Competition of Social Opinions on Two Layer Networks

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Abstract: We investigate a model for the competition between two-layer opinion, where the first layer is opinion formation and the second layer is decision making, on multilayer networks. Networks show the two interacting social sectors, the civilians, and representatives. Layer A is civilian opinion layer consists of four states (-2, -1, +1, +2). These states describe the level of influence of opinion dynamics with reinforcement parameter γ. The layer B is the decision making layer consists of only two states (+1, -1). This layer can influence the decision dynamics with the probability in which decision is proportional to the number of interaction with the opposite opinion population raised to the power of β. Starting with a polarized competition case, layer A is all positive and layer B is all negative. In this paper, we create new models by changing the network structure, and compare these models with the pre-existing model. Then we investigate the condition in which the layer A influence the layer B and the conditions to make consensus in the interconnected network. This study could help to analyze social networks, such as legalization of social issues and prediction of vote results.

Key Words: Complex Network, Interconnected Networks, Modeling and Simulation, Python, Social Network Analysis, Language Competition Dynamics, Opinion Dynamics, Consensus

1 Introduction

Active study of complex systems has contributed to analyze natural or social phenomena, and the theory of complex network has been applied in real world such as climate forecast, social networks, and traffic control systems. [1-5] So far, the researches have been conducted focusing on interactions of a single layer or on the dynamics of each network. However, there are many systems in the world that cannot be explained only on a single layer. They must be described as multiple layer system, that consists of two or more layers and has the interaction between layers. These complex networks are known as “Multilayer complex network” [6]. Multilayer networks theories provide a solid and powerful approach to the study of complex phenomena. For example, interdependencies between different networks of a multilayer structure can make cascades of failure events that can dramatically increase the fragility of these systems. That can help to understand spreading of diseases, opinions and ideas, when a single layer is unable to explain those phenomena. [7] In the real world, the multilayer network theory has already applied to transportation system and social network analysis. [8]

Recently, researchers show their interest in modeling social network dynamics, including opinion dynamics, election models and game theory approaches. As the multiplex systems are prominent properties of social networks, which expose the fundamental importance of research on social network dynamics with multilayer networks. In this paper, opinion dynamics are discussed by analyzing the dynamics of a two-layer competing social network, based on the previous research. [9-12] The model consists of interconnected two layer networks. One layer has the function of social opinion and its own dynamics. The dynamics of the social opinion layer is a kind of opinion dynamics which are also known as M-model[12], that includes compromise function and persuasion function. The other layer also has the function of decision-making and its own dynamics. The dynamics of the decision making layer is the language competition dynamics that are also called as Abrams-Strogatz model[13-14]. And, the initial condition of the two layers is assumed to be in opposite states, social opinion layer has all positive states, decision making layer has all negative states.

As the result of previous research, interconnected competition of the social network have been researched by finding the threshold or critical point for consensus. [9-11] It has been proved that the system can make positive consensus, negative consensus or coexistence parts in interconnected competition of the social network. [9] And it is shown that the number of external degree is very important to change the state of layers. [10] We develop the previous modeling and research to find out the characteristics of interconnected networks. By switching the network structure of each layer, such as changing the number of nodes or the number of edges, we can see how the consensus or coexistence states change and what conditions make the social consensus. This can help to explain social networks phenomena, such as conflict between social opinion and the congress. Therefore, this research could be used as a tool for analyzing legislation problems and predicting decision-making system.

The paper is organized as follows. In section 2, the Basic Model is introduced and the dynamics, that is applied to each layer, are described. In section 3, the simulation results for the base model and revised models are presented. In section 4 the characteristics of each model are described through the comparison and analysis. Finally, in section5 the simulation results will be summarized and our findings are concluded.

2 Modeling

The Basic Model of this interconnected network consists of two layers that are called layer A and layer B, individu-

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ally. Each layer consists of random regular network that has N nodes with k internal edges on the same layer. Each node of a layer connects with a random node on the other layer. That means all the node has only 1 external undirected edge. Dynamics of layer A follows opinion dynamics, called as M-model. This dynamics is applied to layer A nodes and layer B nodes that are connected with layer A nodes. Each layer A node updates its state step by step. But, layer B nodes do not change their states, because layer B nodes is applied to decision making dynamics. The state of each node is represented by integer number j and k. And the maximum state of nodes is M. If k and j are connected on networks (it includes internal and external edges), the dynamics is like this formula.[10]

i) Compromise : if they have opposite orientations, their states become more moderate with probability q :

$$if \quad j<0 \quad and \quad k>0 \Rightarrow (j,k)\to(j^r,k^l) \quad with \quad prob.q$$
$$if \quad j>0 \quad and \quad k<0 \Rightarrow (j,k)\to(j^l,k^r) \quad with \quad prob.q$$

