t_0$$

$$v\,(t) = \; = \; v\,(t) + \int a\,(t)\,dt = \int \begin{bmatrix} r\,cos\,(t) \\ r\,sin\,(t) \end{bmatrix} dt = \begin{bmatrix} r\,sin\,(t) \\ -r\,cos\,(t) \end{bmatrix}$$

$$x\,(t) = x\,(t_0) + \int v\,(t)\,dt = \begin{bmatrix} -r\,cos\,(t) \\ -r\,sin\,(t) \end{bmatrix}$$

$$v\,(t_1) = \begin{bmatrix} r\,sin\,(t_1) \\ -r\,cos\,(t_1) \end{bmatrix} - \begin{bmatrix} r\,sin\,(t_0) \\ -r\,cos\,(t_0) \end{bmatrix}$$

$$x\,(t_1) = x\,(t_0) + \begin{bmatrix} -r\,cos\,(t_1) \\ -r\,sin\,(t_1) \end{bmatrix} - \begin{bmatrix} -r\,cos\,(t_0) \\ -r\,sin\,(t_0) \end{bmatrix}$$

3.2 b)

Euler integration takes the first derivative of a quantity and adds it to the current value to approximate the next value.

$$x_{n+1} = x_n + \dot{x_n}\Delta t$$

By comparison with the taylor expansion we know that the euler method approximates the value with a single step error of $O\left(\Delta t^2\right)$. Are approximate $x(t)$ from $x\left(t_0\right)$ we apply the euler method h times with the step size of $\frac{t-t_0}{h}$ for an accumulated error of $O\left(h\right)$over the approximation. For particle kinematics we need to apply the euler method twice. Once to integrate acceleration to velocity and then again to integrate velocity to the position.

$$\vec{v}_{n+1} = \vec{v}_n + \vec{a}_n\Delta t$$

$$\vec{x}_{n+1} = \vec{x}_n + \vec{v}_{n+1}\Delta t$$

3.3 c)

Assuming we start with $t = 0$

$$\vec{v}_{n+1} = 0 + \begin{bmatrix} 1\cos\left(0\right) \\ 1\sin\left(0\right) \end{bmatrix} \pi = \begin{bmatrix} \pi \\ 0 \end{bmatrix}$$

$$\vec{x}_{n+1} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} \pi \\ 0 \end{bmatrix} \pi = \begin{bmatrix} 1+\pi^2 \\ 0 \end{bmatrix}$$

3.4 d)