If $j=\pm1$ and $k=\mp1$, one switches orientation at random:

$$(\pm1,\mp1)\to\begin{cases}(+1,+1) & with \quad prob.q/2\\(-1,-1) & with \quad prob.q/2\end{cases}$$

ii) Persuasion : if they have the same orientation, their states become more extreme with probability p :

$$if \quad j<0 \quad and \quad k<0 \Rightarrow (j,k)\to(j^l,k^l) \quad with \quad prob.p$$
$$if \quad j>0 \quad and \quad k>0 \Rightarrow (j,k)\to(j^r,k^r) \quad with \quad prob.p$$

Here, k^r and k^l denote the right and left neighboring states of k, defined as

$$k^r=\begin{cases}1, & for \quad k=-1\\M, & for \quad k=M\\k+1, & otherwise\end{cases} \qquad k^l=\begin{cases}1, & for \quad k=-1\\M, & for \quad k=M\\k+1, & otherwise\end{cases} \tag{1}$$

As the initial condition for Basic Model, the dynamics on layer A has M-model dynamics with $M=2$. If $M=2$, the opinion state of each node can have four possible value such as S^A = -2, -1, +1, and +2, where the sign of S^A represents its opinion orientation and its absolute value $|S^A|$ measures the intensity of its opinion. So, $|S^A|=2$ represents to positive or negative extremists, while $|S^A|=1$ correspond to moderate opinions of each side. But, as the initial condition, nodes of layer A has only positive values, half nodes of the layer are $+1$ and the others are $+2$. In case of interaction between layer A node and layer B node, layer A node follows the above formula, but the state of layer B node does not change. In other words, the state of layer B affects layer A, but this dynamics does not affect state of layer B node. For example, one of layer A node, $S_i=+2$ is connected with one of layer B node, $S_j=-1$. In this case, S_i will change into $S_i=+1$ with prob.q. But S_j will not change. That means layer B node states have influence on layer A nodes. The dynamics of layer B follow language competition dynamics as the decision-making dynamics. Language competition dynamics follow the formula below. The state of node i in layer B changes with this probability [13, 14]

$$P_B(S_i\to -S_i)=\left\{\frac{n^{-S_i}}{i_i+e_i}\right\}^{\beta} \tag{2}$$

i_i is the number of internal edges and e_i is the number of external edges. n^{-S_i} is the number of neighbors of i with opposite state $-S_i$. $\beta(\geq 0)$ is the volatility exponent that measures how prone a node change state. This formula means that the more the nodes are connected with the opposite state, the easier the nodes can be changed into the opposite state. The states of layer B nodes can be +1 and -1. But, as an initial condition, the states are only -1. By this formula, the state of layer A has influence on change of layer B node states. In summary, states of layer A nodes are positive states of +1 and +2 as an initial condition, and the states of layer B node, which is connected with layer A, affects the state change of the node on layer A. Inversely, states of layer B nodes are negative states of -1 as an initial condition, and the states of nodes, which is connected with a node of layer B, have influence on the change of layer B nodes. Under these conditions, we try to find out how consensus happen and what conditions make consensus happen.

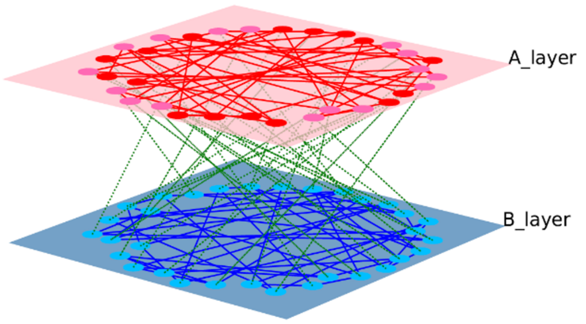

Fig. 1: Competing Interconnected Network

3 Simulation Result

We organized interconnection networks in Section 2. Each layer consists of a random regular network with each node having the same degrees.[15] Each node consists of five internal edges and one external edge. Layer A dynamics has the probability p and probability q denoted by M-model. We observe how the state changes by switching the probability p and probability q. To simply represent the probability p and probability q together, we set the value p+q=1, and γ = p/q. γ represents the tendency of opinion such as extreme or moderate.[9] For layer B dynamics, by switching β, we investigate the change of states, where γ scale is 0 to 2, and β scale is 0 to 3, basically. But, β scale depends on the total number of degrees. So, when the number of degrees is changed, the β scale would be adjusted properly to Basic Model scale. To implement the interconnected dynamics, one step consists of two layers dynamics, where layer A dynamics and layer B dynamics are carried out step by step. Basically, 30 steps are taken. And these procedure is repeated by 100 times to calculate the average states change of each layer. The 'Average state of layer A and B' is calculated by summing the average of layer A state and the average of layer B state. With 'Average state of layer A and B', we can check whether the consensus happen or not in accordance with γ and β changing. If the positive consensus happens, it would be close to the value of 3, summation of layer A node average state(+2) and layer B node average state(+1) , and if the negative consensus happens, it would be close to the value of -3, summation of layer A node(-2) and layer B node(-1). The value between 3 and -3 is not on the consensus yet, so the states of positive and negative are on the coexistence part. First, the Basic Model is simulated, and then other simulations would be implemented with revised network structure.

3.1 Basic Model Result

Basic model simulation result is shown in Fig2 and Fig3. Fig.2(a) shows that when β increases, it normally tends to make positive and negative consensus. But, when γ is very low, it cannot make positive consensus. On the other hand, when γ is large enough, it can make positive consensus. β is large enough, it can change into negative consensus. When both of γ and β are large enough, it has coexistence for positive and negative states. In Fig.2(b), As γ increases, it normally tends to make positive consensus. When β is very low, it cannot make consensus. When β is also large enough, it cannot make positive consensus and has a coexistence part. Fig.3 shows the states of two layers according to γ and β. The X-axis is the γ and the Y-axis is the β, and the Z-axis represents the average summation of the nodes' states in layer A and B. In Fig.3, The closer the color is to be blue,

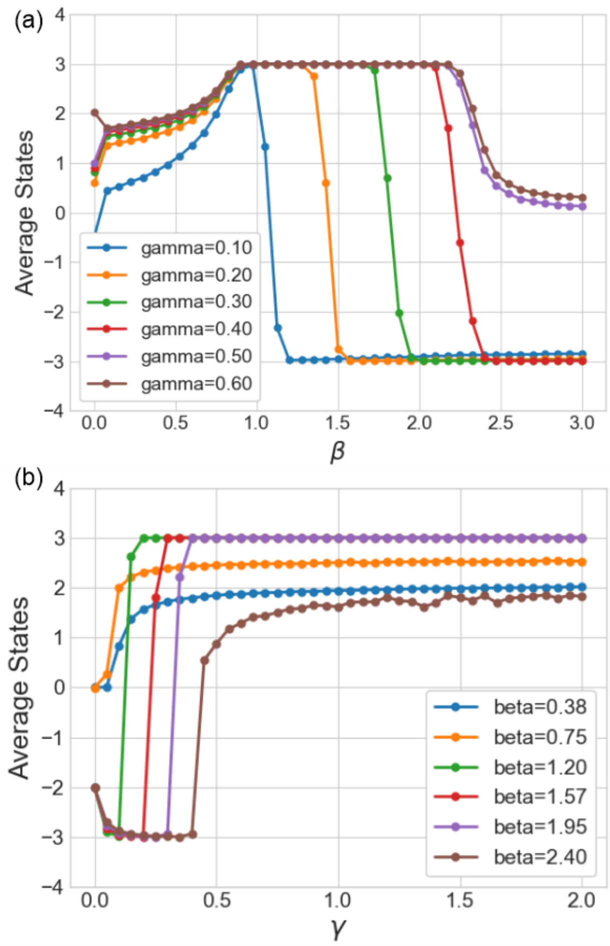

Fig. 2: (a) β-Average state of layer A and B chart according to some γ values. (b) γ-Average state of layer A and B chart according to some β values.

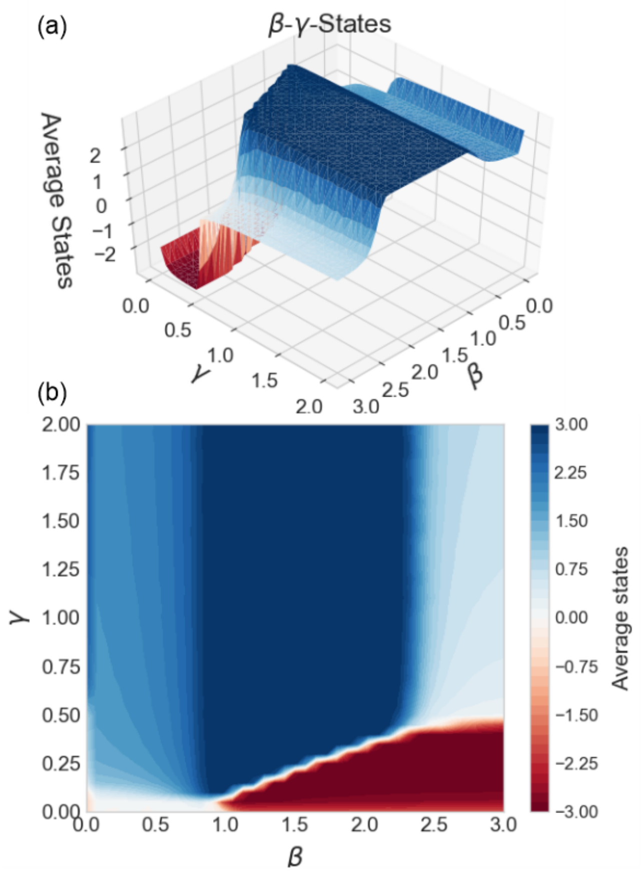

Fig. 3: Basic model 3D result (1). (a) γ-β-Average state of layer A and B chart at angle (45, 45). (b) γ-β-Average state of layer A and B chart at angle (90, 0)

the more it has positive consensus, the closer the color is red, the more it has negative consensus. A light and white areas do not make consensus and have coexistence with positive and negative states. This chart has two areas for coexistence, when β is very low or very high. Which means, it has two transition parts such as transition from coexistence to consensus, and transition from consensus to coexistence. In Fig.4, the fractions of positive state nodes are investigated on each layer. Fig.4 (a) shows the result of layer A. Layer A has bigger positive area than coexistence area. As the β increases, the γ has to be increased for the positive consensus. As the parameter of layer A dynamics, the γ is operating properly. Fig.4 (b) shows the result of layer B. Layer B has various states as γ and β change. But, Almost layer B states depends on the power of β. When β is very small, layer B states are almost in the coexistence area. When β is very large, layer B state are almost negative. When β is in the middle($0.8 < \beta < 2.25$), the states are in the consensus area. As the parameter of layer B dynamics, the β is also operating properly. Fig.4 (c) is the result of the summation of Fig.4 (a) and Fig.4 (b), which chart is similar as Fig.3 result. Through these charts, we can see that there is the difference between two coexistence parts where β is very small and β is very large. The coexistence with small β is due to inner competition of layer B. The coexistence with large β is due to outer competition between layer A and layer B

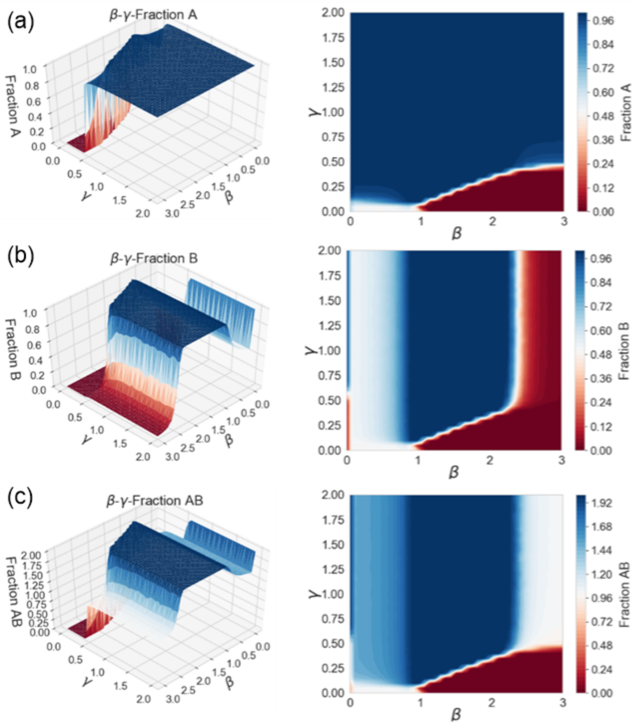

Fig. 4: Basic model 3D result (2) (a) γ-β-Fraction A chart. 'Fraction A' means the fraction of positive states in the layer A. (b) γ-β-Fraction B chart. 'Fraction B' means the fraction of positive states in the layer B. (c) γ-β-Fraction AB chart. 'Fraction AB' means the summation of Fraction A and Fraction B.

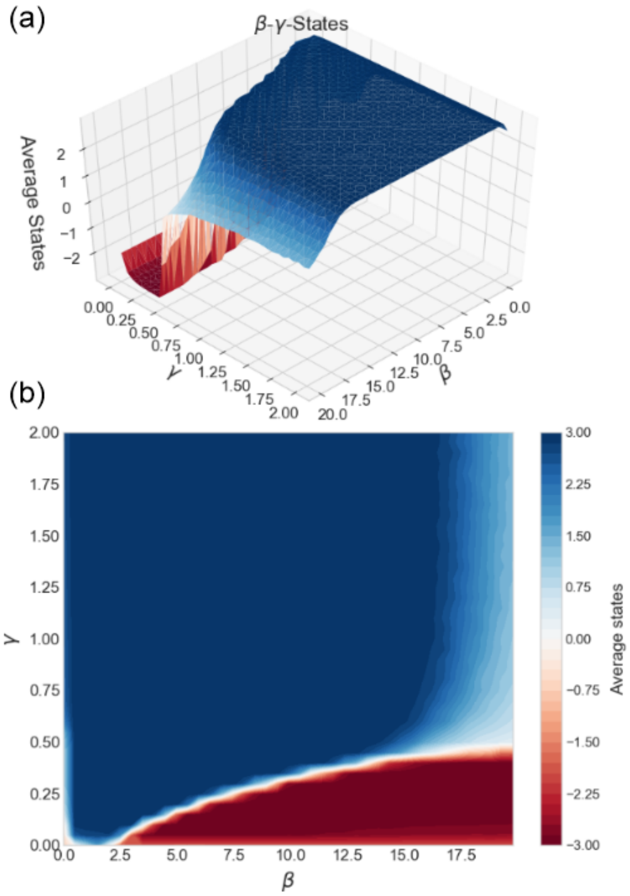

Fig. 5: Leader Model 3D result. (a) γ-β-Average state of layer A and B chart at angle(45, 45). (b) γ-β-Average state of layer A and B chart at angle(90, 0)

3.2 Leader Model Result

The Leader Model is the revised model where the number of layer B nodes is decreased from 2048 to 2048 by 1/16, and increase the number of external edges by 16 times. In other words, each layer A node has one external edge, but each layer B node has 16 external edges. That is, as social network relation, we can analyze that one node of layer B represents the 16 nodes of layer A. γ scale is same as the Basic Model. But, β scale depends on the number of degrees. So the β scale is adjusted to same values as the Basic Model. Fig.5 shows the Leader Model simulation results. Comparing with Basic Model, white and light parts are decreased remarkably. And it has only 1 transition part where β is large enough. It shows that Leader Model has more consensus parts than Basic Model.

3.3 Different Structural Networks Model Result

So far, each layer of the interconnected network consists of random regular network that has the same number of edges for each node. Now, the simulation would be implemented with Barabasi-Albert network[16, 17]. For this simulation, each layer has 2048 nodes, 1 external edges for each node. The internal edges depend on Barabasi-Albert network(BA). So it has total 10,215 edges for each layer(BA is set up as average 5 internal edges), it has more 5,095 edges than the random regular network(RR) that has total 5,120 internal edges(RR is set up as 5 internal edges for each node). The simulation would be carried out, with switching each layer to RR or BA network structure. In Fig.6 (a)

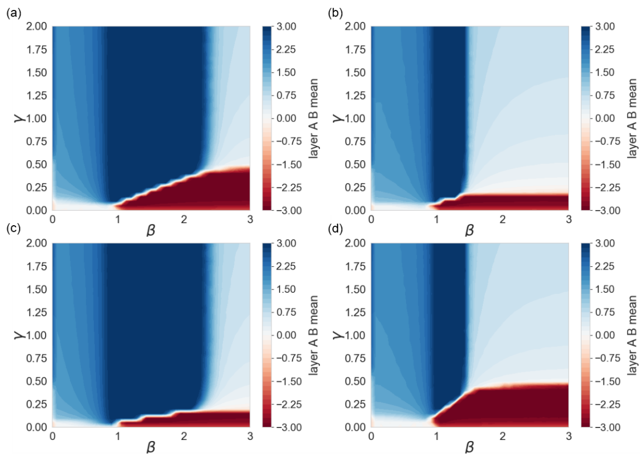

Fig. 6: The simulation results for interconnected network with Different Structural Networks (a) layer A(RR)-layer B(RR), (b) layer A(BA)-layer B(BA), (c) layer A(BA)-layer B(RR), (d) layer A(RR)-layer B(BA)

and (b), (b) result has more white area(coexistence) than (a) result. That means increasing internal edges in both layers makes more coexistence area.In Fig.6 (c), comparing with (a), the red area(negative consensus) is decreased and the blue area(positive consensus) is increased. Since layer A is BA network that has more internal edges, it seems to have more tendency to keep layer A states.In Fig.6 (d), comparing with (a), the red area and white area are increased. Since layer B is BA network that has more internal edges, it seems to have more tendency to keep layer B states. In order to confirm the effect of the number of internal edges on the consensus, The internal edges of each node change to 2 or 5. The simulations are carried out with only RR networks that has total 5,120(when the number of internal edges are 5)or 2048(when the number of internal edges are 2) internal edges on each layer. Fig.7 shows the simulation result with changing the number of internal edges. As we see

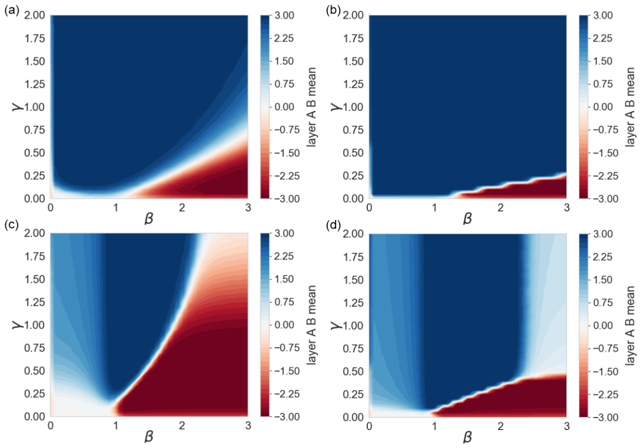

Fig. 7: Consensus Comparison according to changing the number of internal edges on layer A and B (a) RR(2048)-RR(2048), (b) RR(5,120)-RR(2048), (c) RR(2048)-RR(5,120), (d) RR(5,120)-RR(5,120)

the result, we can find out the number of internal edges affecting consensus. In Fig.7 (b) RR(5,120)-RR(2048), it has decreased red area(negative consensus) and increased blue area(positive consensus). As the internal edges on layer A increase, the red area get smaller and the blue area get larger. That means that it has more tendency for layer A than layer B. Inversely, in Fig.7 (c) RR(2048)-RR(5,120), it has increased red area(negative consensus) and decreased blue area(positive consensus). As the internal edges on layer B increase, the red area get larger and the blue area get smaller. That means that it has more tendency for layer B than layer A. In order to compare BA network with RR network that has the similar number of edges, the internal edges of RR network are increased to the similar number of BA network internal edges. The simulation was carried out with RR network that has total 10,240 internal edges on layer A. Layer B has RR network with 5,120 internal edges. The result is shown in Fig.8. In Fig.8 (a), BA network simulation result has similarity with RR network that has similar number of internal edges with BA network. Therefore, we can analyze that the number of edges have influence on making consensus. In other words, the network with more internal edges tends to make consensus for that network state.

3.4 Dynamics steps-increased Model Result

The Basic Model was implemented by 30 steps of interconnected dynamics that includes opinion dynamics and decision-making dynamics. In this simulation, the number of steps was increased from 30 to 100 steps. Increasing the number of steps in interconnected dynamics means the net-

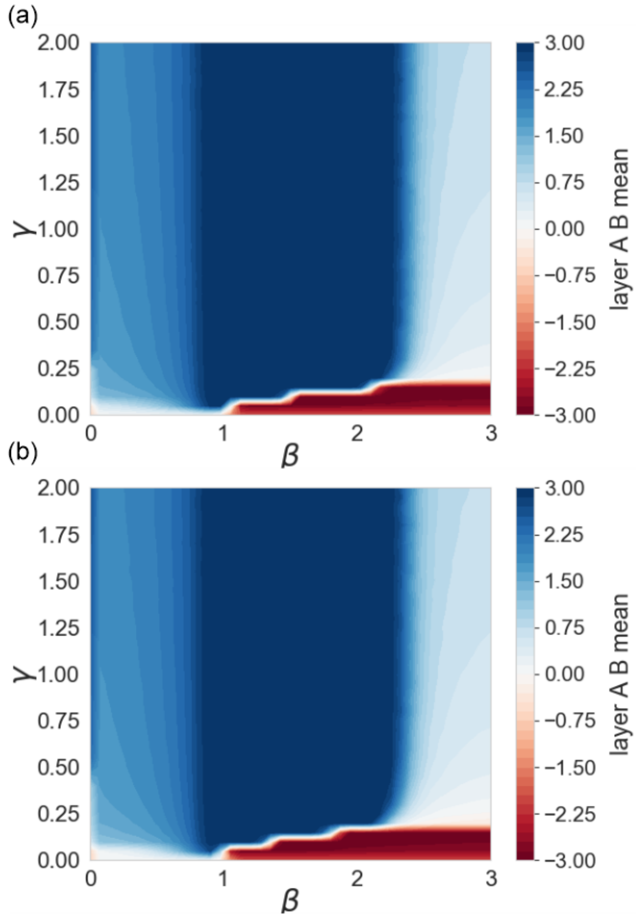

Fig. 8: Comparison between BA network and RR network (a) RR(10,240)-RR(5,120), (b) BA(10,215)-RR(5,120)

works keeps interconnected dynamics for enough time to obtain the result. The result of increasing the number of steps to 100 is as follows. In Fig.9 (d), Dynamics steps-increased

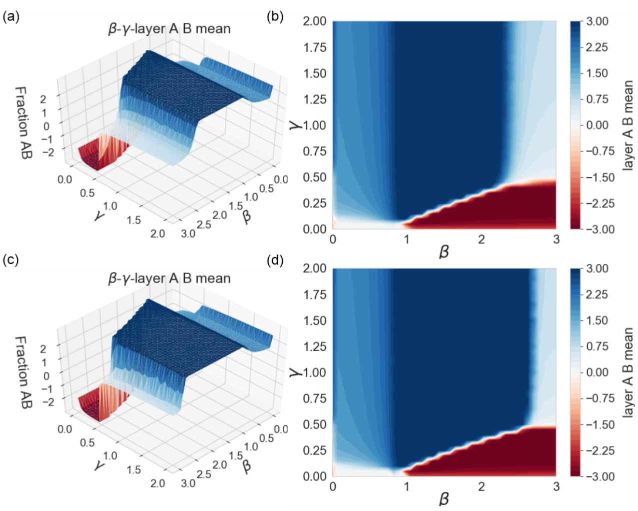

Fig. 9: The comparison between Basic Model(30 times dynamics) ; (a),(b) and Dynamics steps-increased Model(100 times) ; (c),(d)

Model has more blue area(positive consensus) than the blue area of Basic Model, on the other hand, the white area is less than the white area of Basic Model. That means consistent social opinion can make consensus with its state. In Fig.10, the charts show that most of ratio on the layer A are approaching to 0 with steps proceeding, except the case that volatility is very large. The ratio on the layer B are divided by several values. All the values except 0 are made when the volatility is very large. Except the case that volatility is very large, most of them are approaching to 0. That means two layers tend to make consensus by proceeding the steps with stable γ and β.

4 Comparison and Analysis

In this section, the simulation results are compared and analyzed with Basic Model to provide what conditions make the consensus on the interconnected networks. When the Leader Model compares with the Basic Model, we can see that the Leader Model has more consensus area than the Basic Model (White area of Leader Model is less than Basic Model), and the Basic Model has two transition parts, on the other hand, the Leader Model has just one transition part. This shows that the Basic Model has transition parts on the conditions when the β is very small or very large. But, in case of the Leader Model, when β is very small, it can make consensus, easily. If β is very large, the consensus are hard to happen in the Leader Model. Next, the Basic Model is compared with Different Structural Networks Models. The total number of internal edges in BA is 10,215, and the internal edges in RR are 5,120. This simulation shows, how the number of internal edges affects interconnected dynamics. First, when we check the BA-RR and RR-BA simulation results, we can see that it makes more consensus with BA network layer states than RR-RR simulation. In other words, one layer with more internal edges seems to have a tendency to maintain its own state. That is, interconnected network tends to make consensus with one layer that has more internal edges. In case of two layers with same internal edges such as RR-RR network and BA-BA network, RR-RR network simulation results have more consensus parts than BA-BA simulation results. we can analyze that BA-BA simulations result has less consensus area, due to having a strong tendency to maintain their both sides. To confirm the effect of internal edges, BA network was compared with RR network that has the similar number of internal edges. The result was almost same. It can be analyzed that the network with more edges have more tendency to keep its states and make consensus for its states. Next, Dynamics steps-increased Model was compared. A model that increases the number of steps seems to have more consensus area for social opinion because γ is stable and consistent. In other words, it means that social opinion is likely to achieve the consensus if it was given enough time with stable positive orientation.(Absolutely, if the volatility is too low or too high, it would not make consensus.)

Through these three comparisons and analysis, we can provide three conditions that increase the likelihood of consensus with social opinion. Firstly, Leader Model shows, if reducing the nodes of the decision-making layer and increasing the external edges, we can increase the probability of social opinion consensus. Secondly, Different Structural Networks Model shows, social opinion layer requires more internal edges than opposite layer in order to make positive consensus. Because increasing the internal edges of social opinion is likely to have more tendency to keep social opinion states. Thirdly, the Dynamics steps-increased Model shows, if the interaction has given enough time, there is a high possibility of consensus with social opinion.

5 Conclusion

In this work, we have researched competing interconnected network dynamics. Layer A is a layer of social opinion and has positive states as an initial condition. It is influenced by opinion dynamics. Layer B is a network representing decision making system and has a negative state as an initial condition. It is influenced by the language competition dynamics. When these two layers were connected and interacted, it showed how their states were changed with switching γ and β. In the Basic Model, the simulation result shows three sections which are the negative consensus, the positive consensus, and the coexistence according to γ and β. Based on this Basic Model, we have presented and simulated Leader Model, Different Structural Networks Model, and Dynamics steps-increased Model. As a result, we found out three conditions that increase the likelihood of consensus with social opinion layer. The first is to reduce nodes in the decision-making layer and increase the external edges like the Leader Model. The second is to increase the internal edges of social opinion, because internal edges have an influence to strengthen and maintain states of the layer. But too many internal nodes can make internal conflicts. The third is to take enough time and keep social opinion with stable and enough γ. More research will be needed to make generalized model and to be applied to real social networks. We think this research can contribute to providing the characteristics of the system and the framework of social networks analysis such as the legalization or social decision-making system, and we hope that it will help the research on interconnected networks and multi-layer networks. As the future work, it would be very interesting to make the generalized model for competing interconnected network.

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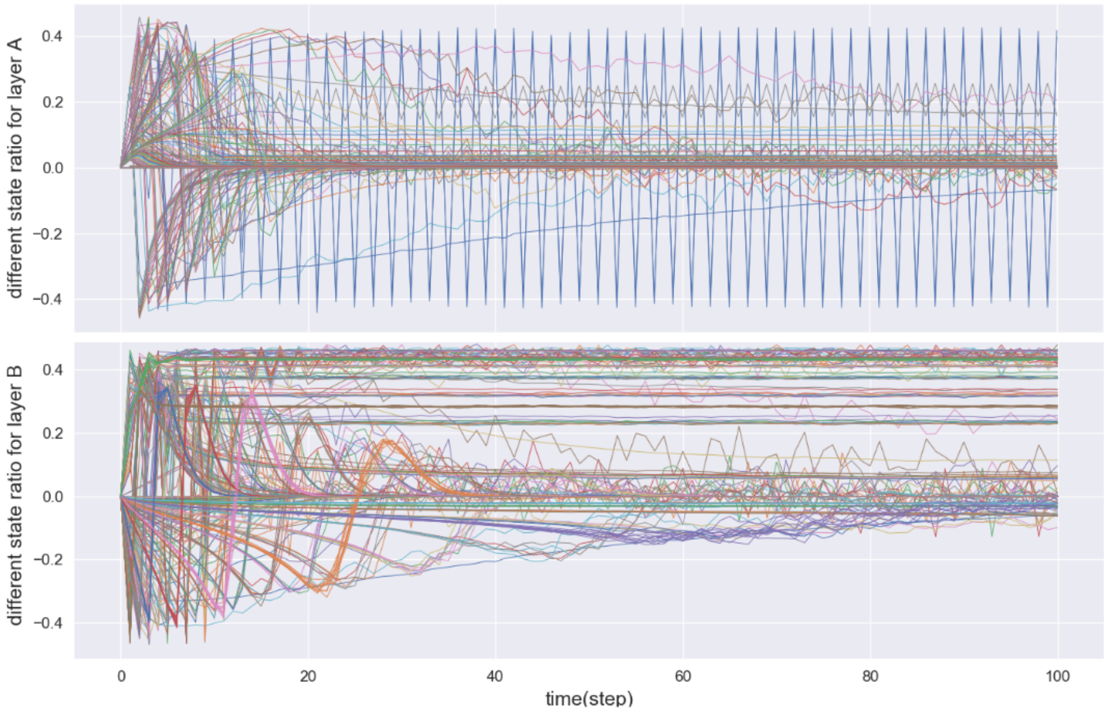

Fig. 10: Time evolution with the fraction of nodes in different states on layer A and layer B, the ratio of different state means the fraction of different state node numbers for each layer. The sign of ratio depends on the major nodes of each layer. For example, if the major nodes on layer A are negative, the sign of that ratio is negative.

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